Studies of neutrino asymmetries generated by ordinary-sterile neutrino oscillations in the early Universe and implications for big bang nucleosynthesis bounds

R. Foot and R. R. Volkas
Research Centre for High Energy Physics, School of Physics, University of Melbourne, Parkville, 3052 Australia

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Ordinary-sterile neutrino oscillations can generate a significant lepton number asymmetry in the early Universe. We study this phenomenon in detail. We show that the dynamics of ordinary-sterile neutrino oscillations in the early Universe can be approximately described by a single integrodifferential equation which we derive from both the density matrix and Hamiltonian formalisms. This equation reduces to a relatively simple ordinary first-order differential equation if the system is sufficiently smooth (static limit). We study the conditions for which the static limit is an acceptable approximation. We also study the effect of the thermal distribution of neutrino momenta on the generation of lepton number. We apply these results to show that it is possible to evade (by many orders of magnitude) the big bang nucleosynthesis (BBN) bounds on the mixing parameters δm^2 and $\sin^2 2\theta_0$ describing ordinary-sterile neutrino oscillations. We show that the large angle or maximal vacuum oscillation solution to the solar neutrino problem does not significantly modify BBN for most of the parameter space of interest, provided that the τ and/or μ neutrinos have masses greater than about 1 eV. We also show that the large angle or maximal ordinary-sterile neutrino oscillation solution to the atmospheric neutrino anomaly does not significantly modify BBN for a range of parameters. [S0556-2821(97)05908-0]

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I. INTRODUCTION

There are three main experimental indications that neutrinos have mass and oscillate. They are the solar neutrino problem [1], the atmospheric neutrino anomaly [2], and the Los Alamos Liquid Scintillation Neutrino Detector (LSND) experiment [3]. It is also possible that dark matter may be connected to neutrino masses [4]. The three experimental anomalies may not all be explained with the three known neutrinos so it is possible that sterile neutrinos exist.

A potential problem with any model which contains sterile neutrinos is that these extra states can contribute to the energy density of the early Universe and spoil the reasonably successful big bang nucleosynthesis (BBN) predictions. For maximally mixed ν_e and ν_e' neutrinos and ν_μ and ν_μ' (or ν_τ and ν_τ') neutrinos (where the primes denote sterile species), the following rather stringent BBN bounds have been obtained [5–8] *assuming that the lepton number asymmetry of the early Universe could be neglected*:

$$|\delta m^2_{ee'}| \lesssim 10^{-8}\ \text{eV}^2, \quad |\delta m^2_{\mu\mu'}|,\ |\delta m^2_{\tau\tau'}| \lesssim 10^{-6}\ \text{eV}^2. \tag{1}$$

Observe that if valid these bounds would rule out the large angle ν_μ-ν_μ' oscillation solution to the atmospheric neutrino anomaly and would restrict much of the parameter space for the maximal oscillation solution of the solar neutrino problem [9,10]. However, these bounds do not hold if there is an appreciable lepton asymmetry in the early Universe for temperatures between $1-30$ MeV [11]. Remarkably, it turns out that ordinary-sterile neutrino oscillations can by themselves create an appreciable lepton number asymmetry [12].

The bound on the effective number of neutrinos N_ν^{eff} present during nucleosynthesis is the subject of some discussion recently. In Ref. [13], it is argued that the current information suggests $N_\nu^{\text{eff}} \simeq 2.1 \pm 0.3$, while other authors dispute this conclusion. For example, in Refs. [14–16], the upper limits $N_\nu^{\text{eff}} < 3.9,\ 4.5,\ 4.0$ are respectively derived. Thus, it may be possible that $N_\nu^{\text{eff}} = 4$ is allowed. In this case note that many of the BBN bounds derived in Refs. [5–8], including the bounds quoted in Eq. (1), need not apply. However, for the present paper we will assume that the bound on the effective number of neutrinos is less than 4. This is useful even if it turns out that $N_\nu^{\text{eff}} > 4$ is allowed. For example, the large angle (or maximal) ordinary-sterile neutrino solutions to the atmospheric and solar neutrino problems may require $N_\nu^{\text{eff}} \sim 5$ if they are to be solved simultaneously. Also note that in the special case of mirror neutrinos [17], the mirror interactions can potentially bring all three mirror neutrinos (as well as the mirror photon and electron-positron pair) into equilibrium (equivalent to about six additional neutrino species) if any one of the mirror neutrinos is brought into equilibrium above the neutrino kinetic decoupling temperature.

The purpose of this paper is twofold. First, we will study the phenomenon of lepton number creation due to ordinary-sterile neutrino oscillations in more detail than in the previous studies [12,18]. For example, we will study the effect of the thermal distribution of neutrino momenta. Using these results we will then study the issue of whether or not the generation of lepton number due to ordinary-sterile neutrino oscillations can reconcile the large angle ordinary-sterile neutrino oscillation solutions to the solar neutrino problem and atmospheric neutrino anomaly with BBN.

The outline of this paper is as follows. In Sec. II, we discuss lepton number generation in the early Universe by ordinary-sterile neutrino oscillations and derive a simple equation describing the evolution of lepton number. We expand the analysis of [12] and discuss in detail the approximations behind this analysis. In Sec. III, we will use the density matrix formalism to derive a more exact equation describing the rate of change of lepton number which is applicable even when the system is changing rapidly (e.g., at the resonance). In the Appendix, we show how the same equation can be derived from the Hamiltonian formalism. Using this equation we derive the region of parameter space where the much simpler equation derived in Sec. II is approximately valid. In Sec. IV the thermal distribution of the neutrino momenta is considered. In Sec. V we study the effect of non-negligible sterile neutrino number densities. We then apply these results to obtain the region of parameter space where large neutrino asymmetries are generated. We also determine the region of parameter space for which ordinary-sterile neutrino oscillations (with $\delta m^2<0$ and for $|\delta m^2| \gtrsim 10^{-4}$ eV^2) are consistent with BBN. Our work improves on previous studies [5–8], because these studies were obtained without taking into account either the neutrino momentum distribution or the result that ordinary-sterile neutrino oscillations create lepton number. In Sec. VI we first briefly review the large angle ordinary-sterile neutrino oscillation solution to the solar neutrino problem. We then show that the generation of lepton number due to ordinary-sterile neutrino oscillations can significantly relax the BBN bounds for this solution to the solar neutrino problem. We also show that the large angle or maximal ν_μ-ν_s oscillation solution to the atmospheric neutrino anomaly is consistent with BBN for a range of parameters. In Sec. VII we conclude.

II. LEPTON NUMBER CREATION FROM NEUTRINO OSCILLATIONS: STATIC APPROXIMATION

Together with Thomson, we showed in [12] that ordinary-sterile neutrino oscillations can create a large lepton asymmetry in the early Universe [19]. A simple differential equation describing the evolution of the lepton number was derived which seemed to work very well. We also checked our results with the more exact density matrix formalism [21]. Further numerical work, and analytical work based on the density matrix formalism, has subsequently been done in [18] which confirms our results.

For ordinary-sterile neutrino two-state mixing, the weak eigenstates (ν_α, ν_s) will be linear combinations of two mass eigenstates (ν_a, ν_b):

$$\nu_\alpha = \cos\theta_0 \nu_a + \sin\theta_0 \nu_b, \quad \nu_s = -\sin\theta_0 \nu_a + \cos\theta_0 \nu_b. \tag{2}$$

Note we will always define θ_0 in such a way so that $\cos 2\theta_0 \geqslant 0$ (this can always be done). We also take the convention that $\delta m^2_{\alpha s} \equiv m_b^2 - m_a^2$. Hence with this convention $\delta m^2_{\alpha s}$ is positive (negative) provided that $m_b > m_a$ $(m_b < m_a)$.

In this section we will for simplicity neglect the effects of the thermal distribution of momentum, and assume that all of the neutrino momenta are the same and equal to the average momentum (i.e., $p = \langle p \rangle \simeq 3.15T$). In Sec. IV we will consider the realistic case where the neutrino spread is given by the Fermi-Dirac distribution. Following [12], we can derive a simple equation for the rate of change of lepton number due to collisions and oscillations. Note that it is possible to identify two distinct contributions to the rate of change of lepton number. First, there are the oscillations between collisions which affect the lepton number of the Universe because neutrinos and antineutrinos oscillate with different matter oscillation lengths and matter mixing angles in the CP asymmetric background. Second, there are the collisions themselves which deplete ν_α and $\overline{\nu_\alpha}$ at different rates. This is because the rates depend on the oscillation probability. The oscillation probability for ordinary-sterile neutrino oscillations is different to the oscillation probability for ordinary-sterile antineutrino oscillations (which is again due to the CP asymmetric background). Generally, the rate of change of lepton number is dominated by collisions in the region where the collision rate is larger than the expansion rate [12]. (A possible exception to this is in the resonance region where the matter mixing angle changes rapidly.) For the case of ν_α-ν_s oscillations (where $\alpha = e, \mu, \tau$), the rate of change of L_{ν_α} due to collisions is governed by the rate equation

$$\begin{aligned}\frac{dL_{\nu_\alpha}}{dt} &= \frac{-n_{\nu_\alpha}}{n_\gamma}\Gamma(\nu_\alpha \to \nu_s) + \frac{n_{\overline{\nu_\alpha}}}{n_\gamma}\Gamma(\overline{\nu_\alpha} \to \overline{\nu_s}) \\ &\quad + \frac{n_{\nu_s}}{n_\gamma}\Gamma(\nu_s \to \nu_\alpha) + \frac{-n_{\overline{\nu_s}}}{n_\gamma}\Gamma(\overline{\nu_s} \to \overline{\nu_\alpha}),\end{aligned} \tag{3}$$

where the n's are number densities and $L_{\nu_\alpha} \equiv (n_{\nu_\alpha} - n_{\overline{\nu_\alpha}})/n_\gamma$ is the lepton number. Using $\Gamma(\nu_\alpha \to \nu_s) = \Gamma(\nu_s \to \nu_\alpha)$ and $\Gamma(\overline{\nu_\alpha} \to \overline{\nu_s}) = \Gamma(\overline{\nu_s} \to \overline{\nu_\alpha})$ (we will justify this in a moment), Eq. (3) simplifies to

$$\frac{dL_{\nu_\alpha}}{dt} = -\left[\frac{n_{\nu_\alpha} - n_{\nu_s}}{n_\gamma}\right]\Gamma(\nu_\alpha \to \nu_s) + \left[\frac{n_{\overline{\nu_\alpha}} - n_{\overline{\nu_s}}}{n_\gamma}\right]\Gamma(\overline{\nu_\alpha} \to \overline{\nu_s}). \tag{4}$$

This equation can be rewritten in the form

$$\begin{aligned}\frac{dL_{\nu_\alpha}}{dt} &\simeq (\mathcal{N}^+_{\nu_\alpha} - \mathcal{N}^+_{\nu_s})[-\Gamma(\nu_\alpha \to \nu_s) + \Gamma(\overline{\nu_\alpha} \to \overline{\nu_s})] \\ &\quad - (\mathcal{N}^-_{\nu_\alpha} - \mathcal{N}^-_{\nu_s})[\Gamma(\nu_\alpha \to \nu_s) + \Gamma(\overline{\nu_\alpha} \to \overline{\nu_s})],\end{aligned} \tag{5}$$

where

$$\mathcal{N}^{\pm}_{\nu_\alpha} \equiv \frac{n_{\nu_\alpha} \pm n_{\overline{\nu_\alpha}}}{2n_\gamma}, \quad \mathcal{N}^{\pm}_{\nu_s} \equiv \frac{n_{\nu_s} \pm n_{\overline{\nu_s}}}{2n_\gamma}. \tag{6}$$

Observe that ordinary-sterile neutrino oscillations do not change the total particle number, from which it follows that

$$\mathcal{N}^-_{\nu_\alpha} + \mathcal{N}^-_{\nu_s} = 0. \tag{7}$$

Using Eqs. (5)–(7), the rate of change of L_{ν_α} due to collisions is given by

$$\begin{aligned}\frac{dL_{\nu_\alpha}}{dt} &\simeq \left(\frac{3}{8} - \mathcal{N}^+_{\nu_s}\right)[-\Gamma(\nu_\alpha \to \nu_s) + \Gamma(\overline{\nu_\alpha} \to \overline{\nu_s})] \\ &\quad - L_{\nu_\alpha}[\Gamma(\nu_\alpha \to \nu_s) + \Gamma(\overline{\nu_\alpha} \to \overline{\nu_s})] + O(L^2_{\nu_\alpha}),\end{aligned} \tag{8}$$

where we have also used $n_{\nu_\alpha} + n_{\overline{\nu_\alpha}} \simeq 3n_\gamma/4 + O(L^2_{\nu_\alpha})$. We will assume for the present that negligible sterile neutrinos are produced, i.e., $n_{\nu_s}, n_{\overline{\nu_s}} \ll n_{\nu_\alpha}, n_{\overline{\nu_\alpha}}$, and hence $\mathcal{N}^+_{\nu_s} \ll 1$.

In order to work out the reaction rates, we can invoke a simple physical picture [22–24]. The oscillations of the neutrino between collisions produce a superposition of states. The collisions are assumed to collapse the wave function into either a pure weak eigenstate neutrino or a pure sterile eigenstate neutrino. In other words, we assume that the collisions are measurements (in the quantum mechanical sense) of whether the state is a sterile or weak eigenstate. The rate of the measurements is expected to be the collision frequency Γ_{ν_α}.

Actually it happens that the above picture is not completely correct. It turns out that it does lead to an accurate description only if the rate of measurement is taken to be *half* of the collision frequency that a pure ν_α state would experience [23]. This applies to both sterile neutrinos and ordinary neutrinos. Thus using this result the reaction rate $\Gamma(\nu_\alpha \to \nu_s)$ is given by *half* the interaction rate of the neutrino due to collisions with the background particles multiplied by the probability (averaged over the neutrinos in the ensemble) that the neutrino collapses to the sterile eigenstate, that is

$$\Gamma(\nu_\alpha \to \nu_s) = \frac{\Gamma_{\nu_\alpha}}{2} \langle P_{\nu_\alpha \to \nu_s} \rangle. \tag{9}$$

The thermally averaged collision frequencies Γ_{ν_α} are

$$\Gamma_{\nu_\alpha} \simeq y_\alpha G_F^2 T^5, \tag{10}$$

where $y_e \sim 4.0, y_{\mu,\tau} \simeq 2.9$ [6], G_F is the Fermi constant ($G_F \simeq 1.17 \times 10^{-11}$ MeV^{-2}), and T is the temperature of the Universe [equations analogous to Eqs. (9) and (10) hold for antineutrinos]. The quantity $P_{\nu_\alpha \to \nu_s}$ is the probability that a neutrino which started off being a pure weak eigenstate, ν_α, as a result of a measurement at time t^*, collapses to the sterile state ν_s when the next measurement is made at time t. The angular brackets denote the average over the interval time between measurements, τ ($\tau \equiv t - t^*$ is the time between measurements). Note that $P_{\nu_\alpha \to \nu_s} = P_{\nu_s \to \nu_\alpha}$, so it follows that $\Gamma(\nu_\alpha \to \nu_s) = \Gamma(\nu_s \to \nu_\alpha)$ (given that the rate of measurement is the same for ordinary and sterile neutrinos [23]) and similarly for the antineutrino rates. In the adiabatic limit,

$$\langle P_{\nu_\alpha \to \nu_s} \rangle \simeq \sin^2 2\theta_m \left\langle \sin^2 \frac{\tau}{2L_m} \right\rangle. \tag{11}$$

The quantities θ_m and L_m are the matter mixing angle and matter oscillation length, respectively. They are related to the vacuum parameters θ_0 and L_0 by [25,26]

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{1 - 2z\cos 2\theta_0 + z^2} \tag{12}$$

and

$$L_m = \frac{L_0}{\sqrt{1 - 2z\cos 2\theta_0 + z^2}}, \tag{13}$$

where $1/L_0 \equiv \Delta_0^p \equiv \delta m^2/2p$. In this equation, $z = 2pV_\alpha/\delta m^2$ where V_α is the effective potential due to the interactions of the neutrinos with matter and p is the neutrino momentum. The effective potential is given by [25]

$$V_\alpha = (-a^p + b^p)\Delta_0^p, \tag{14}$$

where the dimensionless variables a^p and b^p are given by

$$a^p \equiv \frac{-\sqrt{2} G_F n_\gamma L^{(\alpha)}}{\Delta_0^p}, \quad b^p \equiv \frac{-\sqrt{2} G_F n_\gamma A_\alpha T^2}{\Delta_0^p M_W^2} \frac{p}{\langle p \rangle}, \tag{15}$$

where $\langle p \rangle \simeq 3.15T$ is the average neutrino momentum, M_W is the W-boson mass and $A_e \simeq 55.0$, $A_{\mu,\tau} \simeq 15.3$ (note that the "p" superscript serves as a reminder that these quantities are neutrino momentum dependent). The function $L^{(\alpha)}$ is given by

$$L^{(\alpha)} = L_{\nu_\alpha} + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + \eta, \tag{16}$$

where η is a small asymmetry term which arises from the asymmetries of baryons and electrons. It is given by [25]

$$\eta = \left(\frac{1}{2} + 2\sin^2\theta_w\right) L_e + \left(\frac{1}{2} - 2\sin^2\theta_w\right) L_P - \frac{1}{2} L_N \simeq \frac{1}{2} L_N, \tag{17}$$

where $\sin^2\theta_w$ is the weak mixing angle and we have used $L_e = L_P \simeq L_N$. Thus η is expected to be of order 10^{-10}. Note that the matter mixing angle θ_m and oscillation length L_m for antineutrino oscillations are obtained from Eqs. (12)–(15) by performing the transformation $L^{(\alpha)} \to -L^{(\alpha)}$ [27].

We denote the thermal average of the variables a^p, b^p by $a \equiv \langle a^p \rangle$, $b \equiv \langle b^p \rangle$. From Eq. (15), they are given approximately by

$$a \simeq \frac{-6.3\sqrt{2} T G_F n_\gamma L^{(\alpha)}}{\delta m^2} \simeq -25 L^{(\alpha)} \left(\frac{\text{eV}^2}{\delta m^2}\right) \left(\frac{T}{\text{MeV}}\right)^4,$$

$$b \simeq \frac{-6.3\sqrt{2} T G_F n_\gamma A_e T^2}{\delta m^2 M_W^2} \simeq -\left(\frac{T}{13\ \text{MeV}}\right)^6 \left(\frac{\text{eV}^2}{\delta m^2}\right) \quad \text{for } \nu_e\text{-}\nu_s \text{ oscillations},$$

$$b \simeq \frac{-6.3\sqrt{2} T G_F n_\gamma A_{\mu,\tau} T^2}{\delta m^2 M_W^2} \simeq -\left(\frac{T}{16\ \text{MeV}}\right)^6 \left(\frac{\text{eV}^2}{\delta m^2}\right) \quad \text{for } \nu_{\mu,\tau}\text{-}\nu_s \text{ oscillations}, \tag{18}$$

where we have used $n_\gamma = 2\zeta(3)T^3/\pi^2 \simeq T^3/4.1$ [$\zeta(3)\simeq 1.202$ is the Riemann zeta function of 3]. The matter mixing angles θ_m, $\overline{\theta_m}$ expressed in terms of the parameters a,b are given by

$$\sin^2 2\theta_m = \frac{s^2}{[s^2+(b-a-c)^2]}, \quad \sin^2 2\overline{\theta_m} = \frac{s^2}{[s^2+(b+a-c)^2]}, \tag{19}$$

where $s\equiv\sin 2\theta_0$, $c\equiv\cos 2\theta_0$. A resonance occurs for neutrinos when $\theta_m=\pi/4$ and for antineutrinos when $\overline{\theta_m}=\pi/4$, which from Eq. (19) implies that $b-a=\cos 2\theta_0$ and $b+a=\cos 2\theta_0$, respectively. In our analysis we will often need to consider the two distinct cases of very small mixing and very large mixing. For small mixing, $\cos 2\theta_0\simeq 1$ and the resonance conditions become $b-a\simeq 1$ and $b+a\simeq 1$. For large mixing, $\cos 2\theta_0\simeq 0$ and the resonance conditions become $a\simeq b$ and $-a\simeq b$.

Using the above analysis, we can derive a simple equation for the rate of change of L_{ν_α}:

$$\begin{aligned}\frac{dL_{\nu_\alpha}}{dt} = {} & \frac{3}{16}\Gamma_{\nu_\alpha}\left[-\sin^2 2\theta_m \left\langle \sin^2\left(\frac{\tau}{2L_m}\right)\right\rangle + \sin^2 2\overline{\theta_m}\left\langle \sin^2\left(\frac{\tau}{2L_m}\right)\right\rangle\right] \\ & -\frac{L_{\nu_\alpha}\Gamma_{\nu_\alpha}}{2}\left[\sin^2 2\theta_m \left\langle \sin^2\left(\frac{\tau}{2L_m}\right)\right\rangle + \sin^2 2\overline{\theta_m}\left\langle \sin^2\left(\frac{\tau}{2L_m}\right)\right\rangle\right].\end{aligned} \tag{20}$$

The function, $\langle \sin^2(\tau/2L_m)\rangle$ is given by

$$\left\langle \sin^2\left(\frac{\tau}{2L_m}\right)\right\rangle = \frac{1}{\omega_0}\int_0^t e^{-\tau/\omega_0}\sin^2\left(\frac{\tau}{2L_m}\right)d\tau, \tag{21}$$

where $\omega_0\equiv 2\tau_0 = 2/\Gamma_{\nu_\alpha}$ is twice the mean time between collisions (of a pure weak eigenstate) and t is the age of the Universe (note that $t\simeq\infty$ is a good approximation because $\omega_0 \ll t$). Evaluating Eq. (21) we find

$$\left\langle \sin^2\left(\frac{\tau}{2L_m}\right)\right\rangle = \frac{1}{2}\left(\frac{\omega_0^2/L_m^2}{1+\omega_0^2/L_m^2}\right), \tag{22}$$

where we have assumed that ω_0 and L_m are approximately constant over the time scale ω_0 (static approximation) [28]. Thus, using Eqs. (22), (13), and (19), we can rewrite Eq. (20) in the form

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{3}{8}\frac{s^2\Gamma_{\nu_\alpha}a(c-b)}{[x+(c-b+a)^2][x+(c-b-a)^2]} + \Delta, \tag{23}$$

where Δ is a small correction term

$$\Delta = -\frac{1}{2}\frac{L_{\nu_\alpha}s^2\Gamma_{\nu_\alpha}[x+(c-b)^2+a^2]}{[x+(c-b+a)^2][x+(c-b-a)^2]}, \tag{24}$$

and x is given by

$$x = s^2 + \frac{1}{4}\Gamma_{\nu_\alpha}^2\left(\frac{2p}{\delta m^2}\right)^2 \simeq s^2 + 2\times 10^{-19}\left(\frac{T}{\text{MeV}}\right)^{12}\left(\frac{\text{eV}^2}{\delta m^2}\right)^2, \tag{25}$$

where we have assumed $p=\langle p\rangle\simeq 3.15T$ in deriving the last part of the above equation. Note that the correction term Eq. (24) is smaller than the main term [Eq. (23)] provided that $|L_{\nu_\alpha}|\ll|a|$. In the region where the correction term is larger than the main term, its effect is to reduce $|L_{\nu_\alpha}|$ such that $L_{\nu_\alpha}\to 0$. From Eq. (18), the condition $|L_{\nu_\alpha}|>|a|$ only occurs for quite low temperatures:

$$\frac{T}{\text{MeV}} \lesssim \frac{1}{3}\left(\frac{|\delta m^2|}{\text{eV}^2}\right)^{1/4}. \tag{26}$$

From the above equation, we see that in the main region of interest ($T\gtrsim 3$ MeV), the correction term is much smaller than the main term provided that $|\delta m^2|\lesssim 10^4$ eV^2. Note that for very large $|\delta m^2|\gtrsim 10^4$ eV^2, the correction term may be important.

