String evolution in open universes

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The velocity-dependent ''one-scale'' model of Martins and Shellard is used to study the evolution of a cosmic string network (and the corresponding loop population) in open universes. It is shown that in this case there is no linear scaling regime and that even though curvature still dominates the dynamics, at late times strings become the main component of the universe. We also comment on the possible consequences of these results. [S0556-2821(97)01008-4]

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Despite the strong theoretical prejudices favoring a flat universe [1,2], there is a fair amount of observational data which suggests the possibility of an open universe, with a present density that could be as low as $\Omega \sim 0.3$—notably, the so-called ''age problem'' [3] and recent measurements of the baryonic content of x-ray clusters [4]. Since in a low-Ω universe structures collapse earlier, observations of galaxies at high redshift would also be easier to explain if the universe is open. It is therefore appropriate to consider how some of the standard cosmological scenarios would change if this possibility turns out to be true.

One relevant case is that of the evolution of a network of cosmic strings [5]. It has been shown [6,7] that defect models normalized to the Cosmic Background Explorer (COBE) in an open universe predict a galaxy power spectrum consistent with that inferred from galaxy surveys without requiring an extreme bias (in general, $\Omega = 1$ models predict more small-scale power than low-Ωh ones). However, these results were established either using *a priori* scaling assumptions for the string network [6] or numerical simulations of texture evolution [7]. Here we study the evolution of a cosmic string network in open universes, using the velocity-dependent ''one-scale'' model of Martins and Shellard [8,9], which provides the first quantitative description of the complete evolution of the large-scale properties of a cosmic string network. This is briefly summarized below, and used to obtain the evolutionary properties of both the long-string and the loop populations in an open universe; these are then compared with the standard flat universe case, and some implications of these results are discussed.

Two different but complementary approaches have been used to study cosmic string evolution. The simplest (although more expensive) is to use large numerical simulations [10]. Among other interesting things, these revealed a significant amount of small-scale structure (or ''wiggles'') on the strings, containing up to one half of the total string energy.

On the other hand, there is always the possibility of using analytic methods—an approach first used by Kibble [11]. Because of the strings' statistical nature, what one really does is ''string thermodynamics,'' that is describing the network by a small number of macroscopic (or ''averaged'') quantities whose evolution equations are derived from the microscopic string equations of motion, and introducing additional ''phenomenological parameters'' if necessary. The first such model providing a quantitative picture of the complete evolution of a string network (and the corresponding loop population) has been recently developed by Martins and Shellard [8,9]; this has the added advantage of being equally applicable to the study of vortex-string evolution in a condensed matter context [12]. Here we will present a very brief description of this model—the reader is referred to the original paper [9] for further details.

Apart from the straightforward definition of the energy of a piece of string, $E = \mu a(\tau) \int \epsilon d\sigma$ (ϵ being the coordinate energy per unit σ), the only other macroscopic quantity in this model is the string rms velocity, defined by

$$v^2 = \int \dot{\mathbf{x}}^2 \epsilon d\sigma / \int \epsilon d\sigma . \tag{1}$$

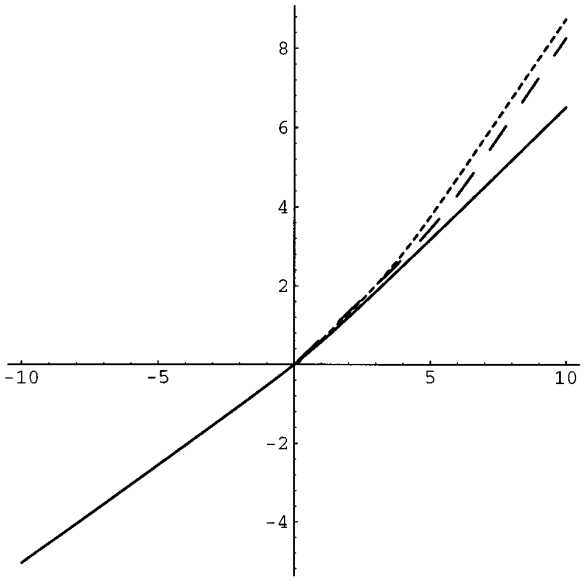

FIG. 1. Log-log plot of the scale factor a [relative to $a(t_{\rm eq}) = 1$] as a function of cosmic time t (relative to $t_{\rm eq}$) for a flat universe (solid line) and open universes with a present density $\Omega_0 = 0.3$ (dashed line) and $\Omega_0 = 0.1$ (dotted line); both of these have $h = 0.6$.

