Radiative Higgs boson decays $H\to f\bar{f}\gamma$

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(Received 4 November 1996)

Higgs boson radiative decays of the form $H\to f\bar{f}\gamma$ are calculated in the standard model using the complete one-loop expressions for the decay amplitudes. Contributions to the radiative width from leptons and light quarks are given. We also present $e\bar{e}$-invariant mass distributions for $H\to e\bar{e}\gamma$, which illustrate the importance of the photon pole contribution and the effects of the box diagrams. [S0556-2821(97)00109-4]

PACS number(s): 13.85.Qk, 14.70.Hp, 14.80.Bn

I. INTRODUCTION

Discussions of searches for intermediate mass Higgs bosons usually concentrate on the decay $H\to\gamma\gamma$ as the discovery mode [1]. Other modes considered include $H\to Z\gamma$ and $H\to b\bar{b}$. The importance of radiative processes led us to consider the class of decays $H\to f\bar{f}\gamma$, where f is a light fermion. The dominant contributions to these decays occur at the one-loop level, and their calculation is related to that of the process $e\bar{e}\to H\gamma$, which we recently completed [2,3].

Typical results for Higgs boson $f\bar{f}\gamma$ decay appear in the calculation of $\Gamma(H\to e\bar{e}\gamma)$, which, for $m_H\gtrsim 100$ GeV, receives a large contribution from the Z pole. Additionally, our calculations show that the photon pole makes a substantial correction to the estimate obtained by simply multiplying the width for $H\to Z\gamma$ by the branching ratio $B(Z\to e\bar{e})$. This feature is common to all light fermions, and we present results for all decays of the type $H\to f\bar{f}\gamma$.

In the next section, we present expressions for the decay amplitudes, the decay matrix element, and results for the fermion-invariant mass distributions and for the widths. This is followed by a discussion. Complete expressions for the various amplitudes are given in the appendices.

II. CALCULATION OF HIGGS BOSON DECAY WIDTHS

Contributions to the decay amplitudes arise from the diagrams illustrated [4] in Fig. 1. They are of two basic types: (a) pole diagrams in which the $f\bar{f}$ emerge from a virtual gauge boson, and (b) box diagrams containing virtual gauge bosons and fermions in the loop. The diagrams are evaluated using the nonlinear gauges discussed in Ref. [2]. In these gauges, the collection of diagrams consists of four separately gauge-invariant contributions, the photon pole, the Z pole, Z boxes, and W boxes. The amplitudes for these contributions are

$$\mathcal{M}^{\gamma}_{\rm pole}=-\frac{\alpha^2 m_W}{\sin\theta_W}\overline{u(p_1)}\gamma_\mu v(p_2)\times\left(\frac{\delta_{\mu\nu}k\cdot(p_1+p_2)-k_\mu(p_1+p_2)_\nu}{m^2_{f\bar{f}}}\right)\times\hat{\epsilon}_\nu(k)\mathcal{A}_\gamma(m^2_{f\bar{f}}), \tag{1}$$

$$\mathcal{M}^{Z}_{\rm pole}=-\frac{\alpha^2 m_W}{\sin^3\theta_W}\overline{u(p_1)}\gamma_\mu(v_f+\gamma_5)v(p_2)\times\left(\frac{\delta_{\mu\nu}k\cdot(p_1+p_2)-k_\mu(p_1+p_2)_\nu}{(m^2_{f\bar{f}}-m^2_Z)+im_Z\Gamma_Z}\right)\times\hat{\epsilon}_\nu(k)\mathcal{A}_Z(m^2_{f\bar{f}}), \tag{2}$$

$$\mathcal{M}^{Z}_{\rm box}=\frac{\alpha^2 m_Z}{4\sin^3\theta_W\cos^3\theta_W}\overline{u(p_1)}\gamma_\mu(v_f+\gamma_5)^2 v(p_2)\times\{[\delta_{\mu\nu}k\cdot p_1-k_\mu(p_1)_\nu]\mathcal{B}_Z(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})+[\delta_{\mu\nu}k\cdot p_2-k_\mu(p_2)_\nu]\mathcal{B}_Z(m^2_{f\bar{f}},m^2_{\bar{f}\gamma},m^2_{f\gamma})\}\hat{\epsilon}_\nu(k), \tag{3}$$

$$\mathcal{M}^{W}_{\rm box}=-\frac{\alpha^2 m_W}{2\sin^3\theta_W}\overline{u(p_1)}\gamma_\mu(1+\gamma_5)^2 v(p_2)\times\{[\delta_{\mu\nu}k\cdot p_1-k_\mu(p_1)_\nu]\mathcal{B}_W(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})+[\delta_{\mu\nu}k\cdot p_2-k_\mu(p_2)_\nu]\mathcal{B}_W(m^2_{f\bar{f}},m^2_{\bar{f}\gamma},m^2_{f\gamma})\}\hat{\epsilon}_\nu(k), \tag{4}$$

where $m^2_{f\bar{f}}=-(p_1+p_2)^2$, $m^2_{f\gamma}=-(k+p_1)^2$, and $m^2_{\bar{f}\gamma}=-(k+p_2)^2$. Here, v_f denotes the $f\bar{f}Z$ vector coupling constant, $v_f=1-4|e_f|\sin^2\theta_W$, and e_f is the fermion charge in units of the proton charge. To calculate the invariant amplitudes $\mathcal{B}_Z(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})$ and $\mathcal{B}_W(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})$, we use the approach of Ref. [2]. Here, too, we find a logarithmic dependence on the fermion mass at intermediate stages of the calculation, but this dependence cancels, enabling us to take the limit of zero fermion mass. Explicit expressions for the invariant amplitudes $\mathcal{A}_\gamma$, $\mathcal{A}_Z$, $\mathcal{B}_Z$, and $\mathcal{B}_W$ are given in the appendices.

