Three-scale factorization theorem and effective field theory: Analysis of nonleptonic heavy meson decays

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We develop a perturbative QCD factorization theorem which is compatible with effective field theory. The factorization involves three scales: an infrared cutoff of order $\Lambda_{\rm QCD}$, a hard scale of the order of the B meson mass, and an ultraviolet cutoff of the order of the W boson mass. Our approach is renormalization-group invariant and moderates the scale-dependent problem in effective field theory. Employing this formalism with nonfactorizable contributions included, we clarify the controversy over the BSW parameters a_2/a_1 for charm and bottom decays. [S0556-2821(97)01109-0]

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Nonleptonic heavy meson decays are difficult to analyze due to the complicated QCD corrections and the multiple characteristic scales they involve. While semileptonic decays involve only conserved currents, nonleptonic decays are described by four-quark current-current operators. For example, the relevant operator for the $B\to D\pi$ decays is

$$H=\frac{4G_F}{\sqrt{2}}V_{cb}V_{ud}^{*}(\overline{c_L}\gamma_\mu b_L)(\overline{d_L}\gamma^\mu u_L). \qquad (1)$$

The QCD corrections will generate operator mixing, characterized by Wilson coefficients, among these operators. The resultant effective Hamiltonian related to Eq. (1) is written as

$$H_{\rm eff}=\frac{4G_F}{\sqrt{2}}V_{cb}V_{ud}^{*}[c_1(\mu)O_1+c_2(\mu)O_2], \qquad (2)$$

with

$$O_1=(\overline{c_L}\gamma_\mu b_L)(\overline{d_L}\gamma^\mu u_L)$$

and

$$O_2=(\overline{d_L}\gamma_\mu b_L)(\overline{c_L}\gamma^\mu u_L).$$

c_1 and c_2 are the Wilson coefficients, whose evolution from the W boson mass M_W down to a lower scale μ is determined by renormalization-group (RG) running [1]. Though Wilson coefficients are μ dependent, physical quantities such as decay amplitudes are not. In principle, the matrix elements of the four-fermion operators contain a μ dependence, which exactly cancels that of the Wilson coefficients. In practical applications, however, various schemes are needed to estimate the hadronic matrix elements, and the estimates are usually μ independent. Hence, the decay amplitudes turn out to be scale dependent. Take exclusive nonleptonic heavy meson decays as an example, to which the conventional approach is the Bauer-Stech-Wirbel (BSW) factorization approximation [2]. It is assumed that nonleptonic matrix elements can be factorized into two matrix elements of (axial) vector currents. Since the currents are conserved, the matrix elements have no anomalous scale dependence. Presumably μ should be set to the dominant scale of the matrix elements. However, the matrix elements involve both the heavy quark scale and the small hadronic scale. Naively setting μ to the heavy quark mass will lose large logarithms associated with the hadronic scale. It is then quite natural that theoretical predictions are sensitive to the scale we choose [3].

To circumvent this problem, a phenomenological approach is adopted to bypass the strong scale dependence. The Wilson coefficients $c_{1,2}$ are regarded as free parameters and determined by experimental data [2]. In this model two equivalent parameters $a_1=c_1+c_2/N_c$ and $a_2=c_2+c_1/N_c$ describe the external and internal W-emission amplitudes, respectively. However, the evaluation of the hadronic form factors usually involve some ansatz [4], and thus the extraction of $a_{1,2}$ is model dependent. It is also found that a negative a_2/a_1 and a positive a_2/a_1 are concluded from the data of charm and bottom decays [2,5], respectively.