Observe that Eq. (23) differs slightly from the equation derived in [12]. The difference is that, in [12], we assumed that $\omega_0^2/L_m^2 \gg 1$ (so that $\langle\sin^2\tau/2L_m\rangle\simeq 1/2$) which is always true except at the very center of the resonance [12]. Also note that in [12] we neglected a factor of 2 which arises because we negligently assumed that the rate of measurement was equal to the rate of collision.

We now pause to review and comment on the assumptions made in deriving Eq. (23). There are five main simplifying assumptions.

(1) We have neglected the thermal spread of the neutrino momenta, and have replaced all momenta by their thermal average $\langle p\rangle\simeq 3.15T$.

(2) We have assumed that $n_{\nu_s}, n_{\overline{\nu_s}} \ll n_{\nu_\alpha}, n_{\overline{\nu_\alpha}}$. If the number densities $n_{\nu_s}, n_{\overline{\nu_s}}$ are non-negligible, then we must multiply the first term on the right-hand side of Eq. (23) by the factor $[n_{\nu_\alpha}-n_{\nu_s}]/n_{\nu_\alpha}$.

(3) We have assumed that the transformation from the vacuum parameters to matter parameters, i.e., $\sin\theta_0\to\sin\theta_m$ and $L_0\to L_m$ diagonalizes the Hamiltonian. This is only strictly true in the adiabatic limit ($|d\theta_m/dt|\ll|\Delta_m|$). In the general case [26],

$$i\frac{d}{dt}\begin{pmatrix}\nu_m^1\\ \nu_m^2\end{pmatrix} = \begin{pmatrix} -\frac{\Delta_m}{2} & -i\frac{d\theta_m}{dt} \\ i\frac{d\theta_m}{dt} & \frac{\Delta_m}{2}\end{pmatrix}\begin{pmatrix}\nu_m^1\\ \nu_m^2\end{pmatrix}, \tag{27}$$

with

$$\frac{d\theta_m}{dt} = \frac{1}{2}\,\frac{\sin 2\theta_0}{(b-a-\cos 2\theta_0)^2+\sin^2 2\theta_0}\,\frac{d(b-a)}{dt}, \tag{28}$$

where $\nu_m^{1,2}$ are the instantaneous matter eigenstates and $\Delta_m \equiv 1/L_m$. Expanding out $\gamma \equiv |(d\theta_m/dt)/\Delta_m|$ we find (neglecting da/dt)

$$\gamma \lesssim 2(4)\times 10^{-8}\left(\frac{\sin^2 2\theta_0}{10^{-6}}\right)^{1/2}\left(\frac{\mathrm{eV}^2}{|\delta m^2|}\right)^{1/2} \quad \text{away from resonance},$$

$$\gamma \simeq 2(4)\left(\frac{10^{-5}}{\sin^2 2\theta_0}\right)\left(\frac{\mathrm{eV}^2}{|\delta m^2|}\right)^{1/2} \quad \text{at the initial resonance where } b=\cos 2\theta_0, a\simeq 0,$$

$$\gamma \simeq 6(9)\times 10^{-4}T^3\left(\frac{10^{-5}}{\sin^2 2\theta_0}\right)\left(\frac{\mathrm{eV}^2}{|\delta m^2|}\right) \quad \text{at the resonance where } |b-a|=\cos 2\theta_0, \tag{29}$$

for ν_e-ν_s ($\nu_{\mu,\tau}$-ν_s) oscillations. However at the initial resonance where $b=\cos2\theta_0$, $a\simeq 0$, L_{ν_α} is created rapidly. The contribution to γ from a rapidly changing L_{ν_α} at this resonance is

$$\gamma \simeq \frac{(0.5)(3)\times 10^2}{\sin^2 2\theta_0}\left(\frac{\mathrm{eV}^2}{|\delta m^2|}\right)^{2/3}\frac{dL_{\nu_\alpha}}{d(T/\mathrm{MeV})}, \tag{30}$$

for ν_e-ν_s ($\nu_{\mu,\tau}$-ν_s) oscillations (and we have assumed that $\cos2\theta_0 \sim 1$). Thus away from the resonance the adiabatic approximation is valid for the parameter space of interest (i.e., for $|\delta m^2| \gtrsim 10^{-4}$ eV^2). However at the resonance the adiabatic approximation may not be valid.

(4) Equation (23) neglects flavor conversion of neutrinos passing through the resonance [the Mikheyev-Smirnov-Wolfenstein (MSW) effect]. Observe that there is not expected to be significant flavor conversion at the initial resonance (where $b\simeq\cos2\theta_0$) due to the MSW effect (even if the system is adiabatic at this resonance) because the frequency of the collisions is such that $\langle \sin^2 \tau/2L_m\rangle \ll 1$ at the center of the initial resonance, for most of the parameter space of interest. Indeed, at the center of the resonance,

$$\begin{aligned}\frac{\omega_0}{2L_m} &= \frac{\sin 2\theta_0}{y_\alpha G_F^2 T^5}\,\frac{\delta m^2}{2p}\\ &\simeq 4\times 10^8 \sin 2\theta_0\left(\frac{\mathrm{MeV}}{T}\right)^6\frac{\delta m^2}{\mathrm{eV}^2}, \simeq 90 \tan 2\theta_0\\ &\quad \text{if } b=\cos 2\theta_0.\end{aligned} \tag{31}$$

Thus, for $\sin^2 2\theta_0 \lesssim 10^{-4}$, $\langle \sin^2\tau/2L_m\rangle \ll 1$. Note however that for temperatures below the initial resonance, the MSW effect may be important if there are neutrinos passing through the resonance.

(5) We have assumed that the rate of change of lepton number is dominated by collisions. There is also a contribution from oscillations between collisions. Oscillations between collisions affect lepton number because the oscillations produce a superposition of states, where the averaged expectation value of the state being a weak eigenstate is $1-\sin^2 2\theta_m\langle \sin^2\tau/2L_m\rangle$ for neutrinos. This probability is generally unequal to the analogous quantity for antineutrinos, which is $1-\sin^2 2\theta_m\langle\sin^2\tau/2L_m\rangle$. It is possible to show [12] that for temperatures greater than a few MeV, the change in lepton number due to the oscillations between collisions is generally smaller than the change due to collisions except possibly at the resonance where $\sin^2 2\theta_m$ is changing rapidly.

The effect of the thermal spread of the neutrino momenta should be to make the creation and destruction of lepton number much smoother. At any given time, only a small fraction of the neutrinos will be at resonance (because the resonance width is much less than the spread of neutrino momenta). Thus, the regions away from resonance may also be important. We will study the effect of the thermal distribution of momenta in Sec. IV.

The second assumption [(2) above] will be approximately valid for much of the parameter space of interest. This is because we are essentially interested in the region of parameter space where the sterile neutrinos do not come into equilibrium with the ordinary neutrinos. We will study the effect of the sterile neutrino number density being nonzero in Sec. V. Assumptions (3) and (5) may not be valid in the resonance region. Note that we will denote assumptions (3) and (5) collectively as the static approximation because in limit where the system is sufficiently smooth they will be valid.

Clearly a more exact treatment of the resonance is desirable, since assumptions (3) and (5) may not be valid there. In Sec. III we will develop a more exact treatment of the resonance region by examining the appropriate equations from the density matrix. As we will show in Sec. III, this treatment leads to the following equation for the rate of change of lepton number:

$$\frac{dL_{\nu_\alpha}}{dt} \simeq \frac{3\beta^2}{8}\int_0^t e^{-\tau/\omega_0}\sin\left[\int_{t-\tau}^t \lambda^+ dt'\right]\sin\left[\int_{t-\tau}^t \lambda^- dt''\right]d\tau, \tag{32}$$

where

$$\beta = \frac{\delta m^2}{2p}\sin 2\theta_0, \tag{33}$$

$$\lambda^{+}=\frac{\delta m^2}{2p}(\cos 2\theta_0 - b),$$

$$\lambda^{-}=\frac{\delta m^2}{2p}a.$$

This equation is valid given assumptions (1), (2), and (4) but does not require assumptions (3) and (5) (above). This equation is an integrodifferential equation and although compact cannot be solved analytically except in various limits. Note that the static limit corresponds to taking $\lambda^{\pm}$ as constant over the time scale ω_0. In this limit Eq. (32) reduces approximately to Eq. (23) as expected. In the Appendix we show how Eq. (32) can also be obtained using the Hamiltonian formalism.

Qualitatively, it turns out that the simplified equation, Eq. (23), gives a reasonable description of the creation of lepton number as the Universe evolves. Assuming that Eq. (23) is valid, we now analyze the behavior of L_{ν_α} as driven by ν_α-ν_s oscillations in isolation. Suppose that all initial asymmetries other than L_{ν_α} can be neglected so that $L^{(\alpha)}\simeq 2L_{\nu_\alpha}$. Notice first of all that for $\delta m^2>0$ it follows from Eq. (18) that b is negative and a has the opposite sign to L_{ν_α}. Thus from Eq. (23) it is easy to see that the point $L_{\nu_\alpha}=0$ is always a stable fixed point. That is, when $L_{\nu_\alpha}>0$ the rate of change dL_{ν_α}/dt is negative, while when $L_{\nu_\alpha}<0$ the rate of change dL_{ν_α}/dt is positive, so L_{ν_α} always tends to zero. [In the realistic case, where the baryon and electron asymmetries are not neglected, $L^{(\alpha)}$ is given by Eq. (16). In this case $L^{(\alpha)}\sim 0$ is an approximate fixed point. Note that even if all of the lepton numbers where initially zero, lepton number would be generated such that $L^{(\alpha)}\simeq 0$, i.e. $2L_{\nu_\alpha}\simeq -\eta$ [see Eq. (16)]. Note that $2L_{\nu_\alpha}$ is only approximately $-\eta$ because of the Δ term proportional to L_{ν_α} in Eq. (23)].

Now consider neutrino oscillations with $\delta m^2<0$. In this case b is positive and a has the same sign as L_{ν_α}. From Eq. (23), $L_{\nu_\alpha}\simeq 0$ is a stable fixed point only when $b>\cos 2\theta_0$. When $b<\cos 2\theta_0$, the point $L_{\nu_\alpha}\simeq 0$ is unstable. (That is, if $L_{\nu_\alpha}>0$, then $dL_{\nu_\alpha}/dt>0$, while if $L_{\nu_\alpha}<0$ then $dL_{\nu_\alpha}/dt<0$.) Since $b\sim T^6$, at some point during the evolution of the Universe b becomes less than $\cos 2\theta_0$ and $L_{\nu_\alpha}=0$ becomes unstable. If $|\delta m^2|\gtrsim 10^{-4}$ eV^2, then this point (where $b=\cos 2\theta_0$) occurs for temperatures greater than about three MeV (assuming $\cos 2\theta_0\simeq 1$). In this region the rate of change of lepton number is dominated by collisions and Eq. (23) is approximately valid. When the critical point where $b=\cos 2\theta_0$ is reached, the lepton asymmetries are small and hence $|a|\ll\cos 2\theta_0\simeq 1$. Equation (23) then implies that dL_{ν_α}/dt is approximately proportional to L_{ν_α}, which leads to a brief but extremely rapid period of exponential growth of L_{ν_α} [12]. Furthermore, note that the constant of proportionality is enhanced by resonances for both neutrinos and antineutrinos at this critical point ($a\simeq 0$, $b=\cos 2\theta_0$). The exponent governing the exponential increase in L_{ν_α} is thus a large number (unless $\sin^2 2\theta_0$ is very small). Note that the critical point $b=\cos 2\theta_0$ occurs when

$$T_c\simeq 13(16)\left(\frac{|\delta m^2|\cos 2\theta_0}{\mathrm{eV}^2}\right)^{1/6}\ \mathrm{MeV}, \tag{34}$$

for the ν_e-ν_s ($\nu_{\mu,\tau}$-ν_s) oscillations we have been focusing on.

As the system passes through this critical temperature, lepton number is rapidly created until $a\gtrsim\cos 2\theta_0 - b$. The resonance at $a=\cos 2\theta_0-b$ acts like a barrier which keeps $a>\cos 2\theta_0-b$ as the temperature falls below T_c. Since the parameter a is proportional to $L_{\nu_\alpha}T^4$, it follows that the lepton number continues to grow approximately like T^{-4} after the resonance as the temperature falls.

As the temperature drops, eventually the oscillations cannot keep up with the expansion of the Universe. For temperatures well below the resonance, $a\simeq\cos 2\theta_0$ (assuming that $L_{\nu_\alpha}>0$ for definiteness). In this region, the rate of change of a due to the oscillations is balanced by the rate of change of a due to the expansion of the Universe. That is,

$$\frac{da}{dt}=\frac{\partial a}{\partial L_{\nu_\alpha}}\frac{\partial L_{\nu_\alpha}}{\partial t}+\frac{\partial a}{\partial t}\simeq 0. \tag{35}$$

Eventually, the rate of change of a due to the expansion of the Universe becomes larger in magnitude than the maximum rate of change of a due to oscillations. At this point, a falls below the resonance point (i.e., $a<\cos 2\theta_0-b$) and the value of L_{ν_α} will be approximately frozen. The point in time when this occurs is thus governed by the equation

$$\frac{\partial a}{\partial L_{\nu_\alpha}}\frac{\partial L_{\nu_\alpha}}{\partial t}\bigg|_{\max}=-\frac{\partial a}{\partial t}. \tag{36}$$

The maximum rate of change of L_{ν_α} occurs at the resonance where $a=\cos 2\theta_0-b$. Using Eq. (23), we can easily evaluate dL_{ν_α}/dt at this point. Assuming that $\cos 2\theta_0\simeq 1$, we find, at the resonance,

$$\frac{dL_{\nu_\alpha}}{dt}=\frac{3}{32}\Gamma_{\nu_\alpha}a, \tag{37}$$

where we have assumed that $x\simeq\sin^2 2\theta_0$, which should be valid since we are in the region of low temperatures $T\sim 3$ MeV [recall that x is defined in Eq. (25)]. Also note that

$$\frac{\partial a}{\partial t}=\frac{\partial a}{\partial T}\frac{dT}{dt}\simeq -\frac{4a}{T}\frac{5.5T^3}{M_P}, \tag{38}$$

where we have used the result that the parameter a is proportional to T^4, and $dT/dt\simeq -5.5T^3/M_P$ (which is approximately valid for 1 MeV$\lesssim T\lesssim$100 MeV, and $M_P\simeq 1.2\times 10^{22}$ MeV is the Planck mass). Thus, using Eqs. (37) and (38), the condition, Eq. (36), can be solved for T. Doing this exercise, and denoting this value of T by T_f, we find

$$T_f \simeq \left[\frac{50|\delta m^2|}{M_P y_\alpha G_F^3} \right]^{1/7} \simeq \left[\frac{|\delta m^2|}{\mathrm{eV}^2} \right]^{1/7} \ \mathrm{MeV}, \tag{39}$$

where we have used Eq. (10), Eq. (18). Thus, we expect L_{ν_α} to evolve like T^{-4} until quite low temperatures of order 1 MeV. Note however that when the momentum distribution is taken into account, the situation is somewhat different. This is because only a small fraction of neutrinos (typically of order 1% or less) will be at the resonance, so that the magnitude of the maximum value of $\partial L_{\nu_\alpha}/\partial t$ will be reduced by a few orders of magnitude. Because of the 1/7 power in Eq. (39), the temperature where L_{ν_α} is approximately frozen, T_f, increases by only a relatively small factor of 2 or 3. Finally recall that for temperatures below the initial resonance, the MSW effect can also contribute significantly. This is because for low temperatures near T_f, there will be a significant number of neutrinos which will be passing through the resonance. For low temperatures, the adiabatic condition is expected to hold [for most of the parameter space of interest, see Eq. (29)]. Also, recall that the oscillations will not be damped by collisions for low temperatures [see Eq. (31)] and thus ordinary neutrinos can be converted into sterile neutrino states simply by passing through the resonance [26]. This effect will help keep $a \simeq 1$ for even lower temperatures.

Clearly these factors (the momentum distribution and the MSW flavor conversion of the neutrinos passing through the resonance) will be important if one wants to know the final magnitude of L_{ν_α}. For example, the final magnitude of L_{ν_e} is very important if one wants to calculate the region of parameter space where the L_{ν_e} is large enough to affect big bang nucleosynthesis through nuclear reaction rates. However, for the application in this paper, the precise value of L_{ν_α} at low temperatures is not required, so we will leave a study of this issue to the future.

In order to illustrate the evolution of L_{ν_α} we take some examples. It is illuminating to compare the evolution expected from the simple Eq. (23) [based on the assumptions (1)–(5) discussed above], with the evolution governed by the more complicated density matrix equations [Eqs. (46), see next section for some discussion of the density matrix formalism]. The evolution of L_{ν_α} as governed by the density matrix equations hold more generally than Eq. (23). This is because they do not require assumptions (2), (3), (4), or (5) (discussed above) to hold. They do still incorporate assumption (1), that is the thermal distribution of the neutrino momentum is neglected.

In Figs. 1 and 2 we plot the evolution of L_{ν_α} for some typical parameters. We consider, for example, $\nu_{\mu,\tau}$-ν_s oscillations. In Fig. 1 we take $\delta m^2 = -1$ eV^2, and $\sin^2 2\theta_0 = 10^{-4}, 10^{-8}$. Figure 2 is the same as Fig. 1 except that $\delta m^2 = -1000$ eV^2 and $\sin^2 2\theta_0 = 10^{-6}, 10^{-9}$. The solid lines are the result of numerically integrating the density matrix equations, while the dashed lines are the results of numerically integrating Eq. (23). We stress that in both the density matrix equations and in Eq. (23), the momentum distribution of the neutrino has been neglected. The effect of the momentum distribution will be considered in detail in Secs. IV and V.

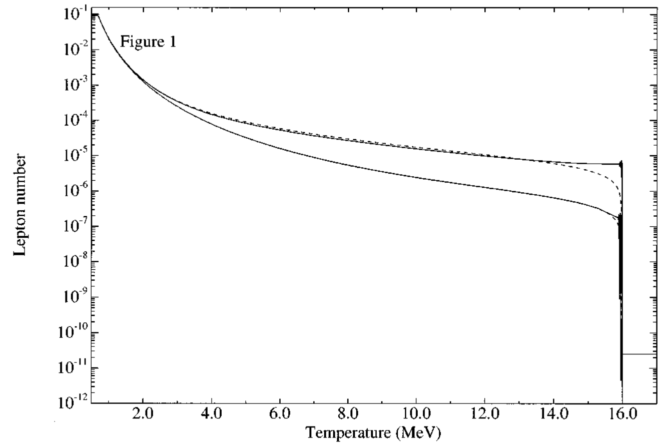

FIG. 1. The evolution of the ν_μ-ν_s(or ν_τ-ν_s) oscillation generated lepton number asymmetry, L_{ν_μ} (or L_{ν_τ}). We have taken by way of example, the parameter choices $\delta m^2 = -1$ eV^2, $\sin^2 2\theta_0 = 10^{-8}$ ($\sin^2 2\theta_0 = 10^{-4}$) for the bottom two curves (top two curves). The solid lines represent the results of the numerical integration of the density matrix equations [Eq. (46)], while the dashed lines result from the numerical integration of Eq. (23).

In the examples in Figs. 1 and 2 the initial lepton asymmetry was taken as zero. The generation of lepton number is independent of the initial lepton number asymmetry provided that it is less than about 10^{-5} [20,11]. This is because, for temperatures greater than the resonance temperature, the oscillations destroy or create lepton number until $L^{(\alpha)} \approx 0$ independently of the initial value of L_{ν_α} (which we denote as L_{init}), provided that $|L_{\mathrm{init}}|$ is less than about 10^{-5}. For $|L_{\mathrm{init}}| \gtrsim 10^{-5}$, the oscillations at temperatures above the resonance temperature are not strong enough to destroy the initial asymmetry. Consequently, L_{ν_α} remains large, and it will become larger due to the oscillations which create lepton number at temperatures below the resonance temperature.

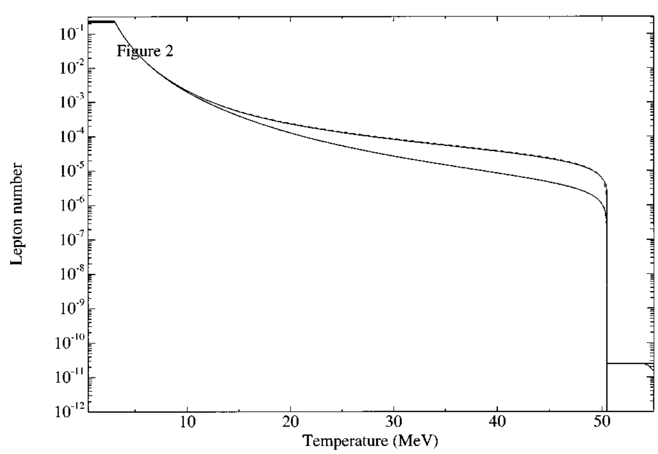

FIG. 2. The evolution of the ν_μ-ν_s (or ν_τ-ν_s) oscillation generated lepton number asymmetry, L_{ν_μ} (or L_{ν_τ}). In this example we have taken the parameter choices, $\delta m^2 = -1000$ eV^2, $\sin^2 2\theta_0 = 10^{-9}$ ($\sin^2 2\theta_0 = 10^{-6}$) for the bottom two curves (top two curves). As in Fig. 1, the solid lines represent the results of the numerical integration of the density matrix equations [Eq. (46)], while the dashed lines result from the numerical integration of Eq. (23).

As the figures show, the behavior expected from Eq. (23) occurs. The main difference arises at the resonance where the magnitude of the lepton number is somewhat larger than expected from Eq. (23). This occurs because assumptions (3) and (5) (discussed above), which lead to Eq. (23) are not valid at this resonance. Actually, in Figs. 1 and 2 we have plotted $|L_{\nu_\mu}|$. Integration of the density matrix equation reveals that in example 1 (but not in example 2, although L_{ν_α} does change sign), the generated lepton number oscillates at the resonance and changes sign a few times (see [18], for a figure illustrating this). Note that this effect can be understood from Eq. (32). To see this, observe that when L_{ν_α} is initially created at the resonance, the parameter λ^- grows very rapidly because it is proportional to L_{ν_α}. The creation of L_{ν_α} may be so rapid that $\int_{t-\tau}^{t}\lambda^- dt'$ is approximately independent of τ when the initial rapid growth of L_{ν_α} occurs. If this happens then, at this instant, Eq. (32) can be simplified to the approximate form

$$\frac{dL_{\nu_\alpha}}{dt}\sim\frac{3\beta^2}{8}\sin\left[\int_{t-\omega_0}^{t}\lambda^- dt'\right]\int_0^t e^{-\tau/\omega_0}\sin\left[\int_{t-\tau}^{t}\lambda^+ dt''\right]d\tau. \tag{40}$$

The oscillations occur because of the factor $\sin\int_{t-\omega_0}^{t}\lambda^- dt'$ which oscillates between ± 1. Note, however, that this oscillation of lepton number would not be expected to occur in the realistic case where the thermal spread of neutrino momenta is considered.