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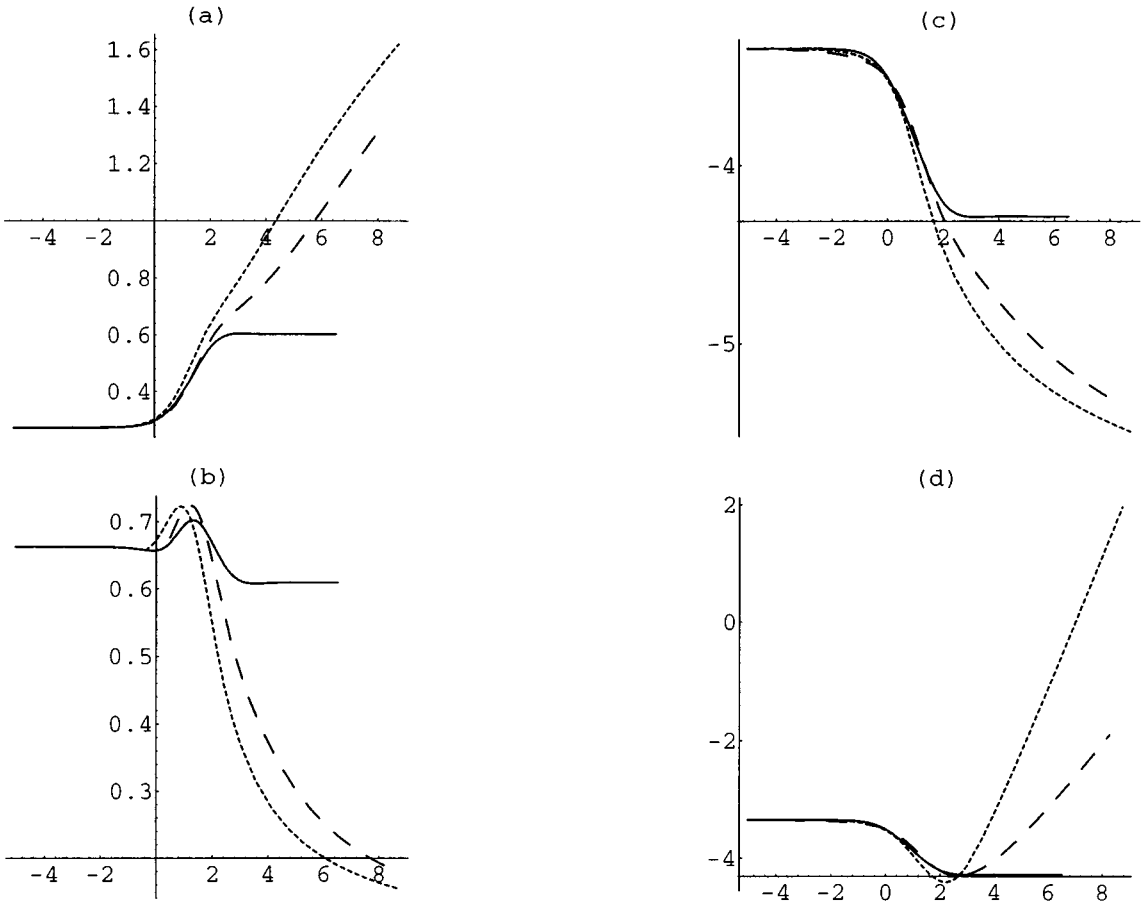

FIG. 2. Properties of a GUT long-string network in a flat universe (solid) and in open universes with a present density $\Omega_0=0.3$ (dashed) and $\Omega_0=0.1$ (dotted), with $h=0.6$ in both cases. Plots represent the ratio L/t (a), the rms string velocity (b), and the log of the ratio of the long-string density to the critical (c) and the background (d) densities. The horizontal axis is labeled in terms of the logarithm of the scale factor [with $a(t_{\rm eq})=1$]; all plotted curves span the period between $10^{-10}t_{\rm eq}$ and $10^{10}t_{\rm eq}$.

