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New gauge interactions and single top-quark production

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Extensions of the standard model that include new W bosons or extended technicolor gauge bosons can predict sizable changes in the rate of single top-quark production, even when constrained to be consistent with precision electroweak data. We analyze the fractional change in the rate of single top-quark production for several classes of models and determine which ones predict an effect visible at the Fermilab Tevatron collider's run 3. [S0556-2821(97)02609-X]

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I. INTRODUCTION

It has been suggested [1] that a sensitive measurement of the Wtb coupling can be made at the Fermilab Tevatron collider by studying single top-quark production through quark-antiquark annihilation ($q\overline{q'} \rightarrow W \rightarrow tb$) [2], and normalizing to the Drell-Yan process ($q\overline{q'} \rightarrow Wq \rightarrow \ell\nu$) to control theoretical systematic uncertainties (e.g., in the initial parton distributions). This method should be more precise than alternative methods involving single top-quark production via W-gluon fusion [3], because there is no similar way to eliminate the uncertainty associated with the gluon distribution function.

In the standard model, the ratio of single top-quark production and Drell-Yan cross sections

$$\frac{\sigma(q\overline{q'} \rightarrow W \rightarrow tb)}{\sigma(q\overline{q'} \rightarrow W \rightarrow \ell\nu)} \equiv R_\sigma^{\mathrm{SM}} \tag{1.1}$$

is proportional to the top-quark decay width $\Gamma(t \rightarrow Wb)$ and, therefore, to $|V_{tb}|^2$. Recent work [4] has shown that with a 30 fb^{-1} data sample from run 3 at the Tevatron with $\sqrt{s}=2$ TeV it should be possible to use single top-quark production to measure $\Delta R_\sigma/R_\sigma$, and hence $|V_{tb}|^2$ in the standard model, to an accuracy of at least ±8%. By that time, the theoretical accuracy in the standard model calculation is projected to become at least this good [5].

Many theories of physics beyond the standard model include new particles or interactions that can contribute to the rate of single top-quark production or the Drell-Yan process, thereby altering the predicted value of R_σ. If the resulting fractional change in the cross-section ratio

$$\frac{R_\sigma - R_\sigma^{\mathrm{SM}}}{R_\sigma^{\mathrm{SM}}} \equiv \Delta R_\sigma / R_\sigma \tag{1.2}$$

is at least 16%, it should be detectable in run 3. By considering the size of $\Delta R_\sigma/R_\sigma$ predicted by different types of new physics, we can assess the likelihood that the measurement of single top-quark production will help distinguish among various classes of models.

This paper focuses largely on models that include new gauge bosons coupled to the ordinary fermions. The models we consider alter R_σ in two distinct ways, each corresponding to the presence of a specific type of extra gauge boson. In models of dynamical electroweak symmetry breaking, exchange of new gauge bosons can make a large direct correction to the Wtb vertex. In models with enlarged weak gauge groups, two sets of W bosons can be present; both sets contribute to the cross sections and mixing between the two sets alters the couplings of the lighter W state to fermions. Sections II and III examine models of dynamical electroweak symmetry breaking with (III) or without (II) extra weak gauge bosons. In Sec. IV, models with light Higgs bosons and extra weak gauge bosons are discussed. The last section summarizes our findings and compares the results to those obtained by others for models of nonstandard physics that do not include new gauge interactions.

II. ORDINARY EXTENDED TECHNICOLOR

In ordinary extended technicolor (ETC) models [6], the extended technicolor gauge group commutes with the weak gauge group. Such models have no extra weak gauge bosons, so that the only effect on R_σ comes from a direct ETC correction to the Wtb vertex.

In order to calculate this correction, we use the methods established for finding how ETC gauge boson exchange alters the Zbb coupling [7]. Recall that the size of the effect on Zbb is set by the top-quark mass. In ordinary ETC models, the top-quark mass is generated by four-fermion operators induced by the exchange of ETC gauge bosons:

$$\mathcal{L}_{4f}^{\mathrm{ETC}} = -\frac{2}{f^2}\left(\xi\,\overline{\psi_L}\gamma^\mu T_L + \frac{1}{\xi}\,\overline{t_R}\gamma^\mu U_R\right) \times \left(\xi\,\overline{T_L}\gamma_\mu \psi_L + \frac{1}{\xi}\,\overline{U_R}\gamma_\mu t_R\right), \tag{2.1}$$

where ξ is a model-dependent Clebsch coefficient; the top-bottom doublet $\psi_L=(t,b)_L$ and the technifermion doublet $T_L=(U,D)_L$ are weak doublets; and the scale f is related (in the absence of fine-tuning) as $f=2M/g$ to the ETC boson's mass and gauge coupling. When the technifermions condense, the LR cross terms in the operator (2.1) produce a top-quark mass [7]