Note that it may be possible to predict the sign of the asymmetry in principle. Assuming that the resonance is smooth enough so that Eq. (23) is valid, the equation governing the evolution of L_{ν_α} has the approximate form (for L_{ν_α} small enough so that $a \ll 1$),

$$\frac{dL_{\nu_\alpha}}{dt}=A(2L_{\nu_\alpha}+\tilde{\eta})-BL_{\nu_\alpha}=(2A-B)L_{\nu_\alpha}+A\tilde{\eta}, \tag{41}$$

where $\tilde{\eta}\equiv\eta+L_{\nu_e}+L_{\nu_\mu}+L_{\nu_\tau}-L_{\nu_\alpha}$ (we have defined $\tilde{\eta}$ such that it is independent of L_{ν_α}). Note that A and B [which can be obtained from Eq. (23), with the B term arising from the Δ term] are complicated functions of time. Observe however that $B>0$ and A is initially less than zero, and at the resonance A changes sign and becomes positive after that. In the region where $2A<B$, the lepton number evolves such that

$$(2A-B)L_{\nu_\alpha}+A\tilde{\eta}\to 0. \tag{42}$$

Thus L_{ν_α} will evolve such that it has a sign opposite to $\tilde{\eta}$ just before the resonance. When $2A>B$, L_{ν_α} will become unstable and grow rapidly. Note that at the point $A=B/2$,

$$\frac{dL_{\nu_\alpha}}{dt}=A\tilde{\eta}. \tag{43}$$

Hence, at the point where the initial rapid creation of L_{ν_α} occurs, the rate of change of L_{ν_α} will be proportional to $\tilde{\eta}$. Thus, we might expect that after the initial resonance the sign of L_{ν_α} will be the same as the sign of the asymmetry $\tilde{\eta}$. This means that L_{ν_α} should change sign at the resonance. Note however that because $\tilde{\eta}$ depends on the initial values of the lepton asymmetries which are unknown at the moment, it seems that the sign of L_{ν_α} cannot yet be predicted. However the above calculation shows that the sign of L_{ν_α} should not depend on statistical fluctuations, as we initially thought likely [12].

Finally we would like to comment on the region of parameter space where significant generation of lepton number occurs. First, we require that $\delta m^2<0$ and that $|\delta m^2| \gtrsim 10^{-4}$ eV^2, so that $T_c^{\alpha s}\gtrsim 3$ MeV. For $|\delta m^2|\lesssim 10^{-4}$ eV^2, lepton number can still be generated but it is dominated by the oscillations between collisions and is oscillatory [18,29]. Note that in the realistic case where the spread of momenta is taken into account, oscillations of lepton number would be smoothed out and may not occur. A numerical study in [18] shows that $\sin^2 2\theta_0 \gtrsim 10^{-11}$ $(\mathrm{eV}^2/|\delta m^2|)^{1/6}$ is also necessary (see also [12] for an approximate analytical derivation). Finally, we must require that $\sin^2 2\theta_0$ be small enough so that the sterile neutrinos do not come into equilibrium. [For example, in the case of ν_α-ν_s oscillations, if there are equal numbers of ν_α and ν_s states then the rates $n_{\nu_\alpha}\Gamma(\nu_\alpha\to\nu_s) = n_{\nu_s}\Gamma(\nu_s\to\nu_\alpha)$, $n_{\bar{\nu}_\alpha}\Gamma(\overline{\nu_\alpha}\to\overline{\nu_s})=n_{\bar{\nu}_s}\Gamma(\overline{\nu_s}\to\overline{\nu_\alpha})$ and from Eq. (3) L_{ν_α} cannot be generated.] We will reexamine the region of parameter space where significant generation of lepton number occurs in Sec. V (where the effects of the Fermi-Dirac momentum distribution of the neutrino will be taken into account).

Note that in [18], it is argued that lepton number generation only occurs provided that $|\delta m^2|\lesssim 100$ eV^2. We have not been able to verify this result, either analytically or numerically. In fact, we have not been able to obtain any significant upper bound on $|\delta m^2|$.

III. LEPTON NUMBER GENERATION DUE TO NEUTRINO OSCILLATIONS: A MORE EXACT TREATMENT

In this section we derive a more general equation describing lepton number generation in the early Universe which can be applied when the system is changing rapidly, as occurs, for instance, at the resonance. The only assumptions that we will make are the assumptions (1), (2), and (4) (discussed in the previous section). That is we will neglect the spread of neutrino momenta and set $p=\langle p\rangle\simeq 3.15T$, and we will also assume that there are negligible numbers of sterile neutrinos generated. In the Appendix an alternative derivation (with the same end result) based on the Hamiltonian formalism is presented. Although not yet realistic because of assumptions (1) and (2), this derivation turns out to be particularly useful because it allows us to work out the region of parameter space where the simple Eq. (23) is approximately valid. As we will show, it turns out that Eq. (23) has a wider applicability than might be expected from the adiabatic condition, Eqs. (29) and (30).

The system of an active neutrino oscillating with a sterile neutrino can be described by a density matrix. See, for example, [21] for details. Below we very briefly outline this formalism and show how it leads to an integrodifferential equation which reduces to Eq. (23) in the static limit.

The density matrices for the neutrino system are given by

$$\rho_\nu = P_0\left(\frac{1+\mathbf{P}\cdot\sigma}{2}\right), \quad \rho_{\bar\nu} = \overline{P_0}\left(\frac{1+\overline{\mathbf{P}}\cdot\sigma}{2}\right), \tag{44}$$

where P_0 and $\overline{P_0}$ are the relative number densities of the mixed neutrino and antineutrino species, and $\mathbf{P}$ and $\overline{\mathbf{P}}$ are the polarization vectors that describe the internal quantum state of the mixed neutrinos in terms of an expansion in the Pauli matrices σ. The number densities of ν_α and ν_s are given by

$$\frac{n_{\nu_\alpha}}{P_0} = \frac{1+P_z}{2}, \quad \frac{n_{\nu_s}}{P_0} = \frac{1-P_z}{2}, \tag{45}$$

with analogous equations for the antineutrinos. The evolution of $P_0, \mathbf{P}$ are governed by the equations [21]

$$\frac{d}{dt}\mathbf{P} = \mathbf{V}\times\mathbf{P} + (1-P_z)\left(\frac{d}{dt}\ln P_0\right)\hat{\mathbf{z}} - \left(D^E + D^I + \frac{d}{dt}\ln P_0\right)(P_x\hat{\mathbf{x}} + P_y\hat{\mathbf{y}}),$$

$$\frac{d}{dt}P_0 = \sum_{i=e,\nu_\beta;\beta\neq\alpha}\langle\Gamma(\nu_\alpha\overline{\nu_\alpha}\to i\bar{i})\rangle(\lambda_i n_i n_{\bar i} - n_{\nu_\alpha}n_{\overline{\nu_\alpha}}), \tag{46}$$

where $\lambda_\nu = 1$ and $\lambda_e = 1/4$, and $\langle\cdots\rangle$ indicates the average over the momentum distributions. The quantity $\mathbf{V}$ is given by

$$\mathbf{V} = \beta\hat{\mathbf{x}} + \lambda\hat{\mathbf{z}}, \tag{47}$$

where β, λ are defined by

$$\beta = \frac{\delta m^2}{2p}\sin 2\theta_0,$$

$$\lambda = \frac{\delta m^2}{2p}(\cos 2\theta_0 - b \pm a), \tag{48}$$

where the $+$ $(-)$ sign corresponds to neutrino (antineutrino) oscillations. The quantities D^E and D^I are quantum damping parameters resulting from elastic and inelastic processes, respectively. According to [21,23], $D^E + D^I = \Gamma_{\nu_\alpha}/2$. The function $\langle\Gamma(\phi\psi\to\phi'\psi')\rangle$ is the collision rate for the process $\phi\psi\to\phi'\psi'$ averaged over the distribution of collision parameters at the temperature T assuming that all species are in equilibrium.

Expanding out Eq. (46), we have

$$\frac{dP_z}{dt} = \beta P_y + (1-P_z)\left(\frac{d}{dt}\ln P_0\right),$$

$$\frac{dP_y}{dt} = \lambda P_x - \beta P_z - P_y/\omega_0,$$

$$\frac{dP_x}{dt} = -\lambda P_y - P_x/\omega_0, \tag{49}$$

where $\omega_0 \equiv 1/[D^E + D^I + (d/dt)\ln P_0] \simeq 1/D$ (where $D \equiv D^E + D^I$). If we make the approximation of setting all of the number densities to their equilibrium values, and also assume that the number of sterile species is small, then $P_z \simeq 1$ and Eq. (49) simplifies to

$$\frac{dP_z}{dt} \simeq \beta P_y,$$

$$\frac{dP_y}{dt} \simeq \lambda P_x - \beta - P_y/\omega_0,$$

$$\frac{dP_x}{dt} \simeq -\lambda P_y - P_x/\omega_0. \tag{50}$$

Strictly speaking, the approximation of setting $P_z = 1 = \text{const}$ can only be valid when βP_z is small enough, so that MSW flavor conversion cannot occur, i.e., when

$$|\beta| \lesssim |\lambda| \quad \text{or} \quad \frac{1}{\omega_0}. \tag{51}$$

It is useful to introduce the complex variable $\tilde{P}(t)$ defined by $\tilde{P} \equiv P_x + iP_y$. It is easy to see that the resulting equation describing the evolution of $\tilde{P}(t)$ is given by

$$i\frac{d\tilde{P}}{dt} = -\lambda\tilde{P} - i\frac{\tilde{P}}{\omega_0} + \beta. \tag{52}$$

The solution to this equation with initial condition $\tilde{P}(0) = 0$ is

$$\tilde{P}(t) = -i\int_0^t \beta(t')e^{(t'-t)/\omega_0}\exp\left(i\int_{t'}^t \lambda\, dt''\right)dt', \tag{53}$$

where ω_0 has been assumed to be approximately constant over the time scale $t-t'$ $(\sim\omega_0)$ which is approximately valid for temperatures above a few MeV where the expansion rate is less than the collision rate [28]. One can easily verify that Eq. (53) is indeed the solution of Eq. (52) by direct substitution. Thus, taking the imaginary part of both sides of Eq. (53), we find that

$$P_y = -\int_0^t \beta e^{(t'-t)/\omega_0}\cos\left[\int_t^{t'}\lambda\, dt''\right]dt'. \tag{54}$$

From Eq. (45) (with $P_0 = n_{\nu_\alpha} + n_{\nu_s} \simeq \frac{3}{8}n_\gamma$ assuming $n_{\nu_s} \ll n_{\nu_\alpha}$), it follows that

$$\frac{dL_{\nu_\alpha}}{dt} \simeq \frac{3}{16}\frac{d}{dt}(P_z - \overline{P_z}), \tag{55}$$

where $\overline{P_z}$ denotes the z component of the polarization vector for antineutrinos. Thus using Eq. (50) the above equation becomes

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{3\beta}{16}(P_y - \overline{P_y}). \tag{56}$$

Note that P_y is given by Eq. (54) and $\overline{P_y}$ is defined similarly to P_y except that we must replace $a\to -a$. Thus, we obtain

$$\frac{dL_{\nu_\alpha}}{dt}\simeq\frac{-3\beta^2}{16}\int_0^t e^{(t'-t)/\omega_0}\left(\cos\left[\int_t^{t'}\lambda\,dt''\right]-\cos\left[\int_t^{t'}\overline{\lambda}dt''\right]\right)dt', \tag{57}$$

where $\lambda=\delta m^2(c-b+a)/2p$, $\overline{\lambda}=\delta m^2(c-b-a)/2p$. Note that we have taken β outside the integral, which is valid for $T\gtrsim 2$ MeV, because β is approximately constant over the interaction time scale $t-t'$ [30]. Changing variables from t' to the variable τ where $\tau\equiv t-t'$, this equation reduces to

$$\frac{dL_{\nu_\alpha}}{dt}=\frac{-3\beta^2}{16}\int_0^t e^{-\tau/\omega_0}\left(\cos\left[\int_{t-\tau}^{t}\lambda\,dt'\right]-\cos\left[\int_{t-\tau}^{t}\overline{\lambda}dt'\right]\right)d\tau, \tag{58}$$

or, equivalently,

$$\frac{dL_{\nu_\alpha}}{dt}=\frac{-3\beta^2\omega_0}{16}\left[\left\langle\cos\int_{t-\tau}^{t}\lambda\,dt'\right\rangle-\left\langle\cos\int_{t-\tau}^{t}\overline{\lambda}dt'\right\rangle\right]. \tag{59}$$

Note that the above equation can be rewritten using a trigonometric identity, so that

$$\frac{dL_{\nu_\alpha}}{dt}=\frac{3\beta^2}{8}\int_0^t e^{-\tau/\omega_0}\sin\left[\int_{t-\tau}^{t}\lambda^+dt'\right]\sin\left[\int_{t-\tau}^{t}\lambda^-dt''\right]d\tau, \tag{60}$$

where $\lambda^{\pm}=(\lambda\pm\overline{\lambda})/2$.

The phenomenon of neutrino oscillations can also be described by the Hamiltonian formalism. We show in the Appendix that this formalism also leads to Eq. (60) under the same assumptions.

In the static limit where $\lambda,\overline{\lambda}$ are approximately constant (over a typical interaction time scale ω_0) it is straightforward to show that Eq. (58) reduces to Eq. (23) with x given by

$$x=\frac{1}{4}\Gamma_{\nu_\alpha}^2\left(\frac{2p}{\delta m^2}\right)^2, \tag{61}$$

rather than by Eq. (25) [note that Eq. (61) reduces to Eq. (25) for most of the parameter space of interest except for quite low temperatures]. This difference is due to the fact that in deriving Eq. (58) we have made the assumption Eq. (51). Because Eq. (23) is much simpler than Eq. (60), it is particularly useful to determine the region of parameter space where the static limit [Eq. (23)] is an acceptable approximation. We now study this issue.

Expand $\lambda_{t'}$ (note that we are here using the notation that λ_x denotes λ evaluated at the point x) in a Taylor series around the point $t'=t$, that is

$$\lambda_{t'}=\lambda_t+[t'-t]\left(\frac{d}{dt'}\lambda\right)_t+\cdots . \tag{62}$$

Using this Taylor series, the integrals $\int_{t-\tau}^{t}\lambda\,dt'$ can be expanded as (with a similar expansion for $\int_{t-\tau}^{t}\overline{\lambda}dt'$),

$$\int_{t-\tau}^{t}\lambda\,dt'=\lambda_t\tau-\frac{\tau^2}{2}\left(\frac{d}{dt'}\lambda\right)_t+\cdots . \tag{63}$$

The static approximation will be valid provided that

$$\left\langle\cos\int_{t-\tau}^{t}\lambda\,dt'\right\rangle-\left\langle\cos\int_{t-\tau}^{t}\overline{\lambda}dt'\right\rangle\simeq\langle\cos\tau\lambda_t\rangle-\langle\cos\tau\overline{\lambda_t}\rangle. \tag{64}$$

Using the expansion Eq. (63), observe that

$$\left\langle\cos\int_{t-\tau}^{t}\lambda\,dt'\right\rangle=\left\langle\cos\tau\left[\lambda_t-\frac{\tau}{2}\left(\frac{d\lambda}{dt'}\right)_t+\cdots\right]\right\rangle. \tag{65}$$

The above equation can be used to determine the region of validity of the static approximation Eq. (64). The region of validity of Eq. (64) depends on the values of the parameters $\lambda,\overline{\lambda}$. There are essentially four regions to consider.

(a) $\omega_0|\lambda_t|$, $\omega_0|\overline{\lambda_t}|\gtrsim 1$. In this region, Eq. (64) is approximately valid provided that

$$\left|\frac{\omega_0}{2}\left(\frac{d\lambda}{dt}\right)_t\right|\lesssim|\lambda_t|,\quad\left|\frac{\omega_0}{2}\left(\frac{d\overline{\lambda}}{dt}\right)_t\right|\lesssim|\overline{\lambda_t}|. \tag{66}$$

(b) $\omega_0|\lambda_t|\simeq 0$, $\omega_0|\overline{\lambda_t}|\gtrsim 1$. In this region, Eq. (64) is approximately valid provided that Eq. (66) holds and

$$\left\langle\cos\int_{t-\tau}^{t}\lambda\,dt'\right\rangle\simeq 1. \tag{67}$$

From Eq. (65) this equation implies that

$$\left|\frac{\omega_0^2}{2}\left(\frac{d\lambda}{dt}\right)_t\right|\lesssim 1. \tag{68}$$

(c) $\omega_0|\overline{\lambda_t}|\simeq 0$, $\omega_0|\lambda_t|\gtrsim 1$. In this region, Eq. (64) is approximately valid provided that Eq. (66) holds and

$$\left|\frac{\omega_0^2}{2}\left(\frac{d\overline{\lambda}}{dt}\right)_t\right|\lesssim 1. \tag{69}$$

(d) $\omega_0|\lambda_t|\simeq 0$, $\omega_0|\overline{\lambda_t}|\simeq 0$. In this region, Eq. (64) can never be a strictly valid approximation because the right-hand side of Eq. (64) is zero at this point. Note however that the static approximation will be acceptable provided that the left-hand side of Eq. (64) is small at this point, which is true if Eq. (68) and Eq. (69) are valid.

Observe that Eqs. (68) and (69) are more stringent than Eq. (66). Evaluating Eq. (68) at the resonance, we find

$$\left|\frac{\omega_0^2}{2}\frac{d}{dt'}\left[\frac{\delta m^2}{2p}(\cos 2\theta_0-b+a)\right]\right|\lesssim 1. \tag{70}$$

For Eq. (69) we only need to replace $a\to -a$ in the above equation. Assuming that there is no accidental cancellation between the various independent terms, Eq. (70) implies

$$\left|\frac{\delta m^2}{2p}\frac{6b}{T}\frac{dT}{dt}\right| \lesssim \frac{\Gamma_{\nu_\alpha}^2}{2},$$
$$\left|\frac{da}{dT}\right| \lesssim \left|\frac{\Gamma_{\nu_\alpha}^2}{2}\frac{2p}{\delta m^2}\frac{dt}{dT}\right|, \tag{71}$$

where we have used $\partial b/\partial T = 6b/T$ and recall that $\omega_0 = 2/\Gamma_{\nu_\alpha}$. In deriving Eq. (71) we have also neglected a term proportional to $(\cos 2\theta_0 - b + a)$ which is less stringent than Eq. (71) because $(\cos 2\theta_0 - b + a) \approx 0$ is just the resonance condition. The first condition in Eq. (71) is satisfied provided that

$$T \gtrsim \left(\frac{11|\delta m^2|\cos 2\theta_0}{M_P y_\alpha^2 G_F^4}\right)^{1/9} \simeq 11\left(\frac{|\delta m^2|\cos 2\theta_0}{\mathrm{eV}^2}\right)^{1/9} \mathrm{MeV}, \tag{72}$$

where we have set $b = \cos 2\theta_0$ (which leads to the most stringent condition) and we have used $dT/dt \simeq -5.5T^3/M_P$. In order to evaluate the second condition in Eq. (71), observe that

$$\frac{da}{dT} = \frac{\partial a}{\partial L_{\nu_\alpha}}\frac{\partial L_{\nu_\alpha}}{\partial T} + \frac{\partial a}{\partial T}. \tag{73}$$

Assuming that there is no accidental cancellation between the two terms on the right-hand side of the above equation, the second term in Eq. (71) implies the following conditions at the resonance:

$$\left|\frac{\partial a}{\partial T}\right| \lesssim \left|\frac{\Gamma_{\nu_\alpha}^2}{2}\frac{2p}{\delta m^2}\frac{dt}{dT}\right|,$$
$$\left|\frac{\partial a}{\partial L_{\nu_\alpha}}\frac{\partial L_{\nu_\alpha}}{\partial T}\right| \lesssim \left|\frac{\Gamma_{\nu_\alpha}^2}{2}\frac{2p}{\delta m^2}\frac{dt}{dT}\right|. \tag{74}$$

Using $\partial a/\partial T \simeq 4a/T$, and $a \simeq 1$, then the first equation above gives approximately the same condition as the first equation in Eq. (71). The second condition in Eq. (74) gives a condition on the rate of change of lepton number at the resonance. Expanding this equation out we find that

$$\left|\frac{\partial L_{\nu_\alpha}}{\partial T}\right| \lesssim \left|\frac{\Gamma_{\nu_\alpha}^2}{2}\frac{dt}{dT}\frac{1}{2\sqrt{2}G_F n_\gamma}\right|$$
$$\simeq \frac{y_\alpha^2 M_P G_F^3 4.1 T^4}{22\sqrt{2}}$$
$$\simeq 4\times 10^{-11}\left(\frac{T}{\mathrm{MeV}}\right)^4 \frac{1}{\mathrm{MeV}}, \tag{75}$$

where we have used $n_\gamma = 2\zeta(3)T^3/\pi^2 \simeq T^3/4.1$. Note that we have also used Eq. (10) for the collision frequency. Thus, for example, if we are interested in studying the region where the lepton number is initially created, then a necessary condition for Eq. (23) to be approximately valid is that the resonance must occur for temperatures satisfying Eq. (72). From Eq. (34) (with $\cos 2\theta_0 \simeq 1$), this implies that

$$|\delta m^2| \gtrsim 9\times 10^{-2}\ \ (5\times 10^{-3})\ \ \mathrm{eV}^2, \tag{76}$$

for ν_e-ν_s ($\nu_{\mu,\tau}$-ν_s) oscillations. The creation of L_{ν_α} must also satisfy Eq. (75) at the resonance. This condition should be checked when using Eq. (23) for self-consistency.

Perhaps surprisingly, there is a significant region of parameter space where the oscillations are not adiabatic at the resonance [i.e., $\gamma \gtrsim 1$ in Eqs. (29) and (30)] but Eqs. (71) are nevertheless satisfied. This is possible because Eqs. (71) are not equivalent to the adiabatic conditions, Eqs. (29) and (30). This is because Eqs. (71) arise from demanding that the total contribution to dL_{ν_α}/dt reduce approximately to the simple Eq. (23). Recall that the total contribution to dL_{ν_α}/dt can be separated into two distinct contributions: from oscillations due to collisions and from oscillations between collisions. The adiabatic condition, on the other hand, is a necessary condition for the contribution of dL_{ν_α}/dt from collisions to reduce to Eq. (23). Thus it turns out that in the region when the system is both nonadiabatic and Eqs. (71) are satisfied, the modification to the equation for dL_{ν_α}/dt from collisions which arises from the nonadiabaticity cancels with the extra contribution to dL_{ν_α}/dt from oscillations between collisions. This type of cancellation is more transparent in the Hamiltonian formalism (see the Appendix).

Finally, to illustrate the analysis of this section, consider the examples given in Figs. 1 and 2. Recall that the solid and dashed lines correspond to the density matrix Eq. (46) and Eq. (23), respectively. Observe that for the example in Fig. 1 (which has $\delta m^2 = -1$ eV^2), Eq. (23) is not a very good approximation at the resonance where the lepton number is initially created (although it is a reasonable approximation for small $\sin^2 2\theta_0$). This is because the lepton number is created so rapidly that Eq. (75) is not valid. However, for the example shown in Fig. 2, where $\delta m^2 = -1000$ eV^2, the temperature where the lepton number is created is much higher. Observe that Eq. (75) is not as stringent for high temperatures and it is therefore not surprising that the static approximation is approximately valid for this case. [Note that the result that the static approximation tends to be a good approximation at high temperatures can also be seen by observing that for high temperatures, $\omega_0 \to 0$, and in this limit, Eq. (64) will be satisfied.]

IV. THE THERMAL MOMENTUM DISTRIBUTION OF THE NEUTRINO

Hitherto we have made the assumption that the neutrinos are monochromatic. This assumption is not expected to hold for the neutrinos in the early Universe. The momentum distribution of these neutrinos will be the usual Fermi-Dirac distribution. Note that the width of the initial resonance in momentum space is much smaller than the spread of neutrino momenta. This means that only a few of the neutrinos will be at resonance at a given time. Also, not all of the neutrinos will be creating lepton number. Neutrinos in the region defined by $b^p > \cos 2\theta_0$ (which includes part of the resonance) destroy lepton number, and those in the region $b^p < \cos 2\theta_0$ create lepton number. The point where net lepton number is created only occurs when the lepton number creating neutrino oscillations dominate over the lepton number destroying oscillations. Recall that in the unphysical case where all of the neutrinos are assumed to be monochromatic, all of the neutrinos enter the resonance at the same time, where they all destroy lepton number if $b > \cos 2\theta_0$, or all create lepton number if $b < \cos 2\theta_0$. Clearly, the effect of the thermal spread of momentum will make the creation of lepton number much smoother. An important consequence of this is that there will be even larger regions of parameter space where the system is smooth enough so that the static approximation is valid and hence Eq. (23) (modified to incorporate the momentum dependence) will be a good approximation.