Explicitly distinguishing between long (or ''infinite'') strings and loops, we can use the fact that the former should be Brownian to define the long-string correlation length as $\rho_\infty \equiv \mu/L^2$. Phenomenological terms must be included for the interchange of energy between long strings and loops. A ''loop chopping efficiency'' parameter (expected to be slightly smaller than unity) is introduced to characterize loop production

$$(d\rho_\infty/dt)_{\rm to\ loops}=\tilde{c}v_\infty\rho_\infty/L\,, \tag{2}$$

and in the particular case of grand-unified-theory (GUT)-scale strings (but not more generally [9]) it can be safely assumed that loop reconnections onto the long-string network are negligible. In particular, this has been confirmed in numerical simulations [10]. Note that it is conceivable that the behavior of $\tilde{c}$ is different in flat and open universes. However, this effect will not be crucial, because the scaling properties do not depend strongly on it [9].

It is then simple to derive the evolution equation for the correlation length L. Since we are only interested in the epoch around radiation-matter equality, we need not be considering frictional forces [9], and we simply have

$$2\ dL/dt=2HL(1+v_\infty^2)+\tilde{c}v_\infty\,. \tag{3}$$

For the case of string loops, the relevant length scale is simply the loop length, which decays due to gravitational radiation, and its evolution equation is

$$d\ell/dt=(1-2v_\ell^2)H\ell-\Gamma' G\mu v_\ell^6, \tag{4}$$

where $\Gamma'\sim 8\times 65$. Then the only other thing that is needed is an assumption on the loop size at formation. In the epoch relevant to this paper, we expect it to be approximately constant and much smaller than the correlation length—we will take $\ell_i=10^{-3}L(t_i)$. Then for any given time, one only has to look at the loops that have formed until then, determine which of them are still around, and add up their lengths to determine the total energy density in the form of loops. This is conveniently expressible in terms of the ratio of the energy densities in loops and long strings

$$\varrho(t)\equiv\frac{\rho_o(t)}{\rho_\infty(t)}=g\tilde{c}L^2(t)\int_{t_c}^{t}\frac{a^3(t')}{a^3(t)}\,\frac{v_\infty(t')}{L^4(t')}\,\frac{\ell(t,t')}{\alpha(t')}\,dt', \tag{5}$$

where g is a Lorentz factor accounting for the fact that loops are usually produced with a nonzero center-of-mass velocity, α is the loop size at formation relative to the correlation length at that time and $\ell(t,t')$ is the length at time t of a loop that was formed at a time t'.

Finally, one can derive an evolution equation for the long string or loop velocity with only a little more than Newton's second law

$$\frac{dv}{dt}=(1-v^2)\left(\frac{k}{R}-2Hv\right); \tag{6}$$

here k is another phenomenological parameter that is related to the presence of small-scale structure on the strings; an appropriate ansatz for it is (refer to [9] for a complete justification)

$$k=\begin{cases}1, & 2HR>\chi,\\ \dfrac{1}{\sqrt{2}}2HR, & 2HR<\chi,\end{cases} \tag{7}$$

where R is the curvature radius of the string (that is, $R=L$ for long strings, but $\ell=2\pi R$ for loops) and χ is a numerically determined coefficient of order unity, whose precise value depends on whether one is using the above ansatz for long strings or loops—see [9] for a complete discussion of this point.

The above quantities are sufficient to quantitatively describe the large-scale characteristics of a cosmic string network around the epoch of equal matter and radiation densities (see [8]). In a more general situation one would need to include the effect of frictional forces [9] due to particle scattering on the strings.

We now come to the issue of this paper. As we already pointed out, we only need to study the behavior of the string network in the transition between the radiation- and the matter-dominated regimes (see Martins and Shellard [9] for a detailed discussion of the early stages of evolution of GUT scale and other cosmic string networks).

