SU(3) breaking in neutral current axial matrix elements and the spin content of the nucleon

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We examine the effects of SU(3) breaking in the matrix elements of the flavor-diagonal axial-vector currents between octet baryon states. Our calculations of K, η, and π loops indicate that the SU(3) breaking may be substantial for some matrix elements and at the very least indicate large uncertainties. In particular, the strange axial matrix element in the proton determined from the measurements of $g_1(x)$ is found to have large uncertainties and might yet be zero. We estimate the strange axial matrix element in the proton to be $-0.35 \lesssim \Delta s \lesssim 0$ and the matrix element of the flavor-singlet current in the proton to be $-0.1 \lesssim \Sigma \lesssim +0.3$ from the E143 measurement of $\int dx g_1(x) = 0.127 \pm 0.004 \pm 0.010$. The up-quark content of the Ξ^- is discussed and its implications for nonleptonic weak processes discussed. We also estimate the matrix element of the axial-vector current coupling to the Z^0 between all octet baryon states. This may be important for neutrino interactions in dense nuclear environments, where hyperons may play an important role. [S0556-2821(97)00709-1]

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One of the more exciting realizations in hadronic physics of the last few years is that the strange quark may play an important role in the structure of the nucleon [1]. While this may seem somewhat unnatural in the context of the most naive quark model, it is perfectly natural from the standpoint of QCD. Matrix elements of the strange vector current must vanish at zero-momentum transfer between states with zero net strangeness; however, matrix elements of the axial-vector current need not. Recent measurements suggest that the matrix element of the strange axial-vector current in the proton is $\Delta s = -0.12 \pm 0.04$ [2]. In addition, one would like to know what fraction of the nucleon spin is carried by the quarks themselves, which is equivalent to determining the matrix element of flavor-singlet axial-vector current in the proton Σ. This is, of course, intimately related to the matrix element of the strange axial-vector current and present analysis suggests that $\Sigma = 0.2 \pm 0.1$ [2], much smaller than the quark model estimate of $\Sigma \sim 0.58$. There have been intense theoretical and experimental efforts to extract Δs and Σ to address the present ''spin crisis'' and such efforts continue (for recent reviews, see [3,4]).

A vital ingredient in the present determination of Σ and Δs is the matrix element of the $j_5^{\mu,8} = \overline{u}\gamma^\mu\gamma_5 u + \overline{d}\gamma^\mu\gamma_5 d - 2\overline{s}\gamma^\mu\gamma_5 s$ axial-vector current in the nucleon, which cannot be measured directly but must be inferred from the approximate SU(3) symmetry observed in nature. The question of SU(3) breaking in the matrix element of the $j_5^{\mu,8}$ current and its impact upon the extraction of Δs and Σ has been previously addressed [5–10]. In [5,6] it was assumed that the breaking in the matrix elements of the axial-vector currents was proportional to the breaking in the octet baryon masses and in [7] a model of the SU(3) breaking was employed. Studies in the Skyrme model [9] suggest that Σ is relatively insensitive to SU(3) breaking while Δs shows significant sensitivity. A more systematic approach was that of [10] in which the breaking was analyzed in the context of the large-N_C limit of QCD. It was found that the matrix element of the $j_5^{\mu,8}$ axial-vector current was substantially reduced from its value in the symmetry limit.

In the limit of flavor SU(3) symmetry the three light quark contributions to the nucleon axial matrix elements are uniquely determined by three low energy observables. In this limit, two of these observables F and D can be extracted from nuclear β decay and from the semileptonic decay of strange hyperons. The third experimental constraint comes from a measurement of the axial singlet current in the nucleon, presently accomplished by measuring the $g_1(x)$ spin-dependent structure function of the nucleon [12,13] and using the SU(3) symmetry to remove the flavor octet contributions. In the real world we know that this symmetry is only approximate, broken by the difference between the mass of the strange quark and of the up and down quarks. Each of the matrix elements of the octet and singlet axial-vector currents will receive SU(3)-breaking contributions, with the leading contributions having the form $m_s \ln m_s$ followed by terms of the form m_s and higher. The leading contributions with nonanalytic dependence on m_s arise from hadronic kaon loops while terms analytic in the strange quark mass do not uniquely arise from such loops and must be fixed by other observables.

In this work we include all terms of the form $m_s \ln m_s$ to the axial matrix elements appearing in hyperon decay and β decay used to determine the axial couplings F, D, C, and H. We use these fits to predict matrix elements relevant for determining Σ, Δs, and for the interaction of neutrinos with hyperons, a situation that may be important at high matter densities [14,15]. Unfortunately, higher order SU(3)-breaking contributions can only be estimated to be of order $M_K^2/\Lambda_\chi^2 \sim 0.25$ (which is not to be confused with a 25% correction to each matrix element). Part of the terms at this order (in fact, a summation to all orders) arises from graphs involving the decuplet of baryon resonances as intermediate states. Such contributions are also present in the flavor-diagonal axial matrix elements with the same uncertainty

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arising from omission of incalculable terms $O(m_s)$ and higher. A further estimate of the incalculable higher order SU(3)-breaking terms is made by evaluating all matrix elements for three different values of the decuplet-octet baryon mass splitting $\Delta_0=0$, 200 MeV, ∞, that enters in at one-loop level. It is clear that our work provides merely an estimate for the size of SU(3) breaking in these matrix elements, however, the terms considered here are formally dominant in the chiral limit.

