Deep-inelastic structure functions in a covariant spectator model

K. Kusaka,[1,*] G. Piller,[2,†] A. W. Thomas,[3,‡] and A. G. Williams[3,‡]
[1]*Department of Physics, Tokyo Metropolitan University, Tokyo 192, Japan*
[2]*Physik Department, Technische Universität München, D-85747 Garching, Germany*
[3]*Department of Physics and Mathematical Physics, University of Adelaide, S.A. 5005, Australia and Institute for Theoretical Physics, University of Adelaide, S.A. 5005, Australia*

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Deep-inelastic structure functions are studied within a covariant scalar diquark spectator model of the nucleon. Treating the target as a two-body bound state of a quark and a scalar diquark, the Bethe-Salpeter equation (BSE) for the bound state vertex function is solved in the ladder approximation. The valence quark distribution is discussed in terms of the solutions of the BSE. [S0556-2821(97)05107-2]

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I. INTRODUCTION

In recent years many attempts have been made to understand the nucleon structure functions measured in lepton deep-inelastic scattering (DIS). Although perturbative QCD is successful in describing the variation of structure functions with the squared momentum transfer, their magnitude and shape is governed by the nonperturbative physics of composite particles, and is, so far, not calculable directly from QCD.

A variety of models has been invoked to describe nucleon structure functions. Bag model calculations for example, which are driven by the dynamics of quarks bound in a nucleon bag, quite successfully describe nonsinglet unpolarized and polarized structure functions (see, e.g., [1,2] and references therein). However, such calculations are not relativistically covariant.

A covariant approach to nucleon structure functions is given by so-called ''spectator models'' [3–5]. Here, the leading twist, nonsinglet quark distributions are calculated from the process in which the target nucleon splits into a valence quark, which is scattered by the virtual photon, and a spectator system carrying baryon number 2/3. Furthermore, the spectrum of spectator states is assumed to be saturated through single scalar and vector diquarks. Thus, the main ingredient of these models are covariant quark-diquark vertex functions.

Until now, vertex functions have been merely parametrized such that the measured quark distributions are reproduced, and no attempts have been made to connect them to some dynamical models of the nucleon. In this work we construct the vertex functions from a model Lagrangian by solving the Bethe-Salpeter equation (BSE) for the quark-diquark system. However, we do not aim at a detailed, quantitative description of nucleon structure functions in the present work. Rather, we outline how to extract quark-diquark vertex functions from Euclidean solutions of the BSE. In this context several simplifications are made. We consider only scalar diquarks as spectators and restrict ourselves to the SU(2) flavor group. The inclusion of pseudovector diquarks and the generalization to SU(3) flavor are relatively straightforward extensions and will be left for future work. It should be mentioned that the quark-diquark Lagrangian used here does not account for quark confinement inside nucleons. However, the use of a confining quark-diquark interaction should also be possible within the scheme that we use.

As an important result of our work we find that the vertex function of the nucleon is highly relativistic even in the case of weak binding. Furthermore, we observe that the nucleon structure function F_1 is determined to a large extent by the relativistic kinematics of the quark-diquark system and is not very sensitive to its dynamics as long as the spectator system is treated as a single particle.

The outline of the paper is as follows. In Sec. II we introduce the spectator model for deep-inelastic scattering. Section III focuses on the scalar diquark model for the nucleon which yields the quark-diquark vertex function as a solution of a ladder BSE. In Sec. IV we present numerical results for the quark-diquark vertex function and the nucleon structure function F_1. Finally, we summarize and conclude in Sec. V.

II. DEEP-INELASTIC LEPTON SCATTERING IN THE SPECTATOR MODEL

Inclusive deep-inelastic scattering of leptons from hadrons is described by the hadronic tensor

$$W^{\mu\nu}(q,P)=\frac{1}{2\pi}\int d^4\xi e^{iq\cdot\xi}\langle P|J^\mu(\xi)J^\nu(0)|P\rangle, \quad (1)$$

where P and q are the four-momenta of the target and exchanged virtual photon, respectively, and J^μ is the hadronic electromagnetic current. In unpolarized scattering processes only the symmetric piece of $W^{\mu\nu}=W^{\nu\mu}$ is probed. It can be expressed in terms of two structure functions F_1 and F_2, which depend on the Bjorken scaling variable, $x=Q^2/2P\cdot q$, and the squared momentum transfer $Q^2=-q^2$:

*Electronic address: kkusaka@phys.metro-u.ac.jp
†Electronic address: gpiller@physik.tu-muenchen.de
‡Electronic address: athomas, awilliam@physics.adelaide.edu.au

$$W^{\mu\nu}(q,P)=\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)F_1(x,Q^2)+\left(P^\mu-q^\mu\frac{P\cdot q}{q^2}\right)\times\left(P^\nu-q^\nu\frac{P\cdot q}{q^2}\right)\frac{F_2(x,Q^2)}{P\cdot q}. \tag{2}$$

In the Bjorken limit ($Q^2, P\cdot q\to\infty$; but finite x) in which we work throughout, both structure functions depend up to logarithmic corrections on x only, and are related via the Callan-Gross relation: $F_2=2xF_1$.

The hadronic tensor $W^{\mu\nu}$ is connected via the optical theorem to the amplitude $T^{\mu\nu}$ for virtual photon-nucleon forward Compton scattering:

$$\frac{1}{\pi}\,\mathrm{Im}T^{\mu\nu}(q,P)=W^{\mu\nu}(q,P). \tag{3}$$

In the Bjorken limit the interaction of the virtual photon with a valence quark from the target leads to a spectator system carrying diquark quantum numbers, i.e., baryon number 2/3 and spin 0 or 1. In the spectator model it is assumed that the spectrum of spectator states can be saturated through a single scalar and pseudovector diquark [3–5]. In the following we will restrict ourselves to contributions from scalar diquarks only. The generalization to include a pseudovector diquark contribution is left for future work. The corresponding Compton amplitude is (Fig. 1)

$$T_S^{\mu\nu}(q,P)=\left\langle\frac{5}{6}+\frac{\tau_3}{2}\right\rangle_N\int\frac{d^4k}{(2\pi)^4 i}\overline{u(P)}\overline{\Gamma}(k,P-k)S(k)\times\gamma^\mu S(k+q)\gamma^\nu S(k)D(P-k)\Gamma(k,P-k)u(P), \tag{4}$$

where the flavor matrix has to be evaluated in the nucleon isospin space. The integration runs over the quark momentum k. The Dirac spinor of the spin-averaged nucleon target with momentum P is denoted by $u(P)$. Furthermore, $S(k)=1/(m_q-\not k-i\epsilon)$ and $D(k)=1/(m_D^2-k^2-i\epsilon)$ are the