The invariant mass distribution $d\Gamma/dm^2_{f\bar{f}}$ is given by

$$\frac{d\Gamma(H\to f\bar{f}\gamma)}{dm^2_{f\bar{f}}}=\frac{1}{256\pi^3}\frac{1}{m^3_H}\int_{(m^2_{\bar{f}\gamma})_{\rm min}}^{(m^2_{\bar{f}\gamma})_{\rm max}}dm^2_{\bar{f}\gamma}\sum_{\rm spin}|\mathcal{M}|^2, \tag{5}$$

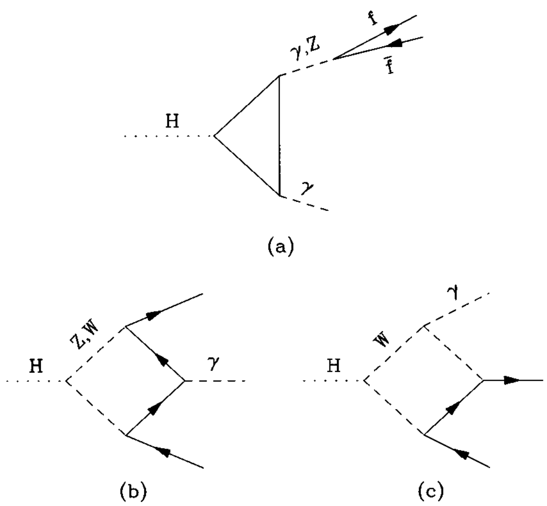

FIG. 1. The diagrams for $H\to f\bar{f}\gamma$ are shown.

with the amplitude $\mathcal{M}$ given by the sum of Eqs. (1)–(4). For an $f\bar{f}\gamma$ final state, the limits on the $dm^2_{\bar{f}\gamma}$ integration are

$$(m^2_{\bar{f}\gamma})_{\min}=m_f^2+\frac{1}{2}(m_H^2-m^2_{f\bar{f}})\left(1-\sqrt{1-\frac{4m_f^2}{m^2_{f\bar{f}}}}\right), \quad (6)$$

$$(m^2_{\bar{f}\gamma})_{\max}=m_f^2+\frac{1}{2}(m_H^2-m^2_{f\bar{f}})\left(1+\sqrt{1-\frac{4m_f^2}{m^2_{f\bar{f}}}}\right). \quad (7)$$

The fermion mass is retained in the phase space integration since, as shown below, there is a $(m^2_{f\bar{f}})^{-1}$ factor associated with the photon pole. Explicitly, $\Sigma_{\rm spin}|\mathcal{M}|^2$ is

$$\sum_{\rm spin}|\mathcal{M}|^2=\frac{\alpha^4 m_W^2 m^2_{f\bar{f}}}{16\sin^6\theta_W\cos^8\theta_W}\{[(m^2_{f\gamma})^2+(m^2_{\bar{f}\gamma})^2][|\widetilde{\mathcal{A}}_\gamma|^2 + 2v_f{\rm Re}(\widetilde{\mathcal{A}}_\gamma\widetilde{\mathcal{A}}_Z^*)+(1+v_f^2)|\widetilde{\mathcal{A}}_Z|^2]+(m^2_{f\gamma})^2 \times[2(1+v_f^2){\rm Re}(\widetilde{\mathcal{A}}_\gamma\widetilde{\mathcal{B}}_Z^*)+4{\rm Re}(\widetilde{\mathcal{A}}_\gamma\widetilde{\mathcal{B}}_W^*) + 2(v_f^3+3v_f){\rm Re}(\widetilde{\mathcal{A}}_Z\widetilde{\mathcal{B}}_Z^*)+4(1+v_e){\rm Re}(\widetilde{\mathcal{A}}_Z\widetilde{\mathcal{B}}_W^*) + (1+6v_f^2+v_f^4)|\widetilde{\mathcal{B}}_Z|^2+4(1+v_f)^2{\rm Re}(\widetilde{\mathcal{B}}_Z\widetilde{\mathcal{B}}_W^*) + 8|\widetilde{\mathcal{B}}_W|^2]+(m^2_{\bar{f}\gamma})^2[m^2_{f\gamma}\leftrightarrow m^2_{\bar{f}\gamma}]\}. \quad (8)$$

Here, $\widetilde{\mathcal{A}}_\gamma$, $\widetilde{\mathcal{A}}_Z$, $\widetilde{\mathcal{B}}_Z$, and $\widetilde{\mathcal{B}}_W$ are

$$\widetilde{\mathcal{A}}_\gamma=4\sin^2\theta_W\cos^4\theta_W\frac{\mathcal{A}_\gamma(m^2_{f\bar{f}})}{m^2_{f\bar{f}}}, \quad (9)$$

$$\widetilde{\mathcal{A}}_Z=4\cos^4\theta_W\frac{\mathcal{A}_Z(m^2_{f\bar{f}})}{(m^2_{f\bar{f}}-m_Z^2)+im_Z\Gamma_Z}, \quad (10)$$

$$\widetilde{\mathcal{B}}_Z=-\mathcal{B}_Z(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma}), \quad (11)$$

$$\widetilde{\mathcal{B}}_W=2\cos^4\theta_W\mathcal{B}_W(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma}). \quad (12)$$

Using our results for the invariant amplitudes, the $dm^2_{\bar{f}\gamma}$ integration can be performed numerically to obtain the invariant mass distribution.

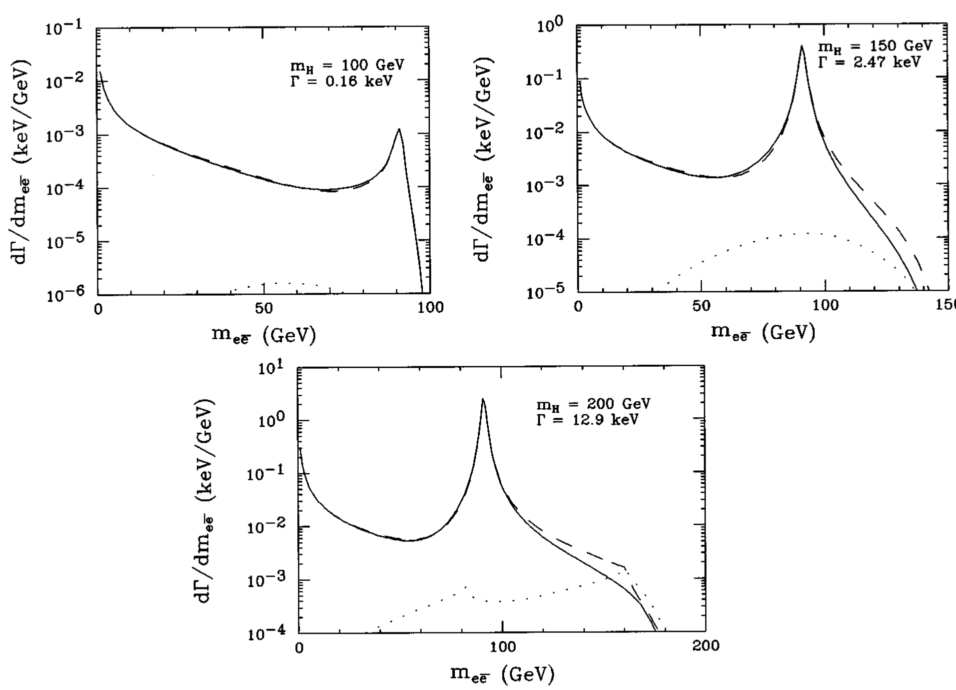

FIG. 2. The invariant mass distribution of the decay mode $H\to e\bar{e}\gamma$ for several Higgs boson masses is shown. The solid line is the full calculation, the dashed line is the pole contribution, and the dotted line is the box contribution.