It is conventional to define the axial matrix elements of the quarks in the proton $|P\rangle$ via

$$2s_\mu \Delta q=\langle P|\overline{q}\gamma_\mu\gamma_5 q|P\rangle, \tag{1}$$

where $q=u,d,s$ denotes the quark flavor, and s_μ is the nucleon spin vector. Any linear combination of the three light quark neutral axial-vector currents can be written in terms of the two diagonal octet generators and the singlet. In deep-inelastic scattering one measures the matrix element of the current,

$$j_5^\mu=\overline{q}\mathcal{Q}^2\gamma^\mu\gamma_5 q, \tag{2}$$

in the proton, where $\mathcal{Q}$ is the light quark charge matrix, given by

$$\mathcal{Q}=\frac{1}{3}\begin{pmatrix} 2 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{pmatrix}. \tag{3}$$

In conjunction with a measurement of the matrix elements of the flavor-diagonal currents,

$$j_5^{\mu,3}=\overline{q}O_3\gamma_\mu\gamma_5 q, \quad j_5^{\mu,8}=\overline{q}O_8\gamma_\mu\gamma_5 q, \tag{4}$$

in the proton, where we use

$$O_3=\begin{pmatrix} 1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}, \quad O_8=\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}, \tag{5}$$

the flavor-singlet or, alternately, the strange quark contribution may be extracted via

$$\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=\frac{9}{2}\mathcal{Q}^2-\frac{3}{4}O_3-\frac{1}{4}O_8, \tag{6}$$

$$\begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}=\frac{3}{2}\mathcal{Q}^2-\frac{1}{4}O_3-\frac{5}{12}O_8. \tag{7}$$

The matrix element of $j_5^{\mu,3}$ in the nucleon is well determined from nuclear β decay via isospin symmetry, leading to

$$\Delta u-\Delta d=g_A=1.2664\pm0.0065, \tag{8}$$

where we have neglected isospin-breaking effects. Unfortunately, we cannot use isospin to relate the matrix element of $j_5^{\mu,8}$ in the proton to any other set of physical observables. We must resort to using flavor SU(3) symmetry as a starting point and systematically determine corrections arising from SU(3) breaking.

Let us begin by discussing the matrix element of the axial-vector currents in the limit of exact SU(3). The matrix elements between baryons in the lowest lying octet of the axial-vector currents transforming as octets under SU(3) are described by the following effective Lagrange density:

$$j_{5,\mathrm{eff}}^{\mu,a}=D\mathrm{Tr}[\overline{B}2s_\mu\{O_a,B\}]+F\mathrm{Tr}[\overline{B}2s_\mu[O_a,B]], \tag{9}$$

where B is the octet of baryon fields

$$B=\begin{pmatrix} \Lambda/\sqrt{6}+\Sigma^0/\sqrt{2} & \Sigma^+ & p\\ \Sigma^- & \Lambda/\sqrt{6}-\Sigma^0/\sqrt{2} & n\\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6}\end{pmatrix}. \tag{10}$$

Also, the matrix element of the singlet current is reproduced by the Lagrange density

$$j_{5,\mathrm{eff}}^{\mu,1}=S\mathrm{Tr}[\overline{B}2s_\mu B]. \tag{11}$$

At the tree level we can determine the parameters F and D by fitting the theoretical expression, linear in F and D, to the observed rates for $n\to pe^-\overline{\nu_e}$, $\Sigma^-\to ne^-\overline{\nu_e}$, $\Xi^-\to\Lambda e^-\overline{\nu_e}$, $\Sigma^-\to\Lambda e^-\overline{\nu_e}$, $\Xi^-\to\Sigma^0 e^-\overline{\nu_e}$, and $\Lambda\to pe^-\overline{\nu_e}$. However, one must keep in mind that we expect deviations between the "best fit" and the experimental results to be at the $\sim$25% level due to the fact that the theoretical expressions have been truncated, and terms of order $O(m_s, m_s\ln m_s, \ldots)$ have been neglected [11,16]. This includes the fit to the experimentally well-measured value of g_A, equal to $D+F$ in the SU(3) limit (i.e., we naively expect to see $D+F$ deviate from g_A at the 25% level in the best fit). In the matrix elements we use to fit the axial couplings the experimental uncertainties are much less than the corresponding theoretical uncertainty. To determine F and D we minimize a χ^2 function

$$\chi^2=\sum_{\mathrm{data}}\frac{(\mathrm{expt}_i-\mathrm{theory}_i)^2}{\sigma^2_{\mathrm{theory}}}, \tag{12}$$

where expt_i denotes an experimental measurement of an axial matrix element, theory_i denotes its theoretical value for given values of F and D, and σ_{theory} denotes the theoretical uncertainty which we somewhat arbitrarily choose to be $\sim$0.2, and equal for all data points, i.e., an unweighted fit. This is in contrast with the fit made by Jaffe and Manohar in [11], the fit of Luty and White [16], and is a more extreme version of a fit made in [10]. The uncertainties we quote for the couplings F and D are found by requiring that $\chi^2<\chi^2_{\mathrm{min}}+2.3$, corresponding to a 68% confidence interval. It is clear that this analysis can only provide an estimate of the uncertainties as the pattern of breaking will not be uncorrelated for these processes. We find that

$$D=0.79\pm0.10,$$

$$F=0.47\pm0.07. \tag{13}$$

FIG. 1. Tree-level contribution to the axial matrix element. The solid square denotes the insertion of the axial-vector current. The labels B and B' denote the incoming and outgoing octet baryons, respectively.

The errors on D and F are highly correlated and one finds that the ''best fit'' value for $D+F$ is 1.26 ± 0.08. Further, the best for $3F-D$ (the tree-level expression for the matrix element of the O_8 current) is 0.65 ± 0.21, in agreement with the central value of 0.60 found in [11]. The values of F and D are in agreement with those found in [10] except the uncertainties found from our somewhat *ad hoc* procedure are larger, but they do represent a reasonable estimate of the true uncertainties.

