Supersymmetric prediction for the muon transverse polarization in the $K^+ \to \pi^0 \mu^+ \nu_\mu$ decay

Marco Fabbrichesi
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy
and Scuola Internazionale Superiore di Studi Avanzati, Via Beirut 2-4, I-34013 Trieste, Italy

Francesco Vissani
Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy
and International Centre for Theoretical Physics, Strada Costiera 11, I-34013 Trieste, Italy
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The muon transverse polarization in the $K^+ \to \pi^0 \mu^+ \nu_\mu$ decay will be measured at the 10^{-4} level in forthcoming experiments. We compare the phenomenological perspectives with the theoretical predictions in supersymmetric extensions of the standard model. In the minimal extension, CP-violating phases lead to a nonzero transverse polarization, that, however, is too small to account for a positive experimental signal. The problems that one encounters when departing from minimal assumptions are discussed. An observable effect is possible if the hypothesis of R-parity conservation is relaxed, but only at the price of assuming a very special pattern for the R-parity-breaking couplings. [S0556-2821(97)05907-9]

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I. INTRODUCTION

A nonvanishing component of the muon polarization, transverse to the decay plane of the $K^+ \to \pi^0 \mu^+ \nu_\mu$ ($K^0_{\mu 3}$) process, would signal CP-violating effects [1], if larger than $\mathcal{P}_\perp \sim 10^{-6}$ (the contribution due to final state interactions [2]). Such a signal would be particularly interesting, since it would imply physics beyond the standard model [3].

Forthcoming experiments at DAΦNE [4], KEK [5], and BNL [6] may push the present limit $\mathcal{P}_\perp < 5 \times 10^{-3}$ [7] by more than one order of magnitude, $\mathcal{P}_\perp < 2.8 \times 10^{-4}$ (from [8]), or perhaps obtain a positive result. For this reason, it is important to state the prediction for this observable in all those models that are potential candidates to describe physics beyond the standard model (see [2,9–14] for earlier studies).

The outline of the present work is the following. Section I is devoted to a phenomenological discussion, including the relationship between the polarization and the invariant form factors, the discussion of the experimental perspectives, and the specification of the effect of strong interactions. We analyze in Section II the size of the transverse polarization in possible extensions of the standard model. We focus in particular on supersymmetric models (see [14–16] for previous analyses): the minimal extension of the standard model and models with explicit R-parity breaking. The last section is devoted to the conclusions.

II. DECAY FORM FACTORS AND MUON POLARIZATION

The form of the invariant amplitude suggested by the Particle Data Group is [17]

$$\mathcal{M} = G_F \sin\theta_C \overline{u(p_\nu)}_L \Big([F_+ (p_K + p_\pi)^a - F_- (p_K - p_\pi)^a]\gamma_a + 2F_S m_K + 2i \frac{F_T}{m_K} \sigma_{ab} p_K^a p_\pi^b \Big) v(p_\mu, \vec{s}_\mu). \quad (1)$$

The leptons in the final state are a left-handed neutrino with four-momentum p_ν and an antimuon with four-momentum p_μ and spin $\vec{s}_\mu$. (Notice that, because of the chirality projector, we can account for an additional contribution $\delta F_T / m_K p_K^a p_\pi^b \sigma^{cd} \epsilon_{abcd}$ to the bracketed term simply by redefining $F_T \to F_T - \delta F_T$.)

In Eq. (1) we distinguish between the contribution of the usual $V-A$ interactions and that of other possible interactions because final state interactions [2] and the standard model sources of CP violation [3] give a negligible contribution to F_S and F_T.

The form factors F_+, F_-, F_S, F_T in Eq. (1) depend in general on the hadronic momentum transferred, $q^2 = (p_K - p_\pi)^2$; CP invariance implies that they are relatively real. They can be calculated once the model and the hadronic matrix elements are specified.

A. Transverse polarization

By the use of the equations of motions we can recast Eq. (1) in the form

$$\mathcal{M} = 2 G_F \sin\theta_C m_K F_+ \left(1 - \xi_T \frac{m_\mu}{m_K}\right) \overline{u(p_\nu)}_L \times \left[\frac{p_K^a}{m_K} \gamma_a + \zeta \right] v(p_\mu, \vec{s}_\mu). \quad (2)$$

The amplitude for scalar interactions is proportional to

$$\zeta = \frac{1}{2} \left[\frac{m_\mu}{m_K} + \left(\frac{m_\mu}{m_K} \xi + 2\xi_S + \frac{2\, p_K \cdot (p_\nu - p_\mu) + m_\mu^2}{m_K^2} \xi_T \right) \times \left(1 - \frac{m_\mu}{m_K} \xi_T \right)^{-1} \right], \quad (3)$$

where we defined

$$\xi=\frac{F_-}{F_+},\quad \xi_S=\frac{F_S}{F_+},\ \text{and}\quad \xi_T=\frac{F_T}{F_+}. \tag{4}$$

We assume that the parameters ξ,ξ_S,ξ_T are constants. Actually, writing Eq. (1) with four form factors makes sense *only if* their dependence on the invariants is to a certain extent specified: Lorentz invariance alone would require just two form factors.