It was shown recently that the perturbative QCD (PQCD) approach based on the full Hamiltonian in Eq. (1) is applicable to heavy meson decays at large recoil [6,7] in the sense that more than half of contributions come from the region with the running coupling constant $\alpha_s<1$. The breakthrough is due to the all-order Sudakov resummation of large radiative corrections, which suppresses contributions from the long-distance region. This formalism, taking into account the evolution from the typical scale of hard subprocesses characterized by the heavy meson mass to a lower hadronic scale, is μ independent for semileptonic decays. However, it cannot be an appropriate tool for nonleptonic decays, because it does not involve the scale M_W. In this paper we shall develop a PQCD formalism based on the effective Hamiltonian in Eq. (2), which further incorporates the evolution from M_W down to the hard scale. This three-scale factorization theorem, as demonstrated below, moderates the scale-setting ambiguity.

We first illustrate the main idea of PQCD factorization theorems by considering one-loop corrections to a generic

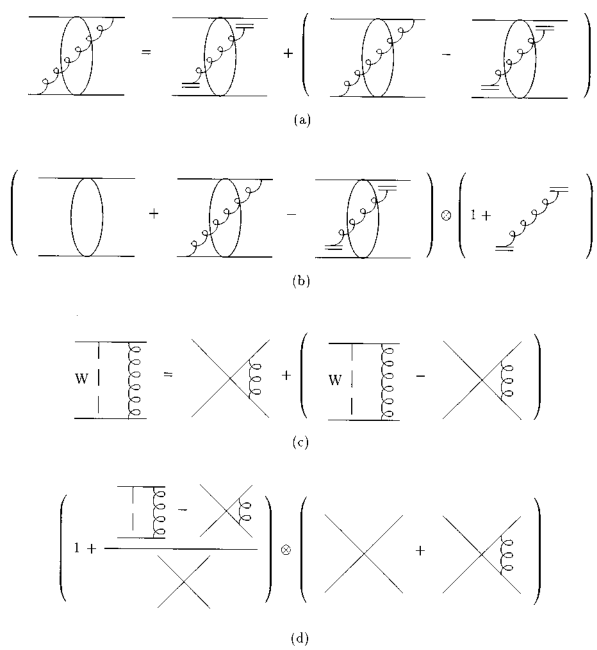

FIG. 1. (a) Separation of infrared and hard $O(\alpha_s)$ contributions in PQCD. (b) $O(\alpha_s)$ factorization into a soft function and a hard scattering amplitude. (c) Separation of hard and harder $O(\alpha_s)$ contributions in an effective field theory. (d) $O(\alpha_s)$ factorization into a ''harder'' function and a hard scattering amplitude.

decay process through a current. These corrections are ultraviolet finite, since the conserved current is not renormalized. However, they also give rise to infrared divergences, when the gluons are soft or collinear to light partons. The factorization is implemented to isolate these infrared divergences. Radiative corrections are classified into reducible and irreducible types. Irreducible corrections contain only single soft logarithms and are absorbed into a soft function U. Reducible corrections, containing double logarithms from the combination of soft and collinear divergences, are absorbed into a wave function $\phi(P,b,\mu)$ and explicitly resummed into a Sudakov factor e^{-s}:

$$\phi(P,b,\mu)=\exp[-s(P,b)]\phi(b,\mu). \tag{3}$$

b is the conjugate variable of the transverse momentum, and $1/b$ can be regarded as an infrared cutoff of order of the hadonic scale. The Sudakov factor strongly suppresses the contributions from the large b region. With Sudakov suppression, the irreducible soft corrections, appearing in the form $1-e^{i\mathbf{l}\cdot\mathbf{b}}$, $\mathbf{l}$ being the loop momentum, cancel asymptotically $(b\to 0)$ [7]. Hence, they will be neglected below (i.e., $U=1$).