A third input required to fix the individual quark axial matrix elements in the proton is measured in deep-inelastic scattering:

$$2s_\mu\int_0^1 dx g_1(x,Q^2)=\frac{1}{2}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)\langle P|\overline{q}Q^2\gamma_\mu\gamma_5 q|P\rangle$$
$$=\frac{1}{9}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)\langle P|\tfrac{3}{4}\overline{q}O_3\gamma_\mu\gamma_5 q$$
$$+\tfrac{1}{4}\overline{q}O_8\gamma_\mu\gamma_5 q+\overline{q}I\gamma_\mu\gamma_5 q|P\rangle, \quad (14)$$

where I is the identity matrix. The two recent measurements of this quantity are

$$\int_0^1 dx g_1(x,Q^2=3\ \ \mathrm{GeV}^2)=0.127\pm0.004\pm0.010, \quad (15)$$

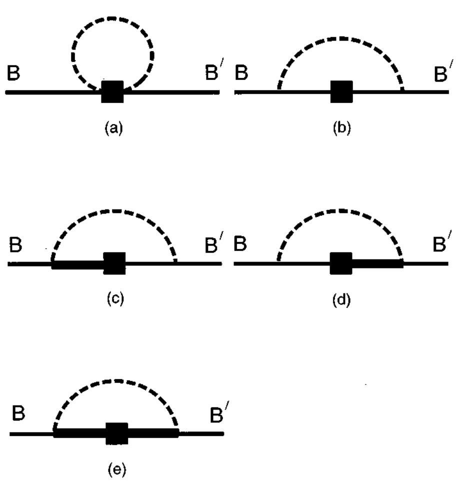

FIG. 2. Loop-level contribution to the axial matrix element. The solid square denotes the insertion of the axial-vector current. The labels B and B' denote the incoming and outgoing octet baryons, respectively. The dashed line denotes a pseudo Goldstone boson. The thicker lines denote decuplet baryon propagators. Graphs of the type (a), (c), and (d) do not arise in the matrix element of the singlet current at one loop.

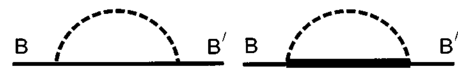

FIG. 3. Loop-level wave function renormalization contributions to the axial matrix element. The solid square denotes the insertion of the axial-vector current. The labels B and B' denote the incoming and outgoing octet baryons, respectively. The dashed line denotes a pseudo Goldstone boson. The thicker lines denote decuplet baryon propagators.

by the E143 Collaboration [13] and

$$\int_0^1 dx g_1(x,Q^2=10\ \ \mathrm{GeV}^2)=0.136\pm0.011\pm0.011, \quad (16)$$

by the Spin Muon Collaboration (SMC) [12]. We choose to use the E143 measurement at $Q^2=3\ \ \mathrm{GeV}^2$ for our evaluations and find, at the tree level,

$$\Delta u+\Delta d+\Delta s=0.10\pm0.10=\Sigma, \quad (17)$$

which, along with the octet matrix elements, allows us to separate the quark contributions

$$\Delta u=0.77\pm0.04, \quad \Delta d=-0.49\pm0.04,$$
$$\Delta s=-0.18\pm0.09. \quad (18)$$

These values are consistent with the analysis of Jaffe and Manohar in [11]. The Q^2 dependence of Σ is very weak [17,18] (see also [11] and [19]) and so we set $\Sigma\sim S$.

We can estimate the leading SU(3) breaking to each axial matrix element in chiral perturbation theory. It is of the form $m_s\ln m_s$ arising from the infrared region of hadronic loops involving K's, η's, and π's and can be computed exactly. Such loop graphs are divergent and require the presence of a local counterterm analytic in the light quark masses which must be fit to data. Some effects of K and η loops on strange quark observables in the nucleon have been considered previously, e.g., [20,21].

For some hyperon decays the axial matrix element is determined from an experimental measurement of the ratio of vector to axial-vector matrix elements. The Ademollo-Gatto theorem [22] protects the vector matrix elements from corrections of the form $m_s\ln m_s$, with leading corrections starting at $O(m_s)$ [23] . Consequently, at the order to which we are working we can consistently ignore deviations of the vector matrix elements due to SU(3) breaking and extract the axial matrix elements from the ratio of axial-vector to vector current matrix elements.

Heavy baryon chiral perturbation theory [24,25] (see also [26]) is used to compute the $O(m_s\ln m_s)$ corrections to the axial matrix elements. This technique is sufficiently well known that we will not go into its details in this work and merely give results of the computation. The Lagrange density for the interaction between the lowest-lying octet and decuplet baryons of four-velocity v_α with the pseudo Goldstone bosons is

TABLE I. The coefficients α^3_{ij} and β^3_{ij} for the flavor-diagonal axial matrix elements ($\Delta_0=0$). The remaining matrix elements are related by isospin to those in the table.