The form (2) is very convenient in evaluating the decay rate (this was originally pointed out in [18]). In bispinorial notation we can write

$$\mathcal{M}\propto\sqrt{2E_\nu 2E_\mu}\phi(-\vec{\beta}_\nu)^\dagger[b_+ + b_-\vec{\beta}_\mu\cdot\vec{\sigma}]\phi(-\vec{s}_\mu), \tag{5}$$

where $\vec{\beta}$ are the velocity vectors for the neutrino and the antimuon, and $\gamma_\mu=1/\sqrt{1-\beta_\mu^2}$ the antimuon Lorentz boost factor (not a γ matrix). The spinors $\phi(\vec{n})$ obey $\phi^\dagger\phi=1$ and $(\vec{n}\vec{\sigma})\phi(\vec{n})=\phi(\vec{n})$. The two coefficients of the amplitude can be expressed as $b_\pm=(\zeta\mp1)(1+\gamma_\mu^{-1})^{\pm1/2}$. All quantities are evaluated in the kaon rest frame. Squaring Eq. (5) we obtain

$$|\mathcal{M}|^2\propto 8E_\nu E_\mu\mathcal{P}\cos^2[\Theta/2], \tag{6}$$

where Θ is the angle between the observed polarization $\vec{s}_\mu$ and the vector $\vec{\mathcal{P}}$,

$$\vec{\mathcal{P}}=[|1-\zeta|^2\gamma_\mu^{-1}-2\mathrm{Re}\zeta(1-\gamma_\mu^{-1})]\vec{\beta}_\nu+[1-|\zeta|^2 + |1+\zeta|^2\vec{\beta}_\nu\cdot\vec{\beta}_\mu(1+\gamma_\mu^{-1})]\vec{\beta}_\mu+\mathrm{Im}\zeta[\vec{\beta}_\nu\times\vec{\beta}_\mu], \tag{7}$$

whose modulus, which enters formula (6) is

$$\mathcal{P}=1+\vec{\beta}_\nu\cdot\vec{\beta}_\mu-2\mathrm{Re}\zeta\gamma_\mu^{-1}+|\zeta|^2(1-\vec{\beta}_\nu\cdot\vec{\beta}_\mu). \tag{8}$$

It is worthwhile emphasizing some aspects of this result, which was originally derived in [19,20].

(a) The probability of a transition, proportional to $|\mathcal{M}|^2$, is small for small lepton energies.

(b) The three terms in Eq. (8) have a clear interpretation: The first (last) describes a left- (right-) handed particle, produced by vector (scalar) interactions, and the second is a typical interference term (ζ parametrizes the relative amount of scalarlike interactions).

(c) The squared cosine factor in Eq. (6) indicates that the antimuon produced in the decay is completely polarized along $\vec{\mathcal{P}}$. The nonpolarized case is recovered upon averaging (according to $\cos^2[\Theta/2]\to1/2$).

(d) For slow antimuons, formula (7) shows that the polarization is parallel to the neutrino velocity vector. This is clearly due to the fact that the neutrino is in a negative helicity state.

(e) The transverse polarization, the last term in Eq. (7), is maximum when the antimuon and the neutrino velocities are orthogonal in the laboratory frame.

B. Phenomenological remarks and experimental perspectives

In the Dalitz plot distributions, the imaginary part of the parameter ζ in Eq. (3) [or equivalently ξ and/or ξ_S and/or ξ_T in Eq. (4)] enters only quadratically [compare with Eq. (8)]. On the other hand, the dependence of Eq. (7) on $\mathrm{Im}\zeta$ is linear and therefore the transverse polarization would give to a large extent independent information on the form factors.