To factorize a one-loop correction, we divide it into two terms as shown in Fig. 1(a). The first term, with an eikonal approximation for fermion propagators, picks up the infrared structure of the full diagram. Being infrared sensitive, it is absorbed into U or ϕ, depending on which type the correction is. The second term, with a soft subtraction, has the same ultraviolet structure as the full diagram, and can be absorbed into a hard scattering amplitude $H(t,\mu)$, where t denotes the typical scale of the hard decay process. We then get the $O(\alpha_s)$ factorization formula shown in Fig. 1(b) with the diagrams in the first parentheses contributing to H. The presence of μ implies that both ϕ and H need renormalization. Let γ_ϕ be the anomalous dimension of ϕ. Then the anomalous dimension of H must be $-\gamma_\phi$, because the full diagram is ultraviolet finit. Using the renormalization group (RG), the convolution of H with ϕ is μ independent as indicated by

$$H(t,\mu)\phi(b,\mu)=H(t,t)\phi(b,1/b)\times\exp\left[-\int_{1/b}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_\phi(\alpha_s(\bar{\mu}))\right]. \tag{4}$$

The contribution characterized by momenta smaller than $1/b$, i.e., the infrared divergence, is absorbed into the initial condition $\phi(b,1/b)$, which is of nonperturbative origin.

Indeed the effective Hamiltonian in Eq. (2) can be constructed in a similar way. Consider a typical one-loop QCD correction to the W-exchange diagram Fig. 1(c). We express the full diagram, which is ultraviolet finite, as two terms as shown in Fig. 1(c). The first term, obtained by shrinking the W boson line into a vertex, corresponds to the local four-fermion operators O_i. It is absorbed into a hard scattering amplitude $H(t,\mu)$, with a typical scale $t\ll M_W$, since gluons involved in this term do not ''see'' the W boson. The second term, characterized by momenta of order M_W, is absorbed into a ''harder'' function $H_r(M_W,\mu)$ (not a scattering amplitude), in which gluons do ''see'' the W boson.

We obtain the $O(\alpha_s)$ factorization formula shown in Fig. 1(d), where the diagrams in the first parentheses contribute to H_r and those in the second parentheses to H. This formula in fact represents a matrix relation because of the mixing between O_1 and O_2. Solving their RG equations, we derive

$$H_r(M_W,\mu)H(t,\mu)=H_r(M_W,M_W)H(t,t)\times\exp\left[\int_{t}^{M_W}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_{H_r}(\alpha_s(\bar{\mu}))\right], \tag{5}$$

where the anomalous dimension γ_{H_r} of H_r is also a matrix. We emphasize that the factorization in Eq. (5) is not complete because of the presence of infrared divergences in H. Without large logarithms, $H_r(M_W,M_W)$ can now be safely approximated by its lowest-order expression $H_r^{(0)}=1$.

We are now ready to contruct a three-scale factorization theorem by combining Eqs. (4) and (5). Consider the decay amplitude up to $O(\alpha_s)$ without integrating out the W boson. We first factorize out the infrared-sensitive wave functions as described above. Though devoid of infrared divergences, the hard part still invloves two scales t and M_W. The factorization in Fig. 1(d) is then employed to separate these two scales, and H_r can be moved out of the hard part, a step valid up to $O(\alpha_s)$. We identify the remaining diagrams, including the four-fermion amplitude and the associated soft subtraction, as H. The anomalous dimension of H is given by $\gamma_H=-(\gamma_\phi+\gamma_{H_r})$. We thus get the three-scale factorization formula

$$H_r(M_W,\mu)H(t,\mu)\phi(b,\mu)=c(t)H(t,t)\phi(b,1/b)\times\exp\left[-\int_{1/b}^{t}\frac{d\overline{\mu}}{\overline{\mu}}\gamma_\phi(\alpha_s(\overline{\mu}))\right], \tag{6}$$

where the exponential factor in Eq. (5) has been identified as the Wilson coefficient $c(t)$. It implies that μ in $c(\mu)$ should be set to the hard scale t. The two-stage evolutions from $1/b$ to t and from t to M_W are both included, and the final expression is μ independent. Note that this μ independence is a direct consequence of the current conservation as stated before.