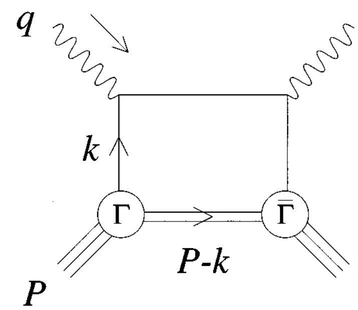

FIG. 1. The diquark spectator contribution to the virtual forward Compton amplitude in the Bjorken limit.

propagators of the quark and diquark, respectively, while Γ is the quark-diquark vertex function. To obtain the hadronic tensor, the scattered quark and the diquark spectator have to be put on mass shell according to Eq. (3):

$$S(k+q)\to i\pi\delta(m_q^2-(k+q)^2)(m_q+\not k+\not q),$$
$$D(P-k)\to i\pi\delta(m_D^2-(P-k)^2). \tag{5}$$

The vertex function Γ for the target, which in our approach is a positive energy, spin-1/2 composite state of a quark and a scalar diquark, is given by two independent Dirac structures:

$$\Gamma(k,P-k)|_{(P-k)^2=m_D^2}=\left(f_1^{\mathrm{on}}(k^2)+\frac{2\not k}{M}f_2^{\mathrm{on}}(k^2)\right)\Lambda^{(+)}(P), \tag{6}$$

where $\Lambda^{(+)}(P)=1/2+\not P/2M$ is the projector onto positive energy, spin-1/2 states and $M=\sqrt{P^2}$ is the invariant mass of the nucleon target. Note that according to the on-shell condition in Eq. (5) the scalar functions $f_{1/2}^{\mathrm{on}}$ will depend on k^2 only.

From Eqs. (3)–(6) we then obtain for the valence quark contribution to the structure function F_1:

$$F_1^{\mathrm{val}}(x)=\left\langle\frac{5}{6}+\frac{\tau_3}{2}\right\rangle_N\frac{1}{16\pi^3}\int_{-\infty}^{k_{\max}^2}\frac{dk^2}{m_q^2-k^2}\left(\left(1-x+\frac{(m_q+M)^2-m_D^2}{m_q^2-k^2}x\right)\frac{f_1^{\mathrm{on}}(k^2)^2}{4}-\left[1+x+\frac{2\,m_q}{M}x+\left(1-\frac{2\,m_q}{M}\right)\frac{(m_q+M)^2-m_D^2}{m_q^2-k^2}x\right]\frac{f_1^{\mathrm{on}}(k^2)f_2^{\mathrm{on}}(k^2)}{2}+\left\{4\frac{m_q^2-k^2}{M^2}+\left[1-\left(\frac{2m_q}{M}\right)^2\right](1+2x)+\left(1-\frac{2\,m_q}{M}\right)^2x+\left(1-\frac{2\,m_q}{M}\right)^2\frac{(m_q+M)^2-m_D^2}{m_q^2-k^2}x\right\}\frac{f_2^{\mathrm{on}}(k^2)^2}{4}\right). \tag{7}$$

The upper limit of the k^2 integral is denoted by

$$k_{\max}^2=x\left(M^2-\frac{m_D^2}{1-x}\right). \tag{8}$$

Note that $k_{\max}^2\to-\infty$ for $x\to 1$. This implies that for any regular vertex function $F_1^{\mathrm{val}}\to 0$ for $x\to 1$ and thus the structure function automatically has the correct support.

Since the spectator model of the nucleon is valence-quark dominated, the structure function F_1^{val} in Eq. (7) is identified with the leading twist part of F_1 at some typical low momentum scale, $\mu^2\lesssim 1\ \mathrm{GeV}^2$. The physical structure function at large $Q^2 \gg \mu^2$ is then to be generated via Q^2 evolution.

It should be mentioned that the Compton amplitude in Eq. (4) and also the expression for the structure function in Eq. (7) contain poles from the quark propagators attached to the quark-diquark vertex functions. From Eq. (8) it follows that these poles do not contribute when $M < m_D + m_q$. This condition is automatically satisfied if the nucleon is considered as a bound state of the quark and diquark, as done in the following.

In the next section we shall determine the vertex function Γ, or equivalently $f_1^{\rm on}$ and $f_2^{\rm on}$ from Eq. (6) as solutions of a ladder BSE.

III. SCALAR DIQUARK MODEL FOR THE NUCLEON

We now determine the vertex function (6) as the solution of a BSE for a quark-diquark system. We start from the Lagrangian

$$\mathcal{L} = \overline{\psi_a}(i\partial\!\!\!/ - m_q)\psi_a + \partial_\mu \phi_a^* \partial^\mu \phi_a - m_D^2 \phi_a^* \phi_a + i\frac{g}{2\sqrt{2}}\epsilon_{bc}^a \psi_b^T C^{-1}\gamma_5 \tau_2 \psi_c \phi_a^* - i\frac{g}{2\sqrt{2}}\epsilon_{bc}^a \overline{\psi_b} C^{-1}\gamma_5\tau_2 \overline{\psi_c^T}\phi_a, \quad (9)$$

where we have explicitly indicated SU(3) color indices but have omitted flavor indices. We restrict ourselves to flavor SU(2), where τ_2 is the symmetric generator which acts on the isodoublet quark field ψ with mass m_q. The charged scalar field ϕ represents the flavor-singlet scalar diquark carrying an invariant mass m_D. Similar Lagrangians have been used recently to describe some static properties of the nucleon, such as its mass and electromagnetic charge (see, e.g., [6–8]).

The nucleon with four-momentum P and spin S is described by the bound state Bethe-Salpeter (BS) vertex function Γ:

$$T_F\langle 0|T\psi_a(x)\phi_b(y)|P,S\rangle = \delta_{ab}S(k)D(P-k)i\Gamma(k,P-k)u(P,S). \quad (10)$$

Here, $u(P,S)$ is the nucleon Dirac spinor and T_F stands for the Fourier transformation.[1] [Again, we have omitted SU(2) flavor indices.]

We will now discuss the integral equation for the vertex function Γ in the framework of the ladder approximation.

A. Ladder BSE

For the following discussion of the integral equation for the vertex function Γ we write the quark momentum as $q + \eta_q P$ and the diquark momentum as $-q + \eta_D P$. The weight factors η_q and η_D are arbitrary constants between 0 and 1 and satisfy $\eta_q + \eta_D = 1$. Within the ladder approximation the BSE for the vertex function of a positive energy, spin-1/2 model nucleon can be written as (see Fig. 2):

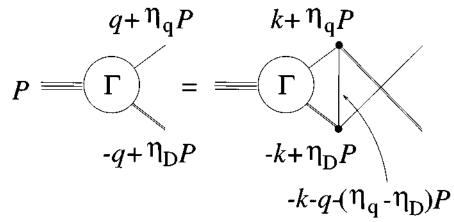

FIG. 2. The Bethe-Salpeter equation for a quark-diquark system in the ladder approximation.

$$\Gamma(q,P)u(P,S) = g^2 \int \frac{d^4k}{(2\pi)^4 i} S[-k-q-(\eta_q-\eta_D)P] \times S(\eta_q P + k)D(\eta_D P - k)\Gamma(k,P)u(P,S), \quad (11)$$

where the flavor and color factors have already been worked out. The scattering kernel is given by a u-channel quark exchange according to the interaction Lagrangian in Eq. (9).