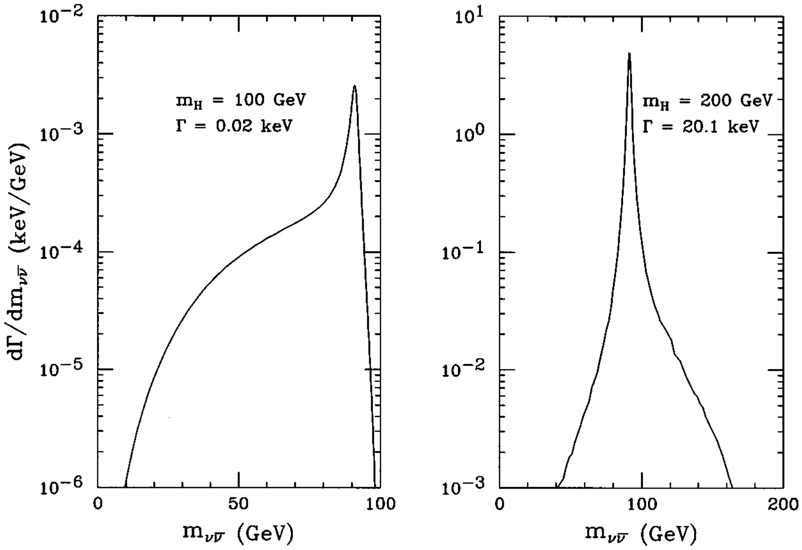

FIG. 3. The invariant mass distribution of the decay mode $H \to \nu\bar{\nu}\gamma$ is shown for two Higgs boson masses.

The invariant mass distribution $d\Gamma(H \to e\bar{e}\gamma)/dm_{e\bar{e}}$ is illustrated in Fig. 2. The striking feature of these distributions is the large peak at small $m^2_{e\bar{e}}$ due to the photon pole. There is no singularity in the physical region since $m^2_{e\bar{e}} \geqslant 4m_e^2$. In fact, as can be seen from Eqs. (6) and (7), the $dm^2_{\bar{e}\gamma}$ integral in Eq. (5) vanishes when $m^2_{e\bar{e}} = 4m_e^2$. Nevertheless, the residual effect of the photon pole is sufficient to contribute $\sim$10–20% of the events in the distribution. It is also evident that the box diagrams make only a small contribution. Curiously, the main effect of the box diagrams is to smooth the distribution by canceling the kinks in the pole contributions at the WW threshold. The invariant mass distribution for the remaining lepton channel $H \to \nu\bar{\nu}\gamma$, which has no contribution from the photon pole, is illustrated in Fig. 3.

The various partial widths can be obtained by integrating Eq. (5). This results in the contributions illustrated in Fig. 4. Also shown in the lepton panel of Fig. 4 is the contribution from the Z pole. The figure clearly shows that the widths are enhanced significantly in the complete calculation, even in the case of neutrino decays. For $m_H \gtrsim 160$ GeV, the up-type quark contributions are basically the same. This is also true for the down-type quarks.

III. DISCUSSION

As can be seen from Fig. (2), the invariant mass distributions are basically determined by the photon and Z pole contributions. The box diagrams make corrections to the high mass side of the distribution where they are of the same order as the pole terms. This being the case, it is possible to obtain a simplified expression for $d\Gamma/dm^2_{f\bar{f}}$ by retaining only the $\mathcal{A}_\gamma$ and $\mathcal{A}_Z$ terms in Eq. (5). After performing the $dm^2_{\bar{f}\gamma}$ integration, one finds

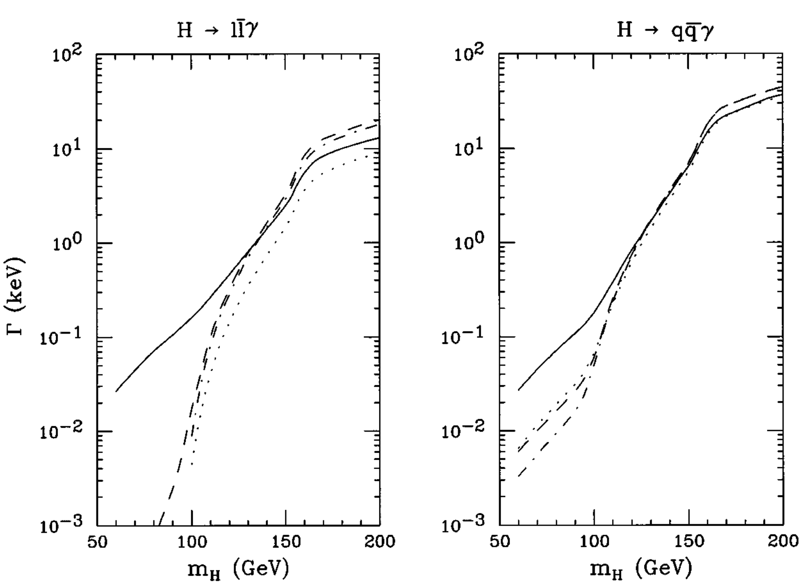

FIG. 4. The partial widths for Higgs boson decay into a lepton pair (left) and quark pair (right) are plotted as a function of the Higgs boson mass. In the left panel, the solid line is the partial width for a charged lepton and the dashed line is the partial width for its neutrino. The dotted line is the Z pole approximation for the charged lepton partial width and the dot-dashed line is the pole approximation to the neutrino partial width. In the right panel, the solid line corresponds to the up quark, the dashed line to the down quark, the dot-dashed line to the strange quark, and the dotted line to the charm quark.

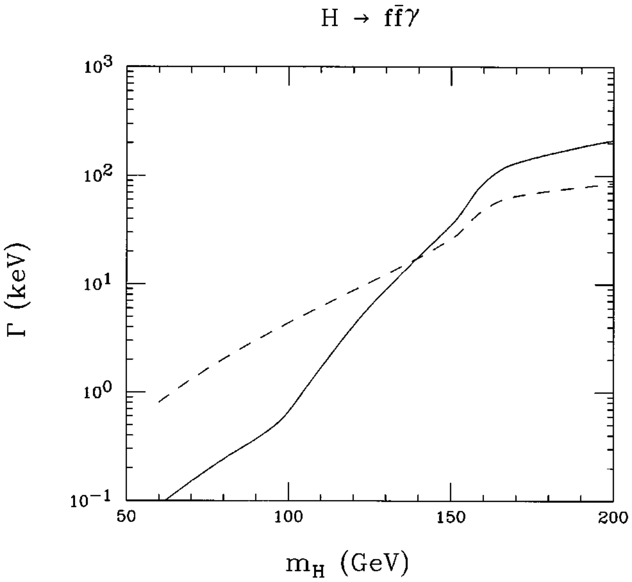

FIG. 5. The partial width $\Gamma(H\rightarrow f\bar{f}\gamma)$, obtained by summing the neutrino, electron, muon, up quark, down quark, and strange quark contributions, is shown as the solid line. For comparison, the dashed line is the partial width $\Gamma(H\rightarrow\gamma\gamma)$.