In the static limit, we can simply rederive Eq. (23), assuming that the neutrino momenta form the usual Fermi-Dirac distribution. In this case, Eq. (5) becomes

$$\begin{aligned} n_\gamma \frac{dL_{\nu_\alpha}}{dt} \simeq & \frac{1}{2}\int [-\Gamma(\nu_\alpha \to \nu_s) + \Gamma(\overline{\nu_\alpha} \to \overline{\nu_s})] \\ & \times (dn_{\nu_\alpha} - dn_{\nu_s} + dn_{\overline{\nu_\alpha}} - dn_{\overline{\nu_s}}) \\ & - \frac{1}{2}\int [\Gamma(\nu_\alpha \to \nu_s) + \Gamma(\overline{\nu_\alpha} \to \overline{\nu_s})] \\ & \times (dn_{\nu_\alpha} - dn_{\nu_s} - dn_{\overline{\nu_\alpha}} + dn_{\overline{\nu_s}}), \end{aligned} \tag{77}$$

where

$$dn_{\nu_\alpha} = \frac{1}{2\pi^2}\frac{p^2 dp}{1+e^{(p-\mu)/T}}, \quad dn_{\overline{\nu_\alpha}} = \frac{1}{2\pi^2}\frac{p^2 dp}{1+e^{(p+\mu)/T}}, \tag{78}$$

and $dn_{\nu_s}, dn_{\overline{\nu_s}}$ are the differential number densities for the sterile and antisterile neutrinos, respectively. In Eq. (78) μ is the chemical potential.

The reaction rates can easily be obtained following a similar derivation as before [Eqs. (9)–(15)], but this time we keep the momentum dependence (rather than setting $p = \langle p \rangle$). Doing this, we find the following equation for the rate of change of lepton number in the static limit:

$$\begin{aligned} \frac{dL_{\nu_\alpha}}{dt} = & \frac{\pi^2}{4\zeta(3)T^3} \\ & \times \int \frac{s^2 \Gamma^p_{\nu_\alpha} a^p (c - b^p)(dn^+_{\nu_\alpha} - dn^+_{\nu_s})}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]} + \Delta, \end{aligned} \tag{79}$$

where Δ is a small correction term

$$\begin{aligned} \Delta \simeq & \frac{-\pi^2}{8\zeta(3)T^3} \\ & \times \int \frac{s^2 \Gamma^p_{\nu_\alpha}[x^p + (a^p)^2 + (b^p - c)^2](dn^-_{\nu_\alpha} - dn^-_{\nu_s})}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]}, \end{aligned} \tag{80}$$

and $dn^\pm_{\nu_\alpha} \equiv dn_{\nu_\alpha} \pm dn_{\overline{\nu_\alpha}}$. Recall that $c \equiv \cos 2\theta_0, s \equiv \sin 2\theta_0$. In these equations note that the quantities, $b^p, a^p, x^p, \Gamma^p_{\nu_\alpha}$ are all functions of momentum of the form

$$b^p = b\frac{p^2}{\langle p \rangle^2}, \quad a^p = a\frac{p}{\langle p \rangle},$$

$$x^p = s^2 + \frac{\Gamma^2_{\nu_\alpha}}{4}\left(\frac{p}{\langle p \rangle}\right)^2\left(\frac{2p}{\delta m^2}\right)^2, \quad \Gamma^p_{\nu_\alpha} = \Gamma_{\nu_\alpha}\frac{p}{\langle p \rangle}, \tag{81}$$

where $a, b, \Gamma_{\nu_\alpha}$ are defined in Eqs. (18) and (10). Equation (79) reduces to Eq. (23), in the limit where all of the neutrino momenta are fixed to $p = \langle p \rangle$. [Note that $a^p = a$ when $p = \langle p \rangle \simeq 3.15T$ and similarly for b^p, x^p, and $\Gamma^p_{\nu_\alpha}$.]

Note that the chemical potential is related to the lepton number by the equation

$$n_\gamma L_{\nu_\alpha} \equiv n_{\nu_\alpha} - n_{\overline{\nu_\alpha}} = \frac{T^3}{6}\left(\frac{\mu}{T}\right) + O(\mu^3). \tag{82}$$

Using Eqs. (78) and (82) we find

$$dn^+_{\nu_\alpha} = \frac{1}{\pi^2}\frac{p^2 dp}{1+e^{p/T}} + O(L^2_{\nu_\alpha}),$$

$$dn^-_{\nu_\alpha} = n_\gamma L_{\nu_\alpha}\frac{6}{\pi^2 T^3}\frac{p^2 e^{p/T} dp}{(1+e^{p/T})^2} + O(L^3_{\nu_\alpha}). \tag{83}$$

Thus substituting the above relations into Eq. (79), we find that

$$\begin{aligned} \frac{dL_{\nu_\alpha}}{dt} = & \frac{\pi^2}{4\zeta(3)T^3} \\ & \times \int_0^\infty \frac{s^2 \Gamma^p_{\nu_\alpha} a^p (c - b^p)}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]} \\ & \times \left[\frac{p^2}{\pi^2(1+e^{p/T})} - \frac{dn^+_{\nu_s}}{dp}\right] dp + \Delta, \end{aligned} \tag{84}$$

where Δ is a small correction term

$$\begin{aligned} \Delta \simeq & \frac{-\pi^2}{8\zeta(3)T^3} \\ & \times \int_0^\infty \frac{s^2 \Gamma^p_{\nu_\alpha}[x^p + (a^p)^2 + (b^p - c)^2]}{[x^p + (c - b^p + a^p)^2][x^p + (c - b^p - a^p)^2]} \\ & \times \left[\frac{12\zeta(3) L_{\nu_\alpha} p^2 e^{p/T}}{\pi^4(1+e^{p/T})^2} - \frac{dn^-_{\nu_s}}{dp}\right] dp. \end{aligned} \tag{85}$$

Equation (84) can be integrated numerically to obtain L_{ν_α} as a function of time (or temperature). We will give some examples in the next section after we discuss how to calculate the sterile neutrino number distributions.

The main effect of the thermal spread of neutrino momenta is to make the generation of lepton number much smoother. From a computational point of view, this is very fortunate. This is because Eq. (84), like Eq. (23), is only valid provided the lepton number generation is sufficiently smooth (see the previous section for a detailed discussion of this point). In particular, Eq. (84) should be a much better approximation to reality at the resonance where significant lepton number is initially generated.

To complete this section, we comment on the rate of change of lepton number due to ordinary-ordinary neutrino oscillations. For definiteness consider ν_e-ν_μ oscillations. The rate of change of $L_{\nu_\mu}-L_{\nu_e}$ is given by

$$\frac{n_\gamma}{2}\frac{d(L_{\nu_\mu}-L_{\nu_e})}{dt} = -\int \Gamma(\nu_\mu\to\nu_e)dn_{\nu_\mu}+\int \Gamma(\overline{\nu_\mu}\to\overline{\nu_e})dn_{\overline{\nu_\mu}} + \int \Gamma(\nu_e\to\nu_\mu)dn_{\nu_e}-\int \Gamma(\overline{\nu_e}\to\overline{\nu_\mu})dn_{\overline{\nu_e}}. \tag{86}$$

Using $\Gamma(\nu_\mu\to\nu_e)=\Gamma(\nu_e\to\nu_\mu)$ (and similarly for the antineutrino rates), Eq. (86) becomes

$$\frac{n_\gamma}{2}\frac{d(L_{\nu_\mu}-L_{\nu_e})}{dt} = -\int_0^\infty \Gamma(\nu_\mu\to\nu_e)\left(\frac{dn_{\nu_\mu}}{dp}-\frac{dn_{\nu_e}}{dp}\right)dp + \int_0^\infty \Gamma(\overline{\nu_\mu}\to\overline{\nu_e})\left(\frac{dn_{\overline{\nu_\mu}}}{dp}-\frac{dn_{\overline{\nu_e}}}{dp}\right)dp, \tag{87}$$

where

$$\frac{dn_{\nu_e}}{dp}=\frac{1}{2\pi^2}\frac{p^2}{1+e^{(p-\mu_1)/T}},\quad \frac{dn_{\overline{\nu_e}}}{dp}=\frac{1}{2\pi^2}\frac{p^2}{1+e^{(p+\mu_1)/T}},$$
$$\frac{dn_{\nu_\mu}}{dp}=\frac{1}{2\pi^2}\frac{p^2}{1+e^{(p-\mu_2)/T}},\quad \frac{dn_{\overline{\nu_\mu}}}{dp}=\frac{1}{2\pi^2}\frac{p^2}{1+e^{(p+\mu_2)/T}}. \tag{88}$$

The chemical potentials $\mu_{1,2}$ are related to the lepton numbers $L_{\nu_{e,\mu}}$ by the equations

$$\frac{\mu_1}{T}=\frac{6n_\gamma}{T^3}L_{\nu_e},\quad \frac{\mu_2}{T}=\frac{6n_\gamma}{T^3}L_{\nu_\mu}, \tag{89}$$

where we have assumed that $\mu_i/T\ll 1$. Using these relations and expanding out Eq. (87) (again assuming that $\mu_i/T\ll 1$) we find to leading order that

$$\frac{d(L_{\nu_\mu}-L_{\nu_e})}{dt}\simeq -\frac{6(L_{\nu_\mu}-L_{\nu_e})}{\pi^2T^3}\int_0^\infty \frac{p^2e^{p/T}}{(1+e^{p/T})^2} \times[\Gamma(\nu_\mu\to\nu_e)+\Gamma(\overline{\nu_\mu}\to\overline{\nu_e})]dp. \tag{90}$$

From the above equation we see that $L_{\nu_\mu}-L_{\nu_e}$ always evolves such that $(L_{\nu_\mu}-L_{\nu_e})\to 0$. Also note that Eq. (90) shows that the rate of change of lepton number due to ordinary-ordinary neutrino oscillations is generally smaller than the rate of change of lepton number due to ordinary-sterile neutrino oscillations, (assuming $L_{\nu_\alpha}\ll 1$). [Actually Eq. (90) has a strength comparable to the correction term Δ for ordinary-sterile neutrino oscillations, Eq. (85), although note that the mixing angle between ordinary neutrinos can be significantly larger than the mixing angle between ordinary and sterile neutrinos.]

For ordinary-ordinary neutrino oscillations, neutral current interactions do not collapse the wave function because they cannot distinguish different neutrino species. Only the charged current interactions can distinguish the neutrino species. For example, for temperatures 1 MeV$\lesssim T\lesssim$30 MeV, there are near-equilibrium number densities of electrons and positrons. The number of muons and antimuons will be much less than the number of electrons and positrons, and we will neglect them (actually these are important for ν_τ-ν_μ oscillations). The rate at which charged current interactions occur is given approximately by $\Gamma_\nu\sim|\Gamma_{\nu_e}-\Gamma_{\nu_\mu}|\simeq y_e' G_F^2T^5$, where $y_e'\sim y_e-y_\mu$ [$\simeq$1.1 see Eq. (10)]. Also note that antineutrino-neutrino [31] and neutrino-neutrino [32] forward scattering amplitudes induce off diagonal contributions to the effective potential. (Note that these contributions do not occur for the effective potential governing ordinary-sterile neutrino oscillations). It would be necessary to include these effects in order to evaluate the reaction rates.

V. THE EFFECTS OF NON-NEGLIGIBLE STERILE NEUTRINO NUMBER DENSITIES AND THE PARAMETER SPACE FOR LARGE LEPTON NUMBER ASYMMETRY GENERATION

In this section we will do three things. We will study the effects of non-negligible sterile neutrino number densities, which can arise for the case of relatively large, or even moderate values of $\sin^2 2\theta_0$. We will examine the parameter space where significant generation of lepton number occurs. Finally, we will obtain the BBN bound on the parameter space for ν_α-ν_s oscillations with $\delta m^2<0$, and with $|\delta m^2|\gtrsim 10^{-4}$ eV^2, $\sin^2 2\theta_0\lesssim 10^{-2}$.

There are several ways in which the creation of lepton number(s) can prevent the sterile neutrinos from coming into equilibrium. One way is that one set of oscillations ν_α-ν_s creates L_{ν_α}. The lepton number L_{ν_α} can then suppress other, independent oscillations such as ν_β-ν_s oscillations (with $\beta\neq\alpha$) for example. A more direct, but less dramatic way in which the creation of lepton number can help prevent the sterile neutrinos from coming into equilibrium, is that the lepton number generated from say ν_α-ν_s oscillations itself suppresses the ν_α-ν_s oscillations [33]. We will examine the latter effect here (some examples of the former effect will be studied in the next section). Previous work [5–8] obtained the BBN bound for large $|\delta m^2|\gtrsim 10^{-4}$ eV^2 (with $\delta m^2<0$) and small $\sin^2 2\theta_0\lesssim 10^{-2}$ which can be approximately parametrized as follows [8]:

$$|\delta m^2|\sin^4 2\theta_0\lesssim 10^{-9}\ \text{eV}^2. \tag{91}$$

This bound arises by assuming that the ν_α-ν_s oscillations do not bring the sterile ν_s state into equilibrium. Note that this bound neglected the creation of lepton number and it also did not include the effects of the distribution of neutrino momentum. However, in the realistic case, the creation of L_{ν_α} (after it occurs) will suppress the ν_α-ν_s oscillations and the actual bound would be expected to be somewhat less stringent than Eq. (91).

To proceed we will need to examine the effects of non-negligible sterile neutrino number densities. The evolution of the number distribution of sterile neutrinos is governed approximately by the rate equation

$$\frac{d}{dt}\left[\frac{dn_{\nu_s}/dp}{dn_{\nu_\alpha}/dp}\right]=\left[\frac{dn_{\nu_\alpha}/dp-dn_{\nu_s}/dp}{dn_{\nu_\alpha}/dp}\right]\Gamma(\nu_\alpha\to\nu_s). \tag{92}$$

A similar equation holds for the number distribution of sterile antineutrinos. Introducing the notation, $z\equiv(dn_{\nu_s}/dp)/(dn_{\nu_\alpha}/dp)$ [for antineutrinos we use the corresponding notation, $\overline{z}\equiv(dn_{\overline{\nu_s}}/dp)/(dn_{\overline{\nu_\alpha}}/dp)$], Eq. (92) becomes

$$\frac{dz}{dt}=(1-z)\Gamma(\nu_\alpha\to\nu_s)=\frac{1-z}{4}\,\frac{\Gamma^p_{\nu_\alpha}s^2}{x^p+(c-b^p+a^p)^2}. \tag{93}$$

The corresponding equation for antineutrinos can be obtained by replacing $z\to\overline{z}$ and $a^p\to-a^p$ in the above equation. In solving the above differential equation, we will assume the initial condition $z=0$. We will also assume that the number densities of the ordinary neutrinos are given by their equilibrium values. Note that the quantity z depends only on the reaction rates and is otherwise independent of the expansion.

From the definition of z, it follows that $dn_{\nu_s}=zdn_{\nu_\alpha}$, $dn_{\overline{\nu_s}}=\overline{z}dn_{\overline{\nu_\alpha}}$. Thus, from Eq. (84),

$$\frac{dL_{\nu_\alpha}}{dt}\simeq\frac{1}{4\zeta(3)T^3}\times\int_0^\infty\frac{s^2\Gamma^p_{\nu_\alpha}a^p(c-b^p)}{[x^p+(c-b^p+a^p)^2][x^p+(c-b^p-a^p)^2]}\times\frac{(1-z^+)p^2dp}{1+e^{p/T}}+\Delta, \tag{94}$$

where Δ is a small correction term

$$\Delta\simeq\frac{1}{8\zeta(3)T^3}\int_0^\infty\frac{s^2\Gamma^p_{\nu_\alpha}[x^p+(a^p)^2+(b^p-c)^2]}{[x^p+(c-b^p+a^p)^2][x^p+(c-b^p-a^p)^2]}\times\frac{z^-p^2dp}{1+e^{p/T}}, \tag{95}$$

with $z^\pm\equiv(z\pm\overline{z})/2$ and we have neglected a small correction term which is proportional to L_{ν_α}. Note that Eq. (93) and Eq. (94) must be solved simultaneously.

For the numerical work, the continuous variable p/T is replaced by a finite set of momenta p_n/T (with $n=0,1,\ldots,N$) and the integral over momentum in Eq. (94) is replaced by the sum of a finite number of terms. Correspondingly, the variable $z(t,p/T)$ is replaced by the set of variables, $z_n(t)$, where the evolution of each variable $z_n(t)$ is

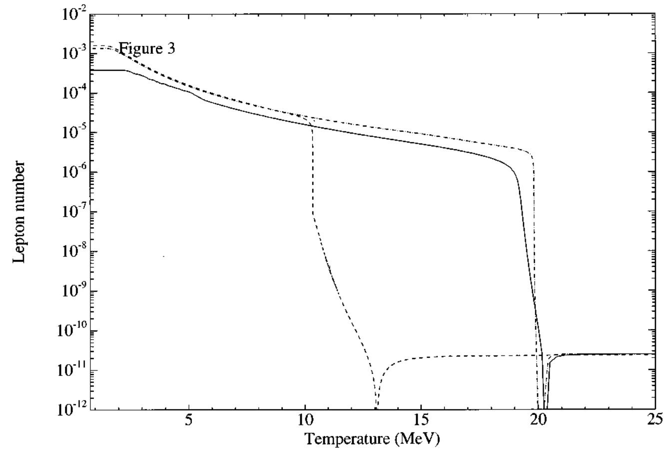

FIG. 3. The evolution of the ν_μ-ν_s (or ν_τ-ν_s) oscillation generated lepton number asymmetry, L_{ν_μ} (or L_{ν_τ}). In this example we have taken the parameter choices $\delta m^2=-1$ eV^2,$\sin^2 2\theta_0=10^{-4}$ (dashed line), $\sin^2 2\theta_0=10^{-6}$ (dash-dotted line) and $\sin^2 2\theta_0=10^{-8}$ (solid line). These curves result from integrating the coupled differential equations, Eqs. (94) and (93), which in contrast to Figs. 1 and 2, incorporate the momentum distribution of the neutrino. They also incorporate the effect of the nonzero number density of the sterile neutrinos which are produced by the oscillations.

governed by the differential equation, Eq. (93) [with $p/T=p_n/T$ for $z=z_n(t)$, $n=0,1,\ldots,N$]. Thus, the single differential equation, Eq. (93) is replaced by a set of N differential equations, one for each momentum step. These differential equations, together with Eq. (94), are coupled differential equations which must be integrated simultaneously.

We now illustrate the creation of lepton number as governed by Eqs. (94) and (93) with some examples. We have numerically integrated Eqs. (94) and (93) for the following parameter choices. In Fig. 3 we have considered $\nu_{\mu,\tau}$-ν_s oscillations with the parameter choice $\delta m^2=-1$ eV^2, $\sin^2 2\theta_0=10^{-4}$ (dashed line), $\sin^2 2\theta_0=10^{-6}$ (dash-dotted line), and $\sin^2 2\theta_0=10^{-8}$ (solid line). Figure 4 is the same as Fig. 3, except that $\delta m^2=-1000$ eV^2, $\sin^2 2\theta_0=10^{-6}$ (dashed line), $\sin^2 2\theta_0=10^{-7}$ (dash-dotted line), and $\sin^2 2\theta_0=10^{-9}$ (solid line). In both examples we have assumed that the ini-

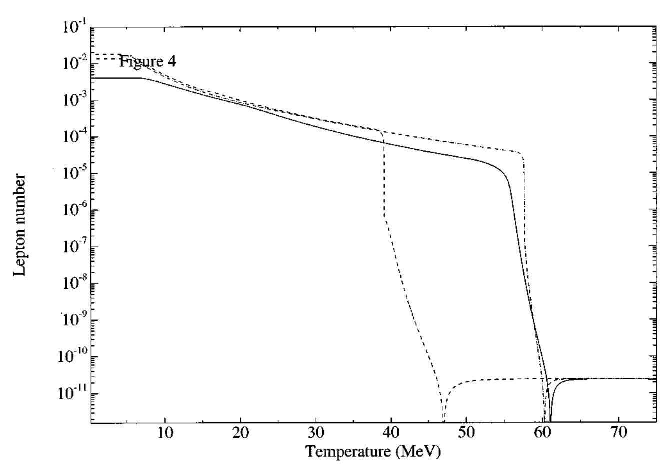

FIG. 4. Same as Fig. 3 except that $\delta m^2=-1000$ eV^2,$\sin^2 2\theta_0=10^{-6}$ (dashed line), $\sin^2 2\theta_0=10^{-7}$ (dash-dotted line), and $\sin^2 2\theta_0=10^{-9}$ (solid line).

tial lepton asymmetry is zero. Recall that the generation of lepton number is essentially independent of the initial lepton number asymmetry provided that it is less than about 10^{-5} (for more discussion about this point see Sec. II). Note that for convenience we have plotted $|L_{\nu_\alpha}|$. The lepton asymmetry L_{ν_α} changes sign at the point where it is created. Before this point L_{ν_α} evolves such that it has the opposite sign to η while for evolution subsequent to the point where L_{ν_α} is initially created, L_{ν_α} has the same sign as η. Recall that this behavior is expected (see the earlier discussion in Sec. II).

In these examples, the generation of lepton number is considerably smoother than in the earlier case where the momentum distribution was neglected (see Figs. 1 and 2). For this reason, it turns out that throughout most of the evolution of L_{ν_α}, the rate of change of L_{ν_α} satisfies the condition Eq. (75) and thus Eq. (94) should be approximately valid (except at quite low temperatures where the MSW effect will be important).

In order to gain insight into the effects of the neutrino momentum distribution, it is useful to compare Figs. 3 and 4 (which incorporate the neutrino momentum distribution) with the Figs. 1 and 2 (where the momentum of all of the neutrinos were set equal to the mean momentum). Qualitatively, there is not a great deal of difference. However there are several very important effects, which we summarize below.

(1) For the examples with relatively small $\sin^2 2\theta_0$, lepton number creation generally begins somewhat earlier (i.e., at a higher temperature) than in the case where momentum distribution is neglected. For the examples shown in Figs. 3 and 4 with $\delta m^2 = -1$ eV^2, $\sin^2 2\theta_0 = 10^{-8}$ (-1000 eV^2, $\sin^2 2\theta_0 = 10^{-9}$), lepton number is created when $T \simeq 20$ MeV ($T \simeq 60$ MeV). This can be compared with the simplistic case where the neutrino momentum distribution was neglected. In this case, we see from Figs. 1 and 2 that lepton number creation begins at $T \simeq 16.0$ MeV ($T \simeq 50$ MeV) for $\delta m^2 = -1$ eV^2 (-1000 eV^2). The fact that the critical temperature increases can be explained rather simply. Note that the neutrino number density distribution peaks at about $p \simeq 2.2T$, which should be compared with the average momentum of about $3.15T$ used in Figs. 1 and 2. Using the former approximation instead of the latter leads the critical temperature to increase by about 12%. This explains qualitatively why the critical temperature increases. Note, however, that the accurate numerical calculations displayed in Figs. 3 and 4 actually show that the temperature increases by more than this (for the examples with small $\sin^2 2\theta_0$) and also that the temperature increase depends on the mixing angle.