Since we are only interested in positive energy solutions, we may write the vertex function as

$$\Gamma(q,P) = \left(a f_1(q,P) + b f_2(q,P) + \frac{q\!\!\!/}{M} f_2(q,P) \right) \Lambda^{(+)}(P). \quad (12)$$

The arguments of the scalar functions $f_\alpha(q,P)$ are actually q^2 and $P\cdot q$, but we use this shorthand notation for brevity. With a and b we denote as yet unspecified scalar functions of q^2 and $P\cdot q$ which will be chosen later for convenience. [The definition of $f_{1/2}^{\rm on}$ in Eq. (6) corresponds to a specific choice of a and b.]

B. Wick rotation

After multiplying the BSE in Eq. (11) with appropriate projectors (which depend on a and b), we obtain a pair of coupled integral equations for the scalar functions $f_1(q,P)$ and $f_2(q,P)$:

$$f_\alpha(q,P) = g^2 \int \frac{d^4k}{(2\pi)^4 i} \tilde{D}_q[-q-k-(\eta_q-\eta_D)P] \times D_q(\eta_q P + k)D_D(\eta_D P - k)K_{\alpha\beta}(q,k,P)f_\beta(k,P), \quad (13)$$

where $D_q(p) \equiv 1/(m_q^2 - p^2 - i\epsilon)$ and $D_D(p) \equiv 1/(m_D^2 - p^2 - i\epsilon)$ are the denominators of the quark and diquark propagators, respectively. The indices α and β stand for the independent Dirac structures of the vertex function Γ, i.e., in the scalar-diquark model they run from 1 to 2 according to Eq. (12). Consequently, the function $K_{\alpha\beta}(q,k,P)$ is a 2×2 matrix, where its explicit form depends on the definition of the scalar functions $f_\alpha(q,P)$. We use a form factor for the quark-diquark coupling which weakens the short range interaction between the quark and the diquark and ensures the existence of solutions with a positive norm. For simplicity,

[1]We use the normalization $\langle P'|P\rangle = 2P^0(2\pi)^3\delta^{(3)}(\vec{P}' - \vec{P})$ and $\Sigma_S u(P,S)\overline{u(P,S)} = \sqrt{P^2} + P\!\!\!/ = M + P\!\!\!/$.

we use a u-channel form factor which can be conveniently absorbed into the denominator of the exchanged quark propagator as follows:

$$D_q(p)\to\widetilde{D}_q(p)\equiv D_q(p)\frac{\Lambda^2}{\Lambda^2-p^2-i\epsilon}. \quad (14)$$

As a next step let us analyze the singularities of the integrand in Eq. (13). For this purpose we choose the nucleon rest frame where $P_\mu=P_\mu^{(0)}\equiv(M,\vec{0})$ and put the weight constants η_q and η_D to the classical values:

$$\eta_q\equiv\frac{m_q}{m_q+m_D}\equiv\frac{1-\eta}{2}, \quad (15)$$

$$\eta_D\equiv\frac{m_D}{m_q+m_D}\equiv\frac{1+\eta}{2}. \quad (16)$$

Here, we have introduced the asymmetry parameter $\eta\equiv(m_D-m_q)/(m_q+m_D)$, such that the invariant quark and diquark mass is given by $m_q=\overline{m}(1-\eta)$ and $m_D=\overline{m}(1+\eta)$, respectively, where $\overline{m}=(m_q+m_D)/2$. In the complex k_0 plane, $D_q(\eta_qP+k)$ and $D_D(\eta_DP-k)$ will be singular for

$$k^0=-\eta_qM\pm E_q(\vec{k})\mp i\epsilon, \quad (17)$$

$$k^0=\eta_DM\pm E_D(\vec{k})\mp i\epsilon, \quad (18)$$

where $E_q(\vec{k})=\sqrt{m_q^2+\vec{k}^2}$ and $E_D(\vec{k})=\sqrt{m_D^2+\vec{k}^2}$. The cuts lie in the second and fourth quadrants of the complex k_0 plane. However, for a bound state, $0<M<m_q+m_D$, a gap occurs between these two cuts which includes the imaginary k_0 axis.

Next, consider the singularities of the exchanged quark propagator:

$$k^0=-q^0+\eta M\pm E_{m_i}(\vec{q}+\vec{k})\mp i\epsilon, \quad (19)$$

where $E_{m_i}(\vec{k})=\sqrt{m_i^2+\vec{k}^2}$ and $m_i=m_q,\Lambda$ for $i=1,2$, respectively. The sum of the second and third terms at the right-hand side (RHS) of Eq. (19) is bound by

$$\eta M+E_{m_i}(\vec{q}+\vec{k})\geqslant(m_D-m_q)\frac{M}{m_q+m_D}+m_i, \quad (20)$$

$$\eta M-E_{m_i}(\vec{q}+\vec{k})\leqslant(m_D-m_q)\frac{M}{m_q+m_D}-m_i. \quad (21)$$

The diquark should be considered as a bound state of two quarks which implies $m_D<2m_q$. Together with $m_i\geqslant m_q$, namely, setting the form factor mass Λ larger than m_q, we have $m_D-m_q+m_i>0$ and $m_D-m_q-m_i<0$. Consequently, we find from Eqs. (20) and (21) $\eta M+E_{m_i}(\vec{q}+\vec{k})>0$ and $\eta M-E_{m_i}(\vec{q}+\vec{k})<0$ for any momenta $\vec{q}$ and $\vec{k}$. Therefore, if $-q^0+\eta M-E_{m_i}(\vec{q}+\vec{k})>0$ or $-q^0+\eta M+E_{m_i}(\vec{q}+\vec{k})<0$, a so-called ''displaced'' pole will occur in the first or third quadrant, respectively. In other words, the displaced pole-free condition is

$$\eta M-E_{m_i}(\vec{q}+\vec{k})<q^0<\eta M+E_{m_i}(\vec{q}+\vec{k}), \quad (22)$$

for any $\vec{k}$. Since $\vec{k}$ is an integration variable, $E_{m_i}(\vec{q}+\vec{k})$ will adopt its minimum value m_q at $\vec{k}=-\vec{q}$ for $i=1$. The above condition, therefore, simplifies to

$$(q^0-\eta M)^2<m_q^2. \quad (23)$$

If q^0 is Wick rotated to pure imaginary values, i.e., $q^\mu\to\widetilde{q}^\mu=(iq^4,\vec{q})$ with real $q^4\in(-\infty,\infty)$, the displaced poles will move to the second and fourth quadrants. Then, after also rotating the momentum $k^\mu\to\widetilde{k}^\mu=(ik^4,\vec{k})$, we obtain the Euclidean vertex functions $f_\beta(\widetilde{k},P^{(0)})$ from the Wick-rotated BSE:

$$\begin{aligned}f_\alpha(\widetilde{q},P^{(0)})=g^2\int\frac{d^4k_E}{(2\pi)^4}&\widetilde{D}_q[-\widetilde{q}-\widetilde{k}-(\eta_q-\eta_D)P^{(0)}]\\ &\times D_q(\eta_qP^{(0)}+\widetilde{k})D_D(\eta_DP^{(0)}-\widetilde{k})\\ &\times K_{\alpha\beta}(\widetilde{q},\widetilde{k},P^{(0)})f_\beta(\widetilde{k},P^{(0)}),\end{aligned} \quad (24)$$

where $d^4k_E=dk^4d^3\vec{k}$.

