

## Reduction of weak interaction rates in neutron stars by nucleon spin fluctuations: Degenerate case

Georg Raffelt and Thomas Strobel

*Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 München, Germany*

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Nucleon spin fluctuations in a dense medium reduce the “naive” values of weak interaction rates (neutrino opacities, neutrino emissivities). We extend previous studies of this effect to the degenerate case which is appropriate for neutron stars a few ten seconds after formation. If neutron-neutron interactions by a one-pion exchange potential are the dominant cause of neutron spin fluctuations, a perturbative calculation of weak interaction rates is justified for  $T \lesssim 3m/(4\pi\alpha_\pi^2) \approx 1$  MeV, where  $m$  is the neutron mass and  $\alpha_\pi \approx 15$  the pion fine-structure constant. At higher temperatures, the application of Landau’s theory of Fermi liquids is no longer justified; i.e., the neutrons cannot be viewed as simple quasiparticles in any obvious sense. [S0556-2821(97)00402-5]

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### I. INTRODUCTION

In a dense nuclear medium the effective neutrino interaction rates are modified by the presence of nucleon-nucleon interactions. While the importance of spatial spin-spin correlations has been recognized for a long time, it had been overlooked that the interaction-induced temporal fluctuations of the spin of a single nucleon can be a more important effect. It reduces the naive neutrino opacities and neutrino emissivities of nuclear matter below their naive values [1–4]. These studies focused on a classical nucleon plasma, i.e., on non-relativistic and nondegenerate conditions which are thought to obtain in the core of a supernova for the first few seconds after collapse. It was found that the spin-fluctuation rate in this environment is so large that it is not possible to calculate weak interaction rates by a perturbative expansion in terms of the nucleon-nucleon interaction potential.

We presently study the same effect for a degenerate medium in order to derive a perturbative expression for the cross-section reduction by nucleon spin fluctuations, and in order to understand the physical conditions of temperature and density where a “naive” calculation of weak interaction rates may be justified. Many attempts have been made to calculate neutrino opacities and emissivities for the physical conditions pertaining to neutron stars because of the obvious importance of these quantities for a theoretical understanding of neutron-star cooling [5]. While many of these works are dedicated to calculating the impact of spatial correlation effects on neutrino interaction rates, none of them appears to have addressed the important issue of nucleon spin autocorrelations.

One may take a somewhat different perspective on the same problem if one notes that these calculations are based on Landau’s theory of interacting Fermi liquids where a “nucleon” is a quasiparticle excitation of the medium [6]. This picture is justified only if the quasiparticles near the Fermi surface do not interact too strongly, i.e.,  $\tau^{-1} \ll T$ , where  $\tau$  is a typical time between collisions and  $T$  is the temperature of the medium. Landau’s condition is based on the observation that at  $T=0$  the Fermi-Dirac distribution is a

step function which, at nonzero temperature, is smeared out over an energy range of approximate width  $T$ . Collisions, on the other hand, introduce an energy uncertainty of order  $\tau^{-1}$  which clearly should be much smaller than  $T$  in order for the Fermi-Dirac distribution to make any sense. When Landau’s condition is violated it is not possible to speak of quasiparticles which obey Fermi statistics. Degeneracy effects ensure that the time between collisions becomes large at low temperatures, so there is no significant restriction in the  $T \rightarrow 0$  limit. For hot neutron-star matter, however, it is not *a priori* obvious that Landau’s condition is satisfied. We were unable to locate any discussion of this problem in the entire literature pertaining to weak interaction rates in neutron stars [5]. Therefore, it is not frivolous to raise the question of how cold the medium in a neutron star has to become before a Fermi-liquid treatment becomes possible.

As previously argued [1–3], the cross-section reduction by nucleon spin fluctuations becomes large when a typical nucleon spin-fluctuation rate is of order the ambient temperature  $T$  or larger. Since nucleons interact by a spin-dependent force, the spin-fluctuation rate is roughly identical with the nucleon collision rate. Therefore, the condition that the spin-fluctuation rate be much less than  $T$  ensures both that the weak interaction rates are not much affected by nucleon spin fluctuations and that Landau’s condition is satisfied.

