Zero temperature chiral phase transition in SU(N) gauge theories

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Recently Appelquist, Terning, and Wijewardhana investigated the zero-temperature chiral phase transition in SU(N) gauge theory as the number of fermions N_f is varied. They argued that there is a critical number of fermions N_f^c, above which there is no chiral symmetry breaking and below which chiral symmetry breaking and confinement set in. They further argued that the transition is not second order even though the order parameter for chiral symmetry breaking vanishes continuously as N_f approaches N_f^c on the broken side. In this note I propose a simple physical picture for the spectrum of states as N_f approaches N_f^c from below (i.e., on the broken side) and argue that this picture predicts very different and *nonuniversal* behavior than is the case in an ordinary second order phase transition. In this way the transition can be *continuous* without behaving conventionally. I further argue that this feature results from the (presumed) existence of an infrared Banks-Zaks fixed point of the gauge coupling in the neighborhood of the chiral transition and, therefore, depends on the long-distance nature of the non-Abelian gauge force. [S0556-2821(97)04608-0]

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Recently Appelquist, Terning, and Wijewardhana [1] have investigated the zero-temperature chiral phase transition in SU(N) gauge theory as the number of massless Dirac fermions N_f is varied. To second order, the β function of such a theory is given by

$$\mu\frac{\partial}{\partial\mu}\alpha(\mu)=\beta(\alpha)\equiv-b\alpha^2(\mu)-c\alpha^3(\mu)-d\alpha^4(\mu)-\cdots, \tag{1}$$

where

$$b=\frac{1}{6\pi}(11N-2N_f), \tag{2}$$

$$c=\frac{1}{24\pi^2}\left(34N^2-10NN_f-3\frac{N^2-1}{N}N_f\right). \tag{3}$$

For a small number of flavors the theory is asymptotically free and one expects QCD-like behavior, with confinement and with the chiral $SU(N_f)_L\times SU(N_f)_R$ symmetry broken to its vectorial subgroup. If the number of flavors is large enough (in perturbation theory, greater than $11N/2$) asymptotic freedom (and hence chiral symmetry breaking and confinement) is lost. For a range of N_f less than $11N/2$ the first term in the β function is negative and the theory is asymptotically free, but there appears (in perturbation theory) to be a nontrivial infrared (Banks-Zaks) fixed point [2] because the second term in the β function is *positive*. For N_f just slightly less than $11N/2$, this fixed point α^* is at weak coupling and the analysis is self-consistent. As N_f is lowered further, the fixed point moves to *larger* coupling.

In vectorlike gauge theories an analysis of the gap equation [3,4] suggests that, in a theory with an approximately constant coupling, chiral symmetry breaking occurs only if the coupling α exceeds a critical value

$$\alpha_c\equiv\frac{\pi}{3C_2(R)}=\frac{2\pi N}{3(N^2-1)}. \tag{4}$$

The authors of [1] suggested that the Banks-Zaks fixed point [2] persists to large coupling, and that the chiral-symmetry-breaking transition in N_f should be associated with the point where $\alpha^*=\alpha_c$. They thereby estimated that

$$N_f^c=N\left(\frac{100N^2-66}{25N^2-15}\right). \tag{5}$$

Furthermore, Appelquist, Terning, and Wijewardhana suggested that the nature of this phase transition is peculiar. Based on a gap equation analysis [5] of the dynamical mass $\Sigma(p)$ of the fermions in the broken phase, they argued that the order parameter for chiral symmetry breaking [which is proportional to $\Sigma(0)$],

$$\Sigma(0)\approx\Lambda\exp\left(\frac{-\pi}{\sqrt{\alpha_*/\alpha_c-1}}\right), \tag{6}$$

goes to zero continuously[1] as $N_f\to(N_f^c)^-$ (and $\alpha^*\to\alpha_c^+$). Here the high-energy scale Λ represents the scale at which the coupling is far enough from the fixed point value to begin to run [1].

