

Is the antiferromagnetic RP^2 model in four dimensions trivial?

H. G. Ballesteros,^{1,*} J. M. Carmona,^{2,†} L. A. Fernández,^{1,‡} V. Martín-Mayor,^{1,§} A. Muñoz Sudupe,^{1,||} and A. Tarancón^{2,¶}

¹*Departamento de Física Teórica I, Facultad de CC. Físicas, Universidad Complutense de Madrid, 28040 Madrid, Spain*

²*Departamento de Física Teórica, Facultad de Ciencias, Universidad de Zaragoza, 50009 Zaragoza, Spain*

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We study the antiferromagnetic RP^2 model in four dimensions. We find a second order transition with two order parameters, one ferromagnetic and the other antiferromagnetic. The antiferromagnetic sector has mean-field critical exponents and a renormalized coupling which goes to zero in the continuum limit. The exponents of the ferromagnetic channel are not the mean-field ones, but the difference can be interpreted as logarithmic corrections. We perform a detailed analysis of these corrections and conclude the triviality of the continuum limit of this model. [S0556-2821(97)05208-9]

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I. INTRODUCTION

The nonperturbative formulation of nonasymptotically free, interacting field theories in four dimensions is yet to be accomplished. The conventional analysis, for $\lambda\phi^4$ and $O(N)$ theories, yields triviality in four dimensions [1]. That is, once the continuum limit is taken, correlation functions factorize as the Wick's theorem prescribes for the Gaussian theory. A possible way, in order to obtain a model with a nontrivial continuum limit, is to introduce antiferromagnetism (AFM). Gallavotti and Rivasseau have considered AFM actions to change the ultraviolet limit of ϕ^4 theories [2]. From a statistical physics point of view, a great variety of AFM models in three dimensions has been studied to obtain different qualitative behavior from that of the corresponding ferromagnetic (FM) models [3]. In four dimensions, recent works have studied the possibility of new universality classes if AFM is added [4].

The AFM RP^2 model has recently been studied in three dimensions, because of its exotic properties [5,6]. For instance, it has a disordered, unfrustrated ground state. Even more, it seems to present a full breaking of the action's $O(3)$ symmetry [6]. Perturbative studies of this spontaneous symmetry breaking (SSB) pattern yield the $O(4)$ universality class [8]. If this prediction holds true in four dimensions, the fate of the model is triviality.

However, this theoretical prediction has been questioned in three dimensions by Monte Carlo (MC) simulations [6]. Therefore, the study of the triviality of this model in four dimensions is very interesting. It also would help to enlighten the situation in three dimensions. We will see, though, that a detailed analysis of the MC simulation of this model indicates the triviality of its continuum limit through the appearance of logarithmic corrections to the divergences of the observables of the theory. A special form of the finite

size scaling (FSS) analysis which includes logarithmic corrections will be used to deal with these effects.

We define the model and observables in Sec. II, where we also describe the techniques we have used to measure the critical exponents. The results of the MC simulation are presented in Sec. III. The model exhibits a phase transition at a negative coupling with two independent order parameters. One of these channels, the staggered one, presents mean-field critical exponents, but the other, ferromagnetic, presents deviations. We show in Secs. III and IV how the discrepancies with the mean-field behavior can be interpreted as logarithmic corrections.

II. THE MODEL

We shall consider the $RP^2 \equiv S^2/Z_2$ (real projective space) spin model in four dimensions. Our basic variable is a three-component normalized spin \mathbf{v}_i , interacting through a gauge Z_2 -invariant action. As a local symmetry cannot be broken, these are effectively RP^2 variables (only the direction of the vectors is relevant). We consider a hypercubic lattice, with first neighbor coupling:

$$S = \beta \sum_{\langle ij \rangle} (\mathbf{v}_i \cdot \mathbf{v}_j)^2, \quad \mathcal{Z} = \int \left(\prod_i d\mathbf{v}_i \right) e^S. \quad (1)$$

The ferromagnetic (positive coupling) model presents a first order transition at $\beta \approx 0.94$. The ground state consists of spins parallel or antiparallel to an arbitrary direction, and the SSB is $SO(3)/SO(2)$. The analysis of the antiferromagnetic counterpart is trickier, given the more complicated nature of the ground state. Let us call a lattice site, labeled by (x, y, z, t) , even or odd according to the parity of $x + y + z + t$. In the ground state, every even or odd spin is parallel or antiparallel to an arbitrary direction, while odd or even spins lie randomly on the perpendicular plane. The corresponding SSB is $SO(3)/SO(2)$, which calls for the $O(3)$ universality class. However, fluctuations induce an interaction between spins on the randomly plane sublattice. In three dimensions, this seems enough to break the remaining $O(2)$ symmetry [6], and to change the universality class.