The main problem in the degenerate case is to identify the quantity which is to be interpreted as the relevant effective spin-fluctuation rate. Because only the spins of nucleons near the Fermi surface have a chance of evolving in a nontrivial way, and because Landau’s condition pertains to the quasiparticles near the Fermi surface, it is clear that we need to define an appropriate effective spin-fluctuation rate for the quasiparticles near the Fermi surface.

The impact of nucleon-nucleon collisions on weak interaction rates is best understood in the language of linear-response theory where the medium is described by the dynamical density and (iso)spin-density structure functions. This method allows for a straightforward calculation of the reduction of weak interaction rates in the perturbative limit where the Landau condition is satisfied, and thus allows for a

delineation of the physical parameters where this treatment is justified.

We will limit ourselves to the simple situation of a non-relativistic, single-species medium, i.e., we will study non-relativistic degenerate neutron matter. This excludes the important Urca processes from consideration which are more difficult to address because they involve two degenerate Fermi seas (protons and neutrons) with vastly different Fermi momenta. We believe that for the present exploratory purposes a simple toy model is best suited to illuminate the issues at hand. Therefore, we shall limit our attention to the neutral-current scattering process  $\nu + n \rightarrow n + \nu$  in nonrelativistic degenerate neutron matter in the presence of interactions which cause the neutron spins to fluctuate.

In Sec. II we introduce the relevant spin-density structure function and derive a simple sum rule which is used in Sec. III to calculate the  $\nu n$  scattering cross-section reduction by neutron spin fluctuations. In Sec. IV we summarize and discuss our result.

## II. DYNAMICAL STRUCTURE FUNCTIONS

### A. Definition

In nonrelativistic neutron matter all weak interaction rates are determined by the dynamical density and spin-density structure functions. In an isotropic medium they are given by [2]

$$S_\rho(\omega, \mathbf{k}) = \frac{1}{n_B V} \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \rho(t, \mathbf{k}) \rho(0, -\mathbf{k}) \rangle,$$

$$S_\sigma(\omega, \mathbf{k}) = \frac{4}{3n_B V} \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \boldsymbol{\sigma}(t, \mathbf{k}) \cdot \boldsymbol{\sigma}(0, -\mathbf{k}) \rangle, \quad (1)$$

where  $n_B$  is the baryon (here neutron) density,  $V$  the volume of the system,  $\mathbf{k}$  the momentum transfer, and  $\omega$  the energy transfer from the weak probe to the neutron medium. Further,  $\rho(t, \mathbf{k})$  is the spatial Fourier transform at time  $t$  of the neutron density operator,  $\rho(x) = \psi^\dagger(x) \psi(x)$ , where  $\psi(x)$  is the neutron field operator, a Pauli two-spinor. Similarly,  $\boldsymbol{\sigma}(t, \mathbf{k})$  is the Fourier transform of the spin-density operator,  $\boldsymbol{\sigma}(x) = \frac{1}{2} \psi^\dagger(x) \boldsymbol{\tau} \psi(x)$ , with  $\boldsymbol{\tau}$  a vector of Pauli matrices. The expectation value  $\langle \cdots \rangle$  is taken over a thermal ensemble so that detailed balance  $S_{\rho, \sigma}(\omega, \mathbf{k}) = S_{\rho, \sigma}(-\omega, -\mathbf{k}) e^{\omega/T}$  is satisfied. Note that a positive  $\omega$  is energy given to the medium by the weak probe.

The energies of the neutrinos which interact with the medium are much smaller than the neutron mass or momenta so that the long-wavelength limit  $\mathbf{k} \rightarrow 0$  is an adequate first approximation. In practice, its validity is questionable if neutron-neutron correlations or collective modes are important which for the moment we shall assume is not the case. Then, the neutrino differential scattering cross section is given by

$$\frac{d\sigma}{d\epsilon_2} = \frac{G_F^2 \epsilon_2^2}{4\pi} \left( C_V^2 \frac{S_\rho(\epsilon_1 - \epsilon_2)}{2\pi} + 3C_A^2 \frac{S_\sigma(\epsilon_1 - \epsilon_2)}{2\pi} \right), \quad (2)$$

where  $\epsilon_{1,2}$  is the energy of the incoming and outgoing neutrino, respectively, and  $S_{\rho, \sigma}(\omega)$  stands for  $\lim_{\mathbf{k} \rightarrow 0} S_{\rho, \sigma}(\omega, \mathbf{k})$ . Further,  $G_F$  is the Fermi constant, and

$C_V = -1$  and  $C_A \approx -1.15$  are the neutral-current weak coupling constants for the neutron [2]. In bulk nuclear matter,  $C_A$  may be suppressed somewhat.