$$
\begin{aligned}
\frac{d\Gamma}{dm^2_{f\bar{f}}}=&\frac{\alpha^4 m_W^2}{(8\pi)^3\sin^6\theta_W m_H^3}\Bigg[\sin^4\theta_W\frac{|\mathcal{A}_\gamma(m^2_{f\bar{f}})|^2}{m^2_{f\bar{f}}}\\
&+2\sin^2\theta_W v_f \mathrm{Re}\left(\frac{\mathcal{A}_\gamma(m^2_{f\bar{f}})\mathcal{A}_Z^*(m^2_{f\bar{f}})}{(m^2_{f\bar{f}}-m_Z^2)-im_Z\Gamma_Z}\right)\\
&+\frac{(1+v_f^2)m^2_{f\bar{f}}|\mathcal{A}_Z(m^2_{f\bar{f}})|^2}{(m^2_{f\bar{f}}-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Bigg](m_H^2-m^2_{f\bar{f}})\\
&\times\sqrt{1-\frac{4m_f^2}{m^2_{f\bar{f}}}}\Bigg[(m_H^2+2m_f^2-m^2_{f\bar{f}})^2\\
&+\frac{1}{3}(m_H^2-m^2_{f\bar{f}})^2\left(1-\frac{4m_f^2}{m^2_{f\bar{f}}}\right)\Bigg].
\end{aligned}
\tag{13}
$$

This expression reproduces the dashed lines in Fig. 2.

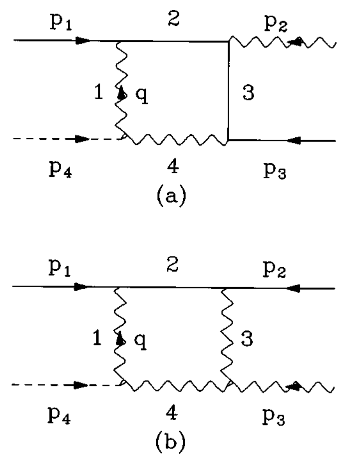

FIG. 6. The numbering schemes used for the computation of $D_0(1,2,3,4)$ in the case of the Z box (a) and the W box (b) are shown.

The total width for $H\rightarrow f\bar{f}\gamma$ is shown in Fig. 5, where the neutrino, electron, muon, up quark, down quark, and strange quark contributions are added. The dashed line in this figure corresponds to $H\rightarrow\gamma\gamma$, and it can be seen that the $f\bar{f}\gamma$ width exceeds the $\gamma\gamma$ width for $m_H \gtrsim 140$ GeV.

The relatively large size of the $H\rightarrow f\bar{f}\gamma$ decay mode compared to $H\rightarrow\gamma\gamma$ decay mode is somewhat unexpected and this could be used to supplement Higgs boson searches which rely on the detection of photons. Our results also show that (occasionally rumored) $e\bar{e}\gamma$ events, where the $e\bar{e}$-invariant mass does not lie in the Z^0 bin, are unlikely to be Higgs boson decays unless the $e\bar{e}$ mass is low. Finally, should the Higgs boson be detected in the intermediate mass range, measurement of these rare decay modes may be of use in determining whether its interactions are those of the standard model or one of its extensions.

ACKNOWLEDGMENTS

This research was supported in part by the U.S. National Science Foundation under Grant No. PHY-93-07980 and by the United States Department of Energy under Contract No. DE-FG013-93ER40757.

APPENDIX A: POLE CONTRIBUTIONS

The amplitudes $\mathcal{A}_\gamma(m^2_{f\bar{f}})$ and $\mathcal{A}_Z(m^2_{f\bar{f}})$ can be expressed in terms of two scalar functions $C_0(m^2_{f\bar{f}},m_H^2,m^2)$ and $C_{23}(m^2_{f\bar{f}},m_H^2,m^2)$, which occur in the Passarino-Veltman decomposition [5] of the loop integrals, as

$$
\mathcal{A}_\gamma(m^2_{f\bar{f}})=-e_f\left\{4\left(6+\frac{m_H^2}{m_W^2}\right)C_{23}(m^2_{f\bar{f}},m_H^2,m_W^2)-16C_0(m^2_{f\bar{f}},m_H^2,m_W^2)-\frac{16}{3}\frac{m_t^2}{m_W^2}[4C_{23}(m^2_{f\bar{f}},m_H^2,m_t^2)-C_0(m^2_{f\bar{f}},m_H^2,m_t^2)]\right\},
\tag{A1}
$$

$$\mathcal{A}_Z(m_{f\bar{f}}^2)=-2I_3\Bigg\{\left(5-\tan^2\theta_W+\frac{m_H^2}{2m_W^2}(1-\tan^2\theta_W)\right)C_{23}(m_{f\bar{f}}^2,m_H^2,m_W^2)+(\tan^2\theta_W-3)C_0(m_{f\bar{f}}^2,m_H^2,m_W^2)$$
$$-\frac{1}{2}\frac{m_t^2}{m_W^2}\frac{1-(8/3)\sin^2\theta_W}{\cos^2\theta_W}[4C_{23}(m_{f\bar{f}}^2,m_H^2,m_t^2)-C_0(m_{f\bar{f}}^2,m_H^2,m_t^2)]\Bigg\}, \tag{A2}$$

with I_3 denoting the third component of the *external* fermion weak isospin and m_t being the top quark mass.

The evaluation of $C_0(m_{f\bar{f}}^2,m_H^2,m^2)$ is straightforward, yielding

$$C_0(m_{f\bar{f}}^2,m_H^2,m^2)=\frac{1}{(m_{f\bar{f}}^2-m_H^2)}\left[C\left(\frac{m_H^2}{m^2}\right)-C\left(\frac{m_{f\bar{f}}^2}{m^2}\right)\right], \tag{A3}$$

where

$$C(\beta)=\int_0^1\frac{dx}{x}\ln[1-\beta x(1-x)-i\varepsilon]=\begin{cases}-2\left[\arcsin\left(\sqrt{\frac{\beta}{4}}\right)\right]^2, & 0\leqslant\beta\leqslant4, \quad \text{(A4)}\\ 2\left[\operatorname{arccosh}\left(\sqrt{\frac{\beta}{4}}\right)\right]^2-\frac{\pi^2}{2}-2i\pi\operatorname{arccosh}\left(\sqrt{\frac{\beta}{4}}\right), & \beta\geqslant4. \quad \text{(A5)}\end{cases}$$