Recently a simple experimental method has been proposed [8] to detect the effects of the polarizations at DAΦNE without the need of a polarimeter. It is based on the fact that the direction of the positron emitted in the antimuon decay is correlated to $\vec{\mathcal{P}}$ and, therefore, to the kinematical variables of the decay that produced the antimuon. There is a different probability of emission above and below the decay plane if ζ has an imaginary part (the direction ''above the decay plane'' is specified by the vector $\beta_\nu\times\beta_\mu$; no definition is needed when the particles are collinear). Recalling that the differential probability $p(\theta)$ of emission of a positron with angle θ, measured from the antimuon polarization direction, is $(1+1/3\cos\theta)d\Omega/4\pi$, where Ω is the solid angle, we find $p(\text{above})-p(\text{below})=\mathcal{P}_\perp/(6\mathcal{P})$, where $\mathcal{P}_\perp=\mathrm{Im}\zeta|\vec{\beta}_\nu\times\vec{\beta}_\mu|$. Therefore the asymmetry rate is

$$a_\perp(E_\nu,E_\mu)=\mathrm{Im}\zeta\frac{G_F^2\sin^2\theta_C}{48\pi^3}m_K\left|F_+\left(1-\xi_T\frac{m_\mu}{m_K}\right)\right|^2 \times|\vec{p}_\nu\times\vec{p}_\mu|dE_\nu dE_\mu. \tag{9}$$

Notice that, according to Eqs. (3) and (7), if the leading contribution to the transverse polarization comes from $\mathrm{Im}\xi$ or $\mathrm{Im}\xi_S$, then $\mathcal{P}_\perp$ points in a given half-space with respect to the decay plane; on the contrary, if $\mathcal{P}_\perp$ is related to $\mathrm{Im}\xi_T$, it can point in both directions because the factor $(E_\nu-E_\mu+m_\mu^2/m_K)$ changes sign [see Eq. (3)].

In this way, DAΦNE can provide a factor of 10 improvement of the current limits, and therefore may reveal transverse polarization effects if [1]

$$\mathrm{Im}\xi_S\ \text{or}\ \mathrm{Im}\xi_T\gtrsim2\times10^{-3}. \tag{10}$$

It is suggestive to compare this figure with the most recent experimental analysis of K_{e3}^0 decays [21], for which the best fit of the Dalitz plot distributions requires $F_S=0.070_{-0.016}^{+0.016}$ and $F_T=0.53_{-0.10}^{+0.09}$. If similar values for $K_{\mu3}^0$ decay parameters are asasumed, an imaginary part as small as 0.5% would lead to an observable signal.

C. Current form factors

The Lorentz-invariant decomposition of the hadronic matrix elements introduces five form factors:

$$\langle\pi^0|\overline{s(0)}\gamma^a u(0)|K^+\rangle=\frac{1}{\sqrt{2}}[f_+(q^2)(p_K+p_\pi)^a - f_-(q^2)(p_K-p_\pi)^a], \tag{11}$$

$$\langle\pi^0|\overline{s(0)}u(0)|K^+\rangle=\sqrt{2}m_K f_S(q^2) \tag{12}$$

[1] See Ref. [8]. Note, however, that the different dependence on the energy in Eq. (3) implied by the tensor form factor requires a different experimental analysis from the case in which one assumes a pure scalar contribution (the one considered in [8]).

$$\langle \pi^0|\overline{s(0)}\sigma^{ab}u(0)|K^+\rangle = \frac{i}{\sqrt{2}m_K} f_T(q^2)(p_K^a p_\pi^b - p_K^b p_\pi^a) + f_T'(q^2)\epsilon^{abcd}(p_K)_a(p_\pi)_b . \quad (13)$$

The specification of these form factors amounts to the description of the nonperturbative effects of strong interactions. Renormalization group factors induced by QCD are of order unity for such a semileptonic decay and we shall ignore them. At the tree level, $V-A$ structure entails only the first operator. The other two are in general present when new interactions are introduced.

The flavor SU(3) symmetry imposes relations on the vector form factors so that $f_+(0)=-1$ and $f_-(q^2)=0$ (see, for instance, the discussion in [22]). For the purpose of describing the transverse polarization, it is adequate to consider such a SU(3)-symmetric limit. It is important to point out that the precise knowledge of the form factors and, in particular, of their momentum dependence is important in *testing* the hypothesis of pure standard model interactions when studying the $K_{\mu 3}$ Dalitz plot distributions.