In a noninteracting medium, the density and spin-density operators remain constant so that the dynamical structure functions are proportional to  $\delta(\omega)$ . In the nondegenerate case, they are  $S_{\rho, \sigma}(\omega) = 2\pi \delta(\omega)$ . To include neutron-neutron anticorrelations induced by the Pauli exclusion principle one evaluates the expectation values in Eq. (1) by normal ordering of the neutron field operators, taking proper account of the anticommutation relations. Then, one arrives at the intuitive result

$$S_{\rho, \sigma}(\omega) = 2\pi \delta(\omega) \frac{1}{n_B} \int \frac{2d^3\mathbf{p}}{(2\pi)^3} f_{\mathbf{p}}(1 - f_{\mathbf{p}}), \quad (3)$$

where  $f_{\mathbf{p}}$  is the occupation number of the neutron field mode  $\mathbf{p}$ . In the nondegenerate limit one may neglect the Pauli blocking factor  $(1 - f_{\mathbf{p}})$  so that one arrives at the previous result if one notes that  $n_B = \int f_{\mathbf{p}} 2d^3\mathbf{p} / (2\pi)^3$ . Here, the factor 2 counts the two neutron spin degrees of freedom.

Even after ‘‘turning on’’ interactions between the neutrons, or between the neutrons and some external potential, the density operator remains constant. The vector current quantity that does fluctuate in the presence of interactions is the neutron velocity which in the nonrelativistic limit is small. Therefore,  $S_\rho(\omega)$  remains proportional to  $\delta(\omega)$ .

However, if the interaction involves a spin-dependent force as expected for neutron-neutron interactions, the spin-density structure function will be broadened because the spin of a given neutron near the Fermi surface will ‘‘forget’’ its initial orientation roughly after the collision time  $\tau$ . The width of  $S_\sigma(\omega)$  roughly represents  $\tau^{-1}$  so that the Landau condition corresponds to the requirement that the width of  $S_\sigma(\omega)$  must be much less than  $T$ . If this is satisfied, the neutrino-scattering rates and thus the neutrino opacities are well approximated by the noninteracting result for  $S_\sigma(\omega)$ . Of course, it may be modified by neutron-neutron correlations or collective modes, effects that were the main focus of many of the previous papers [5].

### B. Normalization

An important general property of the dynamical structure functions is their normalization. If one integrates both sides of Eq. (1) over  $d\omega$ , the term  $e^{i\omega t}$  yields  $\delta(t)$  so that the time integral can be trivially done. Then, the normalization for the spin-density case is

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_\sigma(\omega) = \frac{4}{3n_B V} \langle \boldsymbol{\sigma}(0, 0) \cdot \boldsymbol{\sigma}(0, 0) \rangle. \quad (4)$$

If one ignores spin-spin correlations, the right-hand side (RHS) is independent of the neutron spins’ evolution. For the sake of argument one may imagine that this evolution is caused by the interaction with some external potential rather than by neutron-neutron collisions so that there is no reason to expect spin-spin correlations.

In this case one may evaluate the RHS of Eq. (4) as above and find

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{\rho,\sigma}(\omega) = \frac{1}{n_B} \int \frac{2d^3\mathbf{p}}{(2\pi)^3} f_{\mathbf{p}}(1-f_{\mathbf{p}}). \quad (5)$$

The occupation numbers are given by a Fermi-Dirac distribution so that the RHS is

$$\frac{1}{n_B} \int \frac{2d^3\mathbf{p}}{(2\pi)^3} \frac{1}{e^{(E-\mu)/T} + 1} \left( 1 - \frac{1}{e^{(E-\mu)/T} + 1} \right), \quad (6)$$

where  $E = \mathbf{p}^2/2m$  is the kinetic energy,  $m$  the neutron quasi-particle effective mass, and  $\mu$  the nonrelativistic neutron chemical potential. Then, Eq. (6) is

$$\frac{1}{n_B \pi^2} \int_0^\infty dp p^2 \frac{e^z}{(e^z + 1)^2}, \quad (7)$$

where  $z \equiv (E - \mu)/T$ . For very degenerate conditions the integrand is strongly peaked near  $z=0$  (the Fermi surface) so that after a transformation of the integration variable to  $z$  one may replace  $p$  with  $p_F$  and one may extend the lower limit of integration to  $-\infty$ . The integral can then be evaluated analytically so that altogether