Process	Coefficients α^3_{ij}	β^3_{ij}
$p\to p$	$D+F$	$\frac{4}{9}(D^3+D^2F+3DF^2-9F^3)-D-F-\frac{20}{81}C^2H+\frac{4}{2}C^2(F+3D)$
$\Sigma^+\to\Sigma^+$	$2F$	$-2F-\frac{2}{9}F(9F^2-D^2)-\frac{50}{27}C^2H+\frac{8}{3}C^2(\frac{13}{9}D-\frac{1}{3}F)$
$\Xi^0\to\Xi^0$	$F-D$	$D-F-\frac{4}{9}(D^3-D^2F+3DF^2+9F^3)-\frac{40}{81}C^2H-\frac{8}{3}C^2(\frac{17}{18}D+\frac{3}{2}F)$
$\Lambda\to\Sigma^0$	$\frac{2}{\sqrt{3}}D$	$-\frac{1}{\sqrt{3}}\left[2D+\frac{2}{9}D(9F^2-17D^2)+\frac{10}{27}C^2H-\frac{16}{3}C^2(D+F)\right]$

$$\mathcal{L}=\mathrm{Tr}[\overline{B}iv\cdot\mathcal{D}B]+D\,\mathrm{Tr}[\overline{B}2s_\mu\{A^\mu,B\}]+F\,\mathrm{Tr}[\overline{B}2s_\mu\{A^\mu,B\}]$$
$$-\overline{T}iv\cdot\mathcal{D}T+\Delta_0\overline{T}T+C(\overline{T}^\mu A_\mu B+\mathrm{H.c.})+H\overline{T}^\mu 2s_\nu A^\nu T_\mu, \tag{19}$$

where $\mathcal{D}$ is the chiral covariant derivative and

$$A_\mu=\frac{i}{2}(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi) \tag{20}$$

is the axial meson field with

$$\xi=\exp\left(\frac{i}{f}M\right), \tag{21}$$

$$M=\begin{pmatrix}\eta/\sqrt{6}+\pi^0/\sqrt{2} & \pi^+ & K^+\\ \pi^- & \eta/\sqrt{6}-\pi^0/\sqrt{2} & K^0\\ K^- & \overline{K^0} & -2/\sqrt{6}\,\eta\end{pmatrix}, \tag{22}$$

and f is the meson decay constant. The axial constants F, D, C, and H have been discussed extensively in the literature and are seen to be consistent with spin-flavor SU(6) relations [24–27] (a claim against this conclusion can be found in [16]). The mass difference between the decuplet and the octet baryons is Δ_0.

The matrix element of an axial-vector current with flavor index a between two octet baryons states B_i and B_j is given by[1]

$$\langle B_i|j_5^{\mu,a}|B_j\rangle=\overline{U}_i\gamma_\mu\gamma_5U_j\left[\alpha^a_{ij}+(\beta^a_{ij}-\lambda_{ij}\alpha^a_{ij})\frac{M_K^2}{16\pi^2f^2}\right.$$
$$\left.\times\ln(M_K^2/\Lambda_\chi^2)+C^a_{ij}(\Lambda_\chi)+\cdots\right], \tag{23}$$

where we will take f to be the kaon decay constant (motivated by previous experience with such corrections, e.g., [29]), $f_K=1.22f_\pi$, and $f_\pi=132$ MeV. In writing the matrix elements this way we have used the Gell-Mann–Okubo mass formula $M_\eta^2=\frac{4}{3}M_K^2$ and set $M_\pi=0$. The coefficients $\alpha^a_{ij},\beta^a_{ij}$ for flavor-off-diagonal currents and for $a=8$ in the proton, along with the wave function renormalization coefficients λ_{ij} have been computed by Jenkins and Manohar [24–26]. The unknown counterterms that contribute at order $O(m_s)$ are denoted by $C^a_{ij}(\Lambda_\chi)$ where we have chosen to renormalize at the scale $\mu=\Lambda_\chi$. As they are unknown quantities, we will set them equal to zero for our discussions, $C^a_{ij}=0$. The coefficients $\alpha^a_{ij},\beta^a_{ij}$, and λ_{ij} are determined from the graphs shown in Figs. 1, 2, and 3, and for $\Delta_0=0$ are given in Tables I–IV. It is simple to include a nonzero value for the decuplet-octet mass difference Δ_0 [16]. For the vertex graphs involving two decuplet states and the wave function graphs one makes the replacement

$$M_K^2\ln\left(\frac{M_K^2}{\Lambda_\chi^2}\right)\to\mathcal{F}\left(\frac{M_K}{\Delta_0}\right), \tag{24}$$

$$\mathcal{F}\left(\frac{M_K}{\Delta_0}\right)=(M_K^2-2\Delta_0^2)\ln\left(\frac{M_K^2}{\Lambda_\chi^2}\right)$$
$$+2\Delta_0\sqrt{\Delta_0^2-M_K^2}\ln\left(\frac{\Delta_0-\sqrt{\Delta_0^2-M_K^2+i\epsilon}}{\Delta_0+\sqrt{\Delta_0^2-M_K^2+i\epsilon}}\right), \tag{25}$$

and for vertex graphs involving one decuplet state and one octet state one makes the replacement

$$M_K^2\ln\left(\frac{M_K^2}{\Lambda_\chi^2}\right)\to\int_0^1dx\,\mathcal{F}\left(\frac{M_K}{(x\Delta_0)}\right). \tag{26}$$

Similar replacements occur for the η-loop graphs. It was shown by Jenkins and Manohar [24–26] that it is important to include the decuplet as a dynamical field otherwise the natural size of local counterterms is set by the decuplet-octet mass splitting and not by Λ_χ. The difference between $\Delta_0\neq0$ and $\Delta_0=0$ is formally higher order in the expansion than we are working; however, setting $\Delta_0\neq0$ does allow one to estimate the size of higher order effects. For our purpose we treat Δ_0 to be the same for all the decuplet-octet mass splittings and we present results for $\Delta_0=0$, 200 MeV, and ∞. The $\Delta_0=\infty$ theory does not correspond to taking the $\Delta_0\to\infty$ limit of $\mathcal{F}(M_K/\Delta_0)$. In this limit the function becomes analytic in the light quark masses and can be absorbed

[1]We have assumed the matrix element is independent of the invariant mass of the lepton pair. This is a reasonable approximation as the energy release in these decays is small.