It is straightforward to see that there is one crucial difference with respect to the case of a flat universe: in an open universe there will no longer be a linear scaling regime. This arises naturally from the fact that in a universe where the scale factor grows as $a\propto t^{\lambda}$ (with $\lambda<1$) the linear regime has the properties

$$\frac{L}{t}=\left[\frac{k(k+\tilde{c})}{4\lambda(1-\lambda)}\right]^{1/2}, \tag{8}$$

$$v=\left[\frac{k(1-\lambda)}{\lambda(k+\tilde{c})}\right]^{1/2}. \tag{9}$$

In an open universe the ''effective'' λ is a variable, increasing from $\lambda=1/2$ in the radiation era to an asymptotic value of $\lambda=1$ (see Fig. 1). For example, if we happen to live in a universe where $\Omega_0\sim 0.3$, we have $\lambda_0\sim 0.8$ today, and $\lambda_0\sim 0.9$ for $\Omega_0\sim 0.1$. In other words, there will be corrections to the simple linear behavior, such that the correlation length L will grow slightly faster than t. Nevertheless, since the horizon size for an $a\propto t^{\lambda}$ universe is

$$d_H=t/(1-\lambda), \tag{10}$$

one can easily show that L will always be smaller than the horizon. On the other hand, the string velocity will decrease with time, and therefore loop formation will gradually switch off.

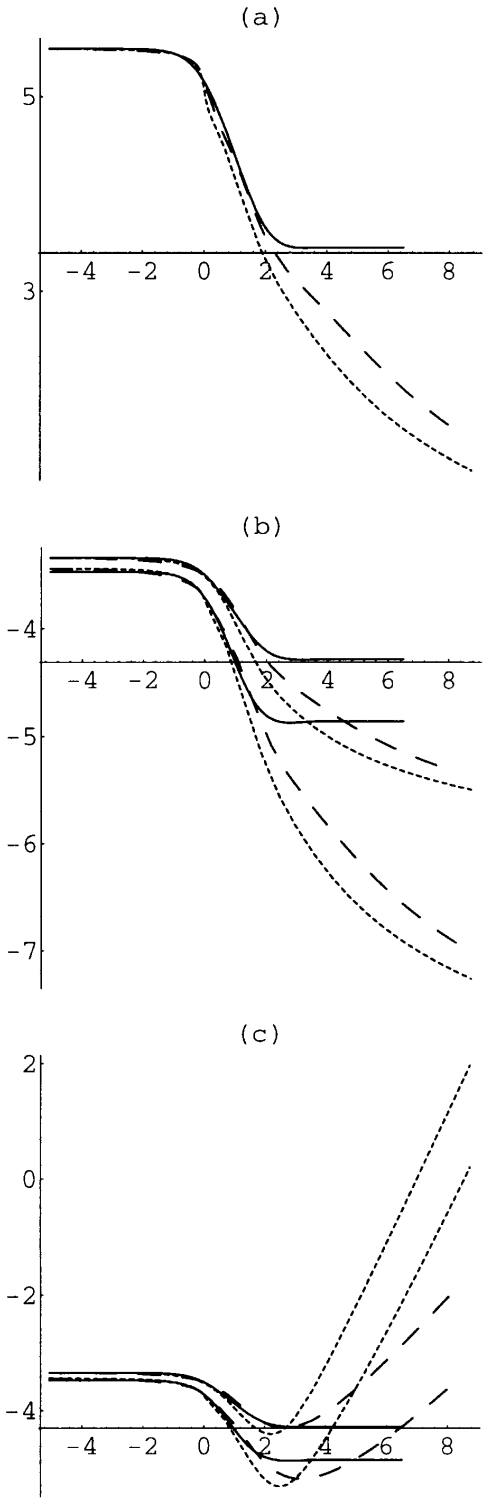

FIG. 3. Properties of the loop population of a GUT string network in a flat universe (solid) and in open universes with a present density $\Omega_0=0.3$ (dashed) and $\Omega_0=0.1$ (dotted), both having $h=0.6$. Plot (a) depicts the log of the number of loops produced per Hubble volume per Hubble time, while (b) shows the log of the ratio of the long-string (upper curve of each pair) and loop (lower curve of each pair) densities to the critical (b) and the background (c) densities. The horizontal axis is labeled in terms of the logarithm of the scale factor [with $a(t_{\rm eq})=1$]; all plotted curves span the period between $10^{-10}t_{\rm eq}$ and $10^{10}t_{\rm eq}$.