The above conclusion is quite natural from the effective field theory approach [8]. An effective field theory is constructed for a scale $\mu<M_W$ by integrating out the W boson at $\mu=M_W$. Matching corrections are determined by the matching condition requiring that the low-energy light-particle Green functions of the two theories be equal. The effective theory is then evolved by RG running from $\mu=M_W$ to a lower scale, ensuring that the amplitudes are μ independent. The scale μ in a continuum effective field theory is actually a scale to separate the long-distance from the short-distance physics with the physics above the scale μ absorbed into the coefficients in the effective Hamiltonian, such as the Wilson coefficients $c_{1,2}(\mu)$. The effective field theory constructed this way has exactly the same low-energy behavior as the full theory, including infrared divergences, physical cuts, etc. Thus the infrared divergences in the decay amplitudes calculated using the effective field theory can be factorized in the same way as the full theory. The factorization formula for the μ-independent amplitude, $c(M_W,\mu)H(t,\mu)\phi(b,\mu)$, is identical to Eq. (6) with the Wilson coefficient c identified as H_r.

We apply the above formalism to the nonleptonic $B(P_1)\to D(P_2)\pi(P_3)$ decays. The decay rate is given by [7]

$$\Gamma=\frac{1}{128\pi}G_F^2|V_{cb}|^2|V_{ud}|^2M_B^3\frac{(1-r^2)^3}{r}|\mathcal{M}|^2, \tag{7}$$

with $r=M_D/M_B$, M_B (M_D) being the B (D) meson mass. In the rest frame of the B meson, P_1 has the components $P_1=(M_B/\sqrt{2})(1,1,\mathbf{0}_T)$. The nonvanishing components of P_2 and P_3 are, respectively, $P_2^+=M_B/\sqrt{2}$, $P_2^-=rM_D/\sqrt{2}$, $P_3^+=0$, and $P_3^-=(1-r^2)M_B/\sqrt{2}$. Let $k_1(k_2)$ be the momentum of the light valence quark in the B (D) meson and k_3 be the momentum of a valence quark in the pion. These k's may be off shell by the amount of their transverse components k_T of order $\Lambda_{\rm QCD}$. We define the momentum fractions x as $x_1=k_1^-/P_1^-$, $x_2=k_2^+/P_2^+$, and $x_3=k_3^-/P_3^-$. To leading power in $1/M_B$, the factorization formula for $\mathcal{M}$ in the transverse configuration space is written as

$$\mathcal{M}=\int_0^1[dx]\int_0^\infty[d^2\mathbf{b}]\phi_B(x_1,b_1,1/b_1)\phi_D(x_2,b_2,1/b_2)\times\phi_\pi(x_3,b_3,1/b_3)c(t)H(x_i,b_i,t)\exp[-S(x_i,b_i)], \tag{8}$$

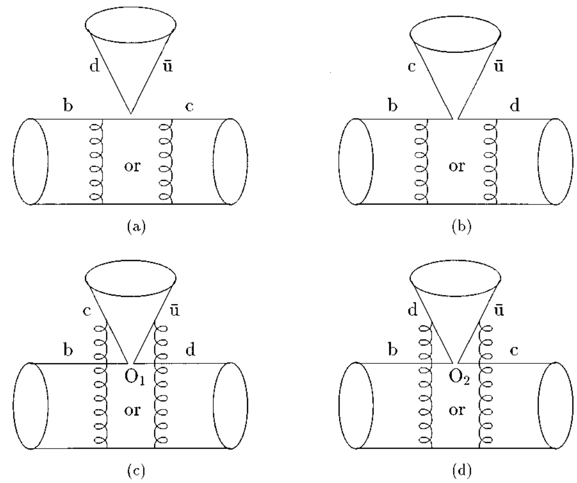

FIG. 2. (a) External W emission. (b) Internal W emission. (c) and (d) Nonfactorizable internal W emissions.

with $[dx]=dx_1dx_2dx_3$ and $[d^2\mathbf{b}]=d^2\mathbf{b}_1d^2\mathbf{b}_2d^2\mathbf{b}_3$. The Sudakov factor e^{-S} is the product of e^{-s} in Eq. (3) and the exponential in Eq. (6) from each wave function. Below we shall neglect the b dependence of the wave functions [6].