TABLE II. The coefficients α^8_{ij} and β^8_{ij} for the flavor-diagonal axial matrix elements ($\Delta_0=0$). The remaining matrix elements are related by isospin to those in the table.

Process	Coefficients α^8_{ij}	β^8_{ij}
$p\to p$	$3F-D$	$3D-9F-\frac{2}{9}(11D^3-27D^2F-27DF^2+27F^3)+4C^2(D-F)$
$\Lambda\to\Lambda$	$-2D$	$6D-\frac{2}{9}D(27F^2-11D^2)+\frac{10}{9}C^2H+\frac{8}{3}C^2(D-3F)$
$\Sigma^+\to\Sigma^+$	$2D$	$-6D+\frac{2}{9}D(D^2+63F^2)-\frac{10}{9}C^2H+\frac{8}{3}C^2(\frac{7}{3}D+F)$
$\Xi^0\to\Xi^0$	$-D-3F$	$3D+9F-\frac{2}{9}(11D^3+27D^2F-27DF^2-27F^3)-\frac{8}{3}C^2(\frac{13}{6}D+\frac{7}{2}F-\frac{10}{9}H)$

TABLE III. The coefficients α^1_{ij} and β^1_{ij} for the flavor-singlet axial matrix elements ($\Delta_0=0$).

Process	Coefficients α^1_{ij}	β^1_{ij}
$p\to p$	S	$-S(5F^2+\frac{17}{9}D^2-\frac{10}{3}FD)-\mathcal{T}\frac{5}{9}C^2$
$\Lambda\to\Lambda$	S	$-S(6F^2+\frac{14}{9}D^2)-\mathcal{T}\frac{10}{9}C^2$
$\Sigma\to\Sigma$	S	$-S(2F^2+\frac{26}{9}D^2)-\mathcal{T}\frac{70}{27}C^2$
$\Xi\to\Xi$	S	$-S(5F^2+\frac{17}{9}D^2+\frac{10}{3}FD)-\mathcal{T}\frac{65}{27}C^2$

TABLE IV. The wave function renormalization coefficients λ_{ij} ($\Delta_0=0$).

Process	λ_{ij}
$N\to N$	$\frac{17}{3}D^2+15F^2-10DF+C^2$
$\Sigma\to\Sigma$	$\frac{26}{3}D^2+6F^2+\frac{14}{3}C^2$
$\Lambda\to\Lambda$	$\frac{14}{3}D^2+18F^2+2C^2$
$\Xi\to\Xi$	$\frac{17}{3}D^2+15F^2+10DF+\frac{13}{3}C^2$
$\Lambda\to\Sigma$	$\frac{20}{3}D^2+12F^2+\frac{10}{3}C^2$

TABLE V. Loop-level axial coupling constants for $\Delta_0=0$, $\Delta_0=200$ MeV, and $\Delta_0=\infty$.

Coupling	Axial-vector coupling constants $\Delta_0=0$	$\Delta_0=200$MeV	$\Delta_0=\infty$
D	0.64 ± 0.05	0.64 ± 0.06	0.59 ± 0.06
F	0.42 ± 0.04	0.34 ± 0.04	0.34 ± 0.04
$\lvert C\rvert$	1.39 ± 0.06	1.37 ± 0.05	1.37 ± 0.06
H	-2.7 ± 0.6	-2.7 ± 0.5	-2.8 ± 0.5

into a renormalization of higher order counterterms. Therefore, the $\Delta_0=\infty$ theory is equivalent to one without contributions from the decuplet (this is also the reason why we can consistently treat the contribution from π loops as negligible). Also, results for $\Delta_0=300$ MeV are little different from those for $\Delta_0=200$ MeV.

Notice that at this order we are forced to introduce an unknown parameter $\mathcal{T}$, the matrix element of the singlet axial-vector current in the decuplet or, equivalently, the strange content of the Δ. It arises in the loop graphs involving decuplet intermediate states (there is no octet to decuplet transition induced by the singlet),

$$j_5^{\mu,1}(\mathbf{10})=\mathcal{T}\,\overline{T}^{abc}_{\alpha}2s_{\mu}T^{\alpha}_{abc}. \tag{27}$$

The value of this constant is unknown and for our calculations we set $\mathcal{T}=0$ (setting $\mathcal{T}=S$ gives virtually identical results). However, this quantity does provide a problem for a systematic inclusion of higher order corrections to the SU(3) limit. Physically, one extracts a linear combination of S and $\mathcal{T}$ at one-loop order and the same linear combination enters in all appropriate observables in the nucleon sector at this order. However, when considering matrix elements between strange hyperons a different linear combination of S and $\mathcal{T}$ will enter.

The axial couplings of the decuplet C and H first contribute to the axial matrix elements of the octet baryons at loop level and hence cannot be well constrained from the semileptonic decays alone. In addition to the β decay and the hyperon decay used for the tree-level fit, we require that the couplings reproduce the strong decays of the Δ,Σ^*, and the Ξ^*. Expressions for these rates at $O(m_s \ln m_s)$ can be found in [27] (tree-level extractions would be sufficient at this order). The procedure for the tree-level fitting was applied to the loop-level fitting, except that we fit four coupling constants instead of the two at the tree level (i.e., $\chi^2<\chi^2_{\rm min}+4.7$). Best fit values for the axial coupling constants are shown in Table V and they are consistent with most previous extractions.[2] The fits to the semileptonic decay matrix elements both at the tree level and the one-loop level are shown in Table VI. Differences between the tree-level and loop-level fits to the semileptonic matrix elements are not great. Neutral current axial matrix elements are estimated at leading order in SU(3) breaking and we present the estimates for O_3, O_8, and the singlet current for each of the octet baryons in Tables VII–IX.