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{\rho,\sigma}(\omega) = \frac{3}{2\eta}. \quad (8)$$

Here,

$$\eta = \frac{E_F}{T} = \frac{p_F^2}{2mT} \quad (9)$$

is the degeneracy parameter in the nonrelativistic and very degenerate limit with  $E_F = p_F^2/2m$  the nonrelativistic Fermi energy.

Therefore, in a noninteracting degenerate medium the structure functions are  $S_{\rho,\sigma}(\omega) = (3/2\eta) 2\pi \delta(\omega)$ . The total scattering cross section of a neutrino with energy  $\epsilon_1$  is then  $\sigma = (3/2\eta)(C_V^2 + 3C_A^2)G_F^2 \epsilon_1^2/4\pi$ .

### C. Perturbative representation of $S_\sigma(\omega)$

If neutrons interact by a spin-dependent force it causes a nontrivial evolution of their spins and thus a nonzero width of  $S_\sigma(\omega)$ . Except in the neighborhood of  $\omega=0$ , where multiple-scattering effects become important,  $S_\sigma(\omega)$  can be calculated on the basis of a bremsstrahlung or medium-excitation amplitude [3,4]. Because for small  $\omega$  the result generically varies as  $\omega^{-2}$ , it is useful to represent it in the form

$$S_\sigma^{\text{brems}}(\omega) = \frac{\Gamma_\sigma}{\omega^2} s(\omega/T) \begin{cases} e^{\omega/T} & \text{for } \omega < 0, \\ 1 & \text{for } \omega > 0. \end{cases} \quad (10)$$

The explicit distinction between positive and negative energy transfers represents the detailed-balance condition. Further,  $s(x)$  is an even function which is normalized such that  $s(0)=1$ . In the classical limit of hard collisions one has  $s(x)=1$  for all  $x$  as discussed in more detail in Refs. [3,7]. In general,  $s(x)$  embodies information about the detailed form of the interaction potential and about quantum corrections to the classical result. In the nondegenerate case,  $\Gamma_\sigma$  has the

interpretation of an average spin rate of change, or conversely,  $\Gamma_\sigma^{-1}$  is the approximate time for a given nucleon spin to relax, i.e., to forget its initial orientation.

Explicit calculations of  $\Gamma_\sigma$  and  $s(x)$  exist for a single-species nuclear medium where the nucleon interaction is modeled by a one-pion exchange (OPE) potential [2]. For a degenerate medium the relevant expressions can be extracted from Ref. [9]:

$$\Gamma_{\sigma,\text{OPE}} = 4\pi \alpha_\pi^2 T^3 / p_F^2, \quad (11)$$

where the neutron Fermi momentum is given by  $n_B = p_F^3/3\pi^2$ ,  $\alpha_\pi \equiv (f_2 m/m_\pi)^2/4\pi \approx 15$  with  $f \approx 1.0$  is the pion fine-structure constant,  $m$  is the neutron mass, and the pion mass has been neglected in the OPE potential. One also finds, from Ref. [9],

$$s_{\text{OPE}}(x) = \frac{(x^2 + 4\pi^2)|x|}{4\pi^2(1 - e^{-|x|})}, \quad (12)$$

which is 1 at  $x=0$  while for  $|x| \gg 1$  it is  $|x|^3/4\pi^2$ .

Sigl [8] has derived an f-sum rule which implies that the integral  $\int S_\sigma(\omega) \omega d\omega$  must exist and thus that  $s(x)$  must be a decreasing function for large  $x$ . This conclusion also pertains to the degenerate case: if the energy transfer  $\omega$  to the medium far exceeds the Fermi energy  $E_F$ , a nucleon is lifted far above the Fermi surface so that degeneracy effects cannot cause a modification of the nondegenerate result. Thus, the degenerate and nondegenerate  $s(x)$  must be identical for  $|x| \gg E_F/T$  apart from a multiplicative factor which arises because of our normalization,  $s(0)=1$ .