The scalar function $C_{23}(m_{f\bar{f}}^2,m_H^2,m^2)$ is expressible as

$$C_{23}(m_{f\bar{f}}^2,m_H^2,m^2)=\frac{1}{2}\frac{1}{(m_{f\bar{f}}^2-m_H^2)}\left\{1+\frac{m_{f\bar{f}}^2}{(m_{f\bar{f}}^2-m_H^2)}\left[B\left(\frac{m_H^2}{m^2}\right)-B\left(\frac{m_{f\bar{f}}^2}{m^2}\right)\right]-\frac{2m^2}{(m_{f\bar{f}}^2-m_H^2)}\left[C\left(\frac{m_H^2}{m^2}\right)-C\left(\frac{m_{f\bar{f}}^2}{m^2}\right)\right]\right\}, \tag{A6}$$

where the function $B(\beta)$ is

$$B(\beta)=\int_0^1 dx\ln[1-\beta x(1-x)-i\varepsilon]=\begin{cases}2\left[\sqrt{\frac{4-\beta}{\beta}}\arcsin\left(\sqrt{\frac{\beta}{4}}\right)-1\right], & 0\leqslant\beta\leqslant4, \quad \text{(A7)}\\ 2\left[\sqrt{\frac{\beta-4}{\beta}}\operatorname{arccosh}\left(\sqrt{\frac{\beta}{4}}\right)-1-\frac{i\pi}{2}\sqrt{\frac{\beta-4}{\beta}}\right], & \beta\geqslant4. \quad \text{(A8)}\end{cases}$$

APPENDIX B: ‘‘Z’’ BOX CONTRIBUTION

The ‘‘Z’’ box diagrams are illustrated in Fig. 1(b), where the internal gauge boson can be either a Z or a W. In addition to these box diagrams, there are two triangle diagrams which make the amplitude gauge invariant. The entire contribution is given by the function $\mathcal{B}_0(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2)$ which is related to the function $\mathcal{B}_Z$ appearing in Eq. (3) as

$$\mathcal{B}_Z(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2)=-e_f\mathcal{B}_0(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2). \tag{B1}$$

In terms of the decomposition of Ref. [5], this function is given by

$$\mathcal{B}_0(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2)=D_0(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2,m_H^2,m_f^2,m_Z^2)+D_{11}(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2,m_H^2,m_f^2,m_Z^2)$$
$$+D_{12}(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2,m_H^2,m_f^2,m_Z^2)+D_{24}(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2,m_H^2,m_f^2,m_Z^2). \tag{B2}$$

When the $D_{\alpha\beta}$ in Eq. (B2) are expanded in terms of scalar integrals, the expression for $\mathcal{B}_0(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2)$ takes the form

$$
\begin{aligned}
\mathcal{B}_0(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma}) = \frac{1}{2}\,\frac{1}{m^2_{f\bar{f}}m^2_{f\gamma}}\Bigg\{&\left(1-\frac{m_Z^2}{m^2_{f\gamma}}\right)[m_Z^2(m^2_{f\gamma}+m^2_{\bar{f}\gamma})-m^2_{f\gamma}m^2_{\bar{f}\gamma}]D_0(1,2,3,4)-\left(1-\frac{m_Z^2}{m^2_{f\gamma}}\right)[m^2_{f\gamma}C_0(1,2,3)\\
&-(m^2_{f\bar{f}}+m^2_{\bar{f}\gamma})C_0(1,3,4)]-\left[\left(1-\frac{m_Z^2}{m^2_{f\gamma}}\right)(m^2_{f\bar{f}}-m^2_{f\gamma})-2m_Z^2\frac{m^2_{f\bar{f}}}{(m^2_{f\bar{f}}+m^2_{f\gamma})}\right]C_0(1,2,4)-m^2_{\bar{f}\gamma}\\
&\times\left(1-\frac{m_Z^2}{m^2_{f\gamma}}\right)C_0(2,3,4)+\frac{2m^2_{f\bar{f}}}{(m^2_{f\bar{f}}+m^2_{f\gamma})}[B_0(1,4)-B_0(2,4)]\Bigg\},
\end{aligned}
\tag{B3}
$$

where we have used the compact notation of Ref. [5] with $m_1=m_4=m_Z$ and $m_2=m_3=m_f$, as illustrated in Fig. 6(a).

Explicitly, we have

$$
\begin{aligned}
D_0(1,2,3,4)=-\frac{1}{\tau}\Bigg\{&\mathrm{Sp}\left(\frac{m^2_{f\gamma}-m_Z^2}{-m_Z^2}\right)-\mathrm{Sp}\left(\frac{-m_Z^2}{m^2_{f\gamma}-m_Z^2}\right)+\mathrm{Sp}\left(\frac{m^2_{\bar{f}\gamma}(m^2_{f\gamma}-m_Z^2)}{\tau}\right)-\mathrm{Sp}\left(\frac{-m^2_{\bar{f}\gamma}m_Z^2}{\tau}\right)-\mathrm{Sp}\left(\frac{m^2_{f\gamma}-m_Z^2}{m^2_{f\gamma}(1-\beta_{Z+})-m_Z^2}\right)\\
&+\mathrm{Sp}\left(\frac{-m_Z^2}{m^2_{f\gamma}(1-\beta_{Z+})-m_Z^2}\right)-\mathrm{Sp}\left(\frac{m^2_{f\gamma}-m_Z^2}{m^2_{f\gamma}(1-\beta_{Z-})-m_Z^2}\right)+\mathrm{Sp}\left(\frac{-m_Z^2}{m^2_{f\gamma}(1-\beta_{Z-})-m_Z^2}\right)+\mathrm{Sp}\left(\frac{m^2_{f\gamma}-m_Z^2}{m_Z^2m^2_{f\gamma}(\delta_0-\delta_2)}\right)\\
&-\mathrm{Sp}\left(\frac{-1}{m^2_{f\gamma}(\delta_0-\delta_2)}\right)+\mathrm{Sp}\left(\frac{m_Z^2m^2_{f\gamma}(\delta_0-\delta_2)}{m^2_{f\gamma}-m_Z^2}\right)-\mathrm{Sp}\left(\frac{m^2_{f\gamma}(\delta_0-\delta_2)}{-1}\right)+\ln\left(\frac{m_f^2}{m_Z^2}\right)\ln\left|\frac{m^2_{f\gamma}-m_Z^2}{-m_Z^2}\right|\\
&+i\pi\left[\theta(m_H^2-4m_Z^2)\ln\left|\frac{\beta_{Z+}-\alpha_0}{\beta_{Z-}-\alpha_0}\right|+\theta(m^2_{f\gamma}-m_Z^2)\left(\ln\left|\frac{m^2_{\bar{f}\gamma}(m^2_{f\gamma}-m_Z^2)}{m^2_{f\gamma}m_Z^2}\right|+\ln\left|\frac{\tau}{m^2_{f\gamma}m^2_{\bar{f}\gamma}}\right|-\ln|m_f^2(\delta_0-\delta_2)|\right)\right]\\
&+m^2_{f\gamma}\leftrightarrow m^2_{\bar{f}\gamma}\Bigg\},
\end{aligned}
\tag{B4}
$$