If we are in a kinematic situation where no displaced poles occur, i.e., Eq. (23) is satisfied, we may obtain the Minkowski space vertex function $f_\alpha(q,P)$ from the Euclidean solution through

$$\begin{aligned}f_\alpha(q,P^{(0)})=g^2\int\frac{d^4k_E}{(2\pi)^4}&\widetilde{D}_q[-q-\widetilde{k}-(\eta_q-\eta_D)P^{(0)}]\\ &\times D_q(\eta_qP^{(0)}+\widetilde{k})D_D(\eta_DP^{(0)}-\widetilde{k})\\ &\times K_{\alpha\beta}(q,\widetilde{k},P^{(0)})f_\beta(\widetilde{k},P^{(0)}).\end{aligned} \quad (25)$$

It should be emphasized that for a given Euclidean solution $f_\beta(\widetilde{k},P^{(0)})$, Eq. (25) is not an integral equation but merely an algebraic relation between $f_\alpha(q,P^{(0)})$ and $f_\beta(\widetilde{k},P^{(0)})$. If, however, displaced poles occur, i.e., Eq. (23) is not satisfied, one needs to add contributions from the displaced poles to the RHS of Eq. (25). This will lead to an inhomogeneous integral equation for the function $f_\alpha(q,P^{(0)})$, where the inhomogeneous term is determined by the Euclidean solution $f_\beta(\widetilde{k},P^{(0)})$.

Since the Euclidean solutions $f_\alpha(\widetilde{q},P^{(0)})$ are functions of $\widetilde{q}^{\,2}=-q_E^2\equiv-[(q_4)^2+|\vec{q}|^2]$, $\widetilde{q}\cdot P^{(0)}=iq_4M$ for a fixed M, it is convenient to introduce four-dimensional polar coordinates:

$$\begin{aligned}q^4&=q_E\cos\alpha_q,\\ q^i&=|\vec{q}|\hat{q}^i,\\ |\vec{q}|&=q_E\sin\alpha_q.\end{aligned} \quad (26)$$

Here, $0<\alpha_q<\pi$ and the three-dimensional unit vector $\hat{q}^i$ is parametrized by the usual polar and azimuthal angles $\hat{q}^i=(\sin\theta_q\cos\phi_q,\sin\theta_q\sin\phi_q,\cos\theta_q)$. In the following we, therefore, consider $f_\alpha(\widetilde{q},P^{(0)})$ as a function of q_E and $\cos\alpha_q$. Furthermore, it is often convenient (and traditional) to factor out the coupling constant g^2 together with a factor $(4\pi)^2$, and define the ''eigenvalue'' $\lambda^{-1}=(g/4\pi)^2$. Then, the BSE in Eq. (24) is solved as an eigenvalue problem for a fixed bound state mass M.

C. O(4) expansion

In the following we will define the scalar functions $f_\alpha(q,P)$ for positive energy ($M>0$) bound states via

$$\Gamma(q,P)=\left[f_1(q,P)+\left(-\frac{P\cdot q}{M^2}+\frac{\not q}{M}\right)f_2(q,P)\right]\Lambda^{(+)}(P), \tag{27}$$

i.e., we now make a specific choice for the scalar functions a and b in Eq. (12). In the rest frame of this model nucleon, this leads to

$$\Phi^{J^P=1/2^+}_{J_3=S/2}(q,P^{(0)})=\Gamma(q,P^{(0)})u(P^{(0)},S)=\begin{pmatrix} f_1(q,P^{(0)}) \\ \dfrac{\vec{q}\cdot\vec{\sigma}}{M}f_2(q,P^{(0)})\end{pmatrix}\chi_S. \tag{28}$$

Here, we have explicitly used the Dirac representation. The Pauli matrices $\vec{\sigma}$ act on the two-component spinor χ_S, where the spin label $S=\pm 1$ is the eigenvalue of σ_3: $\sigma_3\chi_S=S\chi_S$. In terms of the O(3) spinor harmonics $\mathcal{Y}^J_{lm}$ [9],

$$\mathcal{Y}^{1/2}_{0S}(\hat{q})=\frac{1}{\sqrt{4\pi}}\chi_S \quad\text{and}\quad \mathcal{Y}^{1/2}_{1S}(\hat{q})=-\hat{q}\cdot\vec{\sigma}\mathcal{Y}^{1/2}_{0S}(\hat{q}), \tag{29}$$

we have

$$\Phi^{J^P=1/2^+}_{J_3=S/2}(q,P^{(0)})=\sqrt{4\pi}\begin{pmatrix} f_1(q,P^{(0)})\mathcal{Y}^{1/2}_{0S}(\hat{q}) \\ -\dfrac{|\vec{q}|}{M}f_2(q,P^{(0)})\mathcal{Y}^{1/2}_{1S}(\hat{q})\end{pmatrix}. \tag{30}$$

From Eq. (30) we observe that f_1 and f_2 correspond to the upper and lower components of the model nucleon, respectively.

After the Wick rotation, as discussed in the previous subsection, the scalar functions f_α become functions of q_E and $\cos\alpha_q$. Therefore, we can expand them in terms of Gegenbauer polynomials $C^1_n(z)$ [10]:

$$f_\alpha(q_E,\cos\alpha_q)=\sum_{n=0}^{\infty} i^n f^n_\alpha(q_E)C^1_n(\cos\alpha_q). \tag{31}$$

We have introduced the phase i^n to ensure that the coefficient functions f^n_α are real. The integral measure in O(4) polar coordinates is

$$\int\frac{d^4k_E}{(2\pi)^4}=\frac{1}{(4\pi)^2}\int_0^\infty dk_E k_E^3\frac{2}{\pi}\int_0^\pi d\alpha_k\sin^2\alpha_k\frac{1}{2\pi}\int d\Omega_{\hat{k}}. \tag{32}$$

Multiplying the BSE in Eq. (24) with the Gegenbauer polynomial $C^1_n(\cos\alpha_q)$ and integrating over the hyperangle α_q, reduces the BSE to an integral equation for the O(4) radial functions f^n_α:

$$\lambda(M)f^n_\alpha(q_E)=\sum_{\beta=1}^{2}\sum_{m=0}^{\infty}\int_0^\infty dk_E\mathcal{K}^{nm}_{\alpha\beta}(q_E,k_E)f^m_\beta(k_E). \tag{33}$$