The scalar form factor can be computed considering the matrix element of the divergence of the vector current and then using the free equations for the quark fields (as described in [9]):

$$f_S = \frac{(m_K^2 - m_\pi^2)f_+ - q^2 f_-}{2m_K(m_s - m_u)} \simeq \frac{m_K}{2m_s} f_+ . \quad (14)$$

This estimation is subject to considerable uncertainty due to the use of the free equations of motion and the value of m_s [we use the central value of the recent determination $m_s \equiv \overline{m_s}(1\ \text{GeV}) = 175 \pm 25$ MeV [23]]. Nevertheless, it is sufficiently accurate for the purpose of estimating the muon transverse polarization.

For completeness, we also present the estimation of the tensor form factors, which was first evaluated in [24]. In the context of the chiral quark model [25], at the leading order, we find

$$f_T \simeq \frac{6m_K M}{\Lambda_\chi^2} \approx 1, \quad f_T' \approx 0, \quad (15)$$

where M is the constituent quark mass, a model-dependent parameter ($M=220$ MeV in the above), and $\Lambda_\chi \approx 1$ GeV is the chiral symmetry-breaking scale.

III. THEORETICAL PREDICTION

Which kind of models can lead to a nonzero transverse polarization, and what are their phenomenological implications? This question will be discussed within the supersymmetric extensions of the standard model. We first examine the minimal extension, where the effect arises at the one-loop level, and then consider R-parity-nonconserving interactions, which can give an effect already at the tree level.

A. Minimal supersymmetric standard model

1. Introduction

The supersymmetrization of the standard model requires enlarging the spectrum of the theory. The squarks and the sleptons are the scalar partners of the quarks and leptons; two Higgs doublets are present, each one paired with a fermionic doublet (Higgsino); similarly, for each gauge boson one fermionic degree of freedom appears (gaugino). According to the usual convention we denote the supersymmetric particles by a tilde: For instance, $\tilde{g}$ is the gluino, $\tilde{q}$ a generic squark. The values of the supersymmetric partners masses depend on the mechanism of supersymmetry breaking—an open question at present.

SU(2) breaking brings in an important parameter, the ratio

$$\tan\beta = \frac{\langle H_2\rangle}{\langle H_1\rangle}, \quad (16)$$

which is bounded by the requirement of perturbative Yukawa couplings to be approximatively between 1 and 50. At the same time, the Higgsinos and the gauginos (except the gluino) mix; the resulting mass eigenstates are called neutralinos ($\tilde{\chi}_i^0$) and charginos ($\tilde{\chi}_i^+$). Another effect of SU(2) breaking is the mixing of the squarks which are partner of the left and the right quarks (called left and right squarks), similarly for the sleptons. We have

$$m^2_{\tilde{u}_i\,\text{LR}} = m_{u_i}(A_{u_i} - \mu^* \cot\beta),$$
$$m^2_{\tilde{d}_i\,\text{LR}} = m_{d_i}(A_{d_i} - \mu^* \tan\beta), \quad (17)$$

where $i=1,2,3$ is the generation index, A_{u_i} and A_{d_i} are parameters of the Higgs-squark-squark, soft-supersymmetry-breaking interactions, and μ is a supersymmetry-conserving mass parameter. These same parameters enter also the interaction vertices of the Higgs boson with the squarks, as we will see below in a noticeable case.

The massive supersymmetric parameters can be complex; in particular, phases can be present in the gaugino masses, in the A and μ parameters defined above.[2] These phases lead to a muon transverse polarization even if there is no new flavor violation that is related to the supersymmetric parameters; our study extends the analysis of [15] to the region in which $\tan\beta$ is large. Then we will comment on the effect of relaxing the minimal hypothesis on supersymmetric flavor violation; the importance of this point has been stressed in [16].

2. Transverse polarization

To evaluate the possibility to have a positive experimental signal, we need an estimation of the muon transverse polarization. In the following an *upper bound* on this effect is given. It shows that the transverse polarization is too small to be detected.

For this sake, let us consider the gluino exchange diagram in Fig. 1(a). The new supersymmetric phases are present in

[2]In the constrained version called *low-energy supergravity model* there may be at most two new phases.