where

$$
\tau=m^2_{f\gamma}m^2_{\bar{f}\gamma}-m_Z^2(m^2_{f\gamma}+m^2_{\bar{f}\gamma}),
\tag{B5}
$$

$$
\alpha_0=1-\frac{m_Z^2}{m^2_{f\gamma}}+m_f^2\delta_0,
\tag{B6}
$$

$$
\beta_{Z\pm}=\frac{1}{2}(1\pm\sqrt{1-4m_Z^2/m_H^2}),
\tag{B7}
$$

with

$$
\delta_0=\frac{m^2_{f\bar{f}}}{\tau},
\tag{B8}
$$

$$
\delta_2=\frac{-1}{(m^2_{f\gamma}-m_Z^2)}.
\tag{B9}
$$

The function $\mathrm{Sp}(y)$ is defined as the real part of the dilogarithm. For real $y>1$, we have

$$
\mathrm{Sp}(y)=\mathrm{Re}\left(-\int_0^1\frac{dx}{x}\ln(1-yx+i\varepsilon)\right).
\tag{B10}
$$

When y is complex, the $i\varepsilon$ may be ignored.

The required C_0 functions are

$$C_0(1,2,3)=\frac{-1}{m_{f\gamma}^2}\left\{\mathrm{Sp}\left(\frac{m_Z^2-m_{f\gamma}^2}{m_{f\gamma}^2}\right)+\mathrm{Sp}\left(\frac{m_{f\gamma}^2}{m_Z^2-m_{f\gamma}^2}\right)-\mathrm{Sp}\left(\frac{(m_Z^2-m_{f\gamma}^2)^2}{m_Z^2m_{f\gamma}^2}\right)-\mathrm{Sp}\left(\frac{m_Z^2m_{f\gamma}^2}{(m_Z^2-m_{f\gamma}^2)^2}\right)+\mathrm{Sp}\left(\frac{m_Z^2}{m_Z^2-m_{f\gamma}^2}\right)-\frac{\pi^2}{6}\right.$$
$$\left.+\ln\left(\frac{m_f^2}{m_Z^2}\right)\ln\left|\frac{m_Z^2}{m_Z^2-m_{f\gamma}^2}\right|-i\pi\theta(m_{f\gamma}^2-m_Z^2)\ln\left|\frac{(m_Z^2-m_{f\gamma}^2)^2}{m_f^2m_{f\gamma}^2}\right|\right\}, \tag{B11}$$

$$C_0(2,3,4)=\frac{1}{m_{\bar{f}\gamma}^2}\left\{\mathrm{Sp}\left(\frac{m_{\bar{f}\gamma}^2-m_Z^2}{m_Z^2}\right)+\mathrm{Sp}\left(\frac{m_Z^2}{m_{\bar{f}\gamma}^2-m_Z^2}\right)+\mathrm{Sp}\left(\frac{m_{\bar{f}\gamma}^2}{m_{\bar{f}\gamma}^2-m_Z^2}\right)+\frac{\pi^2}{6}+\ln\left(\frac{m_f^2}{m_Z^2}\right)\ln\left|\frac{m_Z^2-m_{\bar{f}\gamma}^2}{m_Z^2}\right|\right.$$
$$\left.+i\pi\theta(m_{\bar{f}\gamma}^2-m_Z^2)\ln\left|\frac{(m_Z^2-m_{\bar{f}\gamma}^2)^2}{m_f^2m_{\bar{f}\gamma}^2}\right|\right\}, \tag{B12}$$

$$C_0(1,2,4)=\frac{-1}{m_{\bar{f}\gamma}^2-m_H^2}\left\{\mathrm{Sp}\left(\frac{\gamma_0}{\gamma_0-\gamma_1}\right)-\mathrm{Sp}\left(\frac{\gamma_0-1}{\gamma_0-\gamma_1}\right)-\mathrm{Sp}\left(\frac{\gamma_0}{\gamma_0-\beta_{Z+}}\right)+\mathrm{Sp}\left(\frac{\gamma_0-1}{\gamma_0-\beta_{Z+}}\right)-\mathrm{Sp}\left(\frac{\gamma_0}{\gamma_0-\beta_{Z-}}\right)+\mathrm{Sp}\left(\frac{\gamma_0-1}{\gamma_0-\beta_{Z-}}\right)\right.$$
$$\left.-\mathrm{Sp}\left(\frac{\gamma_0-1}{\gamma_0}\right)+\frac{\pi^2}{6}+i\pi\left[\theta(m_H^2-4m_Z^2)\ln\left|\frac{\beta_{Z+}-\gamma_0}{\beta_{Z-}-\gamma_0}\right|-\theta(m_{\bar{f}\gamma}^2-m_Z^2)\ln\left|\frac{\gamma_1-\gamma_0}{-\gamma_0}\right|\right]\right\}, \tag{B13}$$

with

$$\gamma_0=\frac{m_Z^2}{m_H^2-m_{\bar{f}\gamma}^2}, \tag{B14}$$

$$\gamma_1=\frac{m_{\bar{f}\gamma}^2-m_Z^2}{m_{\bar{f}\gamma}^2}. \tag{B15}$$

The expression for $C_0(1,3,4)$ is obtained from $C_0(1,2,4)$ by the interchange $m_{f\gamma}^2\leftrightarrow m_{\bar{f}\gamma}^2$.

Finally, the required B_0 functions are

$$B_0(1,4)=\Delta+2+\beta_{Z+}\ln\left(\frac{\beta_{Z+}-1+i\varepsilon}{\beta_{Z+}}\right)+\beta_{Z-}\ln\left(\frac{\beta_{Z-}-1-i\varepsilon}{\beta_{Z-}}\right), \tag{B16}$$

$$B_0(2,4)=\Delta+2-\left(1-\frac{m_Z^2}{m_{\bar{f}\gamma}^2}\right)\left[\ln\left|1-\frac{m_{\bar{f}\gamma}^2}{m_Z^2}\right|-i\pi\theta(m_{\bar{f}\gamma}^2-m_Z^2)\right], \tag{B17}$$

where Δ is

$$\Delta=\pi^{(n/2-2)}\left(\frac{m_Z^2}{\mu^2}\right)^{(n/2-2)}\Gamma\left(2-\frac{n}{2}\right), \tag{B18}$$

and μ is introduced to preserve the dimensions of the regularized integrals.

The cancellation of the $\ln(m_f^2)$ dependence in Eqs. (B4), (B11), and (B12) can be checked by substituting the explicit expressions into Eq. (B3).