Here, $\lambda(M)$ is the eigenvalue which corresponds to a fixed bound state mass M. Furthermore, note that the integral kernel

$$\begin{aligned}\mathcal{K}^{nm}_{\alpha\beta}(q_E,k_E)=&(-i)^n i^m\frac{2}{\pi}\int_0^\pi d\alpha_q\sin^2\alpha_q C^1_n(\cos\alpha_q)\\ &\times\frac{2}{\pi}\int_0^\pi d\alpha_k\sin^2\alpha_k C^1_m(\cos\alpha_k)\frac{1}{2\pi}\int d\Omega_{\hat{k}}k_E^3\\ &\times\tilde{D}_q[-\tilde{q}-\tilde{k}-(\eta_q-\eta_D)P^{(0)}]\\ &\times K_{\alpha\beta}(\tilde{q},\tilde{k},P^{(0)})D_q(\eta_q P^{(0)}+\tilde{k})\\ &\times D_D(\eta_D P^{(0)}-\tilde{k})\end{aligned} \tag{34}$$

is real, so that we can restrict ourselves to real O(4) radial functions f^n_α.

To close this section we shall introduce normalized O(4) radial functions. Since the scalar functions $f_1(q,P)$ and $f_2(q,P)$ correspond to the upper and lower components of the model nucleon, respectively, one may expect that $f_2(q,P)$ becomes negligible when the quark-diquark system forms a weakly bound state. Thus, one can use the relative magnitude of the two scalar functions, $f_2(q,P)/f_1(q,P)$, as a measure of relativistic contributions to the model nucleon. To compare the magnitude of the Wick-rotated scalar functions $f_1(\tilde{q},P)$ and $f_2(\tilde{q},P)$, we introduce normalized O(4) radial functions. Recall the O(4) spherical spinor harmonics [11,12]:

$$\begin{aligned}\mathcal{Z}_{njlm}(\alpha,\theta,\phi)=&\left[\frac{2^{2l+1}(n+1)(n-l)!}{\pi(n+l+1)!}\right]^{1/2}\\ &\times l!(\sin\alpha)^l C^{1+l}_{n-l}(\cos\alpha)\mathcal{Y}^j_{lm}(\theta,\phi).\end{aligned} \tag{35}$$

The integers n and l denote the O(4) angular momentum and the ordinary O(3) orbital angular momentum, respectively. The half-integer quantum numbers j and m stand for the usual O(3) total angular momentum and the magnetic quantum number. We rewrite the Wick-rotated solution

$\Phi^{J^P=1/2^+}_{J_3=S/2}(\tilde{q},P^{(0)})$ in terms of the spinor harmonics $\mathcal{Z}_{njlm}(\alpha,\theta,\phi)$ and define the normalized O(4) radial functions $F_n(q_E)$ and $G_n(q_E)$ as

$$\Phi^{J^P=1/2^+}_{J_3=S/2}(\tilde{q},P^{(0)}) \equiv \sqrt{2}\pi\begin{pmatrix} \sum_{n=0}^{\infty} i^n F_n(q_E)\mathcal{Z}_{n(1/2)\ 0S}(\alpha_q,\hat{q}) \\ \sum_{n=1}^{\infty} i^{n-1} G_n(q_E)\mathcal{Z}_{n(1/2)\ 1S}(\alpha_q,\hat{q}) \end{pmatrix}. \tag{36}$$

The extra factor $\sqrt{2}\pi$ is introduced for convenience. The normalized O(4) radial functions F_n and G_n are then linear combinations of the f^n_α:

$$F_n(q_E)=f^n_1(q_E), \tag{37}$$

$$G_n(q_E)=-\frac{q_E}{2M}\sqrt{n(n+2)}\left(\frac{f^{n-1}_2(q_E)}{n}+\frac{f^{n+1}_2(q_E)}{n+2}\right). \tag{38}$$

Equivalently, we can express the Wick-rotated scalar functions as

$$f_1(q_E,\cos\alpha)=\sum_{n=0}^{\infty} i^n F_n(q_E)C^1_n(\cos\alpha), \tag{39}$$

$$f_2(q_E,\cos\alpha)=-\sum_{n=1}^{\infty}\frac{2M}{q_E}\frac{i^{n-1}}{\sqrt{n(n+2)}}G_n(q_E)C^2_{n-1}(\cos\alpha). \tag{40}$$

D. Euclidean solutions

In this subsection we present our results for the integral equation in Eq. (33). For simplicity, we considered the quark and diquark mass to be equal: $m_q=m_D=\overline{m}$. In this case the kernel $\mathcal{K}^{nm}_{\alpha\beta}$ can be evaluated analytically in a simple manner, since for $\eta=0$ the denominator of the propagator for the exchanged quark does not depend on the nucleon momentum P. We fixed the scale of the system by setting the mass $\overline{m}$ to unity. The ''mass'' parameter in the form factor was fixed at $\Lambda=2\overline{m}$.

We solved Eq. (33) as follows. First, we terminated the infinite sum in Eq. (31) at some fixed value $n_{\max}$. Then, the kernel in Eq. (34) for the truncated system becomes a finite matrix with dimension $(2\times n_{\max})^2$. Its elements are functions of q_E and k_E. Next, we discretized the Euclidean momenta and performed the integration over k_E numerically together with some initially assumed radial functions f^n_α. In this way new radial functions and an ''eigenvalue'' λ associated with them were generated. The value of λ was determined by imposing the normalization condition on f^n_α such that the resultant valence-quark distribution is properly normalized. We then used the generated radial functions as an input and repeated the above procedure until the radial functions and λ converged.

Note that our normalization differs from the commonly used one [13], since we are going to apply the vertex function only to processes with the diquark as a spectator, i.e., we do not consider the coupling of the virtual photon directly to the diquark. The ordinary choice of normalization would not lead to an integer charge for the model nucleon in the spectator approximation. We, therefore, normalize the valence-quark distribution itself.

Regarding Eq. (33) as an eigenvalue equation, we found the ''eigenvalue'' λ (coupling constant) as a function of M^2, varying the latter over the range $0.85\overline{m^2}\leq M^2\leq 1.99\overline{m^2}$. The eigenvalue λ was stable, i.e., independent of the number of grid points and the maximum value of k_E. Furthermore, for a weakly bound state, $M>1.6\overline{m}$, the solutions were independent of the choice of the starting functions. However, for a strongly bound state, $M<1.4\overline{m}$, we found that the choices of the starting functions were crucial for convergence. A possible reason of this instability for a strongly bound system is because of the fact that we did not use the O(4) eigenfunctions F_n and G_n in numerical calculations but the functions f^n_α defined in Eq. (31). Since strongly bound systems, $M\sim 0$, are approximately O(4) symmetric, a truncated set of functions f^n_α may be an inappropriate basis for numerical studies of the BSE.