The loop-level extractions of the quark contributions to the proton spin are shown in Table X, along with the tree-level result. It is evident that the up- and down-quark contributions are insensitive to the SU(3) breaking. In contrast, the strange quark content is very sensitive to the breaking; however, all the determinations agree within the uncertainties. Further, the matrix element of the singlet current in the proton extracted from the E143 measurement of $\int dx g_1(x)=0.127\pm0.004\pm0.010$ appears to be compatible with zero in each of the determinations, as it is at the tree

[2] We do not agree with the extraction of F and D presented in [16] and insufficient details are given for us to be able to determine the reason for the disagreement.

TABLE VI. Tree- and loop-level evaluations of matrix elements of the axial-vector current. Superscripts a,b denote $\Delta_0=0$ and 200 MeV, respectively.

Process	Axial matrix elements Tree level	Loop level[a]	Loop level[b]	Experimental [10,28]
$n\to pe^-\overline{\nu_e}$	1.26 ± 0.08	1.24 ± 0.11	1.16 ± 0.09	1.2664 ± 0.0065
$\Sigma^-\to ne^-\overline{\nu_e}$	0.31 ± 0.10	0.35 ± 0.13	0.31 ± 0.1	0.341 ± 0.015
$\Xi^-\to\Lambda e^-\overline{\nu_e}$	0.27 ± 0.09	0.25 ± 0.14	0.29 ± 0.10	0.306 ± 0.061
$\Lambda\to pe^-\overline{\nu_e}$	-0.90 ± 0.07	-1.01 ± 0.11	-0.98 ± 0.09	-0.890 ± 0.015
$\Sigma^-\to\Lambda e^-\overline{\nu_e}$	0.64 ± 0.06	0.64 ± 0.06	0.57 ± 0.06	0.602 ± 0.014
$\Xi^-\to\Sigma^0 e^-\overline{\nu_e}$	0.89 ± 0.06	0.89 ± 0.06	1.00 ± 0.09	0.929 ± 0.112
$\Delta\to N\pi$	-1.70 ± 0.07	-1.76 ± 0.13	-1.75 ± 0.11	-2.04 ± 0.01
$\Sigma^*\to\Lambda\pi$	-1.70 ± 0.07	-1.76 ± 0.14	-1.77 ± 0.12	-1.71 ± 0.03
$\Sigma^*\to\Sigma\pi$	-1.70 ± 0.07	-1.50 ± 0.18	-1.52 ± 0.15	-1.60 ± 0.13
$\Xi^*\to\Xi\pi$	-1.70 ± 0.07	-1.64 ± 0.12	-1.65 ± 0.09	-1.42 ± 0.04

TABLE VII. Tree-level and loop-level estimates of the matrix elements of the O_3 axial current. The matrix element in the proton is not shown as it is fixed by isospin to g_A. Similarly, the matrix element for the Λ-Σ transition is not shown as it is related to the matrix element for $\Sigma^-\to\Lambda$ by isospin. Also, the matrix element between Λ states vanishes by isospin. Superscripts a, b, c denote $\Delta_0=0$, 200 MeV, and ∞, respectively.

Process	Tree level	O_3 Loop level[a]	Loop level[b]	Loop level[c]
$\Sigma^+\to\Sigma^+$	0.95 ± 0.12	0.70 ± 0.23	1.09 ± 0.12	0.98 ± 0.14
$\Xi^0\to\Xi^0$	-0.31 ± 0.10	-0.35 ± 0.17	-0.18 ± 0.10	-0.36 ± 0.13

TABLE VIII. Tree-level and loop-level estimates of the matrix elements of the O_8 axial current. The matrix element between Λ and Σ states vanishes by isospin. Superscripts a,b,c denote $\Delta_0=0$, 200 MeV, ∞, respectively.

Process	Tree level	O_8 Loop level[a]	Loop level[b]	Loop level[c]
$p\to p$	0.65 ± 0.21	0.78 ± 0.24	0.45 ± 0.20	0.60 ± 0.22
$\Sigma^+\to\Sigma^+$	1.56 ± 0.15	1.63 ± 0.26	1.58 ± 0.18	1.78 ± 0.23
$\Lambda\to\Lambda$	-1.56 ± 0.15	-1.83 ± 0.28	-2.08 ± 0.20	-1.88 ± 0.21
$\Xi^0\to\Xi^0$	-2.21 ± 0.17	-2.31 ± 0.40	-2.79 ± 0.30	-2.81 ± 0.35

TABLE IX. Tree-level and loop-level estimates of the matrix elements of the singlet axial current extracted from the E143 measurement of $\int dx g_1(x)=0.127\pm0.004\pm0.010$. The matrix element between Λ and Σ states vanishes by isospin. Superscripts a,b,c denote $\Delta_0=0$, 200 MeV, ∞, respectively. We have set $\mathcal{T}=0$ in the loop-level calculations.

Process	Tree level	I Loop level[a]	Loop level[b]	Loop-level[c]
$p\to p$	0.10 ± 0.11	0.08 ± 0.12	0.16 ± 0.11	0.11 ± 0.13
$\Sigma^+\to\Sigma^+$	0.10 ± 0.11	0.13 ± 0.19	0.15 ± 0.10	0.14 ± 0.15
$\Lambda\to\Lambda$	0.10 ± 0.11	0.10 ± 0.16	0.18 ± 0.13	0.13 ± 0.15
$\Xi^0\to\Xi^0$	0.10 ± 0.11	0.14 ± 0.22	0.18 ± 0.13	0.16 ± 0.18

TABLE X. Tree-level and loop-level estimates of the individual quark contributions to the proton spin extracted from the E143 measurement of $\int dx g_1(x)=0.127\pm0.004\pm0.010$. Superscripts a,b,c denote $\Delta_0=0$, 200 MeV,∞, respectively. We have set $\mathcal{T}=0$ in the loop-level calculations.