Without large logarithms, H can be reliably treated by perturbation theory. To leading order in α_s, the hard part for the decay $B^-\to D^0\pi^-$ consists of four sets of diagrams shown in Fig. 2. The diagrams in Fig. 2(a) correspond to the external W emission [2,3], while those in Fig. 2(b) to the internal W emission. They have been calculated using the PQCD formalism in [6,7] without including the Wilson coefficients. Denote their contributions to the amplitude $\mathcal{M}$ as $\mathcal{M}_a$ and $\mathcal{M}_b$. It is easy to find that the Wilson coefficients associated with $\mathcal{M}_a$ and $\mathcal{M}_b$ are respectively a_1 and a_2. Readers are referred to [7] for the complete formulas of $\mathcal{M}_a$ and $\mathcal{M}_b$.

Diagrams in Figs. 2(c) and 2(d) are absent in the factorization approximation and will be called the nonfactorizable diagrams. Figure 2(c) leads to

$$\mathcal{M}_c=32\sqrt{2N_c}\pi\mathcal{C}_F\sqrt{r}M_B^2G_F\int_0^1[dx]\int_0^\infty b_1db_1b_2db_2\phi_B(x_1)\times\phi_D(x_2)\phi_\pi(x_3)\bigg[\alpha_s(t_1)\frac{c_1(t_1)}{N_c}e^{-S_c^{(1)}(x_i,b_i)}\times[x_1-x_2-x_3(1-r^2)]h_c^{(1)}(x_i,b_i)+\alpha_s(t_2)\frac{c_1(t_2)}{N_c}\times e^{-S_c^{(2)}(x_i,b_i)}[1-(x_1+x_2)(1-r^2)]h_c^{(2)}(x_i,b_i)\bigg]. \tag{9}$$

The functions $h_c^{(j)}$, $j=1$ and 2, are given by

$$h_c^{(j)}=[\theta(b_1-b_2)K_0(AM_Bb_1)I_0(AM_Bb_2)+\theta(b_2-b_1)\times K_0(AM_Bb_2)I_0(AM_Bb_1)]\times\begin{pmatrix}K_0(B_jM_Bb_2) & \text{for } B_j\geqslant 0\\ \frac{i\pi}{2}H_0^{(1)}(|B_j|M_Bb_2) & \text{for } B_j\leqslant 0\end{pmatrix}, \tag{10}$$

with $A^2=x_1x_3(1-r^2)$, $B_1^2=(x_1+x_2)r^2-(1-x_1-x_2)x_3(1-r^2)$, and $B_2^2=(x_1-x_2)x_3(1-r^2)$. The Sudakov exponent $S_c^{(j)}$ is written as

$$S_c^{(j)}=s(x_1P_1^+,b_1)+s(x_2P_2^+,b_2)+s((1-x_2)P_2^+,b_2)+s(x_3P_3^-,b_3)+s((1-x_3)P_3^-,b_3)-\frac{1}{\beta_1}\sum_{i=1}^{3}\ln\frac{\ln(t_j/\Lambda)}{-\ln(b_i\Lambda)}, \tag{11}$$

with $b_3=b_2$, $\beta_1=(33-2n_f)/12$, and n_f the number of flavors. The scale t_j is chosen as $t_j=\max(AM_B,|B_j|M_B,1/b_1,1/b_2)$. The amplitude $\mathcal{M}_d$ is obtained from Fig. 2(d) accordingly. The amplitudes for the decay $\overline{B^0}\to D^+\pi^-$ can be derived in a similar way. However, it is found that only the external W-emission contribution, the same as $\mathcal{M}_a$, is important.