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$$m_t \approx \frac{g^2 4\pi f_Q^3}{M^2}, \tag{2.2}$$

where the numerator contains an estimate of the technifermion condensate (using dimensional analysis [8]) and f_Q is the Goldstone boson decay constant associated with the technifermions which help provide a mass to the top quark. In a one-doublet technicolor model, $f_Q = v = 250$ GeV.

The purely left-handed piece of operator (2.1) affects the the Zbb, Ztt, and Wtb vertices. As shown in [7], that left-handed interaction is equivalent to

$$\frac{\xi^2}{2}\frac{g^2 f_Q^2}{M^2}\bar{\psi}_L\left[\frac{e}{\sin\theta\,\cos\theta}\not{Z}\frac{\tau_3}{2} + \frac{e}{\sqrt{2}\sin\theta}(\not{W}^+\tau^+ + \not{W}^-\tau^-)\right]\psi_L . \tag{2.3}$$

Hence the Wtb coupling is shifted by (taking $V_{tb}=1$)

$$(\delta g)^{\mathrm{ETC}} = -\frac{\xi^2}{2}\frac{g^2 f_Q^2}{M^2}\frac{e}{\sqrt{2}\sin\theta} = -\frac{\xi^2}{2}\frac{m_t}{4\pi f_Q}\frac{e}{\sqrt{2}\sin\theta}. \tag{2.4}$$

The effect of the shifted coupling on the ratio of cross sections R_σ is

$$\frac{\Delta R_\sigma}{R_\sigma} \approx \frac{2}{g}[(\delta g)^{\mathrm{ETC}}] \approx -5.6\%\,\xi^2\left(\frac{250\ \mathrm{GeV}}{f_Q}\right)\left(\frac{m_t}{175\ \mathrm{GeV}}\right). \tag{2.5}$$

Since ξ^2 is generally of order 1, this lies well below the projected sensitivity of the Tevatron's run 3. Ordinary extended technicolor models, then, do not predict a visible change to the rate of single top-quark production.

Note that operator (2.3) also induces a fractional shift in R_b [7]:

$$\frac{\Delta R_b}{R_b} \approx \frac{2}{g}[(\delta g)^{\mathrm{ETC}}] \approx -5.6\%\,\xi^2\left(\frac{250\ \mathrm{GeV}}{f_Q}\right)\left(\frac{m_t}{175\ \mathrm{GeV}}\right), \tag{2.6}$$

of the same size as $\Delta R_\sigma / R_\sigma$. The value [9] of R_b (0.2179 ± 0.0012) measured at the CERN e^+e^- collider LEP lies close enough to the standard model prediction (0.2158) that a 5% reduction in R_b is excluded at better than the 10σ level. Moreover, attempts to increase $\Delta R_\sigma / R_\sigma$ in ordinary technicolor models may cause the predicted value of R_b to deviate still further from the measured value.[1]

An interesting extension of ordinary extended technicolor models are topcolor-assisted technicolor models [11] in which technicolor is responsible for most of the electroweak symmetry breaking and new strong dynamics coupled to the top and bottom quarks generates most of the top-quark mass. The ETC sector of such models will have an effect on R_σ of the form described above—but the size of the effect is modified by the differing values of f_Q and m_t^{ETC} (the part of the top-quark mass contributed by the ETC sector). Using typical [11] values $f_Q \sim 240$ GeV and $m_t^{\mathrm{ETC}} \sim 1$ GeV we find that ETC-induced shift $\Delta R_\sigma / R_\sigma$ is a fraction of a percent. Exchange of the new "coloron" gauge bosons between the t and b quarks can additionally modify the Wtb vertex; extrapolating from the results of [12], which considered similar effects on the $Zb\bar{b}$ vertex, we estimate that this contributes at most a few percent to R_σ at the momentum transfers where most of the single top production occurs. Thus topcolor-assisted technicolor models do not predict a visible alteration of R_σ.