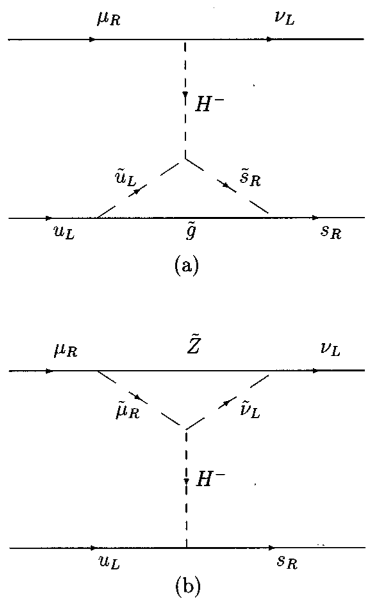

FIG. 1. Two Feynman diagrams inducing scalar-type four-fermion interactions in the minimal supersymmetric extension of the standard model.

the gluino-squark loop, which induces the effective coupling between the u and s quarks and the charged Higgs field H^-:

$$\frac{g_s^2}{(4\pi)^2}\left(V_{us}\frac{m_s}{v}A_s\tan\beta\right)\frac{m_{\tilde{g}}}{m_{\tilde{q}}^2}[\overline{s(x)}P_L u(x)]H^-(x)\mathcal{I}\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\right), \tag{18}$$

where V is the Cabibbo-Kobayashi-Maskawa matrix, g_s the strong coupling, $v=174$ GeV, and

$$\mathcal{I}(x)=\frac{8}{3}\frac{1-x+x\ln x}{(1-x)^2}, \tag{19}$$

having taken the two squark masses equal. In formula (18) we only kept the part of the H^--$\tilde{q}$ coupling that grows with $\tan\beta$; the gluino mass $m_{\tilde{g}}$ provides the chirality flip, and the squark mass $m_{\tilde{q}}$ gives the correct dimension. The exchange of charged Higgs boson H^- therefore leads to the effective operator

$$\sin\theta_C G_S\cdot[\overline{s(x)}P_L u(x)][\overline{\nu_\mu}(x)P_R\mu(x)], \tag{20}$$

where

$$G_S=\frac{1}{m_{H^-}^2}\frac{g_s^2}{(4\pi)^2}\frac{m_\mu m_s}{v^2}\frac{A_s m_{\tilde{g}}}{m_{\tilde{q}}^2}\tan^2\beta\,\mathcal{I}\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\right). \tag{21}$$

The coupling of the charged Higgs boson with the lepton has brought in a second factor $\tan\beta$, and this is the origin of the enhancement of this type of diagram.

Accordingly to previous discussion and using Eq. (14) we find

$$\xi_S=\frac{G_S}{2\sqrt{2}G_F}\frac{f_S}{f_+}$$
$$\sim 6\times10^{-5}\left(\frac{A_s m_{\tilde{g}}}{m_{\tilde{q}}^2}\right)\left(\frac{100\text{ GeV}}{m_{H^-}}\right)^2\left(\frac{\tan\beta}{50}\right)^2\mathcal{I}\left(\frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2}\right). \tag{22}$$

Keeping the squark mass fixed, the overall numerical factor in front of Eq. (22) can be slightly increased if the mass ratio $m_{\tilde{g}}/m_{\tilde{q}}$ lies approximatively between 0.5 and 10, with a maximum of a factor 1.5 if the mass ratio is around 2. The estimation in Eq. (22) shows that nonzero contributions to the form factor F_S are possible in the minimal supersymmetric extension of the standard model if the supersymmetric parameters A_s or $m_{\tilde{g}}$ are complex. However, it also shows why it is difficult to expect a positive signal in the next generation searches of transverse polarization. In fact, in the numerical estimation (22), the transverse polarization can attain relatively large values only if we assume

(a) quite light supersymmetric masses, (b) large values of $\tan\beta$, and (c) large supersymmetric phases (in the gluino masses or in A_s).

We recall two possible unpleasant features of the large $\tan\beta$ scenario: First, it usually implies fine-tuning in the parameters of the scalar sector; second, amplitudes depending on the Yukawa parameters (like those for "dimension-5" proton decay or those for a $b\to s\gamma$ transition) may become too large in this limiting case. [Notice, for the following discussion, that $m_{\tilde{b}\,\mathrm{LR}}^2$ defined in Eq. (17) is expected to be large, of the order of $m_b\times\mu\tan\beta$: Cancellations with the A_b-term contribution would imply color-breaking minima in the scalar potential.] Furthermore, let us remark that the parameter A_s is not free from experimental constraints. In fact the gluino s quark loop will generate an electric dipole moment for the s-quark; this effect is further amplified by a $\tan\beta$ factor (present in the left-right s-squarks mixing).

Let us consider now the diagram in Fig. 1(b), which is obtained from Fig. 1(a) by replacing u with ν, s with μ, and $\tilde{g}$ with $\tilde{Z}$. The loop is at the leptonic end, and the phases now appear in the soft-breaking leptonic parameter A_μ and in the Z-ino mass. The contribution of this diagram has the same quadratic behavior in $\tan\beta$ discussed in Eq. (21). The amplitude is a few times smaller, due to the weak gauge coupling replacing the strong one. However, the limits from muon electric dipole moment are much weaker. For this reason, the leading effect might be related to neutralino exchange diagrams.