Explicit calculations of  $s(x)$  for various assumptions concerning the neutron interaction potential and for various degrees of neutron degeneracy are left for a future study [10].

### D. Physical estimate of $\Gamma_\sigma$

It will turn out that the  $\nu n$  scattering cross-section reduction is primarily sensitive to the neutron spin-fluctuation rate  $\Gamma_\sigma$ . Therefore, it is useful to understand on physical grounds its overall magnitude and its scaling with temperature and density. To this end we assume that neutrons scatter with a velocity-independent cross section  $\sigma_n$  which is caused by a spin-dependent force such that the neutron spin is flipped in a typical collision. If the interaction is approximated by an OPE potential, on dimensional grounds the cross section is estimated to be  $\sigma_n \approx \alpha_\pi^2/m^2$ . We will assume that the scattering is either due to a random collection of external scattering centers with a density  $n_c$ , or due to collisions with the other neutrons with a density  $n_B$ .

If the neutrons are nondegenerate they move with a typical thermal velocity  $v \approx (3T/m)^{1/2}$ . By assumption, the spin-fluctuation rate is roughly equivalent to the collision rate so that  $\Gamma_\sigma \approx n_c \langle \sigma_n v \rangle \approx n_c \sigma_n (3T/m)^{1/2}$ . With the above estimate for  $\sigma_n$  and with the other neutrons being the scattering centers ( $n_c = n_B$ ), one finds that  $\Gamma_\sigma$  scales as  $\alpha_\pi^2 T^{1/2} m^{-5/2}$ . This agrees with an explicit calculation which yields  $4\sqrt{\pi}$  in the numerical factor [2].

Next, we consider degenerate neutrons for which a typical velocity is  $p_F/m$ . If they interact with external scattering centers, the collision rate for neutrons near the Fermi surface

is about  $n_c \sigma_n p_F/m$ . However, only the scattering of neutrons with an energy  $E$  within about a distance  $T$  from the Fermi surface is not blocked by degeneracy effects. This is an approximate fraction,  $T/E_F = 1/\eta$ , of all neutrons. Therefore, the spin-fluctuation rate averaged over all neutrons is  $\Gamma_\sigma \approx n_c \sigma_n (p_F/m)(T/E_F)$ .

Finally, if the scattering is among degenerate neutrons we have  $n_c = n_B$  and a typical relative velocity  $p_F/m$ . The average collision rate among neutrons is reduced by several factors of  $\eta^{-1} = T/E_F$ . Two such factors arise because each initial-state neutron must have an energy within about  $T$  of the Fermi surface. One further factor arises because one final-state neutron must also lie near the Fermi surface; energy-momentum conservation then ensures that the other one satisfies this condition as well. Altogether, we thus find  $\Gamma_\sigma \approx \sigma_n n_B (p_F/m)(T/E_F)^3$ . With  $n_B = p_F^3/3\pi^2$  and  $\sigma_n \approx \alpha_\pi^2/m^2$  we thus recover Eq. (11) apart from the numerical coefficient. This  $\Gamma_\sigma$  is the spin-fluctuation rate averaged over all neutrons. The spin-fluctuation rate of those neutrons which lie near the Fermi surface is larger by a factor  $\eta$ .

### III. CROSS-SECTION REDUCTION

#### A. General result

We may now proceed to calculate the  $\nu n$  scattering cross section in the presence of spin fluctuations of the degenerate neutrons. To this end we begin with the total axial current-scattering cross section  $\sigma_A$  of a neutrino with energy  $\epsilon_1$ . In the structure-function language it is the  $d\epsilon_2$  integral of the axial part of Eq. (2) or, equivalently,

$$\sigma_A = \frac{3C_A^2 G_F^2}{4\pi} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_\sigma(\omega) (\epsilon_1 - \omega)^2 \Theta(\epsilon_1 - \omega). \quad (13)$$

The problem with this expression is that it diverges if one uses the perturbative expression  $S_\sigma^{\text{brems}}(\omega)$  instead of the full but unknown  $S_\sigma(\omega)$ . Following the treatment of the nondegenerate case [3] we note, however, that Eq. (13) can still be evaluated on the basis of  $S_\sigma^{\text{brems}}(\omega)$  without knowledge of the detailed low- $\omega$  behavior if one includes the sum rule equation (8).