III. NONCOMMUTING EXTENDED TECHNICOLOR

In "noncommuting" extended technicolor models, the gauge groups for extended technicolor and for the weak interactions do not commute. In other words, $\mathrm{SU}(2)_L$ is partially embedded in the ETC gauge group and ETC gauge bosons carry weak charge. As a result the models include both ETC gauge bosons and an extra set of weak gauge bosons [13].

The pattern of gauge symmetry breaking required in noncommuting ETC models generally involves three scales (rather than just two as in ordinary ETC) to provide masses for one family of ordinary fermions:

$$\begin{tikzcd}
G_{\mathrm{ETC}} \otimes \mathrm{SU}(2)_{\mathrm{light}} \otimes \mathrm{U}(1)' \arrow[d, "f"] \\
G_{\mathrm{TC}} \otimes \mathrm{SU}(2)_{\mathrm{heavy}} \otimes \mathrm{SU}(2)_{\mathrm{light}} \otimes \mathrm{U}(1)_Y \arrow[d, "u"] \\
G_{\mathrm{TC}} \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \arrow[d, "v"] \\
G_{\mathrm{TC}} \otimes \mathrm{U}(1)_{\mathrm{em}} .
\end{tikzcd}$$

The $\mathrm{SU}(2)_{\mathrm{heavy}}$ gauge group (a subgroup of G_{ETC}) is effectively the weak gauge group for the third generation, while the $\mathrm{SU}(2)_{\mathrm{light}}$ is the weak gauge group for the two light generations. Keeping the two SU(2) groups distinct at high energies allows a range of fermion masses to be generated. The two SU(2)'s break to a diagonal $\mathrm{SU}(2)_L$ subgroup [which we identify with $\mathrm{SU}(2)_{\mathrm{weak}}$] at the scale u, thereby preserving the observed low-energy universality of the weak interactions. The final electroweak symmetry breaking is accomplished dynamically at the weak scale v.

The two simplest possibilities for the $\mathrm{SU}(2)_{\mathrm{heavy}} \times \mathrm{SU}(2)_{\mathrm{light}}$ transformation properties of the order parameters that mix and break the SU(2) groups are [13]

$$\langle\varphi\rangle \sim (2,1)_{1/2}, \quad \langle\sigma\rangle \sim (2,2)_0, \quad \text{``heavy case,''} \tag{3.1}$$

and

$$\langle\varphi\rangle \sim (1,2)_{1/2}, \quad \langle\sigma\rangle \sim (2,2)_0, \quad \text{``light case,''} \tag{3.2}$$

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[1] A recent effective-Lagrangian analysis of a nonstandard contribution to the Zbb and Wtb vertices [10] similarly finds that a large shift in R_b is the price of a visible shift in R_σ.

where order parameter $\langle\varphi\rangle$ breaks $\mathrm{SU}(2)_L$ while $\langle\sigma\rangle$ mixes $\mathrm{SU}(2)_{\text{heavy}}$ with $\mathrm{SU}(2)_{\text{light}}$. We refer to these two possibilities as ''heavy'' and ''light'' according to whether $\langle\varphi\rangle$ transforms nontrivially under $\mathrm{SU}(2)_{\text{heavy}}$ or $\mathrm{SU}(2)_{\text{light}}$. In the heavy case [13], the technifermion condensate responsible for providing mass for the third generation of quarks and leptons is also responsible for the bulk of electroweak symmetry breaking (as measured by the contribution made to the W and Z masses). In the light case, the physics responsible for providing mass for the third generation *does not* provide the bulk of electroweak symmetry breaking.