APPENDIX C: W BOX CONTRIBUTION

The box diagrams with W's in the loop are shown in Figs. 1(b) and 1(c). The non-gauge-invariant portions of these diagrams are again canceled by triangle diagrams in which the Higgs boson decays into an $f\bar{f}$ pair through a WWf triangle and the photon is emitted from one of the fermions. The invariant amplitude $\mathcal{B}_W(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2)$ appearing in Eq. (4) can be expressed as

$$\mathcal{B}_W(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2)=-2I_3[\mathcal{B}_1(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2)+\mathcal{B}_2(m_{f\bar{f}}^2,m_{\bar{f}\gamma}^2,m_{f\gamma}^2)]+e_i\mathcal{B}_0'(m_{f\bar{f}}^2,m_{f\gamma}^2,m_{\bar{f}\gamma}^2), \tag{C1}$$

where, as in Appendix A, I_3 is the third component of the external fermion weak isospin. Here, e_i is the charge of the *internal* fermion in units of the proton charge and the prime denotes the replacement $m_Z\rightarrow m_W$ in Eq. (B2). Notice that the arguments $m_{f\gamma}^2$ and $m_{\bar{f}\gamma}^2$ are interchanged in $\mathcal{B}_2$. In terms of the $D_{\alpha\beta}$ of Ref. [5], we have

$$\mathcal{B}_1(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})=D_0(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma},m^2_H,m^2_f,m^2_W)+D_{11}(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma},m^2_H,m^2_f,m^2_W)$$
$$+D_{13}(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma},m^2_H,m^2_f,m^2_W)+D_{25}(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma},m^2_H,m^2_f,m^2_W), \tag{C2}$$

$$\mathcal{B}_2(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})=-D_{12}(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma},m^2_H,m^2_f,m^2_W)+D_{13}(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma},m^2_H,m^2_f,m^2_W)$$
$$+D_{26}(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma},m^2_H,m^2_f,m^2_W). \tag{C3}$$

The decomposition into scalar functions takes the form

$$\mathcal{B}_1(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})=\frac{1}{2}\,\frac{1}{m^2_{f\bar{f}}m^2_{f\gamma}}\Bigg\{\Bigg[\frac{(m^2_{f\bar{f}}+m^2_{f\gamma}-m^2_W)}{m^2_{f\gamma}}[m^2_W(m^2_{f\gamma}+m^2_{\bar{f}\gamma})-m^2_{f\bar{f}}m^2_{\bar{f}\gamma}]-2m^2_Wm^2_{f\bar{f}}\Bigg]D_0(1,2,3,4)$$
$$+(m^2_{f\bar{f}}+m^2_{f\gamma}-m^2_W)\Bigg[-\frac{m^2_{f\bar{f}}}{m^2_{f\gamma}}C_0(1,2,3)+\frac{(m^4_{f\bar{f}}+2m^2_{f\bar{f}}m^2_{f\gamma}-m^4_{f\gamma})}{(m^2_{f\bar{f}}+m^2_{f\gamma})m^2_{f\gamma}}C_0(1,2,4)$$
$$+\frac{(m^2_{f\gamma}+m^2_{\bar{f}\gamma})}{m^2_{f\gamma}}C_0(1,3,4)-\frac{m^2_{\bar{f}\gamma}}{m^2_{f\gamma}}C_0(2,3,4)\Bigg]+\frac{2m^2_{f\bar{f}}}{(m^2_{f\gamma}+m^2_{\bar{f}\gamma})}[B_0(1,3)-B_0(1,4)]$$
$$+\frac{2m^2_{f\bar{f}}}{(m^2_{f\bar{f}}+m^2_{f\gamma})}[B_0(2,4)-B_0(1,4)]\Bigg\}, \tag{C4}$$

$$\mathcal{B}_2(m^2_{f\bar{f}},m^2_{f\gamma},m^2_{\bar{f}\gamma})=\frac{1}{2}\,\frac{1}{m^2_{f\bar{f}}m^2_{\bar{f}\gamma}}\Bigg\{\frac{(m^2_{\bar{f}\gamma}-m^2_W)}{m^2_{\bar{f}\gamma}}[m^2_{f\bar{f}}m^2_{\bar{f}\gamma}+m^2_W(m^2_{f\gamma}+m^2_{\bar{f}\gamma})]D_0(1,2,3,4)+(m^2_{\bar{f}\gamma}-m^2_W)\Bigg[\frac{m^2_{f\bar{f}}}{m^2_{\bar{f}\gamma}}C_0(1,2,3)$$
$$-\frac{(m^2_{f\bar{f}}+m^2_{f\gamma})}{m^2_{\bar{f}\gamma}}C_0(1,2,4)+\frac{(m^2_{f\gamma}+m^2_{\bar{f}\gamma})}{m^2_{\bar{f}\gamma}}C_0(1,3,4)-C_0(2,3,4)\Bigg]+\frac{2m^2_{f\bar{f}}}{(m^2_{f\gamma}+m^2_{\bar{f}\gamma})}[B_0(1,4)-B_0(1,3)]\Bigg\}. \tag{C5}$$

The numbering is defined in Fig. 6(b), with $m_1=m_3=m_4=m_W$ and $m_2=m_i=0$.

In this case, the scalar function $D_0(1,2,3,4)$ is

$$D_0(1,2,3,4)=\frac{-1}{m^2_{f\bar{f}}m^2_{\bar{f}\gamma}}\,\frac{1}{(\beta_{0+}-\beta_{0-})}\Bigg\{\mathrm{Sp}\left(\frac{\beta_{0+}}{\beta_{0+}-m^2_W/m^2_{\bar{f}\gamma}}\right)-\mathrm{Sp}\left(\frac{\beta_{0+}-1}{\beta_{0+}-m^2_W/m^2_{\bar{f}\gamma}}\right)-\mathrm{Sp}\left(\frac{\beta_{0+}}{\beta_{0+}-\beta_{W+}}\right)+\mathrm{Sp}\left(\frac{\beta_{0+}-1}{\beta_{0+}-\beta_{W+}}\right)$$
$$-\mathrm{Sp}\left(\frac{\beta_{0+}}{\beta_{0+}-\beta_{W-}}\right)+\mathrm{Sp}\left(\frac{\beta_{0+}-1}{\beta_{0+}-\beta_{W-}}\right)+\mathrm{Sp}\left(\frac{\beta_{0+}}{\beta_{0+}-\gamma_+}\right)-\mathrm{Sp}\left(\frac{\beta_{0+}-1}{\beta_{0+}-\gamma_+}\right)+\mathrm{Sp}\left(\frac{\beta_{0+}}{\beta_{0+}-\gamma_-}\right)-\mathrm{Sp}\left(\frac{\beta_{0+}-1}{\beta_{0+}-\gamma_-}\right)$$
$$+i\pi\Bigg[-\theta(m^2_{\bar{f}\gamma}-m^2_W)\ln\left|\frac{\beta_{0+}-1}{\beta_{0+}-m^2_W/m^2_{\bar{f}\gamma}}\right|+\theta(m^2_H-4m^2_W)\ln\left|\frac{\beta_{W+}-\beta_{0+}}{\beta_{W-}-\beta_{0+}}\right|-\theta(m^2_{f\bar{f}}-4m^2_W)\ln\left|\frac{\gamma_+-\beta_{0+}}{\gamma_--\beta_{0+}}\right|\Bigg]$$
$$+\beta_{0+}\to\beta_{0-}\Bigg\}, \tag{C6}$$