On the other hand, based on an analysis of the Bethe-Salpeter equation for the quark-antiquark scattering amplitude, Appelquist, Terning, and Wijewardhana argue that in the symmetric phase (close to the transition, when N_f is just above N_f^c) there are no light scalar resonances. Indeed, since the theory with $N_f\gtrsim N_f^c$ is presumed to be in a conformally

[1]For momenta below $\Sigma(0)$, the fermions can be integrated out and no longer contribute to the β function. Therefore, strictly speaking, for any $N_f<N_f^c$ there is no fixed point. Nonetheless, for N_f close to N_f^c the coupling remains close to α^* for a large range of momenta, in a manner reminiscent of ''walking technicolor'' [6].

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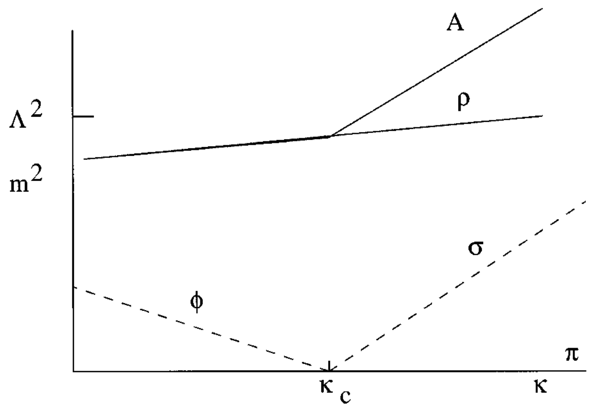

FIG. 1. Spectrum of bosonic excitations in the NJL model for couplings close to κ_c. Chiral symmetry is broken for $\kappa > \kappa_c$, and preserved for $\kappa < \kappa_c$. The σ and π in the broken phase combine to be the scalar ϕ multiplet in the unbroken phase. The vector and axial-vector resonances form one chiral multiplet, and are therefore degenerate, in the unbroken phase.

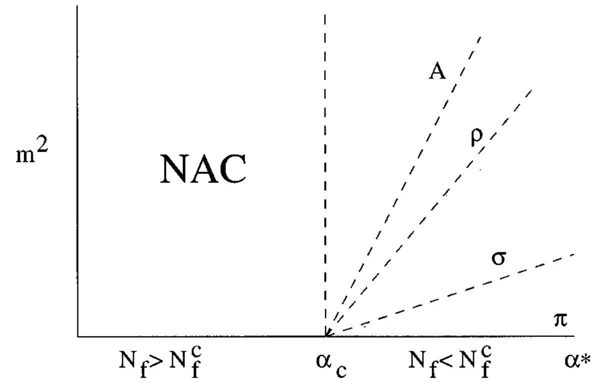

FIG. 2. Spectrum of bosonic excitations in SU(N) gauge theory for N_f close to N_f^c. For $N_f < N_f^c$, chiral symmetry is broken. For $N_f \gtrsim N_f^c$, the theory is assumed to be in a conformally invariant ''non-Abelian Coulomb'' (NAC) phase [2] which has no isolated single-particle states.

invariant ''non-Abelian Coulomb'' phase [2], it cannot have *any* isolated states. They then concluded that the phase transition is not second order *even though the order parameter changes continuously.*

In this work I propose a simple physical picture for the spectrum of states as N_f approaches N_f^c from below (i.e., on the broken side) and argue that this picture predicts very different and *nonuniversal* behavior than is the case in an ordinary second-order phase transition. In this way the transition can be *continuous* without behaving conventionally [7]. I further argue that this feature results from the (presumed) existence of an infrared Banks-Zaks fixed point of the gauge coupling in the neighborhood of the chiral transition and therefore depends on the long-distance nature of the non-Abelian gauge force.