A. Direct ETC effects on the *Wtb* vertex

A priori, it appears that the Wtb vertex may be affected by both ETC gauge boson exchange and weak gauge boson mixing. However, a closer look at the operator that gives rise to the top-quark mass demonstrates that there are no direct ETC contributions to the Wtb vertex of order $m_t/4\pi v$ in noncommuting ETC models. The left-handed third-generation quarks $\psi_L=(t,b)_L$ and right-handed technifermions $T_R=(U,D)_R$ are doublets under $\mathrm{SU}(2)_{\text{heavy}}$ while the left-handed technifermions are $\mathrm{SU}(2)_{\text{heavy}}$ singlets. The four-fermion interaction whose left-right interference piece gives rise to the top-quark mass may be written as [13]

$$\mathcal{L}_{4f}^{\text{nc-ETC}}=-\frac{2}{f^2}\left(\xi\,\overline{\psi_L}\gamma^\mu U_L+\frac{1}{\xi}\,\overline{t_R}\gamma^\mu T_R\right)\times\left(\xi\,\overline{U_L}\gamma_\mu\psi_L+\frac{1}{\xi}\,\overline{T_R}\gamma_\mu t_R\right), \tag{3.3}$$

where ξ is a model-dependent Clebsch coefficient. This is the operator that can potentially alter couplings between the weak bosons and the third-generation quarks by an amount of order $m_t/4\pi v$. However, because the left-left piece of this operator includes (t_l,b_l,U_L) but not D_L and because its purely right-handed piece contains (t_R,U_R,D_R) but not b_R, this operator does *not* contribute to the Wtb vertex.

This is in contrast to the result for R_b where a similar operator involving electrically neutral currents does affect the $Zb\bar{b}$ coupling [13].

B. Extra weak gauge bosons in noncommuting ETC

The extra set of weak gauge bosons in noncommuting ETC models affects R_σ both because there are now two W bosons participating in the scattering process and because gauge boson mixing alters the light W boson's couplings to fermions. We summarize here the properties of the W bosons (mass, couplings, width) that are directly relevant to calculating $\Delta R_\sigma/R_\sigma$. Further details are in [13].

The electromagnetic gauge group $\mathrm{U}(1)_{\text{em}}$ is generated by $Q=T_{3l}+T_{3h}+Y$ and the associated photon eigenstate can be written as

$$A^\mu=\sin\theta\,\sin\phi\;W_{3l}^\mu+\sin\theta\,\cos\phi\;W_{3h}^\mu+\cos\theta\;X^\mu, \tag{3.4}$$

where θ is the weak angle and ϕ is an additional mixing angle. In terms of the electric charge and these mixing angles, the gauge couplings of the original $\mathrm{SU}(2)_{\text{heavy}}\times\mathrm{SU}(2)_{\text{light}}\times\mathrm{U}(1)_Y$ gauge groups are

$$g_{\text{light}}=\frac{e}{s\sin\theta},\quad g_{\text{heavy}}=\frac{e}{c\sin\theta},\quad g'=\frac{e}{\cos\theta}, \tag{3.5}$$

where $s\equiv\sin\phi$ and $c\equiv\cos\phi$.

It is convenient to discuss the W mass eigenstates in the rotated basis

$$W_1^\pm=s\;W_l^\pm+c\;W_h^\pm,\quad W_2^\pm=c\;W_l^\pm-s\;W_h^\pm, \tag{3.6}$$

so the gauge covariant derivatives separate into standard and nonstandard parts

$$D^\mu=\partial^\mu+ig(T_l^\pm+T_h^\pm)W_1^{\pm\,\mu}+ig\left(\frac{c}{s}T_l^\pm-\frac{s}{c}T_h^\pm\right)W_2^{\pm\,\mu}+\cdots, \tag{3.7}$$

with $g\equiv e/\sin\theta$. By diagonalizing the mass matrix of the W bosons in the limit where $u^2/v^2\equiv x$ is large, we can find the form of the light and heavy mass eigenstates W^L and W^H. For the heavy case of noncommuting ETC, we have

$$W^L\approx W_1+\frac{cs^3}{x}\,W_2,\quad W^H\approx W_2-\frac{cs^3}{x}\,W_1. \tag{3.8}$$

In the light case, we have mass eigenstates

$$W^L\approx W_1-\frac{c^3s}{x}\,W_2,\quad W^H\approx W_2+\frac{c^3s}{x}\,W_1. \tag{3.9}$$

In either case, the mass of the heavy W boson is approximately given by

$$M_{W^H}\approx\sqrt{\frac{x}{sc}}\,M_W, \tag{3.10}$$

where M_W is the tree-level standard model mass of the W boson. The tree-level (pole) width of the heavy W boson is

$$\Gamma_{W^H}=\frac{g^2}{12\pi^2}\left(\frac{2c^2}{s^2}+\frac{s^2}{c^2}\right)M_{W^H}. \tag{3.11}$$