The wave functions are chosen as [7]

$$\phi_\pi(x)=\frac{5\sqrt{6}}{2}f_\pi x(1-x)(1-2x)^2,$$

$$\phi_{B,D}(x)=\frac{N_{B,D}}{16\pi^2}\frac{x(1-x)^2}{M_{B,D}^2+C_{B,D}(1-x)}, \tag{12}$$

with $f_\pi=132$ MeV the pion decay constant. $N_B=650.212$ GeV^3 and $C_B=-27.1051$ GeV^2 correspond to $f_B=200$ MeV. N_D is determined by $f_D=220$ MeV, and C_D is fixed by data for the decay $\overline{B^0}\to D^+\pi^-$. All other parameters are referred to [7].

The experimental data of the branching ratios are $\mathcal{B}_0=\mathcal{B}(\overline{B^0}\to D^+\pi^-)=(3.08\pm0.85)\times10^{-3}$ and $\mathcal{B}_-=\mathcal{B}(B^-\to D^0\pi^-)=(5.34\pm1.05)\times10^{-3}$ [9]. Our predictions using the full Hamiltonian in Eq. (1), i.e., $c_1=1$ and $c_2=0$, are $\mathcal{B}_0=3.08\times10^{-3}$ and $\mathcal{B}_-=5.10\times10^{-3}$. If the three-scale factorization formula is employed, we obtain $\mathcal{B}_0=3.08\times10^{-3}$ and $\mathcal{B}_-=5.00\times10^{-3}$, differing from the previous results only by 2%. We claim that the main theoretical uncertainty comes from higher corrections to the hard part. They are estimated to be 15–20 % using the value of $\alpha_s(t)/\pi$ evaluated at the scale t below which half of the contributions have been accumulated. A careful observation reveals that when the evolution of the Wilson coefficients is included, the amplitude $\mathcal{M}_b$, proportional to $a_2(t)$, becomes smaller, while $\mathcal{M}_c$, proportional to $c_1(t)/N_c$, becomes larger. The two changes cancel each other, and the total decay rate remains almost the same. $\mathcal{M}_d$ is less important because of the pair cancellation between the two diagrams in Fig. 2(d). Our calculation indicates that the nonfactorizable contribution $\mathcal{M}_c$ is substantial and the limit of the BSW factorization approximation. That is why the naive choice of $a_{1,2}=a_{1,2}(M_B)$ in the BSW model fails to explain the data.

Applying the three-scale factorization theorem to the modes $D\to K\pi$, we obtain the predictions $\mathcal{B}(\overline{D^0}\to K^+\pi^-)=4.05\%$ and $\mathcal{B}(D^-\to K^0\pi^-)=2.67\%$, consistent with the data $(4.01\pm0.14)\%$ and $(2.74\pm0.29)\%$, respectively. With the running scale t reaching below the c quark mass, $\mathcal{M}_b$ becomes more negative and overcomes the positive contribution of $\mathcal{M}_c$. This explains the observed destructive interference of the external and internal W emissions absent in the B meson decays. Hence, nonfactorizable diagrams play an important role in the explanation of the heavy meson decay data. It is clear that a PQCD formalism based on the original Hamiltonian without Wilson coefficients [7,10] cannot account for this change of sign in the charm decays. From the above analysis, we suggest that $c_{1,2}$ could be regarded as Wilson coefficients as they originally are, instead of as fitting parameters in the BSW model. That is, the controversy over the extraction of a_2/a_1 from the bottom and charm decays does not exist in our theory.

The scale dependence of our formalism can be tested by substituting $2t$ for t in the factorization formula. It is found that the prediction decreases by only 5%. In the conventional effective field theory, the substitution of M_b by $2M_b$ for the argument of $c_{1,2}$ results in a 10–20 % difference [1]. Hence, the scale-setting ambiguity is moderated in the three-scale factorization theorems. Our formalism provides a more sophisticated choice of the scale and is expected to give more definitive predictions when it is applied to inclusive nonleptonic B meson decays. This subject will be discussed elsewhere.

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