Before moving to the case of a long-range force, let us review the familiar behavior of the chiral phase transition in a model with short-range interactions, the Nambu–Jona-Lasinio (NJL) model [8]. Here the fundamental interactions are modeled by chirally invariant local four-fermion operators

$$\mathcal{L} = -\frac{4\pi\kappa}{\Lambda^2}\left[\bar{\psi}\gamma_\mu \frac{\lambda^a}{2}\psi\right]^2, \tag{7}$$

where the $\lambda^a/2$ are the generators of SU(N) ''color'' and the ψ are the N_f flavors of fermions. This interaction is attractive in the chiral-symmetry-breaking channel, but is suppressed by a (large) energy scale Λ. In the limit where the NJL coupling κ is small, chiral symmetry remains unbroken. When the coupling is large, the chiral-symmetry-breaking scale (as characterized by the value of the F constant—the analogue of f_π in QCD—or by Σ, the momentum-independent dynamical mass of the fermion) is of order Λ. There is a critical value κ_c, estimated to be $\pi/3$ in the large-N limit, below which chiral symmetry is unbroken and above which it is broken. If the transition between these two regimes is *smooth*, as it is in the gap equation in the fermion-bubble approximation, the dynamical mass of the fermion goes smoothly to zero as $\kappa \to (\kappa_c)^+$, and remains identically zero for $\kappa \leqslant \kappa_c$.

In the case of the Nambu–Jona-Lasinio model, the spectrum of bosonic excitations [9] is shown in Fig. 1. Note that, aside from the light scalar-multiplet ϕ on the unbroken side and the σ and Goldstone bosons π on the broken side, *all* other ''excitations'' have a mass of order Λ. The reason for this is that the intrinsic physical scale of the interactions is *always* of order Λ. Near $\kappa = \kappa_c$, the scalar states are *anomalously light* due to the ''fine-tuning'' [10] of the NJL interaction.

This picture should be contrasted with the analogous change of the spectrum of particles near the chiral phase transition in non-Abelian gauge theory as $N_f \to (N_f^c)^-$. Because of the infrared fixed point, the high-energy scale Λ [see Eq. (6)] is no longer relevant. In this case the *only* relevant dynamical scale is the magnitude of dynamical mass $\Sigma(0)$. All other scales, the F constant, the confinement scale, etc., are of the same order of magnitude [1]. Therefore, if $\Sigma(0) \to 0$ continuously as $N_f \to (N_f^c)^-$, we expect the spectrum of bosonic excitations to be as shown in Fig. 2. Note that as $N_f \to (N_f^c)^-$, the entire spectrum collapses to zero mass. So long as the Banks-Zaks fixed point persists[2] in the non-Abelian gauge theory, all high-energy scales are irrelevant. Therefore, unlike the NJL model, if $\Sigma(0) \to 0$ the mass of all excitations must also tend to zero.

What are the implications of this behavior? In the case of the NJL model near the critical coupling, it is appropriate to ''integrate out'' all higher-mass states and the critical behavior of the theory is determined entirely by the infrared behavior of the corresponding scalar field theory.[3] We therefore

[2]More properly, since we are in the broken phase where no true fixed point exists, so long as the coupling remains close to α^* over a large range of momenta.

[3]In some cases the resulting scalar theory cannot have a second-order transition, but will have a fluctuation-induced first-order transition instead [11]. If the transition is driven to be first order, all relevant dimensional quantities will be [12,13] of order Λ.

expect similar behavior in *any* $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)$ chirally invariant four-dimensional field theory with *short-range interactions* near the phase boundary: i.e., we expect the behavior to be *universal*.

In contrast, in the case of non-Abelian gauge theory with N_f just below N_f^c, we *cannot* reduce the theory to an effective low-energy scalar theory. In principle *all* ''higher'' resonances contribute to any correlation function and the behavior is *nonuniversal*. In this case it is plausible that the order parameter changes continuously as $N_f \to (N_f^c)^-$, even though the theory with $N_f \gtrsim N_f^c$ has no light scalar states.

Finally, I note that arguments similar to those presented here may cast light on the discrepancies between the gap equation [14,15] and the effective scalar field-theory [16] analyses of the chiral phase transition in QED3.

In this work I have proposed a simple physical picture for the spectrum of states in $\mathrm{SU}(N)$ gauge theory as the number of flavors N_f approaches the critical number N_f^c from below (i.e., from the broken side). This picture predicts very different and *nonuniversal* behavior than is the case in a conventional second-order phase transition. I have further argued that this feature results from the (presumed) existence of a Banks-Zaks fixed point of the gauge coupling in the neighborhood of the chiral transition and therefore depends on the long-distance nature of the non-Abelian gauge force.

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