(2) For the examples with large $\sin^2 2\theta_0$, the point where significant generation of lepton number is created occurs much later than in the examples with small $\sin^2 2\theta_0$. The reason for this is that for large $\sin^2 2\theta_0$, the number density of sterile neutrinos is larger. In the region before significant lepton number is generated, $a \simeq 0$ and all of the neutrino oscillations with $b^p < \cos 2\theta_0$ have already passed through the resonance while the neutrino oscillations with $b^p > \cos 2\theta_0$ have yet to pass through the resonance. Since the creation of sterile neutrinos is dominated by the oscillations at the resonance, it follows that the sterile neutrino number distribution with momenta in the region where $b^p < \cos 2\theta_0$ will be much greater than for sterile neutrinos with momenta in the region where $b^p > \cos 2\theta_0$. Thus, from Eq. (94), the lepton number creating oscillations (with $b^p < \cos 2\theta_0$) are suppressed if the number density of sterile neutrinos is non-negligible, as occurs for large $\sin^2 2\theta_0$. The lepton number destroying oscillations (with $b^p > \cos 2\theta_0$), on the other hand, are not suppressed because the number of sterile neutrinos with $b^p > \cos 2\theta_0$ are negligible.

(3) The creation of lepton number is considerably smoother in the realistic case. For instance, in the example where $\delta m^2 = -1$ eV^2, $\sin^2 2\theta_0 = 10^{-8}$, in the realistic case (shown in Fig. 3), $|L_{\nu_\mu}|$ ranges from 10^{-10} to 10^{-6} in about $\Delta T \approx 1$ MeV, whereas in the unrealistic case where the neutrino momentum distribution was neglected (shown in Fig. 1), $|L_{\nu_\mu}|$ ranges from 10^{-10} to 10^{-6} in about $\Delta T \approx 0.005$ MeV.

(4) At low temperatures, the lepton number gets ''frozen'' at an earlier time. For example, in the case where $\delta m^2 = -1$ eV^2 and $\sin^2 2\theta_0 = 10^{-8}$, with momentum dependence (Fig. 3), the final value for the lepton number is $\sim 4 \times 10^{-4}$, whereas in the unrealistic case without the neutrino momentum distribution, the final value for the lepton number for this example (Fig. 1) is $\sim 10^{-1}$. As discussed briefly in Sec. II, this effect is expected because the temperature where the lepton number gets frozen occurs when the rate of change of the variable a due to the expansion of the Universe dominates over the rate of change of a due to neutrino oscillations. In the realistic case where the momentum distribution is taken into account, the maximum value of the rate of change of a due to neutrino oscillations is suppressed because only a small fraction of the neutrinos will be at the resonance.

This last point suggests that the momentum distribution cannot be ignored if one is interested in finding out the precise final value of the lepton number generated. However, note that Eq. (94) does not incorporate flavor conversion due to the MSW effect [see assumption (4) in Sec. II for some discussion about this point]. The effect of the MSW flavor conversion should be to keep $a \simeq 1$ for lower temperatures. This means that the final value of L_{ν_α} should be significantly larger than suggested by Figs. 3 and 4. This effect will need to be incorporated if one wants to calculate the precise value of the final lepton number generated. [The precise value of the final lepton number can be obtained by numerically integrating the density matrix equations Eqs. (46) suitably modified to incorporate the neutrino momentum distribution.] In particular, if one is interested in working out the region of parameter space where the electron lepton number is large enough to affect BBN through nuclear reaction rates, then the final value of the electron lepton number is very important [34,35].

Note that we can check Eq. (91) by numerically integrating Eq. (93) and Eq. (94) assuming for definiteness that $\rho_{\nu_s}/\rho_{\nu_\alpha} \lesssim 0.6$ (where the ρ's are the energy densities). This leads to the following constraint on $\delta m^2, \sin^2 2\theta_0$:

$$\sin^2 2\theta_0 \lesssim 2(4) \times 10^{-5} \left[\frac{\mathrm{eV}^2}{|\delta m^2|} \right]^{1/2}, \tag{96}$$

for ν_e-ν_s ($\nu_{\mu,\tau}$-ν_s) oscillations. Thus, we see that Eq. (91) turns out to be a good approximation after all. This is basically due to the result that the creation of a non-negligible number of sterile neutrinos has the effect of delaying the point where significant lepton number is created [see point (2) above].

Finally, the region of parameter space where significant neutrino asymmetries are generated by ordinary sterile neutrino oscillations can be obtained by integrating Eq. (94) and Eq. (93). The result of this numerical work is that significant neutrino asymmetry ($|L_{\nu_\alpha}| \gtrsim 10^{-5}$) is generated by ordinary-sterile neutrino oscillations for the following region of parameter space:

$$6(5)\times 10^{-10}\left[\frac{\mathrm{eV}^2}{|\delta m^2|}\right]^{1/6} \lesssim \sin^2 2\theta_0 \lesssim 2(4)\times 10^{-5}\left[\frac{\mathrm{eV}^2}{|\delta m^2|}\right]^{1/2}$$

and

$$|\delta m^2| \gtrsim 10^{-4}\ \mathrm{eV}^2, \tag{97}$$

for ν_e-ν_s ($\nu_{\mu,\tau}$-ν_s) oscillations. Note that we have assumed that $\rho_{\nu_s}/\rho_{\nu_\alpha} \lesssim 0.6$ [Eq. (96)]. In the general case where no bound on $\rho_{\nu_s}/\rho_{\nu_\alpha}$ is assumed, the upper bound on $\sin^2 2\theta_0$ is considerably weaker. For example, $\nu_{\mu,\tau}\nu_s$ oscillations with $\delta m^2 = -1\ \mathrm{eV}^2$, $\sin^2 2\theta_0 = 10^{-4}$ violate the bound Eq. (96) but still generate a significant neutrino asymmetry, as illustrated in Fig. 3. (For this particular example, we found that $\rho_{\nu_s}/\rho_{\nu_\alpha} \simeq 0.86$.)

The parameter space in Eq. (97) can be compared with previous work where the momentum dependence was neglected [18,12]. As we have mentioned above, the upper bound on $\sin^2 2\theta_0$ which assumes a BBN bound of $\rho_{\nu_s}/\rho_{\nu_\alpha} \lesssim 0.6$, is not modified much when the momentum distribution of the neutrino is incorporated. For the lower limit of $\sin^2 2\theta_0$, the effect of the momentum dependence is to reduce the region of parameter space by nearly two orders of magnitude.

Finally, it may be possible for significant neutrino asymmetries to be generated for $|\delta m^2| \lesssim 10^{-4}\ \mathrm{eV}^2$, however the mechanism of production of these asymmetries is dominated by oscillations between collisions (rather than the mechanism of collisions) and tend to be oscillatory [18,29,33].

VI. CONSISTENCY OF THE MAXIMAL VACUUM OSCILLATION SOLUTIONS OF THE SOLAR AND ATMOSPHERIC NEUTRINO PROBLEMS WITH BBN

We now turn to another application of the phenomenon of lepton number creation due to ordinary-sterile neutrino oscillations. First, in the context of a simple explanation of the solar neutrino problem which involves large angle ν_e-ν_s oscillations, we will determine the conditions under which the lepton number produced from ν_α-ν_s oscillations can suppress the oscillations ν_β-ν_s (where $\beta \neq \alpha$). This allows the BBN bounds on ordinary-sterile neutrino oscillations to be evaded by many orders of magnitude, as we will show. We begin by briefly reviewing the maximal vacuum oscillation solution to the solar neutrino problem [9,10].

One possible explanation of the solar neutrino problem is that the electron neutrino oscillates maximally (or near maximally) with a sterile neutrino (which we here denote as ν_e' rather than as ν_s in order to remind the reader that this sterile neutrino is approximately maximally mixed with ν_e) [9,36]. We will denote the δm^2 for ν_e-ν_e' oscillations by $\delta m^2_{ee'}$. As is well known, for a large range of parameters [37]

$$3\times 10^{-10}\ \mathrm{eV}^2 \lesssim |\delta m^2_{ee'}| \lesssim 10^{-3}\ \mathrm{eV}^2, \tag{98}$$

maximal vacuum oscillations imply that the flux of electron neutrinos from the sun will be reduced by a factor of 2 for all neutrino energies relevant to the solar neutrino experiments. We will call this scenario the ‘‘maximal vacuum oscillation solution’’ to the solar neutrino problem. It is a very simple and predictive scheme which can either be ruled out or tested more stringently with the *existing experiments*. Importantly, it also makes definite predictions for the new experiments, Sudbury Neutrino Observatory (SNO), Superkamiokande, and Borexino. Our interest in this scheme is also motivated by the exact parity symmetric model (see [17] for a review of this model). This model predicts that ordinary neutrinos will be maximally mixed with mirror neutrinos (which are approximately sterile as far as ordinary matter is concerned) if neutrinos have mass [17]. If we make the assumption that the mixing between the generations is small (as it is in the quark sector) then the exact parity symmetric model predicts that the three known neutrinos will each be (to a good approximation) maximal mixtures of two mass eigenstates. There are also other interesting models which predict that the electron neutrino is approximately maximally mixed with a sterile neutrino [38]. The maximal mixing of the electron neutrino (ν_e) and the sterile neutrino will reduce the solar neutrino flux by an energy independent factor of 2 for the large range of parameters given in Eq. (98). This leads to definite *predictions* for the expected solar neutrino fluxes for the existing experiments. In [9], we compared these predictions with the existing experiments. We summarize the results of that exercise in Table I which we have updated to include the most recent data [39].

TABLE I. Summary of the predictions for the chlorine and gallium experiments assuming (1) standard electroweak theory (i.e., no new physics), (2) that the electron neutrino oscillates maximally into a sterile state (maximal mixing model), and (3) the experimental measurements. All numbers are in units of SNU.

Prediction/Expt.	Chlorine	Gallium
Standard electroweak theory	4.5 ± 0.5	123^{+8}_{-6}
Maximal mixing model	3.7 ± 0.4	65^{+7}_{-4}
Experiment	2.78 ± 0.35	71 ± 7

Note that in Table I, the Kamiokande experiment has been used as a measurement of the Boron flux [40–42]. This is a sensible way to analyze the data (but not the only way of course) because the flux of neutrinos coming from this reaction chain is difficult to reliably calculate due primarily to uncertainties in nuclear cross sections [43]. Clearly, the simple energy independent flux reduction by a factor of 2 leads to predictions which are in quite reasonable agreement with the data. If the minimal standard model had given such good predictions, few would have claimed that there was a solar neutrino problem.

Note that the maximal vacuum oscillation solution is distinct from the ''just so'' large angle vacuum oscillation solution [46]. In the ''just so'' solution, the electron neutrino oscillation length is assumed to be about equal to the distance between the earth and the sun (which corresponds to $|\delta m^2| \simeq 10^{-10}$ eV^2). In this case the flux of neutrinos depends sensitively on δm^2 and it is possible to fit the data to the free parameters δm^2, $\sin^2 2\theta_0$ [46]. The advantage of doing this is that a good fit to the data can be obtained (however this is not so surprising since there are two free parameters to adjust). The disadvantage is that fine-tuning is required and predictivity is lost because of the two free parameters. The maximal mixing solution on the other hand assumes maximal mixing and that δm^2 is in the range Eq. (98). For this parameter range there is an energy independent flux reduction by a factor of 2. The advantage of this possibility is that it does not require fine-tuning and it is predictive. A consequence of this is that it is testable with the existing experiments. The disadvantage of this scenario is that it does not give a perfect fit to the data. However, in our opinion the predictions are in remarkably good agreement with the data given the simplicity and predictivity of the model.

With the range of parameters in Eq. (98) there is a potential conflict with BBN [44,45]. For maximally mixed ν_e and ν_e' neutrinos, the following rather stringent BBN bound has been obtained *assuming that the lepton number asymmetry could be neglected* [5–8]:

$$|\delta m^2_{ee'}| \lesssim 10^{-8} \ \text{eV}^2. \tag{99}$$

This bound arises by requiring that the sterile neutrinos do not significantly modify the successful BBN calculations. For temperatures above the kinetic decoupling temperature the requirement that the sterile neutrinos do not come into equilibrium implies the bound $|\delta m^2_{ee'}| \lesssim 10^{-6}$ eV^2. Smaller values of $\delta m^2_{ee'}$ in the range $10^{-8} \lesssim |\delta m^2_{ee'}|/\text{eV}^2 \lesssim 10^{-6}$ can be excluded because the oscillations deplete the number of electron neutrinos (and antineutrinos) after kinetic decoupling (so that they cannot be replenished). The depletion of electron neutrinos increases the He/H primordial abundance ratio. This is because the temperature where the ratio of neutrons to protons freezes out is increased if there are less electron neutrinos around. For $|\delta m^2_{ee'}| \lesssim 10^{-8}$ eV^2, the oscillation lengths are too long to have any significant effect on the number densities of electron neutrinos during the nucleosynthesis era. If the bound in Eq. (99) were valid then it would restrict much of the parameter space for the maximal vacuum oscillation solution of the solar neutrino problem. However, this bound does not hold if there is an appreciable lepton number asymmetry in the early Universe for temperatures between 1 and 100 MeV [11]. This is because the generation of significant lepton number $L^{(e)}$ implies that the quantity $a_{ee'}$ [which is the a parameter defined in Eq. (18) with $\delta m^2 = \delta m^2_{ee'}$] is very large thereby suppressing the oscillations [note that for $a_{ee'} \gg 1$, $\sin^2 2\theta_m \ll \sin^2 2\theta_0$, see Eq. (19)]. We will now show in detail how the creation of lepton number can relax the BBN bound Eq. (99) by many orders of magnitude.

We will assume that the various oscillations can be approximately broken up into the pairwise oscillations ν_e-ν_e', ν_μ-ν_e', and ν_τ-ν_e'. We will denote the various oscillation parameters in a self-evident notation,

$$b_{\alpha e'}, a_{\alpha e'} \ \text{ for } \nu_\alpha - \nu_e' \ \text{ oscillations}, \tag{100}$$

where $\alpha = e, \mu, \tau$. We will denote the mixing parameters, δm^2, $\sin^2 2\theta_0$ appropriate for ν_α-ν_e' oscillations by $\delta m^2_{\alpha e'}$, $\sin^2 2\theta_0^{\alpha e'}$. Note that lepton number cannot be created by ν_α-ν_e' oscillations until $b_{\alpha e'} < \cos 2\theta_0^{\alpha e'}$. Recall that the b parameter is inversely proportional to δm^2 [see Eq. (18)]. Thus, the earliest point during the evolution of the Universe where lepton number can be created due to ordinary-sterile neutrino oscillations occurs for oscillations which have the largest $|\delta m^2|$. Note that these oscillations must satisfy the bound in Eq. (96) if we demand that the sterile neutrino energy density be small enough so that BBN is not significantly modified. Note that the ν_e-ν_e' oscillations have very small $|\delta m^2_{ee'}| \lesssim 10^{-3}$ eV^2 [37], and $\cos 2\theta_0^{ee'} \sim 0$ (assuming maximal or near maximal mixing), and thus these oscillations themselves cannot produce significant lepton number. However, the δm^2 for ν_τ-ν_e' or ν_μ-ν_e' oscillations can have much larger $|\delta m^2|$ (and they should also have $\delta m^2 < 0$ if $m_{\nu_\mu}, m_{\nu_\tau} > m_{\nu_e'}$) [47]. We will assume for definiteness that $m_{\nu_\tau} > m_{\nu_\mu} > m_{\nu_e'}$ so that $|\delta m^2_{\tau e'}| > |\delta m^2_{\mu e'}|$ and the ν_τ-ν_e' oscillations create L_{ν_τ} first (with L_{ν_μ}, L_{ν_e} assumed to be initially negligible). If $m_{\nu_\mu} > m_{\nu_\tau}$ then we only need to replace ν_τ by ν_μ in the following analysis.

Thus, we will consider the system comprising ν_τ, ν_e, and ν_e' (and their antiparticles). Our analysis will be divided into two parts. First, we will calculate the condition that the $L^{(e)}$ created by ν_τ-$\nu_{e'}$ oscillations survives without being subsequently destroyed by ν_e-ν_e' oscillations. We will then establish the conditions under which $L^{(e)}$ is created early enough and is large enough to suppress the ν_e-ν_e' oscillations so that only a negligible number of ν_e' are produced.

For simplicity we will first analyze the system neglecting the momentum distribution of the neutrino. This is useful because under this assumption it turns out that this system can be approximately solved analytically as we will show. We will then consider the realistic case where the spread of momenta is taken into consideration.

It is important to observe that the generation of L_{ν_τ} also leads to the generation of $L^{(e)}$ [through Eq. (16)]. If we assume that negligible L_{ν_e} is generated, then $L^{(e)} \simeq L^{(\tau)}/2$. However ν_e-ν_e' oscillations can potentially generate L_{ν_e} such that $L^{(e)} \to 0$. (Recall that $L^{(e)} \simeq 0$ is an approximately stable fixed point for the ν_e-ν_e' system for temperatures greater than a few MeV.) The effect of the ν_e-ν_e' oscillations will be greatest when the ν_e-ν_e' oscillations are at resonance. If negligible L_{ν_e} is generated, then $|a_{ee'}| \simeq R|a_{\tau e'}|/2$ and $|b_{ee'}| \simeq R(A_e/A_\tau)|b_{\tau e'}|$ (where $R \equiv |\delta m^2_{\tau e'}/\delta m^2_{ee'}|$). Hence the ν_e-ν_e' resonance condition ($a_{ee'} = b_{ee'}$) will be satisfied when

$$|a_{\tau e'}| = 2(A_e/A_\tau)|b_{\tau e'}|. \tag{101}$$

Recall that the ν_τ-ν_e' oscillations generate L_{ν_τ} such that

$$a_{\tau e'} \simeq 1 - b_{\tau e'}, \tag{102}$$

where we have assumed that $\cos 2\theta_0^{\tau e'} \sim 1$ and that $L_{\nu_\tau} > 0$ for definiteness. Observe that Eqs. (101) and (102) imply that the system inevitably passes through the ν_e-ν_e' resonance. This event will occur when

$$|b_{\tau e'}| \simeq \frac{A_\tau}{A_\tau + 2A_e}. \tag{103}$$

Using the definition of $b_{\tau e'}$ which can be obtained from Eq. (18), the above equation can be solved for the ν_e-ν_e' resonance temperature

$$T_{\rm res}^{ee'} \simeq \left[\frac{|\delta m^2_{\tau e'}| M_W^2 4.1}{6.3\sqrt{2} G_F A_\tau} \frac{A_\tau}{A_\tau + 2A_e} \right]^{1/6} \simeq 11 \left(\frac{|\delta m^2_{\tau e'}|}{\mathrm{eV}^2} \right)^{1/6} \ \mathrm{MeV}. \tag{104}$$

Thus, when $T = T_{\rm res}^{ee'}$, the ν_τ-ν_e' oscillations have created enough $L^{(e)}$ so that the ν_e-ν_e' oscillations will be at the resonance, assuming that negligible L_{ν_e} has been generated. In general the ν_e-ν_e' resonance temperature depends on both L_{ν_e} and L_{ν_τ}. The ν_e-ν_e' resonance condition $a_{ee'} = b_{ee'}$ implies that the resonance temperature for ν_e-ν_e' oscillations is related to L_{ν_e} and L_{ν_τ} by the equation

$$T_{\rm res}^{ee'} = \sqrt{\frac{M_W^2}{A_e} L^{(e)}} \simeq \sqrt{\frac{M_W^2}{A_e}(2L_{\nu_e} + L_{\nu_\tau})}, \tag{105}$$

where we have neglected the small baryon and electron asymmetries and a possible μ neutrino asymmetry (we will discuss the effects of the μ neutrino later). Thus the resonance temperature will change when L_{ν_e} and L_{ν_τ} change due to oscillations.

Let us consider the rate of change of the quantity $(T_{\rm res}^{ee'} - T)$,

$$\frac{d(T_{\rm res}^{ee'} - T)}{dt} = \frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_e}} \frac{\partial L_{\nu_e}}{\partial t} + \frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_\tau}} \frac{\partial L_{\nu_\tau}}{\partial t} - \frac{dT}{dt}, \tag{106}$$

evaluated at the temperature $T = T_{\rm res}^{ee'}$. Note that the first term on the right-hand side of Eq. (106) represents the rate of change of $T_{\rm res}^{ee'}$ due to ν_e-ν_e' oscillations (and its sign is negative), while the second term is the rate of change of $T_{\rm res}^{ee'}$ due to ν_τ-ν_e' oscillations (and the sign of this term is positive). The third term in Eq. (106) is the rate of change of $(T_{\rm res}^{ee'} - T)$ due to the expansion of the Universe $(-dT/dt \simeq 5.5 T^3/M_P)$ (this term is also positive in sign). Observe that if $d(T_{\rm res}^{ee'} - T)/dt > 0$, then the system passes through the resonance without significant destruction of $L^{(e)}$. If on the other hand, $d(T_{\rm res}^{ee'} - T)/dt \lesssim 0$, then the position of the resonance moves to lower and lower temperatures and $L^{(e)} \to 0$. Thus, a sufficient condition that $L^{(e)}$ survives without being destroyed by ν_e-ν_e' oscillations is that

$$\frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_e}} \frac{\partial L_{\nu_e}}{\partial t} + \frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_\tau}} \frac{\partial L_{\nu_\tau}}{\partial t} > \frac{dT}{dt}. \tag{107}$$

To evaluate $\partial L_{\nu_\tau}/\partial t$, observe that

$$\frac{\partial L^{(\tau)}}{\partial t} = 2\frac{\partial L_{\nu_\tau}}{\partial t} + \frac{\partial L_{\nu_e}}{\partial t} \simeq \frac{-4L^{(\tau)}}{T} \frac{dT}{dt}, \tag{108}$$

where we have assumed that $L^{(\tau)} \sim T^{-4}$ for $T = T_{\rm res}^{ee'}$. Of course, this latter assumption only holds provided that the ν_e-ν_e' resonance does not occur while L_{ν_τ} is still growing exponentially. However, for $\sin^2 2\theta_0^{\tau e'}$ sufficiently large, the ν_e-ν_e' resonance can occur during the rapid exponential growth phase of L_{ν_τ}. If this happens then the rate at which the ν_τ-ν_e' oscillations move the system away from the ν_e-ν_e' resonance is much more rapid. Consequently, the region of parameter space where $L^{(e)}$ survives without being destroyed by ν_e-ν_e' oscillations is significantly larger in this case (this effect will be illustrated later on when we study the system numerically).

Using Eq. (108), Eq. (107) can be written in the form

$$\frac{3}{4} \frac{\partial L_{\nu_e}}{\partial t} \frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_e}} \gtrsim \left[1 + \frac{L^{(\tau)}}{T} \frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_e}} \right] \frac{dT}{dt}, \tag{109}$$

where we have used the relation $\partial T_{\rm res}^{ee'}/\partial L_{\nu_e} = 2\partial T_{\rm res}^{ee'}/\partial L_{\nu_\tau}$ which is easily obtainable from Eq. (105).