C. Results

Using the information on the mass, width, and couplings of the W bosons from the previous sections, we found the size of $\Delta R_\sigma/R_\sigma$ in both the heavy and light cases of noncommuting ETC. Details of the calculation are given in the Appendix. We used results from [13] to fix the 95% C.L. experimental constraints on the model from low-energy and LEP precision electroweak measurements; these are stronger than limits from direct searches [14] for heavy weak bosons at Fermilab. Physically speaking, the constraints tell us the lightest possible value of M_{W^H} for any given value of $\sin^2\phi$, i.e., the value of M_{W^H} yielding the largest $\Delta R_\sigma/R_\sigma$.

By checking the maximum $\Delta R_\sigma/R_\sigma$ in the experimentally allowed region for heavy case noncommuting ETC, we find that $|\Delta R_\sigma/R_\sigma|$ never exceeds 9%. This means that the shift in the rate of single top-quark production is never large enough to be clearly visible at TeV33.

Repeating the exercise for the light case of noncommuting ETC leads to a very different conclusion. The pattern of

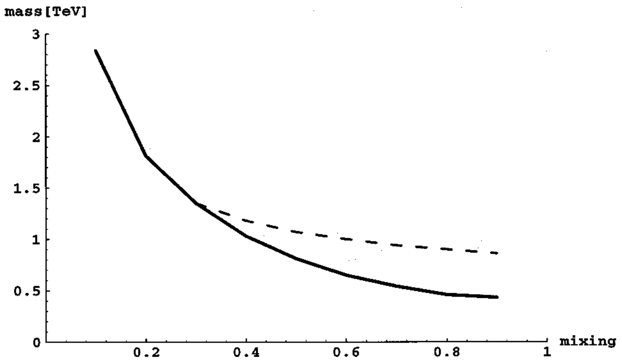

FIG. 1. Region (shaded) where light-case noncommuting ETC models predict a visible increase ($\Delta R_\sigma / R_\sigma \geqslant 16\%$) in single top-quark production at TeV33. The dark line marks the lower bound (at 95% C.L.) on the mass of the heavy weak bosons M_{W^H} (as a function of mixing parameter $\sin^2\phi$) by electroweak data [13]. Below the dashed line, the predicted value of $\Delta R_\sigma / R_\sigma \geqslant 24\%$.

shifts in the predicted values of various electroweak observables has been found [13] to allow the extra weak bosons in light noncommuting ETC to be as light as 400 GeV. Since lighter extra bosons produce larger shifts in R_σ, there is a significant overlap between the experimentally allowed portion of parameter space and the region in which $|\Delta R_\sigma / R_\sigma| \geqslant 16\%$, as shown in Fig. 1. In fact, the predicted fractional shift in R_σ is greater than 24% for much of this overlap region. More precisely, the shift in R_σ is towards values exceeding R_σ^{SM}, so that noncommuting ETC models with the ''light'' symmetry breaking pattern predict a visible *increase* in the rate of single top-quark production.

What allows the corrections to single top-quark production to be relatively large in noncommuting ETC models is the fact that there is no direct ETC effect on the Wtb vertex to cancel the contributions from weak gauge boson mixing. This is in contrast to the calculation of R_b, where such a cancelation does occur. Hence within the context of these models it is possible for R_b to have a value close to the standard model prediction while R_σ is visibly altered.

IV. MODELS WITH EXTENDED WEAK GAUGE GROUPS

There are also models with extended electroweak gauge groups (but no technicolor sector) that predict an R_σ that differs from the standard model value.[2] The analysis of weak gauge boson mixing presented in Sec. III B can be adapted to these models.

A. Topflavor

A recently introduced model known as topflavor [17,18] has the same $SU(2)_{\text{heavy}} \times SU(2)_{\text{light}} \times U(1)_Y$ electroweak gauge group as noncommuting ETC (without an underlying ETC sector). Again, the third generation of fermions couples to $SU(2)_{\text{heavy}}$ while the first and second generations couple to $SU(2)_{\text{light}}$. The simplest forms of the symmetry breaking sector include a scalar which transforms as $(2,2)_0$ and one which is a doublet under only one of the SU(2) groups. As in noncommuting ETC, there are therefore ''heavy'' and ''light'' cases of topflavor according to whether the second scalar transforms as a doublet under $SU(2)_{\text{heavy}}$ or $SU(2)_{\text{light}}$ (i.e., according to whether the same order parameter gives mass to the weak gauge bosons and the heavy fermions). The phenomenology of the heavy case is explored in [17] and that of the light case is discussed in [18,19].