In view of the negative result we will not proceed in the discussion and regard Eq. (22) as an upper bound on the effect. Such an upper bound is rather robust in the sense that the final number is small, no matter what loop diagram we consider.

3. Supersymmetric flavor violations and transverse polarization

Recently, a supersymmetric scenario which makes room for observable effects in the next generation of transverse polarization experiments has been proposed [16]. The scenario relies on the sources of flavor violation that are provided by the supersymmetric parameters: This leads to the possibility that the gluino couplings with the u and the s quarks involve the third family squarks, those with the largest couplings with the charged Higgs boson [compare with Eq. (18)]. Three major assumptions have to be satisfied:

(a) $V_{32}^{D_L}$, which quantifies the mixing of the ''left'' squark $\tilde{s}_L$ with b_L in the gluino coupling, and $V_{31}^{U_R}$, analogously defined, are order unity and carry large phases; (b) the masses of the gluino and of the squarks are close to their present experimental value; (c) $\tan\beta$ is large. Unfortunately, this set of assumptions becomes problematic as soon as we consider the rate of the $b\to s\gamma$ transition, which is known to be fairly well reproduced by the standard model amplitude alone.

To make this point explicit, let us consider the gluino-bottom-squark diagram for this transition. The gluino mass provides the chirality flip, and one $m^2_{\tilde{b}\,\mathrm{LR}}$ insertion in the bottom-squark line allows one to construct the dipole operator. Let us compare this contribution with that of the standard model:

$$\frac{\mathcal{M}_{\tilde{g}}}{\mathcal{M}_W} = \frac{(e\alpha_s/4\pi)(m_{\tilde{g}}/m_{\tilde{b}}^2)[V_{32}^{D_L}(m^2_{\tilde{b}\,\mathrm{LR}}/m^2_{\tilde{b}})V_{33}^{D_R}]\mathcal{F}_{\tilde{g}}(m^2_{\tilde{b}}/m^2_{\tilde{g}})}{(e\alpha_W/4\pi)(m_b/m_W^2)(V_{ts}V_{tb})\mathcal{F}_W(m_t^2/m_W^2)}. \tag{23}$$

The loop functions $\mathcal{F}$ were computed in [26]; assuming, consistently with [16], that $m_{\tilde{b}}\sim m_{\tilde{g}}$, the functions amount to a factor close to unity. From Eq. eq. (23) we come to the estimate

$$\frac{\mathcal{M}_{\tilde{g}}}{\mathcal{M}_W}\sim 10^3\times\left(\frac{V_{32}^{D_L}}{1/\sqrt{2}}\right)\left(\frac{V_{33}^{D_R}}{1/\sqrt{2}}\right)\left(\frac{\tan\beta}{50}\right)\left(\frac{\mu m_{\tilde{g}} m_W^2}{m_{\tilde{b}}^4}\right), \tag{24}$$

where we assumed $V_{ts}\sim V_{cb}$. Therefore this contribution may trigger a $b\to s\gamma$ transition 1×10^6 times faster than that of the standard model, unless we fine-tune $V_{33}^{D_R}$ to be sufficiently small.

A way out suggested in [16] is that the chargino contribution, whose importance has been emphasized in the literature [27], cancels the gluino amplitude and accordingly makes the bounds discussed above less stringent. It is, however, hard to justify such a precise cancellation, since different parameters enter the two amplitudes. Moreover, it is not clear if it is possible to implement such a cancellation without suppressing the transverse polarization effect as well. Finally, in a purely phenomenological scenario, like the one considered, one would expect the dominance of the gluino exchange amplitude over all the other ones, chargino exchange included, since no smallness factors are attached to the gluino couplings.[3]

[3]A different situation happens in the models where flavor-changing gluino couplings are induced radiatively by the usual Yukawa couplings and, therefore, are strongly suppressed—see, for instance, [26].

In conclusion, we feel that it is difficult to account for a transverse polarization within the context of the minimal supersymmetric extension of the standard model even after making allowence for family mixing and flavor violations.