To this end we note that for degenerate free neutrons the  $\nu n$  scattering cross section is  $\sigma_{A,\text{free}} = (3/2\eta)(3/4\pi)C_A^2 G_F^2 \epsilon_1^2$  as stressed after Eq. (9). Therefore, the interaction-induced modification  $\delta\sigma_A \equiv \sigma_A - \sigma_{A,\text{free}}$  is given by

$$\frac{\delta\sigma_A}{\sigma_{A,\text{free}}} = -1 + \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{2\eta S_\sigma(\omega)}{3} \frac{(\epsilon_1 - \omega)^2 \Theta(\epsilon_1 - \omega)}{\epsilon_1^2}. \quad (14)$$

Then, we may proceed analogously to Ref. [3] and replace  $-1$  with an integral over the structure function by virtue of the sum rule equation (8),

$$\frac{\delta\sigma_A}{\sigma_{A,\text{free}}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{2\eta S_\sigma(\omega)}{3} \left[ \frac{(\epsilon_1 - \omega)^2 \Theta(\epsilon_1 - \omega)}{\epsilon_1^2} - 1 \right]. \quad (15)$$

For small  $\omega$  the integrand varies effectively as  $\omega^2 S_\sigma(\omega)$  because the term linear in  $\omega$  switches sign at the origin. Therefore, to lowest order we may substitute  $S_\sigma(\omega) \rightarrow S_\sigma^{\text{brems}}(\omega)$ , provided we interpret the remaining integral by its principal part.

This result becomes more transparent if we consider the reduction of an average cross section rather than one for a fixed initial-state neutrino energy. To this end we use a Maxwell-Boltzmann distribution of neutrino energies at the same temperature  $T$  which characterizes the ambient neutron medium. The thermally averaged free cross section is found to be  $\langle \sigma_{A,\text{free}} \rangle = (3/2\eta)(9/\pi)C_A^2 G_F^2 T^2$ . Because Eq. (15) is fully analogous to the corresponding result of Ref. [3] apart from an overall factor  $2\eta/3$ , we may conclude without further calculations that

$$\frac{\delta\langle \sigma_A \rangle}{\langle \sigma_{A,\text{free}} \rangle} = -\frac{2\eta}{3} \int_0^\infty \frac{dx}{2\pi} \tilde{S}_\sigma(x) G(x), \quad (16)$$

where  $x = \omega/T$ ,

$$G(x) = 1 - (1 + x + \frac{1}{6}x^2)e^{-x} = \frac{1}{3}x^2 + \mathcal{O}(x^3), \quad (17)$$

and  $\tilde{S}_\sigma(x) \equiv TS_\sigma(xT)$ .

As in Ref. [3] the  $x^2$  behavior of  $G(x)$  at small  $x$  allows us to replace  $\tilde{S}_\sigma(x)$  to lowest order with the perturbative  $\tilde{S}_\sigma^{\text{brems}}(x)$ . Therefore, with the representation equation (10) and with  $\gamma_\sigma \equiv \Gamma_\sigma/T$ , we find for the cross-section reduction

$$\frac{\delta\langle \sigma_A \rangle}{\langle \sigma_{A,\text{free}} \rangle} = -\frac{2\eta}{3} \frac{\gamma_\sigma}{2\pi} \int_0^\infty dx x^{-2} G(x) s(x). \quad (18)$$

The integral expression is  $5/6$  for the classical approximation  $s(x) = 1$ . In general, the integral will be a numerical expression of order unity. Its precise value for a variety of assumptions concerning the cause for the neutron spin fluctuations will be studied elsewhere [10].

Equation (18) shows that the expansion parameter which defines the perturbative regime is  $2\eta\gamma_\sigma/3$ , as opposed to the nondegenerate case where it was found to be  $\gamma_\sigma$ . In both cases,  $\gamma_\sigma$  is defined to be the spin-fluctuation rate averaged over all neutrons of the medium. However, in the degenerate case only the neutrons near the Fermi surface participate in collisions; it is *their* spin-fluctuation rate which reduces the  $\nu n$  scattering cross section. The quantity  $2\eta\gamma_\sigma/3$  corrects for this effect. It is to be interpreted as an effective spin-fluctuation rate for the neutrons near the Fermi surface, in agreement with our estimates of Sec. II D.