The analysis of topflavor is similar to that of noncommuting extended technicolor. The calculated value of $\Delta R_\sigma / R_\sigma$ is the same since the weak sectors of the two models are identical. It is the experimental constraints on the models' parameter spaces that differ (since the noncommuting ETC model contains parameters not present in topflavor).

We can find a lower bound on the allowed value of the heavy W mass in heavy-case topflavor by realizing that the extra W boson causes a fractional shift in $R_{\mu\tau}$, just as in noncommuting ETC [13]

$$(\Delta R_{\mu\tau})^{\text{topflavor}}_{\text{heavy}} = -2/x. \tag{4.1}$$

Since current experiment [20] requires $|\Delta R_{\mu\tau}| \leqslant 1.8\%$ at the 2σ level, we can apply Eq. (3.10) to find the lower bound

$$M_{W^H} \geqslant 10.5\ M_W / sc \tag{4.2}$$

on the heavy W boson's mass. When this bound is satisfied, the value of $|\Delta R_\sigma / R_\sigma|$ always lies below[3] 13.5%, so that the change in the rate of single top-quark production is not likely to be visible at the Tevatron.

The current experimental constraints for the light case of topflavor have been explored in [18]. When the constraints are expressed as a lower bound on the mass of the extra weak bosons (as a function of mixing parameter $\sin^2\phi$), they appear stronger than those on noncommuting ETC. In other words, the shape of the exclusion curve is similar to that shown in Fig. 1, but lies above it, with the lowest allowed value of M_{W^H} being about 1.1 TeV. As a result, the change in the rate of single top-quark production in the light case of topflavor always lies below about 13%. Again, this is unlikely to be observable.

B. Ununified standard model

The ununified standard model [21] also sports an extended weak gauge group with two SU(2) components and a single U(1). However, in this case, the quarks transform according to one non-Abelian group $[SU(2)_q]$ and the leptons

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[2] The left-right symmetric model [15] is not among them. In the limit of no mixing between the left- and right-handed W bosons, R_σ would have the standard model value. The experimentally allowed mixing is small so that including mixing should not qualitatively alter the conclusion. The extra W boson in the alternative left-right model [16] carries lepton number and would not contribute to single top production.

[3] This maximum fractional shift in R_σ is obtained when $\sin^2\phi$ is at its minimum value of 0.034. A smaller value of $\sin^2\phi$ would make g_{light} large enough to break the light fermions' chiral symmetries. The critical value of the coupling is estimated using the results of a gap-equation analysis of chiral symmetry breaking in the ''rainbow'' approximation [23]; see [13] for further details.

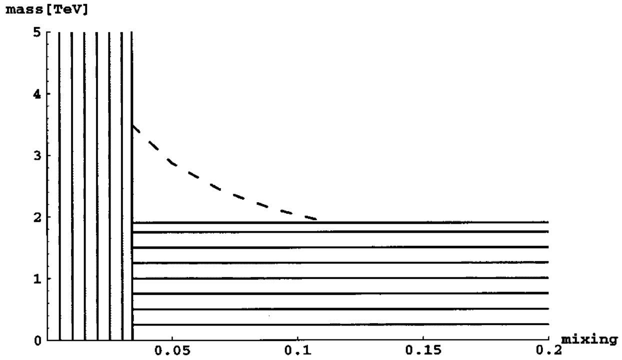

FIG. 2. Region (shaded) where the ununified standard model predicts a visible decrease ($\Delta R_\sigma / R_\sigma \leqslant -16\%$) in single top-quark production at TeV33. Below the dashed line, the predicted decrease is $\Delta R_\sigma / R_\sigma \leqslant -24\%$. The horizontally hatched region marks the lower bound on the mass of the heavy weak bosons M_{W^H} (for small mixing parameter $\sin^2\phi$) from electroweak data [13]. In the vertically hatched region, the chiral symmetries of the fermions would be broken by a strong SU(2) coupling.

according to the other [$\mathrm{SU}(2)_\ell$]. In order to preserve the experimentally verified relationship between the leptonic and semileptonic weak interactions that holds in the standard model, the symmetry breaking sector must be of the ''light'' type in which no new low-energy charged current interactions between a leptonic and a hadronic current occur. The simplest possibility is therefore to have one scalar that transforms as a (2,2) under the two SU(2) groups and another that is an $\mathrm{SU}(2)_\ell$ doublet, but an $\mathrm{SU}(2)_q$ singlet.