Note that the power-law dependence of the scale factor obviously changes in the transition between radiation and matter domination, even in the case of a flat universe. In particular, there will of course be a departure from linear scaling while the transition is taking place, with L/t growing from 0.27 to 0.6 (approximately).

In particular, we can easily find the solution to the averaged equations of motion in the limit, where $\lambda=1$,

$$L \propto t(\ln t)^{1/2}, \quad v \propto (\ln t)^{-1/2}; \tag{11}$$

note that in this limit the correlation length grows more slowly than the horizon (which goes like $d_H \propto t \ln t$).

In order to get quantitative results, we must solve the averaged evolution equations described earlier numerically. To these we must add a further equation—the Friedmann equation—specifying how the scale factor (and hence the Hubble parameter) evolves in the transition between the radiation- and the matter-dominated epochs:

$$H^2+\frac{K}{a^2}=\frac{8\pi G}{3}(\rho_{\rm rad}+\rho_{\rm mat}+\rho_{\rm string}), \tag{12}$$

where $K=0$ for a flat universe and $K=-1$ for an open universe; note that we should at least consider the possibility of the string density becoming a non-negligible source for the Friedmann equation.

Figures 2 and 3 contrast the evolution of the long-string and loop populations in a flat universe and in open universes with $\Omega_0=0.3$ and $\Omega_0=0.1$ (we have assumed that $h=0.6$ in both cases); note that the present epoch corresponds to $a_0/a_{t_{\rm eq}} \sim 2.3\times 10^4 \Omega_0 h^2$.

As was first discussed in [9], even in the case of a flat universe the transition from the radiation to the matter epoch is a very slow process, lasting about eight orders of magnitude in time. In the case of an open universe, apart from the differences we already expected, the most interesting result is that, although the string density always decreases with respect to the critical density (as one would also expect), at a redshift around $z \sim \Omega_0^{-1}$ (which is approximately when curvature has started to dominate the dynamics) the string density has started to grow relative to the background density. In fact, in a $\Omega_0=0.3$ universe strings will become the main component of the universe in about seven orders of magnitude in time, whereas if we had $\Omega_0=0.1$ this would only take about four orders of magnitude in time.

It is also interesting to point out that, although having $\Omega_0=0.3$ or $\Omega_0=0.1$ does not yield very significant differences in the values of the long-string correlation length or velocity, it does produce very significant differences in the ratio of the long-string and loop densities to that of the background.

Moreover, one should note that despite the significant drop in the number of loops produced [see Fig. 3(a)], the ratio of the energy densities in loops and long strings decreases rather more slowly. This is because loops are slightly larger at formation and, since the average long-string velocity is decreasing, so is that of large enough loops (note that we assume that the rms loop velocity at formation is equal to the rms long-string velocity at that time). Consequently, loops will live longer, since the redshift and gravitational radiation terms in Eq. (4) are velocity dependent.

In this paper we presented the first discussion of cosmic string evolution in open universes, in the context of the generalized ''one-scale'' model of Martins and Shellard [9]. We have shown that there is no linear scaling regime in an open universe, and that although the string density always decreases with respect to the critical density, it has been increasing relative to that of the background from $z \sim \Omega_0^{-1}$, and it will become the main component of the universe sometime in the future.

These differences with respect to the standard (flat universe) case only become significant fairly late in the matter-dominated epoch, so with respect to the string-seeded structure formation scenario we should only expect changes on very large scales—that one can easily estimate to be larger than the scales of the largest existing surveys.

On the other hand, such large scales are of course relevant when one is comparing the cosmic microwave background anisotropies produced by cosmic strings to COBE data—thereby normalizing the string mass per unit length [13]—since this essentially involves an integration from the present time to the surface of last scattering. Thus the changes in the string network properties discussed in the present paper can significantly alter this normalization. This issue will be discussed in a forthcoming publication.

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