We found that the eigenvalue λ converges quite rapidly when $n_{\max}$, the upper limit of the O(4) angular momentum, is increased. This stability of our solution with respect to $n_{\max}$ is independent of M. We observe that contributions to the eigenvalue λ from $f^n_\alpha(q_E)$ with $n>4$ are negligible. This dominance of the lowest O(4) radial functions has also been observed in the scalar-scalar-ladder model [14] and utilized as an approximation for solving the BSE in a generalized fermion-fermion-ladder approach [15].

To compare the magnitude of the two scalar functions $f_1(\tilde{q},P)$ and $f_2(\tilde{q},P)$ we show in Fig. 3 the normalized O(4) radial functions F_n and G_n. As the dependence of λ on $n_{\max}$ suggests, radial functions with O(4) angular momenta $n>4$ are quite small compared to the lower ones. Together with the fast convergence of λ, this observation justifies the truncation of Eq. (33) at $n=n_{\max}$. Note that even for very weakly bound systems ($\sim$0.5% binding energy), the magnitude of the ''lower-component'' $f_2(\tilde{q},P)$ remains comparable to that of the ''upper-component'' $f_1(\tilde{q},P)$. This suggests that the spin structure of relativistic bound states is nontrivial, even for weakly bound systems. So-called ''nonrelativistic'' approximations, in which one neglects the nonleading components of the vertex function [$f_2(\tilde{q},P)$ in our model], are, therefore, only valid for extremely weak binding, $2\overline{m}\rightarrow M$ only.

E. Analytic continuation

In the previous subsection we obtained the quark-diquark vertex function in Euclidean space. Its application to deep-inelastic scattering, as discussed in Sec. II, demands an analytic continuation to Minkowski space. Here, the scalar functions f_α, which determine the quark-diquark vertex function through Eq. (12), will depend on the Minkowski space momenta q^2 and $P\cdot q$.

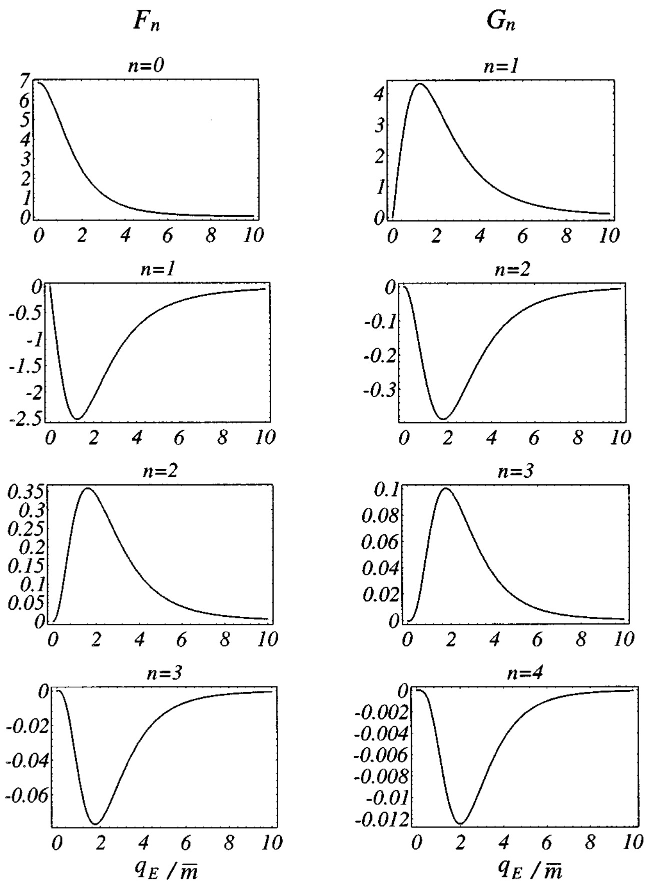

FIG. 3. The normalized O(4) radial functions F_n and G_n from Eq. (36) for $M=1.8\overline{m}$ as functions of the Euclidean momentum q_E.

Recall that our Euclidean solution is based on the expansion of the scalar functions f_α in terms of Gegenbauer polynomials in Eq. (31). This expansion was defined in Sec. III C for real hyperangles α_q, with $-1<\cos\alpha_q<1$. Consequently, the infinite sum over the O(4) angular momenta n in Eq. (31) is absolutely convergent for pure imaginary energies q^0. Now, we would like to analytically continue q^0 to physical, real values. The Euclidean hyperangle α_q is defined in Euclidean space such that

$$\cos\alpha_q=\frac{q^4}{\sqrt{-q^2}}. \tag{41}$$

In Minkowski space, $\cos\alpha_q$ is then purely imaginary $(\cos\alpha_q=-iq^0/\sqrt{-q^2})$ for spacelike q, and real $(\cos\alpha_q=-q^0/\sqrt{q^2})$ if q is timelike. Note that the angular momentum sum (31) converges even for complex values of $\cos\alpha_q$ as long as $|\cos\alpha_q|<1$. Then, an analytic continuation of f_α to Minkowski space is possible. In terms of the Lorentz invariant scalars q^2 and $P\cdot q$ we obtain

$$z=\cos\alpha_q=-\ \mathrm{sgn}(q^2)\frac{P\cdot q}{\sqrt{q^2M^2}}. \tag{42}$$

Then, the convergence condition for the sum over the O(4) angular momenta in Eq. (31) reads

$$(P\cdot q)^2<M^2|q^2|. \tag{43}$$

Even if Eq. (43) is satisfied, the radial functions f_α^n themselves may contain singularities which prevent us from performing the analytic continuation by numerical methods. However, note that the Euclidean solutions for f_α^n are regular everywhere on the imaginary q^0 axis. Consequently, the RHS of the ''half-Wick-rotated'' equation (25) contains no singularities if the displaced pole-free condition in Eq. (23) is met. Therefore, in Minkowski space, the radial functions f_α are regular everywhere in the momentum region where the displaced pole-free condition (23) and the convergence condition (43) are satisfied. Here, the analytic continuation to Minkowski space is straightforward. Recall the normalized O(4) radial functions F_n and G_n from Eqs. (37) and (38), which are linear combinations of f_α^n. Writing them as $F_n(q_E)=q_E^n\widetilde{F}_n(q_E^2)$ and $G_n(q_E)=q_E^n\widetilde{G}_n(q_E^2)$, we find for the scalar functions $f_\alpha(q^2,P\cdot q)$ from Eqs. (39) and (40):

$$f_1(q^2,P\cdot q)=\sum_{n=0}^{\infty}\frac{\widetilde{F}_n(-q^2)}{M^n}(\sqrt{q^2M^2})^nC_n^1(z), \tag{44}$$

$$f_2(q^2,P\cdot q)=-\sum_{n=1}^{\infty}\frac{2}{\sqrt{n(n+2)}}\frac{\widetilde{G}_n(-q^2)}{M^{n-2}}\times(\sqrt{q^2M^2})^{n-1}C_{n-1}^2(z). \tag{45}$$

Note that the Gegenbauer polynomials $C_n^1[C_{n-1}^2]$ together with the square root factors $(\sqrt{q^2M^2})^n$ $[(\sqrt{q^2M^2})^{n-1}]$ are nth $[(n-1)\text{th}]$ order polynomials of q^2, M^2, and $P\cdot q$ and contain, therefore, no $\sqrt{q^2M^2}$ factors. Since in the kinematic region under consideration, $f_1(q^2,P\cdot q)$ and $f_2(q^2,P\cdot q)$ are regular, it is possible to extrapolate $\widetilde{F}_n(-q^2)$ and $\widetilde{G}_n(-q^2)$ numerically from spacelike q^2 to timelike q^2 as necessary.