The extra weak gauge boson mixing angle ϕ_{uum} in this model is conventionally defined so that $\sin\phi_{uum} \leftrightarrow \cos\phi_{\text{NC-ETC}}$. Otherwise, the formalism developed earlier for the analysis of noncommuting ETC carries through; explicit expressions for the top-bottom and leptonic cross sections are in the Appendix.

A fit of the ununified standard model to precision electroweak data [22] has found a 95% C.L. lower bound of just under 2 TeV on the masses of the heavy W and Z bosons.[4] Keeping this in mind, and restricting the value of $\sin^2\phi_{uum}$ to exceed the critical value of 0.034, we checked for an intersection between the experimentally allowed parameter space and the region of visible alteration of the Wtb vertex.

We find a small region in the $\sin^2\phi_{uum} - M_{W^H}$ plane, the shaded triangle in Fig. 2, which is allowed by experiment and in which $\Delta R_\sigma / R_\sigma \leqslant -16\%$. Elsewhere in the model's experimentally allowed parameter space, the shift in R_σ is too small to be reliably detected by an experimental precision of $\pm 8\%$. Note that since the shift is negative, it is distinct from that predicted by models like noncommuting ETC which have an $\mathrm{SU}(2)_{\text{heavy}} \times \mathrm{SU}(2)_{\text{light}}$ group structure.

Furthermore, R_b has essentially the standard model value in the region where $\Delta R_\sigma / R_\sigma$ is large. One may calculate the shift in R_b by repeating the analysis of Sec. III B for the Z bosons and finding how $Zq\bar{q}$ couplings are altered. The result [22] is that $\Delta R_b / R_b \approx -0.052 (M_W / M_{W^H})^2 / c^2$. Since $c^2 \geqslant 0.83$ and $M_{W^H} \gtrsim 2$ TeV in the region in question, $|\Delta R_b / R_b| \lesssim 10^{-4}$. Qualitatively this is because no factor of the top-quark mass enters to enhance the shift in R_b as can happen in ETC models.

V. DISCUSSION

Measuring the rate of single top-quark production in run 3 at the Tevatron offers a promising opportunity to test models of electroweak physics. We have shown here that models with extra W bosons can predict an alteration of R_σ that would be visible to experiment, provided that the new W bosons weigh less than a few TeV.

In particular we found interesting results for models with an $\mathrm{SU}(2)_{\text{heavy}} \times \mathrm{SU}(2)_{\text{light}}$ weak gauge group and an electroweak symmetry breaking condensate charged under the *light* rather than the *heavy* SU(2). In such models, the value of R_σ can be greatly increased above the standard model prediction. Hence the value of R_σ provides a valuable test of the dynamical symmetry breaking models involving noncommuting extended technicolor. If the measurement attains a greater precision than assumed here, it may also be possible to test the related model with fundamental scalars known as topflavor.

The predicted increase in R_σ is not only visible, but distinctive. As we have seen, other models with extra weak bosons that can alter R_σ predict either a shift that is too small to be seen (e.g., ordinary ETC, topcolor-assisted technicolor, left-right-symmetric model, heavy-case noncommuting ETC, or topflavor) or a shift towards a lower value of R_σ (e.g., the ununified standard model). This trend continues when models including other kinds of nonstandard physics are examined. Adding a fourth generation of quarks would tend to reduce $|V_{tb}|$ and, thus, R_σ. The extra scalar bosons in two-Higgs-doublet models [24] have been found [25] to reduce R_σ by an amount not greater than 15%. The electroweak contributions in the minimal supersymmetric standard model [26] likewise alter R_σ by no more than $\pm 10\%$ [27] (the sign varies over the model's parameter space).