Note that the most stringent condition occurs at the ν_e-ν_e' resonance temperature, Eq. (104). We are primarily interested in relatively large values of $|\delta m^2_{\tau e'}| \gtrsim 10^{-1}$ eV^2, which means that $T_{\rm res}^{ee'} \gtrsim 8$ MeV. Thus, from Sec. III, we are in a region of parameter space where we expect Eq. (23) to be valid. [In particular, note that since $T_{\rm res}^{ee'}$ is not at the point where the lepton number is initially created, Eq. (75) should also be valid.] Thus, from Eq. (23) we can obtain the rate of change of L_{ν_e} due to ν_e-ν_e' oscillations, at the ν_e-ν_e' resonance (where $b - a - c = 0$). We find

$$\frac{\partial L_{\nu_e}}{\partial t} \simeq -\frac{3}{8} \frac{\sin^2 \theta_0^{ee'}}{\Gamma_{\nu_e} T^2} \left[\frac{\delta m^2_{ee'}}{6.3} \right]^2. \tag{110}$$

Note that from Eq. (105), we have

$$\frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_e}} = \frac{M_W^2}{A_e T_{\rm res}^{ee'}} = \frac{T_{\rm res}^{ee'}}{L^{(e)}}. \tag{111}$$

Thus,

$$\frac{L^{(\tau)}}{T_{\rm res}^{ee'}} \frac{\partial T_{\rm res}^{ee'}}{\partial L_{\nu_e}} = \frac{L^{(\tau)}}{L^{(e)}} \simeq 2. \tag{112}$$

Hence the sufficient condition that $L^{(e)}$ survives without being destroyed by ν_e-ν_e' oscillations can be obtained by substituting Eqs. (110)–(112) into Eq. (109). Doing this exercise we find

$$|\delta m^2_{ee'}| \lesssim \lambda |\delta m^2_{\tau e'}|^{11/12}, \tag{113}$$

where λ is given by

$$\lambda \simeq \frac{12.6 G_F}{\sqrt{\sin^2 2\theta_0^{ee'}} M_W}\left(\frac{8y_e A_e 5.5}{3M_P}\right)^{1/2}\left(\frac{T^{ee'}_{\rm res}}{(|\delta m^2_{\tau e'}|)^{1/6}}\right)^{11/2}$$
$$\simeq \frac{12.6 G_F}{\sqrt{\sin^2 2\theta_0^{ee'}} M_W}\left(\frac{8y_e A_e 5.5}{3M_P}\right)^{1/2}$$
$$\times\left(\frac{4.1 M_W^2}{6.3\sqrt{2} G_F A_\tau}\frac{A_\tau}{A_\tau + 2A_e}\right)^{11/12}, \tag{114}$$

and we have used Eq. (104). Thus, putting the numbers in, we find

$$\frac{|\delta m^2_{ee'}|}{\mathrm{eV}^2} \lesssim 6\times 10^{-7}\left(\frac{|\delta m^2_{\tau e'}|}{\mathrm{eV}^2}\right)^{11/12}, \tag{115}$$

where we have assumed maximal mixing (i.e., $\sin^2 2\theta_0^{ee'} \simeq 1$). Thus provided this condition holds, $L^{(e)}$ will not be destroyed significantly by ν_e-ν_e' oscillations (under the assumption that the neutrino thermal momentum distribution can be neglected; we will study the effects of the momentum distribution in a moment) and the system moves quickly away from the ν_e-ν_e' resonance. While this condition was derived as a sufficient condition, it turns out to be a necessary one as well. This is because if Eq. (107) were not valid, then ν_e-ν_e' oscillations would create L_{ν_e} rapidly enough such that $\partial(T^{ee'}_{\rm res} - T)/\partial t < 0$. This would mean that the ν_e-ν_e' resonance temperature would move to lower and lower temperatures where the rate of change of L_{ν_e} [from Eq. (110)] would be even larger (as it is proportional to $1/T^7$) and the expansion rate slower. Thus if the condition Eq. (107) were not satisfied initially, it could certainly not be satisfied for lower temperatures.

In the above system consisting of ν_τ, ν_e, and ν_e' (and their antiparticles) discussed above, observe that we have neglected the effects of ν_τ-ν_e oscillations. As discussed in the previous section, the effect of these oscillations is to make $(L_{\nu_\tau} - L_{\nu_e})$ tend to zero. Since these oscillations cannot prevent L_{ν_τ} from being generated the effect of incorporating them should only increase the allowed region of parameter space [48].

The effect of the muon neutrino can also only increase the allowed region of parameter space. The effect of ν_μ-ν_e' oscillations will be to create L_{ν_μ} provided that $\delta m^2_{\mu e'} < 0$. The effects of ν_μ are completely analogous to the effects of the ν_τ neutrino, and we can replace ν_τ with ν_μ in the above analysis. This means that it is only necessary that either $\delta m^2_{\tau e'}$ or $\delta m^2_{\mu e'}$ (or both) satisfy Eq. (115).

Hitherto we have examined the system neglecting the thermal distribution of neutrino momenta. We now study the realistic case where the thermal distribution of neutrino momenta is taken into consideration. We first estimate approximately the effects of the momentum distribution analytically and then we will perform a more accurate numerical study.

The previous calculation assumes that all of the neutrinos have a common momentum and thus they all enter the resonance at the same time. In the realistic case, only a small fraction (less than about 1% as we will show) of the neutrinos are at resonance at any given time. Note that the ν_τ-ν_e' oscillations are not affected greatly by this consideration, since as we showed in Sec. V, the momentum spread does not prevent L_{ν_τ} from being created (and it still satisfies approximately $L^{(\tau)} \sim T^4$ after it is initially created). On the other hand, the effect of the neutrino momentum distribution on the ν_e-ν_e' oscillations is very important. This is because the ν_e-ν_e' oscillations cannot destroy $L^{(e)}$ as efficiently as before. In fact, Eq. (110) will be reduced by a factor which is about equal to the fraction of neutrinos at the resonance. In principle, one should solve Eq. (109) at the point where electron neutrinos of momentum $p = \gamma T$ are at resonance and then calculate the minimum of the value of λ [see Eqs. (113) and (114)] over the range of all possible values of γ. For simplicity we will make a rough approximation and assume that the minimum of λ occcurs when neutrinos of average momentum are at resonance (i.e., assume $\gamma \simeq 3.15$). (Note that later on we will do a more accurate numerical calculation).

To calculate the fraction of ν_e neutrinos at the ν_e-ν_e' resonance we need to calculate the width of the resonance in momentum space. We will denote this width by Δp. From Eq. (84) it is easy to see that the width of the resonance is governed approximately by the equation

$$\Delta p \left|\frac{\partial(b^p_{ee'} - a^p_{ee'})}{\partial p}\right| \simeq 2\sqrt{x_{ee'}}, \tag{116}$$

where we have assumed maximal mixing (i.e., $\cos 2\theta_0^{ee'} \simeq 0$). Note that from the momentum dependence of b^p, a^p [see Eq. (81)], it follows that

$$\frac{\partial b^p_{ee'}}{\partial p} = \frac{2b^p_{ee'}}{p}, \quad \frac{\partial a^p_{ee'}}{\partial p} = \frac{a^p_{ee'}}{p}, \tag{117}$$

and hence

$$\frac{\partial(b^p_{ee'} - a^p_{ee'})}{\partial p} = \frac{2b^p_{ee'}}{p} - \frac{a^p_{ee'}}{p} \simeq \frac{a^p_{ee'}}{p}, \tag{118}$$

where we have used the result that $b^p_{ee'} \simeq a^p_{ee'}$ at the resonance (note that we have assumed that $L^{(e)} > 0$ for definiteness). Note that we are essentially interested in evaluating the maximum value of the fraction of neutrinos at the resonance. This maximum fraction should occur approximately when $p \sim \langle p \rangle$. Thus, from the previous analysis, the resonance for neutrinos of average momentum occurs when

$$a_{ee'} \simeq \frac{a_{\tau e'}}{2} \frac{\delta m^2_{\tau e'}}{\delta m^2_{ee'}} \simeq \frac{1}{2} \frac{\delta m^2_{\tau e'}}{\delta m^2_{ee'}}. \tag{119}$$

Thus, from Eqs. (118) and (119) and Eq. (116) the width of the resonance in momentum space becomes

$$\Delta p \approx 4p\sqrt{\langle x_{ee'}\rangle}\left[\frac{|\delta m^2_{ee'}|}{|\delta m^2_{\tau e'}|}\right]. \tag{120}$$

Recall that $\langle x_{ee'}\rangle$ is defined in Eq. (25), and is given by

$$\langle x_{ee'}\rangle = \sin^2 2\theta_0^{ee'} + \Gamma^2_{\nu_e}\frac{\langle p\rangle^2}{(\delta m^2_{ee'})^2}$$
$$\simeq \sin^2 2\theta_0^{ee'} + \frac{y_e^2 G_F^4 (3.15)^2 T^{12}}{(\delta m^2_{ee'})^2}. \tag{121}$$

Expanding out $\langle x_{ee'}\rangle$ at the temperature $T \approx T^{ee'}_{\rm res}$ [defined in Eq. (104)], we find

$$\langle x_{ee'}\rangle = \sin^2 2\theta_0^{ee'} + \left[\frac{y_e G_F M_W^2 4.1}{2\sqrt{2}A_\tau}\frac{A_\tau}{A_\tau + 2A_e}\right]^2\left[\frac{\delta m^2_{\tau e'}}{\delta m^2_{ee'}}\right]^2$$
$$\simeq 1.2\times 10^{-5}\left[\frac{\delta m^2_{\tau e'}}{\delta m^2_{ee'}}\right]^2, \tag{122}$$

where the last part follows provided that $|\delta m^2_{ee'}| \lesssim 10^{-3}|\delta m^2_{\tau e'}|$. Thus, using the above equation, Eq. (120) simplifies to

$$\frac{\Delta p}{p} \simeq \frac{2y_e G_F M_W^2 4.1}{\sqrt{2}A_\tau}\frac{A_\tau}{A_\tau + A_e} \simeq 1.4\times 10^{-2}. \tag{123}$$

Thus it is clear that only a small fraction of neutrinos will be at the resonance. We denote the fraction of electron neutrinos at the ν_e-ν_e' resonance by $\Delta n_\nu/n_\nu$. Note that $\Delta n_\nu/n_\nu$ is given approximately by the equation

$$\frac{\Delta n_\nu}{n_\nu} \simeq \frac{\Delta p}{n_\nu}\frac{dn_\nu}{dp}. \tag{124}$$

Using $n_\nu = \frac{3}{4}\zeta(3)T^3/\pi^2$, and

$$\frac{dn_\nu}{dp} = \frac{1}{2\pi^2}\frac{p^2}{1+e^{p/T}}, \tag{125}$$

we find

$$\left.\frac{\Delta n_\nu}{n_\nu}\right|_{\rm max} \approx \frac{2y_e G_F M_W^2 4.1}{\sqrt{2}A_\tau}\frac{A_\tau}{A_\tau + 2A_e}\frac{\langle p/T\rangle^3}{1.5\zeta(3)(1+e^{\langle p\rangle/T})}$$
$$\simeq \frac{1.4\times 10^{-2}}{1.5\zeta(3)}\frac{\langle p/T\rangle^3}{1+e^{\langle p\rangle/T}} \simeq 1.0\times 10^{-2}. \tag{126}$$

The effect of the momentum spread is thus to reduce the number of neutrinos at the resonance by the above factor. Multiplying the right-hand side of Eq. (110) by this fraction and repeating the same steps which lead to Eq. (115) we find that Eq. (115) is weakened by the factor $\sqrt{\Delta n_\nu/n_\nu} \simeq 10^{-1}$. In other words the effect of the neutrino momentum distribution is to increase the allowed region of parameter space for which ν_e-ν_e' oscillations do not destroy the $L^{(e)}$ asymmetry created by ν_τ-ν_e' oscillations. This region of parameter space is given approximately by

$$|\delta m^2_{ee'}| \lesssim \frac{\lambda}{\sqrt{\Delta n_\nu/n_\nu}}|\delta m^2_{\tau e'}|^{11/12}, \tag{127}$$

where λ is given in Eq. (114) and $\sqrt{\Delta n_\nu/n_\nu}$ is given in Eq. (126). Putting the numbers in, the above condition can be written in the form

$$\frac{|\delta m^2_{ee'}|}{\mathrm{eV}^2} \lesssim 6\times 10^{-6}\left(\frac{|\delta m^2_{\tau e'}|}{\mathrm{eV}^2}\right)^{11/12}. \tag{128}$$

We now check this result by doing a more accurate numerical study of this problem.

The rate of change of L_{ν_e} and L_{ν_τ} due to the ν_τ-ν_e', ν_e-ν_e' oscillations can be obtained from Eq. (94). This leads to the following coupled differential equations:

$$\frac{dL_{\nu_e}}{dt} \simeq \frac{1}{4\zeta(3)T^3}\int_0^\infty \frac{\sin^2 2\theta_0^{ee'}\Gamma^p_{\nu_e}a^p_{ee'}(\cos 2\theta_0^{ee'} - b^p_{ee'})}{[x^p_{ee'} + (\cos 2\theta_0^{ee'} - b^p_{ee'} + a^p_{ee'})^2][x^p_{ee'} + (\cos 2\theta_0^{ee'} - b^p_{ee'} - a^p_{ee'})^2]}\frac{(1-z^+)p^2dp}{(1+e^{p/T})}$$
$$+ \frac{1}{8\zeta(3)T^3}\int_0^\infty \frac{\sin^2 2\theta_0^{ee'}\Gamma^p_{\nu_e}[x^p_{ee'} + (a^p_{ee'})^2 + (b^p_{ee'} - \cos 2\theta_0^{ee'})^2]}{[x^p_{ee'} + (\cos 2\theta_0^{ee'} - b^p_{ee'} + a^p_{ee'})^2][x^p_{ee'} + (\cos 2\theta_0^{ee'} - b^p_{ee'} - a^p_{ee'})^2]}\frac{z^- p^2 dp}{1+e^{p/T}},$$

$$\frac{dL_{\nu_\tau}}{dt} \simeq \frac{1}{4\zeta(3)T^3}\int_0^\infty \frac{\sin^2 2\theta_0^{\tau e'}\Gamma^p_{\nu_\tau}a^p_{\tau e'}(\cos 2\theta_0^{\tau e'} - b^p_{\tau e'})}{[x^p_{\tau e'} + (\cos 2\theta_0^{\tau e'} - b^p_{\tau e'} + a^p_{\tau e'})^2][x^p_{\tau e'} + (\cos 2\theta_0^{\tau e'} - b^p_{\tau e'} - a^p_{\tau e'})^2]}\frac{(1-z^+)p^2dp}{1+e^{p/T}}$$
$$+ \frac{1}{8\zeta(3)T^3}\int_0^\infty \frac{\sin^2 2\theta_0^{\tau e'}\Gamma^p_{\nu_\tau}[x^p_{\tau e'} + (a^p_{\tau e'})^2 + (b^p_{\tau e'} - \cos 2\theta_0^{\tau e'})^2]}{[x^p_{\tau e'} + (\cos 2\theta_0^{\tau e'} - b^p_{\tau e'} + a^p_{\tau e'})^2][x^p_{\tau e'} + (\cos 2\theta_0^{\tau e'} - b^p_{\tau e'} - a^p_{\tau e'})^2]}\frac{z^- p^2 dp}{1+e^{p/T}}. \tag{129}$$

These equations are coupled differential equations because $a^p_{ee'}$ and $a^p_{\tau e'}$ depend on both L_{ν_e} and L_{ν_τ}. Recall that $z^{\pm}=(z\pm\overline{z})/2$. From Eq. (93) the z parameter, which is related to the number of sterile neutrinos produced, is governed by

$$\frac{dz}{dt}=\frac{1}{4}(1-z)\left[\frac{\Gamma^p_{\nu_e}\sin^2 2\theta_0^{ee'}}{x^p_{ee'}+(\cos 2\theta_0^{ee'}-b^p_{ee'}+a^p_{ee'})^2}+\frac{\Gamma^p_{\nu_\tau}\sin^2 2\theta_0^{\tau e'}}{x^p_{\tau e'}+(\cos 2\theta_0^{\tau e'}-b^p_{\tau e'}+a^p_{\tau e'})^2}\right], \qquad (130)$$

and the evolution of $\overline{z}$ is governed by an equation similar to the above (but with $a^p\to -a^p$).

The above equations can be integrated numerically (following the proceedure mentioned in Sec. V). Doing this, we can find the region of parameter space where the $L^{(e)}$ asymmetry created by the ν_τ-ν_e' oscillations does not get destroyed by the ν_e-ν_e' oscillations. We will solve Eq. (129) and Eq. (130) under the assumption that $\sin^2 2\theta_0^{ee'}\simeq 1$ (i.e., the ν_e-ν_e' oscillations are approximately maximal). Performing the necessary numerical work, we find that $L^{(e)}$ is created by ν_τ-ν_e' oscillations and not subsequently destroyed by ν_e-ν_e' oscillations for the region of parameter space shown in Fig. 5. For definiteness we have taken two illustrative choices for $\sin^2 2\theta_0^{\tau e'}$, $\sin^2 2\theta_0^{\tau e'}=10^{-8},10^{-6}$. Note that in our numerical work, we have studied the region $10^{-1}\lesssim|\delta m^2_{\tau e'}|/\mathrm{eV}^2\lesssim 10^3$. Of course, there will be parameter space outside this region where the $L^{(e)}$ created by ν_τ-ν_e' oscillations is not destroyed by ν_e-ν_e' oscillations. However, one should keep in mind that there is a rather stringent cosmology bound, $m_{\nu_\tau}\lesssim 40$ eV [49] (which implies that $|\delta m^2_{\tau e'}|\lesssim 1600$ eV^2). This bound assumes that the neutrino is approximately stable, which is expected given the standard model interactions. Of course, if there are new interactions beyond the standard model, then it is possible to evade this cosmology bound [50].

Observe that the region of parameter space where $L^{(e)}$ survives is somewhat larger than our analytical estimate Eq. (128). This is partly because the point where L_{ν_τ} is created occurs at a significantly higher temperature than the analytical estimate (see Sec. V for some discussion about this point). Note that the quantity $\lambda/\sqrt{\Delta n_\nu/n_\nu}\propto T_{\mathrm{res}}^{11/2}/T_{\mathrm{res}}^3\sim T_{\mathrm{res}}^{5/2}$. Thus, the result that the lepton number is created at a higher temperature than our analytic estimate can easily lead to a significant increase in the parameter space. More importantly, for large $\sin^2 2\theta_0^{\tau e'}$ the magnitude of L_{ν_τ} created by ν_τ-ν_e' oscillations is considerably larger before the growth of L_{ν_τ} is cut off by the nonlinearity of the differential equation governing its evolution (compare the solid line with the dashed or dash-dotted lines in Figs. 3 or 4). Recall that our analytical estimate assumed that the creation of $L^{(e)}$ due to ν_τ-ν_e' oscillations had already passed the rapid exponential growth phase at the point where the destruction of $L^{(e)}$ due to ν_e-ν_e' oscillations reached a maximum. While this latter assumption is generally true for small values of $\sin^2 2\theta_0^{\tau e'}$, it is not true for larger values. In this case, the rate of change of $T_{\mathrm{res}}^{ee'}$ due to ν_τ-ν_e' oscillations will be much larger than our analytical estimate. Consequently, the allowed region of parameter space is increased. Thus the result that the allowed region of parameter space for $\sin^2 2\theta_0^{\tau e'}=10^{-6}$ is significantly larger than the allowed region for $\sin^2 2\theta_0^{\tau e'}=10^{-8}$ is not unexpected.

Having established the condition that the ν_e-ν_e' oscillations do not destroy the $L^{(e)}$ which is created by the ν_τ-ν_e' oscillations (or ν_μ-ν_e' oscillations), we must also check that the magnitude of $L^{(e)}$ is large enough to invalidate the bound in Eq. (99).

For $\delta m^2_{ee'}$ in the range $|\delta m^2_{ee'}|\gtrsim 10^{-6}$ eV^2, the bound Eq. (99) arises by requiring that the ν_e-ν_e' oscillations do not bring the ν_e' sterile neutrino into equilibrium above the kinetic decoupling temperature (~ 3 MeV). The sterile neutrino ν_e' will not be brought into equilibrium provided that the rate of ν_e' production is approximately less than the expansion rate H, i.e.,

$$\Gamma(\nu_e\to\nu_e')/H\simeq\frac{1}{4}\Gamma_{\nu_e}\sin^2 2\theta_m^{ee'}/H\lesssim 1, \qquad (131)$$

where we have used Eq. (9) with $\langle\sin^2\tau/2L_m\rangle\simeq 1/2$ [51]. Recall that we are primarily interested in the region 1 MeV $\lesssim T\lesssim 100$ MeV, where $H\simeq 5.5T^2/M_P$. Using Eq. (19) with $a\simeq 0$, the above equation can be rewritten in the form

$$\frac{y_e G_F^2 M_P \sin^2 2\theta_0^{ee'}T^3}{22[b^2_{ee'}+1]}\lesssim 1, \qquad (132)$$

where we have assumed large mixing, i.e., $\cos 2\theta_0^{ee'}\ll 1$. Recall that $b_{ee'}$ can be obtained from Eq. (18). Obtaining the maximum of the left-hand side of Eq. (132) leads approximately to the bound $|\delta m^2|\lesssim 10^{-6}$ eV^2.

In the case where L_{ν_τ} is created by ν_τ-ν_e' oscillations, the situation is very different. The lepton number L_{ν_τ} is created at the temperature when $b_{\tau e'}\simeq 1$ (assuming that $\cos 2\theta_0^{\tau e'}\sim 1$). Denoting this temperature by $T_c^{\tau e'}$, then as per Eq. (34)

$$T_c^{\tau e'}\approx 16\left(\frac{|\delta m^2_{\tau e'}|}{\mathrm{eV}^2}\right)^{1/6}\ \mathrm{MeV}. \qquad (133)$$

The evolution of this system can be divided into two regions, the region before lepton number creation (i.e., $T>T_c^{\tau e'}$), and the region after the lepton number creation (i.e., $T<T_c^{\tau e'}$). In the region before the lepton number is created, $a_{\tau e'}\simeq 0$ and Eq. (132) holds. We will obviously be interested in the parameter space where $\delta m^2_{\tau e'}$ is sufficiently large [recall that $\delta m^2_{\tau e'}$ is related to $T_c^{\tau e'}$ by Eq. (133) above] so that L_{ν_τ} is created at some point above the kinetic decoupling temperature $T_{\mathrm{dec}}\simeq 3$ MeV, of ν_e. Let us assume that $|\delta m^2_{\tau e'}|$ is large enough so that $b^2_{ee'}\gg 1$ for temperatures $T>T_c^{\tau e'}$ (which corresponds approximately to, $|\delta m^2_{\tau e'}|>|\delta m^2_{ee'}|$). In this case, $b^2_{ee'}+1\simeq b^2_{ee'}$ and Eq. (132) can be rewritten in the form

$$T^9 \gtrsim \left(\frac{4.1\delta m^2_{ee'}M_W^2}{6.3\sqrt{2}G_F A_e}\right)^2 \frac{\sin^2 2\theta_0^{ee'} y_e G_F^2 M_P}{22}, \qquad (134)$$

where we have used Eq. (18) with $n_\gamma \simeq T^3/4.1$. Observe that the most stringent condition occurs for $T=T_c^{\tau e'}$. Thus taking $T=T_c^{\tau e'}$ [Eq. (133)], we find that the sterile neutrino ν_e' will not come into equilibrium with the ordinary neutrinos for temperatures above $T_c^{\tau e'}$ provided that

$$\frac{|\delta m^2_{\tau e'}|}{\text{eV}^2} \gtrsim 15\left[\frac{|\delta m^2_{ee'}|}{\text{eV}^2}\right]^{4/3} [\sin^2 2\theta_0^{ee'}]^{2/3}. \qquad (135)$$

Assuming maximal ν_e-ν_e' oscillations (i.e., $\sin 2\theta_0^{ee'}\simeq 1$) and assuming $|\delta m^2_{ee'}|\lesssim 10^{-3}$ eV^2 [37], Eq. (135) implies that

$$|\delta m^2_{\tau e'}| \gtrsim 10^{-3} \text{ eV}^2. \qquad (136)$$

Thus provided that this constraint is satisfied, the sterile neutrino, ν_e' will not come into equilibrium for temperatures greater than the temperature where L_{ν_τ} is created, $T_c^{\tau e'}$.