We conclude that a ‘‘naive’’ perturbative calculation of neutrino interaction rates in a degenerate neutron medium is possible if  $\eta\Gamma_\sigma \ll T$  while a significant correction arises if  $\eta\Gamma_\sigma \gtrsim T$ . This latter case corresponds to a situation where the collision rate of neutrons near the Fermi surface is not small relative to  $T$ , in violation of Landau’s condition for the applicability of a Fermi-liquid treatment.

In the nondegenerate case it was reasonable to extrapolate the behavior of the cross section  $\langle \sigma_A \rangle$  into the nonperturbative regime by virtue of an explicit ansatz for the low- $\omega$  behavior of  $S_\sigma(\omega)$  which incorporated the equivalent of the

sum rule equation (8). In the present case such an extrapolation is far more problematic because the derivation of the sum rule itself was based on the assumption that neutrons can be treated as quasiparticles which follow a thermal Fermi-Dirac distribution. In the nonperturbative regime this assumption is not justified so that in the present case the sum rule has a weaker standing than it did in the nondegenerate case where we did not need to invoke the anticommutation relations for the nucleon fields.

### B. Numerical result for OPE potential

If neutron-neutron collisions are the primary cause for neutron spin fluctuations, and if one models the interaction by an OPE potential, we may use Eq. (11) to estimate  $\Gamma_\sigma$ . In this case we find

$$\frac{2\eta}{3} \frac{\Gamma_{\sigma,\text{OPE}}}{T} = \frac{4\pi}{3} \alpha_\pi^2 \frac{T}{m} = 1.00 \frac{T}{\text{MeV}}, \quad (19)$$

where we have used the vacuum neutron mass for the numerical estimate. This result does not depend on the density (or Fermi momentum) which fortuitously cancels as explained by the physical arguments in Sec. II D. If the neutron spin fluctuations were caused by the interaction with a distribution of external scattering centers,  $\Gamma_\sigma$  would depend on their density as well as on the neutron Fermi momentum.

Of course, if neutron-neutron interactions are the primary cause for neutron spin fluctuations one would also expect significant spin-spin correlations which we have ignored. However, in order to study spin-spin correlations in the framework of a Fermi liquid theory one would need to assume that Landau's condition is satisfied which is not the case in any obvious sense when  $\eta\Gamma_\sigma \gtrsim T$ . Therefore, we believe that the usual calculations of neutrino opacities in hot degenerate neutron-star matter are applicable only for  $T \lesssim 1$  MeV.

### IV. SUMMARY

We have derived an expression for the  $\nu n$  scattering cross-section reduction in degenerate neutron matter caused by neutron spin fluctuations. We have used the linear-response theory approach of Ref. [3], but undoubtedly one would reach the same result by the direct perturbative method of Ref. [4].

In a neutron star, these spin fluctuations will be caused by a spin-dependent  $nn$  interaction potential. Therefore, in general, spin-spin correlations will also be important which may cause further modifications of the scattering cross section. Many of the previous papers which deal with weak interaction rates in neutron stars [5], were dedicated to an analysis of these latter effects. We stress, however, that these calculations were based on the assumption that Landau's condition is satisfied which is roughly equivalent to the requirement that the autocorrelation function of a single nucleon spin near the Fermi surface is narrow on a scale set by the ambient temperature  $T$ .

If neutron-neutron interactions are modeled by a one-pion exchange potential we estimate that the usual perturbative calculations are justified for  $T \lesssim 1$  MeV, a temperature which is reached very quickly in a neutron star after formation. Therefore, the long-term cooling history remains unaffected. Of course, a calculation of the long-term cooling history does not require knowledge of the neutrino opacity anyway as at late times neutrinos are no longer trapped. Roughly speaking, then, the neutrino opacities matter only for a short period after formation of a neutron star. However, precisely for this period the  $\nu n$  scattering rate cannot be calculated by straightforward perturbative techniques on the basis of first principles.

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