B. *R*-parity breaking models

1. Introduction

The requirement of gauge invariance permits the following renormalizable interactions in the superpotential:

$$(Y^E_{jk}H_1+\lambda_{ijk}L_i)L_jE^c_k+(Y^D_{jk}H_1+\lambda'_{ijk}L_i)Q_jD^c_k + (\mu H_1+\mu_i L_i)H_2 - Y^U_{jk}H_2Q_jU^c_k+\lambda''_{ijk}D^c_iD^c_jU^c_k. \tag{25}$$

Besides the interactions of quarks (Q,U^c,D^c) and leptons (L,E^c) with Higgs (H_1,H_2) superfields and the μ term, we have the *R-parity-breaking* interactions, parametrized by λ, λ', λ'', and μ_i, which have no correspondence in the standard model Lagrangian and break either the lepton (λ,λ',μ_i) or the baryon (λ'') number. We consider strict baryon number conservation $(\lambda''=0)$ in the following, in order to avoid strong matter stability bounds [28,29].

Since the R-parity-breaking couplings are *a priori* complex quantities, as remarked in [30], it is important to ask whether they can manifest themselves in a large $\mathcal{P}_\perp$. Before answering this question we must recall some relevant information on the model under consideration.

Let us assume a generic pattern of the R parity-breaking couplings λ, λ', and μ_i. We fix the basis in the H_1, L_i four-dimensional space in two steps: (1) We redefine the Higgs superfield in such a way that μ_i terms are absent; (2) we further rotate the three lepton superfields in order to make the lepton mass matrix diagonal (analogously for the quark superfields).

In general, R-parity-breaking interactions of scalars induce vacuum expectation values of sneutrino fields $\tilde{\nu}_i$ [31,32]. In fact the similarity between the scalar leptons and the usual Higgs doublets is almost complete in the model under consideration.

Supersymmetry manifests itself by providing relations among the various interactions; for instance, $\lambda'_{ijk}L_iQ_jD^c_k$ describes at the same time the interactions of quarks with the slepton $\tilde{l}_i(x)$ (which in this context can be thought of as a Higgs doublet) and the interactions in which the squarks behave as leptoquarks. This implies that this model is more predictive, and more constrained, than a multi-Higgs-doublet (or a leptoquark) model.

The Yukawa Y^D,Y^E interactions are fixed by the tree-level condition

$$Y^D_{jk}=\frac{M^D_{jk}}{\langle H_1\rangle}-\lambda'_{ijk}\frac{\langle\tilde{\nu}_i\rangle}{\langle H_1\rangle},$$

$$Y^E_{jk}=\frac{M^E_{jk}}{\langle H_1\rangle}-2\lambda_{ijk}\frac{\langle\tilde{\nu}_i\rangle}{\langle H_1\rangle}, \tag{26}$$

to reproduce the observed fermion masses (the contribution to M^E_{jk} from mixing with gauginos is neglected). The λ and λ' couplings are also subject to experimental bounds from various processes [28,33]. In particular the couplings that trigger flavor-changing neutral current transitions are quite strongly constrained.

Neutrino masses can be induced in this model by two mechanisms: due to the mixing of neutrinos and Z-inos, caused by the sneutrino vacuum expectation values, but also by fermion-scalar loops with two R-parity-breaking vertices [31]. We will be interested in this second type of contribution in the following. Let us therefore recall that the coupling λ'_{ijj} gives

$$(\delta m_{\nu_i})^{\mathrm{loop}} \sim \frac{3}{8\pi^2}\frac{(\lambda'_{ijj})^2}{m^2_{\tilde{d}_j}} m_{d_j} m^2_{\tilde{d}_j\,\mathrm{LR}}, \tag{27}$$

where the factor of 3 is for color; this factor is absent for loops induced by λ couplings. A glance at Eq. (17) shows that there are two fermion mass insertions in Eq. (27): This must be so, since the gauge-invariant operator for neutrino mass is of the form neutrino-neutrino-Higgs-Higgs, and the Higgs field is coupled to fermion masses.

2. The transverse polarization

Let us assume that the couplings in the interactions

$$\lambda'_{322} L_3 Q_2 D^c_2 \ \text{ and } \ \lambda_{322} L_3 L_2 E^c_2 \tag{28}$$

in Eq. (25) are not small, whereas the other R-parity-breaking couplings are suppressed to obey the experimental bounds. Let us further assume that the quark doublet is $Q=(V^{\dagger}U,D)$, in order to avoid sneutrino mediated flavor-changing interactions (the bound of which makes the estimate of $\mathcal{P}_{\perp}$ obtained in [14] one order of magnitude smaller than the sensitivity of the next generation experiments). The couplings in Eq. (28) induce, after integrating away the slepton $\tilde{e}_3$, the effective operator