*Electronic address: hector@lattice.fis.ucm.es

§Electronic address: carmona@sol.unizar.es

†Electronic address: laf@lattice.fis.ucm.es

‡Electronic address: victor@lattice.fis.ucm.es

||Electronic address: sudupe@lattice.fis.ucm.es

¶Electronic address: tarancon@sol.unizar.es

A. Definition of observables

The natural RP^2 variables are given by the traceless tensorial field \mathbf{T}_i

$$\mathbf{T}_i^{\alpha\beta} = v_i^\alpha v_i^\beta - \frac{1}{3} \delta^{\alpha\beta}, \quad (2)$$

whose lattice Fourier transform will be represented by $\hat{\mathbf{T}}$:

$$\hat{\mathbf{T}}_p = \sum_r \exp(-ip \cdot r) \mathbf{T}_r. \quad (3)$$

We will work in a L^4 lattice with periodic boundary conditions. We define two order parameters, according to the discussion of the ground state above, the intensive staggered (ferromagnetic) magnetization, as the sums of tensors on even sites minus (plus) those on odd sites or, equivalently,

$$\mathbf{M}_s = \frac{1}{L^4} \hat{\mathbf{T}}_{(\pi, \pi, \pi, \pi)}, \quad \left(\mathbf{M} = \frac{1}{L^4} \hat{\mathbf{T}}_{(0,0,0,0)} \right). \quad (4)$$

As no spontaneous symmetry breaking can occur on a finite lattice, in a MC simulation one needs to measure $O(3)$ -invariant operators. For the magnetization and the susceptibility, we define

$$M = \langle \sqrt{\text{tr} \mathbf{M}^2} \rangle, \quad \chi = L^4 \langle \text{tr} \mathbf{M}^2 \rangle, \quad (5)$$

and analogously with the staggered observables.

A very useful quantity for a triviality study is the Binder cumulant. For this model, we define

$$V_M = \frac{5}{2} \left(\frac{7}{5} - \frac{\langle (\text{tr} \mathbf{M}^2)^2 \rangle}{\langle \text{tr} \mathbf{M}^2 \rangle^2} \right), \quad (6)$$

which, in the infinite volume limit, becomes 1 in the broken phase and 0 in the symmetric one. The cumulant for the staggered magnetization is defined analogously.

Another very interesting quantity is the second momentum correlation length defined as [9]

$$\xi_L = \left(\frac{\chi/F - 1}{4 \sin^2(\pi/L)} \right)^{1/2}, \quad (7)$$

where F is the mean value of the trace of $\hat{\mathbf{T}}$ squared at minimal momentum ($2\pi/L$ in any of the four directions). For ξ_s we use χ_s and F_s , analogously defined from $\hat{\mathbf{T}}$ at momentum $(2\pi/L + \pi, \pi, \pi, \pi)$ and permutations.

The field theoretical definition of the renormalized coupling constant can now be introduced

$$g_R = V_M (L/\xi_L)^d, \quad (8)$$

where d is the dimension of the lattice. We will consider the renormalized couplings associated with the two different sectors.

In addition, we measure the energy, which is needed for the spectral density method [10], invaluable for extrapolating MC measures to a neighborhood of the critical coupling.