Finally, we are interested in the quark-diquark vertex function as it appears in the handbag diagram for deep-inelastic scattering. Therefore, we need the functions f_α for on-shell diquarks only. The squared relative momentum q^2 and the Lorentz scalar $P\cdot q$ are then no longer independent but related by

$$P\cdot q=-\frac{m_q+m_D}{2m_D}\left[-q^2+\left(\frac{m_D}{m_q+m_D}\right)^2[(m_q+m_D)^2-M^2]\right]. \tag{46}$$

Then, f_1 and f_2 from Eqs. (44) and (45) are functions of the squared relative momentum q^2 only.

In Sec. II the parametrization (6) for the Dirac matrix structure of the vertex function was more convenient to use. The corresponding functions f_α^{on} which enter the nucleon structure function in Eq. (7) are given by

$$f_1^{\text{on}}(k^2)=f_1(q^2,P\cdot q)+\frac{m_D^2-k^2}{2M^2}f_2(q^2,P\cdot q), \tag{47}$$

$$f_2^{\text{on}}(k^2)=\frac{1}{2}f_2(q^2,P\cdot q). \tag{48}$$

Here, the arguments q^2 and $P\cdot q$, of the scalar functions f_α on the RHS should be understood as functions of k^2 through Eq. (46), together with the relation

$$q^2=\frac{m_D}{m_q+m_D}\left[k^2-m_q\left(\frac{M^2}{m_q+m_D}-m_D\right)\right]. \qquad (49)$$

As already mentioned, the procedure just described yields radial functions $f_\alpha^{\rm on}$ in Minkowski space only in the kinematic region where the conditions, Eqs. (23) and (43), are met. These are satisfied for weakly bound states $M^2 \lesssim (m_D+m_q)^2$ at moderate values of $|k^2|$. On the other hand, the nucleon structure function in Eq. (7) at small and moderate values of x is dominated by contributions from small quark momenta $|k^2|<m_q^2$. Consequently, the Minkowski space vertex function obtained in the kinematic region specified by the displaced pole-free condition (23) and the convergence condition (43) determines the valence-quark distribution of a weakly bound nucleon at small and moderate x.

In the case of strong binding, $M^2 \ll (m_q+m_D)^2$, or at large x, the nucleon structure function is dominated by contributions from large spacelike k^2. Here, the above analytic continuation to Minkowski space is not possible and the sum over the O(4) angular momenta in Eq. (31) should be evaluated first. In principle, this is possible through the Watson-Sommerfeld method [16–18], where the leading power behavior of $f_\alpha(q^2,P\cdot q)$ for asymptotic $P\cdot q$ can be deduced by solving the BSE at complex O(4) angular momenta [19], or by assuming conformal invariance of the amplitude and using the operator product expansion technique as outlined in Ref. [18].

However, the use of the operator product expansion is questionable here, since in the quark-diquark model, which is being used, we have introduced a form factor for the quark-diquark coupling and our model does not correspond to an asymptotically free theory. Existence of the form factor also makes the analysis of complex O(4) angular momenta complicated. Therefore, a simpler approach is used. It can be shown from a general analysis that BS vertex functions which satisfy a ladder BSE are regular for spacelike k^2, when one of the constituent particles is on mass shell. Furthermore, from the numerical solution studied in the previous section, we found that the magnitude of the O(4) partial wave contributions to the function $f_\alpha^{\rm on}$ decreases reasonably fast for large O(4) angular momenta n, except at very large k^2. We, therefore, use the expansion formulas (44) and (45) with an upper limit on $n \leqslant n_{\rm max}$ to evaluate $f_\alpha^{\rm on}$ defined by Eqs. (47) and (48) as an approximation. Nevertheless, this application of BS vertex functions to deep-inelastic scattering emphasizes the need to solve Bethe-Salpeter equations in Minkowski space from the very beginning, as has been done recently for scalar theories without derivative coupling [20].

IV. NUMERICAL RESULTS

In this section we present results for the valence contribution to the nucleon structure function F_1 from Eq. (7), based on the numerical solutions discussed above. First, we show in Fig. 4 the physical, on-shell scalar functions $f_\alpha^{\rm on}$ for a bound state mass $M=1.8\overline{m}$. The maximal O(4) angular momentum is fixed at $n_{\rm max}=4$. Figure 4 demonstrates that the magnitude of $f_1^{\rm on}$ and $f_2^{\rm on}$ is quite similar, even for a weakly bound quark-diquark system. Furthermore, we find that for weakly bound states ($M \gtrsim 1.8\overline{m}$), the dependence of $f_\alpha^{\rm on}$ on $n_{\rm max}$ is negligible in the region of moderate, spacelike $-k^2 \lesssim 5\,\overline{m}^2$. However, for larger spacelike values of k^2 the convergence of the O(4) expansion in Eq. (31) decreases for any M^2, and numerical results for fixed $n_{\rm max}$ become less accurate.

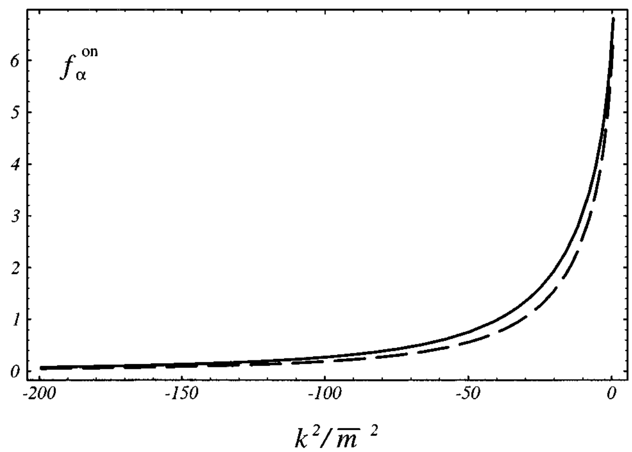

FIG. 4. The on-shell scalar functions $f_1^{\rm on}$ (solid) and $-f_2^{\rm on}$ (dashed) as a function of the quark momentum k^2 for $M=1.8\overline{m}$ and $n_{\rm max}=4$.