We now need to check that the lepton number created is sufficient to suppress ν_e-ν_e' oscillations for temperatures less than $T_c^{\tau e'}$. Demanding that the interactions do not bring the sterile neutrino into equilibrium with the ordinary neutrinos, that is again imposing the inequality Eq. (131), but this time for $T<T_c^{\tau e'}$ where there is significant creation of $L^{(e)}$ [11], we find that

$$\frac{y_e G_F^2 M_P \sin^2 2\theta_0^{ee'} T^3}{22[(b_{ee'}\pm a_{ee'})^2+1]} \lesssim 1, \qquad (137)$$

where the $-$ $(+)$ signs correspond to ν_e-ν_e' ($\overline{\nu_e}$-$\overline{\nu_e'}$) oscillations. Note that Eq. (137) is only required to be satisfied for $T>T_{\rm dec}\simeq 3$ MeV (since we only need to require that the sterile neutrinos do not come into equilibrium before kinetic decoupling of the electron neutrinos occurs). Once $L^{(e)}$ is created at $T=T_c^{\tau e'}$ (where $b_{\tau e'}=\cos 2\theta_0^{\tau e'}\simeq 1$), its magnitude will rise according to the constraint $a_{\tau e'}\gtrsim 1$ (assuming for definiteness that $L^{(e)}>0$). Note that the quantities $b_{ee'}, a_{ee'}$ are related to $b_{\tau e'}, a_{\tau e'}$ as follows:

$$\frac{b_{ee'}}{b_{\tau e'}} = \frac{A_e}{A_\tau}\frac{\delta m^2_{\tau e'}}{\delta m^2_{ee'}}, \quad \frac{a_{ee'}}{a_{\tau e'}} \simeq \frac{1}{2}\frac{\delta m^2_{\tau e'}}{\delta m^2_{ee'}}. \qquad (138)$$

After the initial resonance $a_{\tau e'}\gtrsim 1$ while $b_{\tau e'}\lesssim 1$ (and quickly becomes much less than one). Thus very soon after the resonance, $a_{\tau e'}\gg b_{\tau e'}$ and hence from Eq. (138), $a_{ee'}\gg b_{ee'}$. As before the most stringent bound occurs when $T\simeq T_c^{\tau e'}$, and Eq. (137) leads to approximately the same bound as before [i.e., Eq. (136)], since at the point $T=T_c^{\tau e'}$, $a_{\tau e'}\approx b_{\tau e'}$.

Finally, we need to check that the oscillations of the ν_e, ν_e' neutrinos do not significantly deplete the number of electron neutrinos for the temperature range,

$$0.7 \text{ MeV} \lesssim T \lesssim T_{\rm dec} \simeq 3 \text{ MeV}. \qquad (139)$$

Electron neutrino oscillations in this temperature range can affect BBN because they will deplete electron neutrinos (and antineutrinos) and thus modify the temperature when the neutron/proton ratio freezes out. This effect is generally small unless $\sin^2 2\theta_m \gtrsim 10^{-2}$ [6,8]. If we demand that $\sin^2 2\theta_m \lesssim 10^{-2}$ for this temperature range, then from Eq. (19) we require $|a|\gtrsim 10$ (for the most stringent case of maximal mixing) for this temperature range (or $|\delta m^2|\lesssim 10^{-8}$ eV^2). Thus, from Eq. (18), $|a|\gtrsim 10$ implies

$$|L^{(e)}| \gtrsim 2\left(\frac{|\delta m^2_{ee'}|}{\text{eV}^2}\right). \qquad (140)$$

Recall that for temperatures $T\gtrsim T_f$ [where T_f is the temperature where the change in a due to the expansion is larger in magnitude to the change in a due to oscillations, see the earlier comments around Eq. (39) for some discussion about this], L_{ν_τ} is created such that $a_{\tau e'}\simeq 1$, from this it follows that

$$L^{(e)} \simeq L_{\nu_\tau} \simeq 2\times 10^{-2}\left(\frac{|\delta m^2_{\tau e'}|}{\text{eV}^2}\right)\left(\frac{\text{MeV}}{T_f}\right)^4. \qquad (141)$$

Combining Eq. (140) and Eq. (141), sufficient lepton number will be generated to suppress the oscillations in the temperature range Eq. (139) provided that

$$|\delta m^2_{ee'}| \lesssim 10^{-2}|\delta m^2_{\tau e'}|\left(\frac{\text{MeV}}{T_f}\right)^4. \qquad (142)$$

Note that the temperature T_f is generally less than about 4 MeV [see Eq. (39) for a discussion about this]. Thus, Eq. (142) will be easily satisfied given the condition Eq. (128).

In summary, a consequence of the creation of L_{ν_τ} by ν_τ-ν_e' oscillations is that the large angle or maximal ν_e-ν_e'

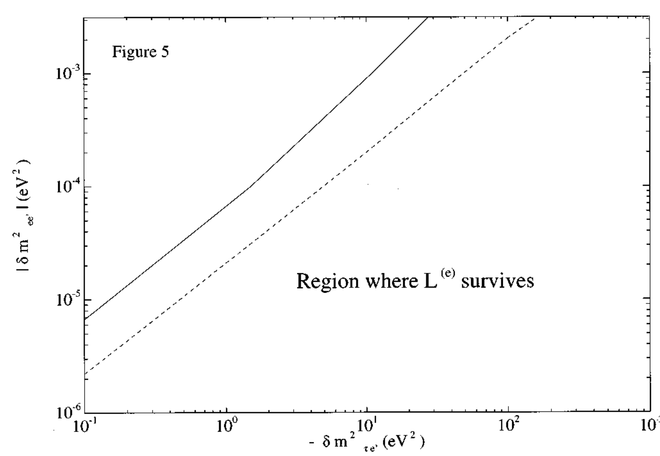

FIG. 5. Region of parameter space in the $-\delta m^2_{\tau e'}$, $|\delta m^2_{ee'}|$, plane (assuming $\sin^2 2\theta_0^{ee'}\simeq 1$) where the $L^{(e)}$ created by ν_τ-ν_e' oscillations does not get destroyed by ν_e-ν_e' oscillations. The solid line corresponds to $\sin^2 2\theta_0^{\tau e'}=10^{-6}$, while the dashed line corresponds to $\sin^2 2\theta_0^{\tau e'}=10^{-8}$. Note that similar results hold for ν_μ-ν_e' oscillations by replacing $\nu_\tau \to \nu_\mu$.

oscillations will not significantly modify BBN provided that $L^{(e)}$ does not get destroyed by ν_e-ν'_e oscillations (see Fig. 5 for some of this region of parameter space) and the condition Eq. (136) holds. Thus, it is clear that the oscillation generated neutrino asymmetry can weaken the rather stringent BBN bound ($|\delta m^2_{ee'}| \lesssim 10^{-8}$ eV^2 for maximal mixing) by many orders of magnitude. A consequence of this is that the maximal ordinary-sterile neutrino oscillation solution to the solar neutrino problem does not significantly modify BBN for a large range of parameters.

While we have focused on a particular scenario, our analysis will be relevant to other models with sterile neutrinos. For example, assume that there is a sterile neutrino which mixes with parameters corresponding to the large angle MSW solution to the solar neutrino problem, that is $\delta m^2 \sim 10^{-5}$ eV^2 and $\sin^2 2\theta_0 \sim 0.7$ [52]. This scenario has been ''ruled out'' (assuming negligible lepton number asymmetry) in [6,8]. However, if the sterile neutrino also mixes slightly with the μ and/or τ neutrino (and such mixing would be expected), then these BBN bounds can be evaded provided that $|\delta m^2_{\tau e'}|$ and/or $|\delta m^2_{\mu e'}| \gtrsim 0.1-1$ eV^2. Note that the evidence for ν_μ-ν_e oscillations found by the LSND collaboration suggests that $|\delta m^2_{\mu e}| \gtrsim 0.3$ eV^2 [3,47]. If this is the case then the large angle MSW solution will not lead to a significant modification to BBN for a large range of values for $\sin^2 2\theta_0^{\mu e'}$.

We now discuss the possibility that the atmospheric neutrino anomaly is due to large angle or maximal muon neutrino-sterile neutrino oscillations. Here, we will denote the sterile neutrino by ν'_μ (this neutrino is expected to be distinct from ν'_e). Note that the possibility that the atmospheric neutrino anomaly is due to large angle or maximal ν_μ-ν'_μ oscillations can be well motivated. For example, the exact parity model [17] predicts that all three ordinary neutrinos mix maximally with mirror neutrinos if neutrinos have mass. (See also Ref. [38] for some other interesting models which can solve the atmospheric neutrino anomaly through maximal ordinary-sterile neutrino oscillations.) The deficit of atmospheric muon neutrinos can be explained if there are ν_μ-ν'_μ oscillations with $\sin^2 2\theta_0 \gtrsim 0.5$ and $10^{-3} \lesssim |\delta m^2_{\mu\mu'}|/\mathrm{eV}^2 \lesssim 10^{-1}$ [2,53]. The best fit occurs for $|\delta m^2_{\mu\mu'}| \simeq 10^{-2}$ eV^2 and $\sin^2 2\theta_0 \simeq 1$ [2]. However, this parameter range is naively inconsistent with BBN [see Eq. (1)] if the lepton number asymmetries are neglected. Can the generation of lepton number by ordinary-sterile neutrino oscillations reconcile this solution to the atmospheric neutrino anomaly with BBN?

To study this issue, consider the system consisting of $\nu_\tau, \nu_\mu, \nu'_\mu$. This system is similar to the ν_τ, ν_e, ν'_e system that we have discussed above. Doing a similar analysis to the above (i.e., replacing ν_e and ν'_e by ν_μ and ν'_μ), we find that the $L^{(\mu)}$ asymmetry created by ν_τ-ν'_μ oscillations will not be destroyed by ν_μ-ν'_μ oscillations provided that

$$|\delta m^2_{\mu\mu'}| \lesssim \frac{\lambda}{\sqrt{\Delta n_\nu / n_\nu}} |\delta m^2_{\tau\mu'}|^{11/12}, \tag{143}$$

where λ and $\Delta n_\nu/n_\nu$ are given by equations similar to Eq. (114) and Eq. (126) except that the replacements

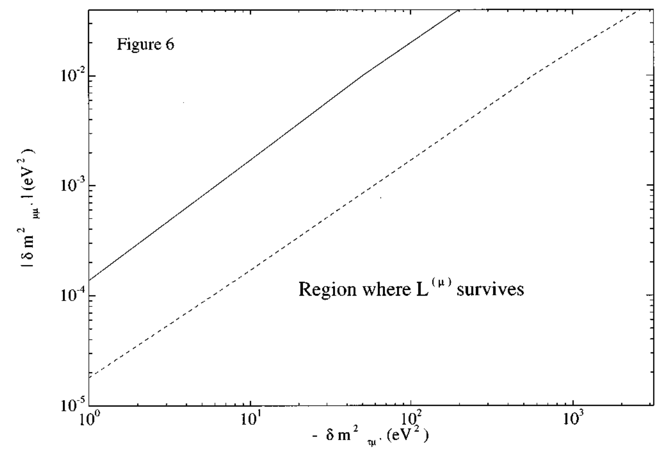

FIG. 6. Region of parameter space in the $-\delta m^2_{\tau\mu'}$, $|\delta m^2_{\mu\mu'}|$ plane where the $L^{(\mu)}$ created by ν_τ-ν'_μ oscillations does not get destroyed by ν_μ-ν'_μ oscillations (assuming $\sin^2 2\theta_0^{\mu\mu'} \simeq 1$). The solid line corresponds to $\sin^2 2\theta_0^{\tau\mu'} = 10^{-6}$, while the dashed line corresponds to $\sin^2 2\theta_0^{\tau\mu'} = 10^{-8}$. Note that similar results hold for ν_τ-ν'_e oscillations if both ν'_μ and ν'_e exist.

$y_e \to y_\mu$, $A_e \to A_\mu$ have to be made. Thus, evaluating the resulting expressions for λ and $\Delta n_\nu/n_\nu$, we find

$$\frac{|\delta m^2_{\mu\mu'}|}{\mathrm{eV}^2} \lesssim 5 \times 10^{-6} \left(\frac{|\delta m^2_{\tau\mu'}|}{\mathrm{eV}^2} \right)^{11/12}. \tag{144}$$

As before, we have made a more accurate numerical study of this problem. If we solve the system of equations Eq. (129) and Eq. (130), with the replacements $\nu_e, \nu'_e \to \nu_\mu, \nu'_\mu$, then we can obtain the region of parameter space where the $L^{(\mu)}$ created by ν_τ-ν'_μ oscillations does not get destroyed by ν_μ-ν'_μ oscillations. We show some of this parameter space in Fig. 6.

If we assume the best fit of the atmospheric neutrino data, then $|\delta m^2_{\mu\mu'}| \simeq 10^{-2}$ eV^2 and $\sin^2 2\theta_0^{\mu\mu'} \simeq 1$. Numerically solving Eqs. (129) and Eq. (130) (with the replacement of ν_e, ν'_e with ν_μ, ν'_μ) assuming the best fit parameters, $|\delta m^2_{\mu\mu'}| \simeq 10^{-2}$ eV^2 and $\sin^2 2\theta_0^{\mu\mu'} \simeq 1$, we again obtain the region of parameter space where the $L^{(\mu)}$ asymmetry is created by ν_τ-ν'_μ oscillations and does not get destroyed subsequently by ν_μ-ν'_μ oscillations. Our results are shown in Fig. 7. As the figure shows, the asymmetry $L^{(\mu)}$ created by ν_τ-ν'_μ oscillations will not be destroyed by ν_μ-ν'_μ oscillations provided that $\delta m^2_{\tau\mu'}$ is quite large, i.e.,

$$|\delta m^2_{\tau\mu'}| \gtrsim 30 \ \mathrm{eV}^2. \tag{145}$$

Recall that our analysis neglects the possible effects of ν_τ-ν_μ oscillations. It may be possible that smaller $\delta m^2_{\tau\mu'}$ are allowed if the ν_τ-ν_μ mixing parameters are large enough.

The requirement that ν_τ-ν'_μ oscillations do not produce too many sterile states implies an upper limit on $\sin^2 2\theta_0^{\tau\mu'}$ [see Eq. (96)]. This upper limit has been shown in the figure (dash-dotted line). Also shown in Fig. 7 (dashed line) is the cosmological energy density bound $|\delta m^2_{\tau\mu'}| \lesssim 1600$ eV^2 [49].

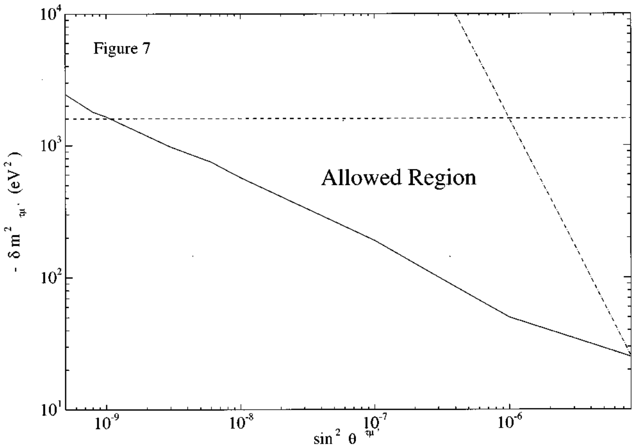

FIG. 7. Region of parameter space ($\sin^2 2\theta_0^{\tau\mu'}$, $-\delta m^2_{\tau\mu'}$) where the $L^{(\mu)}$ created by ν_τ-ν'_μ oscillations does not get destroyed by ν_μ-ν'_μ oscillations. This region which in the figure is denoted by the ''allowed region'' is all of the parameter space above the solid line. We have assumed that $\sin^2 2\theta_0^{\mu\mu'} \simeq 1$ and $|\delta m^2_{\mu\mu'}| = 10^{-2}$ eV^2 (which is the best fit to the atmospheric neutrino data). Also shown (the dashed line) is the cosmology bound $m_{\nu_\tau} \lesssim 40$ eV (which implies $|\delta m^2_{\tau\mu'}| \lesssim 1600$ eV^2), which is required if the neutrino is sufficiently long lived. The dash-dotted line is the BBN bound, Eq. (96).

Recall that the differential equations, Eq. (129) are only valid provided that Eq. (75) holds. [We also require Eq. (76) to hold for $\delta m^2 = \delta m^2_{\tau e'}$, which is clearly valid for the region of parameter space studied.] Note that in our numerical work we found that the condition Eq. (75) was approximately valid for the points in the allowed region of the figures except for the region with relatively large values of $\sin^2 2\theta_0 \gtrsim 10^{-6}$. It would be a useful exercise to check our analysis by performing a more accurate study using the density matrix equations, Eq. (46), modified to incorporate the neutrino momentum distribution.

The $\sin^2 2\theta_0^{\tau\mu'}$ dependence shown in Figs. 6 and 7 can be understood qualitatively as follows. For small $\sin^2 2\theta_0^{\tau\mu'}$ ($\lesssim 10^{-8}$), the creation of $L^{(\mu)}$ is sluggish which has the effect of delaying the point where the destruction of $L^{(\mu)}$ by ν_μ-ν'_μ oscillations reaches its maximum rate. As mentioned earlier, for lower temperatures the rate at which ν_μ-ν'_μ oscillations destroy $L^{(\mu)}$ increases, which has the effect of reducing the allowed parameter space. For larger values of $\sin^2 2\theta_0^{\tau\mu'}$ ($\gtrsim 10^{-8}$), the maximum rate at which the ν_μ-ν'_μ oscillations destroy $L^{(\mu)}$ occurs during the time when $L^{(\mu)}$ is still growing exponentially. In this case the system moves rapidly away from the ν_μ-ν'_μ resonance region. Consequently, the allowed region of parameter space is significantly increased.

Observe that from Eq. (135), this lepton number will easily be sufficiently large and created early enough to prevent the ν'_μ sterile neutrino from coming into equilibrium given Eq. (145). Thus, the large angle or maximal muon-sterile neutrino oscillation solution to the atmospheric neutrino anomaly is in fact consistent with BBN for a significant range of parameters. Note that the condition Eq. (145) implies quite large τ neutrino masses, $m_{\nu_\tau} \gtrsim 6$ eV. Note that if the neutrinos are approximately stable (which would be expected unless some new interactions exist [50]) then there is a stringent cosmology bound of $m_{\nu_\tau} \lesssim 40$ eV [49]. Although this parameter space is not so big, it can be well motivated from the point of view of dark matter (since stable tau neutrinos with masses in the range 6 eV$\lesssim m_{\nu_\tau} \lesssim 40$ eV could provide a significant fraction of the matter in the Universe).

If we add the ν'_μ sterile neutrino to the ν_τ, ν_e, ν'_e system we considered earlier (in connection to the large angle ordinary-sterile neutrino oscillation solution to the solar neutrino problem), then ν_τ-ν'_μ oscillations will also generate $L^{(e)}$ in a similar manner to the way in which ν_τ-ν'_e oscillations generated $L^{(e)}$. Consequently, the bounds on $\delta m^2_{\tau e'}$, $\sin^2 2\theta_0^{\tau e'}$, can alternatively be considered as bounds on $\delta m^2_{\tau\mu'}$, $\sin^2 2\theta_0^{\tau\mu'}$. Of course, we only need to require that either $\delta m^2_{\tau e'}$, $\sin^2 2\theta_0^{\tau e'}$ or $\delta m^2_{\tau\mu'}$, $\sin^2 2\theta_0^{\tau\mu'}$ satisfy the bounds derived. Similarly, we can add the ν'_e sterile neutrino to the $\nu_\tau, \nu_\mu, \nu'_\mu$ system and analogous reasoning leads to the conclusion that the bounds on $\delta m^2_{\tau\mu'}$, $\sin^2 2\theta_0^{\tau\mu'}$ can alternatively be considered as bounds on $\delta m^2_{\tau e'}$, $\sin^2 2\theta_0^{\tau e'}$. Observe that with $\delta m^2_{\tau e'}$, $\sin^2 2\theta_0^{\tau e'}$ or $\delta m^2_{\tau\mu'}$, $\sin^2 2\theta_0^{\tau\mu'}$ in the range identified in Fig. 7 (where the atmospheric neutrino anomaly is explained by large angle ν_μ-ν'_μ oscillations without significantly modifying BBN) the solar neutrino problem can also be solved for the entire parameter space [Eq. (98)], without significantly modifying BBN. Alternatively one can argue that the present data may allow the ν'_μ to come into equilibrium with the ordinary neutrinos and still be consistent with BBN [15] and thus we only require the less stringent bounds given in Fig. 5. Clearly this is a possibility at the moment. Note however, that for the case of the exact parity symmetric model [17], where the mirror neutrinos interact with themselves, this way out is not possible. This is because if the mirror muon neutrino is brought into equilibrium above the kinetic decoupling temperature (which is about 5 MeV for muon neutrinos) then the mirror weak interactions will bring all three mirror neutrinos together with the mirror photon and mirror electron-positron into equilibrium (which would lead to about nine effective neutrino degrees of freedom during nucleosynthesis). For the case of mirror neutrinos it seems to be necessary to ensure that the mirror muon neutrino is not brought into equilibrium in the first place.

Note that in our previous analysis, we have assumed that the sterile neutrino is truly sterile and does not interact with the background. In the special case of mirror neutrinos, the mirror neutrinos are expected to interact with the background because they interact with themselves [54]. In general the effective potential describing coherent forward scattering of the neutrino with the background has the form $V = V_\alpha - V'_s$. For truly sterile neutrinos, $V'_s = 0$ (as has been assumed hitherto). For mirror neutrinos V'_s is nonzero. Denoting the mirror neutrinos by ν'_β, then for the case of ν_α-ν'_β oscillations we will denote the effective potential by

$$V = V_\alpha - V'_\beta, \tag{146}$$

where V_α is given by Eq. (14) and V'_β is the effective potential due to the interactions of the mirror neutrinos with the background. The mirror effective potential V'_β can be expressed in an analogous way to V_α, that is there is a part which is proportional to mirror lepton number and a part which is independent of mirror lepton number,

$$V'_\beta = (-a'^p + b'^p)\Delta_0^p. \tag{147}$$

If the number of mirror neutrinos is much less than the number of ordinary neutrinos then $b' \simeq 0$. [Note that the b part of the effective potential is proportional to the number densities of the background particles. This dependence is not given in Eq. (15) since for this equation the number densities were set equal to their equilibrium values.] The parameter a' has the form

$$a'^p \equiv \frac{-\sqrt{2}G_F n_\gamma L'^{(\beta)}}{\Delta_0^p}, \tag{148}$$

where $L'^{(\beta)}$ is given

$$L'^{(\beta)} = L_{\nu'_\beta} + L_{\nu'_e} + L_{\nu'_\mu} + L_{\nu'_\tau} + \eta', \tag{149}$$

where $L_{\nu'_\beta}$ are the mirror lepton numbers, which are defined by $L_{\nu'_\beta} \equiv (n_{\nu'_\beta} - n_{\bar{\nu}'_\beta})/n_\gamma$ (note that n_γ is the number density of *ordinary* photons) and η' is a function of the mirror baryon-electron number asymmetries [which is defined analogous to Eq. (17)]. We will assume that η' is small and can be approximately neglected. Since ordinary + mirror lepton number is conserved (and we will assume that it is zero), it follows that

$$L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} + L_{\nu'_e} + L_{\nu'_\mu} + L_{\nu'_\tau} = 0. \tag{150}$$

From the above equation, it follows that a' is expected to be of the same order of magnitude as a. In the case of the $\nu_\tau, \nu_\mu, \nu'_\mu$ system, the effect of the mirror-neutrino effective potential can be accounted for by simply replacing $L^{(\mu,\tau)}$ in V_α by

$$\begin{aligned} L^{(\mu)} &\to L^{(\mu)} - L'^{(\mu)} \simeq 2L_{\nu_\mu} + L_{\nu_\tau} - 2L_{\nu'_\mu} \simeq 4L_{\nu_\mu} + 3L_{\nu_\tau}, \\ L^{(\tau)} &\to L^{(\tau)} - L'^{(\mu)} \simeq 2L_{\nu_\tau} + L_{\nu_\mu} - 2L_{\nu'_\mu} \simeq 4L_{\nu_\tau} + 3L_{\nu_\mu}, \end{aligned} \tag{151}$$

where we have used $L_{\nu_e'} \simeq L_{\nu'_\tau} \simeq 0$ and Eq. (150). Thus, from the above equation, assuming that negligible L_{ν_μ} is produced, we see that $|a_{\mu\mu'}| \simeq \frac{3}{4}R|a_{\tau\mu'}|$ (where $R \equiv |\delta m^2_{\tau\mu'}/\delta m^2_{\mu\mu'}|$). The factor of 3/4 replaces the factor of 1/2 that we had earlier (for the case where ν'_μ or ν'_e were sterile neutrinos). This difference will increase the region of allowed parameter space, because it will make $T_{\rm res}^{\mu\mu'}$ closer to the point where L_{ν_τ} is initially created. At this point $\partial L_{\nu_\tau}/\partial t$ can be significantly enhanced because it is very close to the resonance (also note that $\partial L_{\nu_\mu}/\partial t$ will be suppressed because it is proportional to $1/T^7$).