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APPENDIX A

Here we present some details of our calculation for the reader's convenience. The cross section for production of a fermion-antifermion pair via exchange of W^L and W^H bosons contains the terms

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[4] This is the bound for zero mixing angle; the bound gets even stronger as $\sin^2\phi_{uum}$ increases.

$$[C_f\, \hat{u}(\hat{u}-m_f^2)]\left[\frac{\alpha}{(\hat{s}-M_W^2)}+\frac{2\beta(\hat{s}-M_{W^H}^2)}{\hat{s}[(\hat{s}-M_{W^H}^2)^2+\Gamma_{W^H}^2M_{W^H}^2]}+\frac{\gamma}{(\hat{s}-M_{W^H}^2)^2+\Gamma_{W^H}^2M_{W^H}^2}\right], \tag{A1}$$

where m_f is m_t for the tb final state and zero for the $l\nu$ final state, C_f is 3 for the tb final state and 1 for the $l\nu$ final state, V_{tb} has been set equal to 1, and multiplicative constants which cancel in the ratio R_σ have been dropped. Here Γ_{W^H} is taken to be the s-dependent width of the heavy weak boson so that the results match correctly onto those from calculations based on four-fermion operators.

The coefficients α, β, and γ are specific to the process (tb or $l\nu$ production) and the model. We write them in terms of the heavy W boson mass M_{W^H} and the weak boson mixing angle ($s\equiv\sin\phi$, $c\equiv\cos\phi$). They have been derived using Eqs. (3.7), (3.8), and (3.9) and dropping terms of order x^{-2} or higher (where $x\equiv u^2/v^2$ is the ratio of mixing and breaking VEV's squared). In the heavy case of noncommuting ETC or topflavor:

$$\alpha^{tb}=-\beta^{tb}=\gamma^{tb}=1+\frac{2(c^2-s^2)}{c^2}\left(\frac{M_W^2}{M_{W^H}^2}\right)\equiv\alpha_h^{tb},$$

$$\alpha^{l\nu}=1+4\frac{M_W^2}{M_{W^H}^2}\equiv\alpha_h^{l\nu},$$

$$\beta^{l\nu}=\frac{c^2}{s^2}+\frac{2(c^2-s^2)}{c^2}\left(\frac{M_W^2}{M_{W^H}^2}\right)\equiv\beta_h^{l\nu},$$

$$\gamma^{l\nu}=\frac{c^4}{s^4}-\frac{4c^2}{s^2}\left(\frac{M_W^2}{M_{W^H}^2}\right)\equiv\gamma_h^{l\nu}. \tag{A2}$$

For the light case of noncommuting ETC or topflavor,

$$\alpha^{tb}=-\beta^{tb}=\gamma^{tb}=1+\frac{2(s^2-c^2)}{s^2}\left(\frac{M_W^2}{M_{W^H}^2}\right)\equiv\alpha_l^{tb},$$

$$\alpha^{l\nu}=1-4\frac{c^2}{s^2}\frac{M_W^2}{M_{W^H}^2},$$

$$\beta^{l\nu}=\frac{c^2}{s^2}-\frac{2c^2}{s^4}\left(\frac{M_W^2}{M_{W^H}^2}\right),$$

$$\gamma^{l\nu}=\frac{c^4}{s^4}-\frac{4c^4}{s^4}\left(\frac{M_W^2}{M_{W^H}^2}\right). \tag{A3}$$

In the ununified standard model:

$$\alpha^{tb}=\alpha_h^{l\nu},\quad \alpha^{l\nu}=\alpha_h^{tb},$$

$$\beta^{tb}=\beta_h^{l\nu},\quad \beta^{l\nu}=-\alpha_h^{tb},$$

$$\gamma^{tb}=\gamma_h^{l\nu},\quad \gamma^{l\nu}=\alpha_h^{tb}. \tag{A4}$$

To find the hadronic cross section for each process, we used MRSDO' structure functions and integrated over center-of-mass energy ($m_t+m_b<\sqrt{\hat{s}}<1\,\mathrm{TeV}$) boost rapidity ($-2.0<Y_{\mathrm{boost}}<2.0$), and center-of-mass scattering angle [to the kinematic limit imposed by the masses and greatest rapidity (±2.0) of the final state particles]. Our results were insensitive to the precise choice of energy and rapidity integration limits.

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