In Fig. 5 the structure function $F_1^{\rm val}$ is shown for various values of M^2 using $n_{\rm max}=4$. The distributions are normalized to unity. One observes that for weakly bound systems ($M=1.99\overline{m}$), the valence-quark distribution peaks around $x\sim 1/2$. On the other hand, the distribution becomes flat if binding is strong ($M=1.2\overline{m}$). This behavior turns out to be mainly of kinematic origin. To see this, remember that $F_1^{\rm val}$ is given by an integral [cf., Eq. (7)] over the squared quark momentum k^2, bounded by $k^2_{\rm max}=x[M^2-m_D^2/(1-x)]$. The latter has a maximum at $x=1-m_D/M$. Therefore, the peak of the valence distribution for weakly bound systems occurs

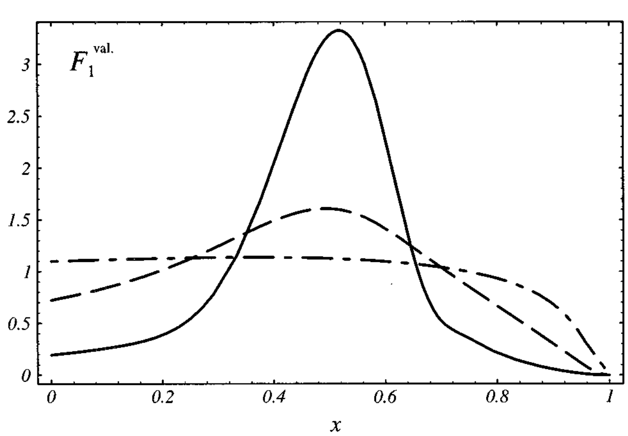

FIG. 5. The valence-quark distribution $F_1^{\rm val}$ from Eq. (7) for different binding for the model proton. The solid, dashed, and dot-dashed lines show the results for weak ($M=1.99\overline{m}$), moderate ($M=1.8\overline{m}$), and strong ($M=1.2\overline{m}$) binding, respectively.

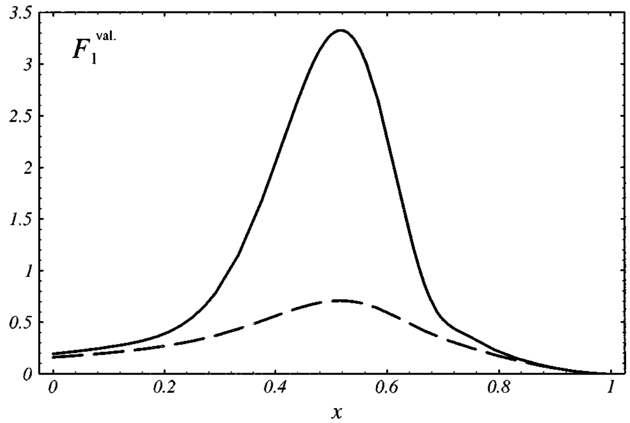

FIG. 6. Contribution of the ''relativistic component'' f_2^{on} to the structure function F_1^{val} from Eq. (7) for a weakly bound model proton ($M=1.99\overline{m}$). The solid line denotes the total valence distribution F_1^{val}. The dashed line shows the contributions to F_1^{val}, which are proportional to f_2^{on}.

at $x\approx 1/2$ for $m_D=m_q$. For a more realistic choice $m_D\sim 2\,m_q$, the valence distribution would peak at $x\sim 1/3$. The more strongly the system is bound, the less k^2_{max} varies with x. This leads to a broad distribution in the case of strong binding. Thus, the global shape of F_1^{val} is determined to a large extent by relativistic kinematics.

Note that successful fits to the measured nucleon valence contribution to F_1 exhibit at low mass scales a significant maximum at $x\approx 1/3$ [21,22]. This behavior is in agreement with our result for F_1^{val} in the case of weak quark-diquark binding. At the present stage of development of the model we hesitate to compare our results with data. For example, we would need to treat vector diquarks explicitly, including their mass difference from the scalar diquarks [23]. Nevertheless, from the results we have obtained we can certainly conclude that in terms of quark-diquark degrees of freedom the nucleon has to be viewed as a weak bound state.

To investigate the role of the relativistic spin structure of the vertex function we discuss the contribution of the ''relativistic'' component f_2^{on} to the nucleon structure function F_1^{val}. Figure 6 shows that the contribution from f_2^{on} is negligible for a very weakly bound quark-diquark state ($M=1.99\overline{m}$). Here, the ''nonrelativistic,'' leading component f_1^{on} determines the structure function. However, even for moderate binding the situation is different. In Fig. 7 one observes that the contribution from the ''relativistic'' component is quite significant for $M=1.8\overline{m}$. Nevertheless, the characteristic x dependence, i.e., the peak of the structure function at $x\approx 1/2$, is still because of the ''nonrelativistic'' component.

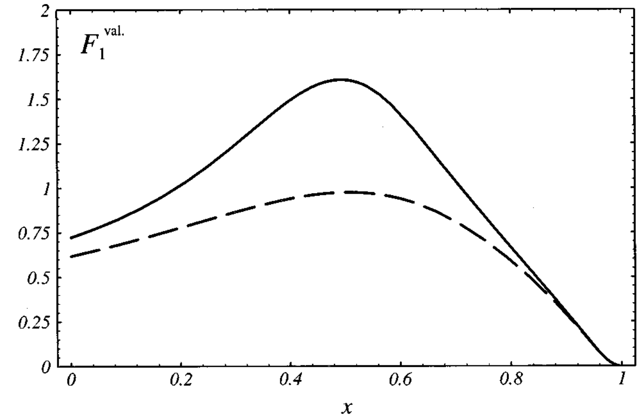

FIG. 7. As in Fig. 6, but for moderate binding for the model proton ($M=1.8\overline{m}$).

V. SUMMARY

The aim of this work was to outline a scheme whereby structure functions can be obtained from a relativistic description of a model nucleon as a quark-diquark bound state. For this purpose we solved the BSE for the nucleon starting from a simple quark-diquark Lagrangian. From the Euclidean solutions of the BSE we extracted the physical quark-diquark vertex functions. These were applied to the spectator model for DIS, and the valence-quark contribution to the structure function F_1 was calculated.

Although the quark-diquark Lagrangian used here is certainly not realistic, and the corresponding BSE was solved by applying several simplifications, some interesting and useful observations were made. We found that the spin structure of the nucleon, seen as a relativistic quark-diquark bound state, is nontrivial, except in the case of very weak binding. Correspondingly, the valence-quark contribution to the structure function is governed by the ''nonrelativistic'' component of the nucleon vertex function only for a very weakly bound state. Furthermore, we observed that the shape of the unpolarized valence-quark distribution is mainly determined by relativistic kinematics and does not depend on details of the quark-diquark dynamics. However, at large quark momenta, difficulties in the analytic continuation of the Euclidean solution for the vertex function to Minkowski space emphasize the need to treat Bethe-Salpeter equations in Minkowski space from the very beginning.

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