Finally, observe that another important feature of mirror neutrinos is that the mirror interactions can potentially bring all three of the mirror neutrinos into equilibrium with themselves as well as the mirror photon and mirror electron positron. [However, the temperature of the mirror particles will generally be less than the temperature of the ordinary particles if the oscillations satisfy Eq. (96).] Detailed studies involving mirror neutrinos will need to incorporate this. We leave a more detailed study of mirror neutrinos to the future [56].

VII. CONCLUSION

In summary, we have studied the phenomenon of neutrino oscillation generated lepton number asymmetries in the early Universe in detail. This extended study clarifies the origin of the approximations adopted in the earlier work [12]. We have also studied the effects of the thermal distribution of the neutrino momenta and non-negligible sterile neutrino number densities.

In the unrealistic case where the neutrino momentum distribution is neglected, the evolution of L_{ν_α} can be approximately described by seven coupled differential equations [Eqs. (46)], which can be obtained from the density matrix. We showed in Sec. III that these equations can be reduced to a single integrodifferential equation (we show in the Appendix that the same equation can be obtained from the Hamiltonian formalism). In general, the density matrix equations cannot be solved analytically, and must be solved numerically. However, if the system is sufficiently smooth (the static limit), then the integrodifferential equation reduces to a relatively simple first-order ordinary differential equation [Eq. (23)]. This equation gives quite a reasonable description of the evolution of L_{ν_α}, except possibly at the initial resonance where significant generation of L_{ν_α} occurs. We show that when the thermal distribution of the neutrino momenta is incorporated several important effects occur. One of these effects is that the creation of lepton number is much smoother. This allows a considerable computational simplification, because it means that the static approximation can be a reasonably good approximation, even at the resonance for a much larger range of parameters. This means that L_{ν_α} can be accurately described by the relatively simple first-order differential equation (modified to incorporate the neutrino momentum distribution). This equation is given by Eq. (84), expressed as a function of the number distribution of sterile states. In Sec. V, we showed that the number distribution of sterile neutrino states approximately satisfied a first order differential equation [Eq. (93)] which must be integrated for each momentum step.

We first applied our analysis to obtain the region of parameter space where large neutrino asymmetries are generated. This region of parameter space is given in Eq. (97). This analysis included the effects of the neutrino momentum distribution which was neglected in earlier studies [12,18]. We also examined the implications of lepton number generation for the BBN bounds for $\delta m^2, \sin^2 2\theta_0$ for ordinary-sterile neutrino mixing. There are two ways in which the creation of lepton number can modify the BBN bounds. One way is where the ν_α-ν_s oscillations themselves produce L_{ν_α} thereby suppressing the number of sterile neutrinos produced from the same oscillations. The other way is where the ν_β-ν_s oscillations create L_{ν_β} which thereby suppresses ν_s production from ν_α-ν_s oscillations. The bound for the former case is given in Eq. (96), while the latter case is studied in Sec. VI, in the context of the maximal vacuum oscillation solutions to the solar and atmospheric neutrino problems. The maximal vacuum oscillation solution of the solar neutrino problem assumes that the electron neutrino is approximately maximally mixed with a sterile neutrino. For a large range of parameter space, the maximal mixing leads to an energy independent factor of 2 reduction in the solar neutrino fluxes. This leads to a reasonably simple predictive solution to the solar neutrino problem which is supported by the experiments. However, most of the parameter space for this solution is inconsistent with standard big bang nucleosynthesis (BBN) if the lepton numbers are assumed to be negligible [5–8]. We showed that there is a large region of parameter space where the oscillations generated lepton number in such a way so as to allow the maximal vacuum oscillation solution to the solar neutrino problem to be solved without significantly modifying BBN. The allowed parameter space is given in Fig. 5. We also showed that there is a range of parameters where the lepton number is generated so that the large angle muon-sterile neutrino oscillation solution to the atmospheric neutrino anomaly does not lead to any significant modification of BBN. This parameter space is illustrated in Figs. 6 and 7.

We finish with a speculation. One of the mysteries of cosmology is the origin of the observed baryon asymmetry of the early Universe. In principle, it may be possible that the baryon asymmetry arises from a lepton number asymmetry. The lepton number asymmetry can be converted into a baryon number asymmetry through sphaleron transitions at or above the weak phase transition. It may be possible that a small lepton number asymmetry arises from the mechanism of ordinary-sterile neutrino oscillations, which is seeded by statistical fluctuations of the background. One interesting feature of this possibility is that the baryon number asymmetry would not be related to the CP asymmetry of the Lagrangian. Instead the origin of matter over antimatter would be due to a statistical fluctuation which is then amplified by neutrino oscillations. However before this speculation can be checked, it would be necessary to work out the effective potential at high temperatures ($T\sim 250$ GeV) and study the phase transition region.

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APPENDIX

The purpose of this appendix is to show that Eq. (60) can be derived from the Hamiltonian formalism. In applying this formalism, we will assume that the rate at which collisions collapse the wave function (i.e., the rate of measurement of whether the state is a weak or sterile eigenstate) is given by the damping frequency which is half of the collision frequency. For further discussion of this point see Sec. II and [23].

The expectation value that an initial weak eigenstate neutrino ν_α has oscillated into a sterile state ν_s after τ seconds will be denoted by $|\psi_s'(t,\tau)|^2$ (where t is the age of the Universe). The average probability that an initial weak eigenstate has oscillated into a sterile state can be obtained by averaging the quantity $|\psi_s'(t,\tau)|^2$ over all possible times τ (weighted by the probability that the neutrino has survived τ seconds since its last ''measurement''). This average has the form

$$\langle|\psi_s'(t)|^2\rangle=\frac{1}{\omega_0}\int_0^t e^{-\tau/\omega_0}|\psi_s'(t,\tau)|^2 d\tau, \tag{A1}$$

where ω_0 is the mean time between measurements. According to [23], $\omega_0=1/D=2/\Gamma_{\nu_\alpha}$. If we denote the analogous quantity for antineutrinos by $\langle|\tilde{\psi}_s'(t)|^2\rangle$, then the rate of change of lepton number can be expressed as

$$\frac{dL_{\nu_\alpha}}{dt}\simeq-\frac{3}{8}\langle\Omega(t)\rangle\frac{\Gamma_{\nu_\alpha}}{2}-\frac{3}{8}\frac{\partial\langle\Omega(t)\rangle}{\partial t}, \tag{A2}$$

where

$$\langle\Omega(t)\rangle=\langle|\psi_s'(t)|^2\rangle-\langle|\tilde{\psi}_s'(t)|^2\rangle. \tag{A3}$$

Note that the first term in Eq. (A2) represents the rate of change of lepton number due to collisions (which produce sterile neutrino states). The second term represents the rate of change of lepton number due to the oscillations between collisions.

In the adiabatic limit, the transformation $\theta_0\to\theta_m$ and $L_0\to L_m$ diagonalizes the Hamiltonian. In this limit, the mean probability $\langle|\psi_s'(t)|^2\rangle$ is given by

$$\langle|\psi_s'(t)|^2\rangle=\sin^2 2\theta_m\left\langle\sin^2\frac{\tau}{2L_m}\right\rangle. \tag{A4}$$

Note that in the static limit, $\partial\langle\Omega(t)\rangle/\partial t=0$ and hence Eq. (20) results. However, in the expanding Universe which is nonstatic, the above equation is not generally valid (although it turns out that it is a good approximation for oscillations away from resonance where the system changes sufficiently slowly and even at some resonance regions which are sufficiently smooth). To calculate the probability $\langle|\psi_s'(t)|^2\rangle$ in the general case, we go back to the fundamental Hamiltonian equations

$$i\frac{d}{dt}\begin{pmatrix}\psi_\alpha\\ \psi_s'\end{pmatrix}=\frac{1}{2p}\mathcal{M}^2\begin{pmatrix}\psi_\alpha\\ \psi_s'\end{pmatrix}, \tag{A5}$$

where

$$\mathcal{M}^2=\frac{1}{2}\left[R_\theta\begin{pmatrix}-\delta m^2 & 0\\ 0 & \delta m^2\end{pmatrix}R_\theta^T+4p\begin{pmatrix}\langle V\rangle & 0\\ 0 & 0\end{pmatrix}\right], \tag{A6}$$

and

$$R_\theta=\begin{pmatrix}\cos\theta_0 & \sin\theta_0\\ -\sin\theta_0 & \cos\theta_0\end{pmatrix},\ \langle H\rangle=\frac{(b\pm a)\delta m^2}{2p}, \tag{A7}$$

where the $-$ $(+)$ sign corresponds to neutrino (antineutrino) oscillations. Expanding out Eq. (A5), we find

$$i\frac{d\psi_\alpha}{dt}=\alpha\psi_\alpha+\frac{\beta}{2}\psi_s', \quad i\frac{d\psi_s'}{dt}=\frac{\beta}{2}\psi_\alpha+\gamma\psi_s', \tag{A8}$$

where

$$\alpha=\frac{\delta m^2}{4p}(2b\pm 2a-\cos 2\theta_0),$$

$$\beta=\frac{\delta m^2}{2p}\sin 2\theta_0, \tag{A9}$$

$$\gamma=\frac{\delta m^2}{4p}\cos 2\theta_0.$$

If we divide the equations (A8) by ψ_α and ψ_s' respectively, then they can be combined into the single differential equation

$$i\frac{d\Psi}{dt}=\lambda\Psi+\frac{\beta}{2}(1-\Psi^2), \tag{A10}$$

where $\Psi\equiv\psi_s'/\psi_\alpha$ and

$$\lambda=\gamma-\alpha=\frac{\delta m^2}{2p}(\cos 2\theta_0-b\pm a). \tag{A11}$$

The $+$ $(-)$ sign in the above equation corresponds to ν_α-ν_s $(\overline{\nu_\alpha}$-$\overline{\nu_s})$ oscillations. Note that $|\psi_s'|^2=|\Psi|^2/(1+|\Psi|^2)$. If the nonlinear term (Ψ^2) can be neglected, then the solution for constant α,β,γ is

$$\Psi(t)=\frac{-\beta}{2\lambda}[1-e^{-i(\lambda)(t-t^*)}], \tag{A12}$$

with boundary condition $\Psi(t^*)=0$. Introducing the variable $\tau\equiv t-t^*$, and evaluating $|\Psi(t,\tau)|^2$ we find

$$|\Psi(t,\tau)|^2=\frac{\beta^2}{\lambda^2}\sin^2\left[\frac{\lambda\tau}{2}\right], \tag{A13}$$

which is approximately $\sin^2 2\theta_m \sin^2\tau/2L_m$ provided that $|\Psi|^2\ll 1$.

In the general case where α,β, and γ are not constant, the general solution is (where we have again neglected the nonlinear Ψ^2 term)

$$\Psi(t)=\frac{-i}{2}\int_{t^*}^{t}e^{i\tilde{\lambda}(t')}\beta(t')dt', \tag{A14}$$

where

$$\tilde{\lambda}(t')\equiv\int_t^{t'}\lambda dt'', \tag{A15}$$

and the boundary condition $\Psi(t^*)=0$ has again been taken. One may easily verify that Eq. (A14) is indeed the solution by directly substituting it into Eq. (A10). The probability that a weak eigenstate at time t^* has oscillated into a sterile eigenstate at time t is thus

$$|\Psi(t)|^2\simeq\frac{\beta^2}{4}\int_{t^*}^{t}\int_{t^*}^{t}\cos\left[\int_{t_2}^{t_1}\lambda dt'\right]dt_1dt_2, \tag{A16}$$

where we have assumed that β is approximately constant over the interaction time scale $t-t^*$, so that it can be taken outside the integral. This step is a good approximation provided $T\gtrsim 2$ MeV [30]. Again defining the quantity $\tau\equiv t-t^*$ (recall that τ is the time between measurements), and averaging $|\Psi(t,\tau)|^2$ over τ, with the appropriate weighting factor, we find that

$$\langle|\Psi(t)|^2\rangle\simeq\frac{\beta^2}{4\omega_0}\int_0^t e^{-\tau/\omega_0}\int_{t-\tau}^{t}\int_{t-\tau}^{t}\cos\left[\int_{t_2}^{t_1}\lambda dt'\right]dt_1dt_2d\tau. \tag{A17}$$

Integrating this equation by parts (with respect to the τ integration), we find

$$\langle|\Psi(t)|^2\rangle\simeq\frac{\beta^2}{2}\int_0^t\int_{t-\tau}^{t}e^{-\tau/\omega_0}\cos\left[\int_{t-\tau}^{t_1}\lambda dt'\right]dt_1d\tau, \tag{A18}$$

where we have used the fact that $e^{-t/\omega_0}\simeq 0$ [55]. The analogous quantity for antineutrinos, $\langle|\tilde{\Psi}(t)|^2\rangle$, can similarly be defined. Recall that the functions $\langle|\Psi(t)|^2\rangle$, $\langle|\tilde{\Psi}(t)|^2\rangle$ are related to the rate of change of lepton number through Eq. (A2):

$$\frac{dL_{\nu_\alpha}}{dt}\simeq-\frac{3}{8}\frac{\langle|\Psi(t)|^2\rangle}{\omega_0}-\frac{3}{8}\frac{\partial\langle|\Psi(t)|^2\rangle}{\partial t}-[\Psi\to\tilde{\Psi}]. \tag{A19}$$

Evaluating $\partial\langle|\Psi(t)|^2\rangle/\partial t$ we find

$$\frac{\partial\langle|\Psi(t)|^2\rangle}{\partial t}=\frac{\beta^2}{2}\int_0^t e^{-\tau/\omega_0}\left(\cos\left[\int_{t-\tau}^{t}\lambda dt'\right]-1\right)d\tau+\frac{\beta^2}{2}\int_0^t e^{-\tau/\omega_0}\lambda_{(t-\tau)}\int_{t-\tau}^{t}\sin\left[\int_{t-\tau}^{t_1}\lambda dt'\right]dt_1d\tau, \tag{A20}$$

where we use the notation that $\lambda_{(t-\tau)}$ denotes λ evaluated at the point $(t-\tau)$. Dividing Eq. (A18) by ω_0 and integrating Eq. (A18) by parts (with respect to the τ integration), we find

$$\frac{1}{\omega_0}\langle|\Psi(t)|^2\rangle=\frac{\beta^2}{2}\int_0^t e^{-\tau/\omega_0}\left(1-\lambda_{(t-\tau)}\int_{t-\tau}^{t}\sin\left[\int_{t-\tau}^{t_1}\lambda dt'\right]dt_1\right)d\tau. \tag{A21}$$

Adding the above two equations and subtracting the analogous term for antineutrinos, we obtain the following rather compact expression for the rate of change of lepton number:

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{-3\beta^2}{16}\int_0^t e^{-\tau/\omega_0} \times \left(\cos\left[\int_{t-\tau}^{t} \lambda dt'\right] - \cos\left[\int_{t-\tau}^{t} \overline{\lambda} dt'\right]\right) d\tau, \quad \text{(A22)}$$

where $\overline{\lambda}$ is defined similarly to λ except that $a \to -a$. Note that the total contribution to the rate of change of lepton number is in fact simpler than either of the two separate contributions coming from collisions and oscillations between collisions. Equation (A22) can be rewritten (using a trigonometric identity)

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{3\beta^2}{8}\int_0^t e^{-\tau/\omega_0} \sin\left[\int_{t-\tau}^{t} \lambda^+ dt'\right] \sin\left[\int_{t-\tau}^{t} \lambda^- dt''\right] d\tau, \quad \text{(A23)}$$

where $\lambda^{\pm} = (\lambda \pm \overline{\lambda})/2$. Note that this is exactly the same equation that we derived in Sec. III [Eq. (60)] from the density matrix equations.

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[28] The condition that ω_0 and L_m are approximately constant over the time scale ω_0 can be expressed by the equations

$$\left|\omega_0 \frac{\partial \omega_0}{\partial t}\right| \lesssim \omega_0, \quad \left|\omega_0 \frac{\partial L_m}{\partial t}\right| \lesssim L_m.$$

Expanding these conditions out, using the fact $\partial\omega_0/\partial t = (\partial\omega_0/\partial T)(\partial T/\partial t)$ (and similarly for $\omega_0 \to L_m$) and $\partial T/\partial t = -HT$ (where H is the Hubble parameter), it is straightforward to show that the above conditions are satisfied provided that $\Gamma_{\nu_\alpha} \gtrsim H$, i.e., for $T \gtrsim 2$ MeV.

[29] See K. Enqvist *et al.*, Nucl. Phys. **B349**, 754 (1991).

[30] The quantity β can be taken outside the integral (to a good approximation) provided that $|\omega_0(\partial\beta/\partial t)| \lesssim |\beta|$. This equation implies that

$$\left|\omega_0 \frac{\partial \beta}{\partial t}\right| = \frac{2}{\Gamma_{\nu_\alpha}} \left|\frac{\partial T}{\partial t}\frac{\partial \beta}{\partial T}\right| \lesssim |\beta|.$$

Using $\partial\beta/\partial T \simeq -\beta/T$, and $\partial T/\partial t = -HT$ (where H is the Hubble parameter), it follows that the above condition holds provided that $\Gamma_{\nu_\alpha} \gtrsim 2H$, i.e., for $T \gtrsim 2$ MeV.

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[35] The relationship between lepton number and effective number of neutrino species comes about as follows. The effect of electron lepton number is to change the prediction of the primordial ^{4}He abundance [34]. Assuming that $L_{\nu_e} \ll 1$, $Y \approx Y(L_{\nu_e}=0) - 0.234\mu/T \simeq Y(L_{\nu_e}=0) - 0.35L_{\nu_e}$ [18]. Using the result that a change in Y can be equivalently expressed as a change in the effective number of neutrino species ($\delta N_\nu^{\rm eff}$) present during nucleosynthesis, through the equation $\delta Y \simeq 0.012\delta N_\nu^{\rm eff}$ [see, e.g., T. P. Walker *et al.*, Astrophys. J. **376**, 51 (1991)], it follows that the change in the effective number of neutrinos is related to L_{ν_e} by the equation, $\delta N_\nu^{\rm eff} \approx -30L_{\nu_e}$.

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[37] Strictly, the lower limit on $|\delta m^2_{ee'}|$ is 7.5×10^{-3} eV^2 [which is the current laboratory bound, see Particle Data Group, R. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996)]. However, maximal mixing of electron neutrinos in the range $10^{-3} \lesssim |\delta m^2_{ee'}|/\text{eV}^2 \lesssim 10^{-2}$ would appear to be inconsistent with the atmospheric neutrino data [2].

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[42] In [41], they used the Kamiokande flux measurement of $[2.89\pm0.22\pm0.35(\text{syst})]\times10^6$ cm^{-2} s^{-1}, and calculated the Chlorine capture rate of $R(^8\text{B},^{37}\text{Cl}) \geqslant 2.94\pm0.40$ solar neutrino units (SNU). However, the latest measurement of the Kamiokande experiment has measured a flux slightly smaller than the value used in [41]. In Table I we updated this prediction by using the latest value of the Kamiokande flux measurement, $\phi_K(^8B) = [2.80\pm0.17\pm0.34(\text{syst})]\times10^6$ cm^{-2} s^{-1} [Y. Suzuki, talk presented at the XVII International Conference on Neutrino Physics and Astrophysics, 1996 (unpublished)]. The capture rates for the other reaction chains were obtained from [39].

[43] For example, the following three predictions for the boron flux have been recently obtained by three different groups (in units of 10^6 cm^{-2} s^{-1}):

$$6.5^{+0.9}_{-1.1}, \ 4.4\pm1.1, \ 2.77,$$

by J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **67**, 781 (1995), S. Turck-Chièze and I. Lopes, Astrophys. J. **408**, 347 (1993), and A. Dar and G. Shaviv, Astrophys. J. **468,** 933 (1996), respectively.

[44] For a review of BBN see, e.g., S. Sarkar, Rep. Prog. Phys. **59**, 1493 (1996).

[45] For the case of the exact parity symmetric model, it is necessary to assume that an initial macroscopic asymmetry between the ordinary and mirror matter exists. This can be arranged through the inflationary scenario proposed by E. W. Kolb, D. Seckel, and M. S. Turner, Nature (London) **314**, 415 (1985) (for example). We will suppose that ordinary matter dominates mirror matter immediately after the big bang. Note that in addition to ordinary-mirror neutrino mass mixing, the mirror world can interact with the ordinary particles through photon-mirror photon mixing [discussed by B. Holdom, Phys. Lett. **166B**, 196 (1985); S. L. Glashow, *ibid.* **167B**, 35 (1985); E. Carlson and S. L. Glashow, Phys. Lett. B **193**, 168 (1986); and independently in a slightly different way by R. Foot, H. Lew, and R. R. Volkas, *ibid.* **272**, 67 (1991)], and Higgs-boson-mirror–Higgs-boson mixing [discussed in the previous paper and in H. Lew (unpublished)]. We assume that these interactions are either zero or small enough so as to preserve the dominance of matter over mirror matter.

[46] See, e.g., S. M. Bilenky and B. Pontecorvo, Phys. Rep. **41**, 225 (1978); V. Barger, R. J. N. Phillips, and K. Whisnant, Phys.

Rev. D **24**, 538 (1981); S. L. Glashow and L. M. Krauss, Phys. Lett. B **190**, 199 (1987). For a recent analysis, see, e.g., E. Calabresu *et al.*, Astropart. Phys. **4**, 159 (1995).

[47] Note that the LSND experiment [3] suggests that $|\delta m^2_{\mu e}| \gtrsim 0.3$ eV^2. This also implies that $|\delta m^2_{\mu e'}| \gtrsim 0.3$ eV^2 because $\delta m^2_{\mu e'} = \delta m^2_{\mu e} + \delta m^2_{ee'}$ and $|\delta m^2_{ee'}| \lesssim 10^{-3}$ eV^2.

[48] In the case where ν_e-ν_τ oscillations are very significant (this should happen when the mixing angle $\sin^2 2\theta_0^{\tau e}$ is relatively large), it may be possible that the rate of change of $(L_{\nu_\tau} - L_{\nu_e})$ can be large enough to make $L_{\nu_\tau} \simeq L_{\nu_e}$. In this case one then has approximately only one differential equation rather than two coupled differential equations. Even in this case, it is possible to show that the $L^{(e)}$ created by ν_τ-ν_e' oscillations cannot get destroyed by ν_e-ν_e' oscillations provided that $\sin^2 2\theta_0^{\tau e'} \gtrsim (\delta m^2_{ee'}/\delta m^2_{\tau e'})^2$.

[49] See, e.g., the review by G. Gelmini and E. Roulet, Rep. Prog. Phys. **58**, 1207 (1995).

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$$\frac{\omega_0}{2L_m} \simeq \frac{2}{\Gamma_{\nu_\alpha}} \frac{\delta m^2}{4p} b \simeq 25 \gg 1.$$

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