	Matrix elements of the light quark axial currents			
Quark flavor	Tree level	Loop level[a]	Loop level[b]	Loop level[c]
Δu	0.77 ± 0.04	0.79 ± 0.04	0.76 ± 0.04	0.77 ± 0.04
Δd	-0.49 ± 0.04	-0.48 ± 0.04	-0.51 ± 0.04	-0.50 ± 0.04
Δs	-0.18 ± 0.09	-0.23 ± 0.10	-0.10 ± 0.09	-0.16 ± 0.10

level. If, instead, one used the SMC measurement of $\int dx g_1(x)=0.136\pm0.011\pm0.011$, the magnitude of Σ is increased by $\sim50\%$. Our loop analysis of the matrix element of O_8 in the proton is in disagreement with the analysis of Dai *et al.* [10] . In the large-N_c limit they find a value of 0.27 ± 0.09, which is smaller by a factor of 2 than our estimates although we do have a large uncertainty.

It is useful to understand what situation must arise in order to recover the naive quark model estimate of $\Sigma\sim+0.58$. We find that if $D=0.66$, $F=0.37$, $C=-1.4$, and $H=-2.6$, then one can reproduce most axial couplings arising in semileptonic rates reasonably well except for $\Sigma^-\to n$, which would have to be 0.50 compared with 0.341 ± 0.015 observed and $\Xi^-\to\Lambda$, which would have to be 0.09 compared with 0.306 ± 0.061 observed. Unless the experimental determinations are many standard deviations away from the true value of these axial couplings it appears unlikely that the naive quark model value of Σ will arise.

We should remind ourselves that the measurements planned to be made at Jefferson Laboratory of the parity-violating component of ep interactions and the Liquid Scintillation Neutrino Detector (LSND) running at Los Alamos measuring νp scattering (see [4] for a comprehensive review) circumvent the need to use SU(3) symmetry to extract the strange content of the nucleon and hence will not rely upon the estimates made here. The axial-vector current that couples to the Z^0 has the flavor structure

$$j_5^{\mu,Z}=\overline{q}O_Z\gamma^\mu\gamma_5 q, \tag{28}$$

$$O_Z=O_3+\frac{1}{3}O_8-\frac{1}{3}I=O_3-\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}, \tag{29}$$

at the tree level in the standard model[3] (electroweak loops induce a nonstrange isoscalar component). As the matrix element of O_3 in the proton is known from g_A by isospin symmetry, a measurement of the Z^0 axial coupling will yield the strange quark content of the nucleon directly. One might suspect that nuclear physics uncertainties would provide an intrinsic limitation to the extraction of Δs and Σ from ν-scattering experiments such as LSND. However, recent work [30] indicates that, in fact, nuclear physics uncertainties can be minimized by forming appropriate combinations of observables. It would appear from our somewhat primitive analysis of SU(3) breaking that the Z^0 measurements are the key to determining the strange quark content of the nucleon.

As an aside we consider the analogue of the strange quark content of the nucleon for the other baryons in the octet. Such quantities could be the ''up-quark'' content of the Ξ^- (with flavor quantum numbers ssd) or the ''down-quark'' content of the Σ^+ (with flavor quantum numbers uus). In the limit of exact SU(3) one can find the individual quark contributions by large SU(3) transformations. For instance, under $s\leftrightarrow u$ we have $p\leftrightarrow\Xi^-$ and hence we expect that the up-quark content of the Ξ^- is equal to the strange quark content of the nucleon. Similarly, under $s\leftrightarrow d$ we have $p\leftrightarrow\Sigma^+$ and we expect that the down-quark content of the Σ^+ is the same as the strange quark content of the nucleon. We can investigate the effects of the $O(m_s \ln m_s)$ SU(3)-breaking terms on these relations simply from our above analysis (we use the $\Delta_0=0$ results). We find that, for the Ξ^- at the loop level,

$$\Delta u_\Xi=-0.18\pm0.14,\quad \Delta d_\Xi=-0.50\pm0.10,$$

$$\Delta s_\Xi=0.83\pm0.12, \tag{30}$$

and, for the Σ^+ at the loop level,

$$\Delta u_\Sigma=0.68\pm0.12,\quad \Delta d_\Sigma=0.05\pm0.12,$$

$$\Delta s_\Sigma=-0.49\pm0.09. \tag{31}$$

The ''wrong''-quark content is about the same for each baryon and is consistent with the results seen in the nucleon sector alone. We note that the ''d''-quark content of the Σ^+ and the ''u''-quark content of the Ξ^- are dominated by the local counterterms F, D, and S, and not by the meson loop graphs. We may make a connection with the nonleptonic interactions between octet baryons and the pseudo Goldstone bosons. It was realized in [31,32] that a nonzero strange axial matrix element in the nucleon may impact nuclear parity violation. Nonstrange operators are suppressed by custodial symmetries of the standard model of electroweak interactions in the limit $\sin^2\theta_w\to0$, while strange operators are not. The strangeness-changing four-quark interaction (ignoring strong interaction corrections) is

[3]We thank M. Musolf for pointing out that this is only a tree-level relation.