with

$$\beta_{0\pm}=\frac{1}{2}\left[1-\frac{m^2_W}{m^2_{f\bar{f}}}-\frac{m^2_W}{m^2_{f\bar{f}}}\frac{m^2_{f\gamma}}{m^2_{\bar{f}\gamma}}\pm\sqrt{\left(1-\frac{m^2_W}{m^2_{f\bar{f}}}-\frac{m^2_W}{m^2_{f\bar{f}}}\frac{m^2_{f\gamma}}{m^2_{\bar{f}\gamma}}\right)^2+4\frac{m^2_W}{m^2_{f\bar{f}}}\frac{m^2_{f\gamma}}{m^2_{\bar{f}\gamma}}}\right], \tag{C7}$$

$$\beta_{W\pm}=\frac{1}{2}(1\pm\sqrt{1-4m^2_W/m^2_H}), \tag{C8}$$

$$\gamma_\pm=\frac{1}{2}(1\pm\sqrt{1-4m^2_W/m^2_{f\bar{f}}}). \tag{C9}$$

The various C_0's are

$$C_0(1,2,3)=\frac{1}{m_{f\bar{f}}^2}\left\{-\mathrm{Sp}\left(\frac{m_W^2}{m_W^2-m_{f\bar{f}}^2\gamma_+}\right)+\mathrm{Sp}\left(\frac{m_W^2-m_{f\bar{f}}^2}{m_W^2-m_{f\bar{f}}^2\gamma_+}\right)-\mathrm{Sp}\left(\frac{m_W^2}{m_W^2-m_{f\bar{f}}^2\gamma_-}\right)+\mathrm{Sp}\left(\frac{m_W^2-m_{f\bar{f}}^2}{m_W^2-m_{f\bar{f}}^2\gamma_-}\right)-\mathrm{Sp}\left(\frac{m_W^2-m_{f\bar{f}}^2}{m_W^2}\right)\right.$$
$$\left.+\frac{\pi^2}{6}+i\pi\left[\theta(m_{f\bar{f}}^2-4m_W^2)\ln\left|\frac{m_W^2-m_{f\bar{f}}^2\gamma_+}{m_W^2-m_{f\bar{f}}^2\gamma_-}\right|\right]\right\}, \tag{C10}$$

$$C_0(1,2,4)=\frac{-1}{m_{\bar{f}\gamma}^2-m_H^2}\left\{\mathrm{Sp}\left(\frac{\lambda_0}{\lambda_0-\lambda_1}\right)-\mathrm{Sp}\left(\frac{\lambda_0-1}{\lambda_0-\lambda_1}\right)-\mathrm{Sp}\left(\frac{\lambda_0}{\lambda_0-\beta_{W+}}\right)+\mathrm{Sp}\left(\frac{\lambda_0-1}{\lambda_0-\beta_{W+}}\right)-\mathrm{Sp}\left(\frac{\lambda_0}{\lambda_0-\beta_{W-}}\right)+\mathrm{Sp}\left(\frac{\lambda_0-1}{\lambda_0-\beta_{W-}}\right)\right.$$
$$\left.-\mathrm{Sp}\left(\frac{\lambda_0-1}{\lambda_0}\right)+\frac{\pi^2}{6}+i\pi\left[\theta(m_H^2-4m_W^2)\ln\left|\frac{\beta_{W+}-\lambda_0}{\beta_{W-}-\lambda_0}\right|-\theta(m_{\bar{f}\gamma}^2-m_W^2)\ln\left|\frac{\lambda_1-\lambda_0}{-\lambda_0}\right|\right]\right\}, \tag{C11}$$

$$C_0(1,3,4)=\frac{-1}{m_{f\bar{f}}^2-m_H^2}\left\{\mathrm{Sp}\left(\frac{1}{\beta_{W+}}\right)+\mathrm{Sp}\left(\frac{1}{\beta_{W-}}\right)-\mathrm{Sp}\left(\frac{1}{\gamma_+}\right)-\mathrm{Sp}\left(\frac{1}{\gamma_-}\right)+i\pi\left[\theta(m_H^2-4m_W^2)\ln\left|\frac{\beta_{W+}}{\beta_{W-}}\right|-\theta(m_{f\bar{f}}^2-4m_W^2)\ln\left|\frac{\gamma_+}{\gamma_-}\right|\right]\right\}, \tag{C12}$$

$$C_0(2,3,4)=\frac{1}{m_{\bar{f}\gamma}^2}\left\{\mathrm{Sp}\left(\frac{m_{\bar{f}\gamma}^2}{m_W^2}\right)+i\pi\theta(m_{\bar{f}\gamma}^2-m_W^2)\ln\left|\frac{m_{\bar{f}\gamma}^2}{m_W^2}\right|\right\}, \tag{C13}$$

with

$$\lambda_0=\frac{m_W^2}{m_H^2-m_{\bar{f}\gamma}^2}, \tag{C14}$$

$$\lambda_1=\frac{m_{\bar{f}\gamma}^2-m_W^2}{m_{\bar{f}\gamma}^2}. \tag{C15}$$

In this case, the B_0's are

$$B_0(1,3)=\Delta+2+\gamma_+\ln\left(\frac{\gamma_+-1+i\varepsilon}{\gamma_+}\right)+\gamma_-\ln\frac{\gamma_--1-i\varepsilon}{\gamma_-}, \tag{C16}$$

$$B_0(1,4)=\Delta+2+\beta_{W+}\ln\left(\frac{\beta_{W+}-1+i\varepsilon}{\beta_{W+}}\right)+\beta_{W-}\ln\left(\frac{\beta_{W-}-1-i\varepsilon}{\beta_{W-}}\right), \tag{C17}$$

$$B_0(2,4)=\Delta+2-\left(1-\frac{m_W^2}{m_{\bar{f}\gamma}^2}\right)\left[\ln\left|1-\frac{m_{\bar{f}\gamma}^2}{m_W^2}\right|-i\pi\theta(m_{\bar{f}\gamma}^2-m_W^2)\right], \tag{C18}$$

and

$$\Delta=\pi^{(n/2-2)}\left(\frac{m_W^2}{\mu^2}\right)^{(n/2-2)}\Gamma\left(2-\frac{n}{2}\right). \tag{C19}$$

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