$$\sin\theta_C G_S[\overline{s(x)} P_L u(x)][\overline{\nu_\mu}(x) P_R \mu(x)], \tag{29}$$

where

$$G_S = \frac{\lambda'_{322}\lambda_{322}}{m^2_{\tilde{e}_3}}. \tag{30}$$

This yields the scalar form factors

$$\xi_S = \frac{G_S}{2\sqrt{2}G_F}\frac{f_S}{f_+} \sim 4\times 10^{-2}\left(\frac{\lambda'_{322}}{0.1}\right)\left(\frac{\lambda_{322}}{0.1}\right)\left(\frac{100\ \mathrm{GeV}}{m_{\tilde{e}_3}}\right)^2. \tag{31}$$

Comparing with Eqs. (4) and (10), we conclude that a phase larger than 1/20 would lead to a positive signal in the next generation searches of transverse polarization.

The couplings above are, however, subject to bounds due to the neutrino masses. Using the estimate in Eq. (27), and asking the τ *neutrino* mass to be 10 eV, we find an upper limit on the couplings considered: $\lambda_{322} \lesssim 0.02$ and $\lambda'_{322} \lesssim 0.03$, where the supersymmetric massive parameters have been assumed around 100 GeV. This implies

$$\mathrm{Im}\xi_S \lesssim 2\times 10^{-3}, \tag{32}$$

which, with a phase of order unity, could still give an observable transverse polarization effect. On the contrary, if the scale of supersymmetry breaking would be one order of magnitude larger, around 1 TeV, it would be improbable to have a detectable effect. Let us notice that a neutrino in the 10 eV range is a hot dark matter candidate and therefore well motivated. There are several possibilities that allow one to relax the bound (32) (but to advocate one or more of them would take us beyond the present phenomenological approach). One can (1) assume a cancellation in the left-right mixing in Eq. (27) to diminish the induced neutrino mass, (2) allow for compensation between the λ and λ' contributions or else partial cancellations of the loop-induced and the sneutrino-vacuum expectation value contributions to m_{ν_τ}, or, (3) finally, impose the weaker bound coming from experimental studies of τ decays, $m_{\nu_\tau} \lesssim 20$ MeV.

The last possibility can be regarded with favor, since it does not rely on cancellations. Accepting this option, one should, however, bear in mind that a neutrino heavier than approximatively 100 eV has to be unstable to avoid cosmological bounds. In view of the above-mentioned difficulties and also of the very specific choice of couplings in Eq. (28), we conclude that there is only a marginal possibility that an observable transverse polarization is related to this kind of models.

C. Other models

Models that can generate a sizable transverse polarization have been discussed in the literature. In particular, models including leptoquarks or new Higgs particles (more, in general, new scalars coupled to fermions) [2,9–14] or also fundamental tensor particles [24] have been considered.

Models with new scalars are probably the most promising candidates to account for a positive signal. In fact, complex part of the form factor of the order of

$$\mathrm{Im}\xi_S \sim 8\times 10^{-3} \tag{33}$$

can give rise to a measurable muon transverse polarization without conflicting with other observables.

IV. CONCLUSIONS

The knowledge of the form factors in the $K^0_{\mu 3}$ process could give important information on CP violation. The experimental signature is provided by the transverse polarization of the muon.

At the level of a purely phenomenological analysis, we stressed the complementarity of this experimental information with that from the analysis of the Dalitz plot distribution; we also pointed out the importance of distinguishing between scalar and tensor form factors.

The discussion of realistic models was focused on the supersymmetric extensions of the standard model. In the minimal model, assuming that the Cabibbo-Kobayashi-Maskawa matrix is the only source of flavor violation, the contribution to the muon polarization is too small, even under optimistic assumptions, as in Eq. (22). We commented on the scenario in which large supersymmetric CP and flavor violations are allowed, and showed the difficulties that arise of reconciling the assumptions required to have a large $\mathcal{P}_{\perp}$ and the observed rate of the $b \to s\gamma$ transition. Observable effects are in principle possible departing from the hypothesis of R-parity conservation, but constraints from the neutrino masses severely restrict the region of parameter space in which this can happen.

In conclusion, assuming that the minimal supersymmetric standard model (possibly with R-parity-breaking interactions) is correct, we expect that a future search of transverse polarization in $K^0_{\mu 3}$ decay should give a null result. On the other hand, a signal of transverse polarization in forthcoming experiments would point to physics different from the standard model and from its straightforward supersymmetric extensions.

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