TABLE XI. Tree-level and loop-level evaluations of the matrix elements of the neutral axial current coupling to the Z^0. Superscripts a,b denote $\Delta_0=0$, 200 MeV, respectively. Isospin relates the matrix element for $\Sigma^-\to\Lambda$ to the value of C_A for the Λ-Σ^0 transition, giving $C_A=0.85\pm0.02$.

Process	$\Delta s=0$ tree level	C_A Tree level	Loop level[a]	Loop level[b]
$p\to p$	$D+F=1.26$	1.43 ± 0.10	1.50 ± 0.10	1.36 ± 0.09
$n\to n$	$-(D+F)=-1.26$	-1.09 ± 0.10	-1.04 ± 0.10	-1.17 ± 0.09
$\Lambda\to\Lambda$	$-(F+D/3)=-0.73$	-0.56 ± 0.07	-0.64 ± 0.11	-0.75 ± 0.09
$\Sigma^+\to\Sigma^+$	$D+F=1.26$	1.44 ± 0.13	1.20 ± 0.25	1.57 ± 0.15
$\Sigma^0\to\Sigma^0$	$D-F=0.34$	0.46 ± 0.04	0.50 ± 0.09	0.48 ± 0.06
$\Sigma^-\to\Sigma^-$	$D-3F=-0.58$	-0.46 ± 0.13	-0.19 ± 0.20	-0.61 ± 0.12
$\Xi^0\to\Xi^0$	$-(D+F)=-1.26$	-1.07 ± 0.11	-1.17 ± 0.20	-1.17 ± 0.13
$\Xi^-\to\Xi^-$	$D-3F=-0.58$	-0.47 ± 0.13	-0.46 ± 0.23	-0.81 ± 0.16

$$\mathcal{H}=\frac{G_F}{\sqrt{2}}V_{us}V_{ud}^{\dagger}\overline{u}\gamma^{\mu}(1-\gamma_5)s\overline{d}\gamma_{\mu}(1-\gamma_5)u, \tag{32}$$

and naively one might not expect this operator to contribute to the weak coupling $\Xi^-\Xi^-K^0$, as there are no up quarks in any of the hadrons. However, SU(3) symmetry relations arising from the observed octet enhancement in these nonleptonic decays gives S and P wave amplitudes

$$\mathcal{A}^{(S)}=\frac{1}{f}(h_D+h_F), \tag{33}$$

$$\mathcal{A}^{(P)}=\frac{(D+F)(h_D+h_F)}{f(M_\Xi-M_\Sigma)}2S\cdot k, \tag{34}$$

where k is the outgoing meson momentum and h_D and h_F are two constants, determined to be $h_D=(-0.58\pm0.21)G_FM_\pi^2f$ and $h_F=(+1.40\pm0.12)G_FM_\pi^2f$ at the tree level [33]. One can also compute these amplitudes in the factorization limit giving

$$\mathcal{A}_{\rm fact}^{(S)}=0, \tag{35}$$

$$\mathcal{A}_{\rm fact}^{(P)}=\frac{G_F}{\sqrt{2}}V_{us}V_{ud}^{\dagger}f(\Delta u_\Xi)2S\cdot k, \tag{36}$$

where Δu is the up-quark contribution to the Ξ^- spin. In order to reproduce the P-wave amplitude computed via octet enhancement we require $\Delta u_\Xi\sim0.05$, a value that is encompassed by our determination. This suggests that the up-quark content of the Ξ^- could lead to a counterterm for the nonleptonic vertex $\sim\mathcal{A}_{\rm fact}^{(P)}$ that is of the same size if not larger than the vertex resulting from the baryon pole graph $\mathcal{A}^{(P)}$.

In systems of density comparable to or greater than that of nuclear matter such as those that arise in ''neutron stars,'' the exact composition of the matter is far from certain. The strange quark is guaranteed to play a role at high enough density, but the question of at what density it becomes important depends crucially on the strong interactions between the nucleons, the strange hyperons, and the mesons. If indeed it is energetically favored for strange baryons to be present in significant number densities then it is necessary to know the interactions of neutrinos with these baryons in order to construct a reasonable model for the evolution of some dense matter systems [14,15]. We present estimates of the axial matrix elements for Z^0 interactions between hyperons in the lowest-lying octet C_A in Table XI. It is clear that some matrix elements are more susceptible to large SU(3)-breaking corrections than others, at least for the corrections that we could estimate. In particular, matrix elements for the Σ^- and Ξ^- appear to be particularly unreliable, with large deviations from the tree-level estimates likely.

In conclusion, we have computed the leading, model-independent SU(3)-breaking contributions to the matrix elements of axial-vector current with flavor structure O_3, O_8 and the flavor singlet. We find that there is a large uncertainty in some matrix elements, and this is probably an indication of comparable uncertainty in all matrix elements from terms we cannot compute.

It is the matrix element of O_8 in the proton that presently impacts the determination of the Δs and Σ in the proton. We find that both quantities are sensitive to SU(3) breaking [in disagreement with [7] where the impact of SU(3) violation on Σ was claimed to be small], and we estimate them to lie in the intervals $-0.1\lesssim\Sigma\lesssim+0.3$ and $-0.35\lesssim\Delta s\lesssim0$ from the E143 measurement of $\int dx g_1(x)=0.127\pm0.004\pm0.010$. The upper limit of this range for Σ is still much less than the naive quark model estimate of $+0.58$ [using the SMC value for $\int dx g_1(x)=0.136\pm0.011\pm0.011$, the upper limit of Σ becomes ~0.35]. Somewhat more pessimistically, we clearly demonstrate that there is a large theoretical impediment to making a more precise determination of Σ and Δs from better measurements of $g_1(x)$. It appears that improvement can only occur from measurements of the Z^0 coupling to nucleons.

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