B. Standard finite size scaling

To study critical exponents, we have used a method especially suited to the measurements of anomalous dimensions [6,7]. Let us consider the mean value of an operator O , measured in a size L lattice, at a coupling value β in the critical region. Let t be the reduced temperature $(\beta - \beta_c)/\beta_c$. The standard FSS formula states that [11]

$$\langle O(L, t) \rangle = \langle O(t) \rangle \mathcal{F}_O[s(L, t)], \quad s(L, t) \equiv \frac{L}{\xi(t)}, \quad (9)$$

where $O(t)$ means $O(\infty, t)$ and \mathcal{F}_O is a smooth function. We suppose that the values of L and $\xi(t)$ are large, so that we ignore scaling corrections in Eq. (9). Now, we have $\langle O(t) \rangle \sim t^{-x_O}$, which is the definition of the critical exponent x_O , and $\xi(t) \sim t^{-\nu}$, and we can write $L^{x_O/\nu} = \langle O(t) \rangle s^{x_O/\nu}$. This allows one to write Eq. (9) as

$$\langle O(L, t) \rangle = L^{x_O/\nu} \mathcal{G}_O(s). \quad (10)$$

Applying Eq. (10) to the correlation length, it gives $\xi(L, t) = L \mathcal{G}_\xi(s)$, so that

$$x(L, t) \equiv \frac{\xi(L, t)}{L} = \mathcal{G}_\xi(s), \quad (11)$$

and $s = \mathcal{G}_\xi^{-1}(x)$. From Eq. (10) we have

$$\langle O(L, t) \rangle = L^{x_O/\nu} \mathcal{G}_O[\mathcal{G}_\xi^{-1}(x)] \equiv L^{x_O/\nu} f_O(x), \quad (12)$$

and we finish up with the useful expression

$$\langle O(L, t) \rangle = L^{x_O/\nu} f_O\left(\frac{\xi(L, t)}{L}\right) + \dots, \quad (13)$$

where the ellipsis stands for possible scaling corrections. Let us denote

$$Q_O = \frac{\langle O(rL, t) \rangle}{\langle O(L, t) \rangle}, \quad (14)$$

we can produce with it a sensible measure of critical exponents:

$$Q_O|_{Q_\xi=r} = r^{x_O/\nu} + \dots. \quad (15)$$

Therefore, from simulations of lattice sizes L and rL , we can extract the critical exponent x_O/ν from the quotient (14), measured at the point where one correlation length is r times the other.

III. THE MONTE CARLO SIMULATION

To simulate the system, we have used a standard three-hits Metropolis algorithm, with an uncorrelated change proposal, achieving approximately a 50% acceptance. The lattice sizes have been $L=4, 6, 8, 10, 12, 16, 20$, and 24 . For the larger sizes, 20 and 24, we have combined Metropolis with an overrelaxed update, described in the Appendix, to decrease the autocorrelation time. The overrelaxed algorithm is not able to decrease the dynamic critical exponent z but, nevertheless, we save total CPU time when compared with

TABLE I. Total number of measures and the corresponding integrated correlation times τ for χ_s . In the larger lattices the data of the overrelaxed simulations (right) are separated from those of Metropolis (left) by slashes.

L	$\tau(\chi_s)$	Measures [$\times 10^3$ $\tau(\chi_s)$]
4	0.644(16)	30
6	1.189(20)	60
8	2.26(3)	64
10	3.65(11)	19
12	5.53(17)	21
16	10.7(2)	39
20	17.0(10)/2.31(11)	5/5
24	25.5(10)/3.53(17)	4/2

the simple Metropolis simulation.

The runs have been distributed over several workstations. We display in Table I the integrated autocorrelation time for χ_s and the number of measurements performed for every lattice size. Every two measurements are separated by ten sweeps, each consisting of either one Metropolis update or one Metropolis plus three overrelaxed updates when we use the latter algorithm.

A. Order parameters

The RP^2 model presents a second order phase transition at $\beta \sim -1.34$. The ferromagnetic and staggered magnetizations defined in Eq. (5) are zero below the transition. To show that they are real order parameters, we should ensure that they do not vanish in the broken phase when $V \rightarrow \infty$. In Fig. 1 we plot the values of M_s and M at $\beta = -1.5$ for the lattice sizes $L = 8, 12$, and 16 . It is clear that both magnetizations reach an asymptotic value different from zero in the thermodynamical limit in the broken phase.

B. Critical exponents

We have calculated the critical exponents ν and η for the two different channels using Eq. (15), which yields $x_\chi = \gamma$

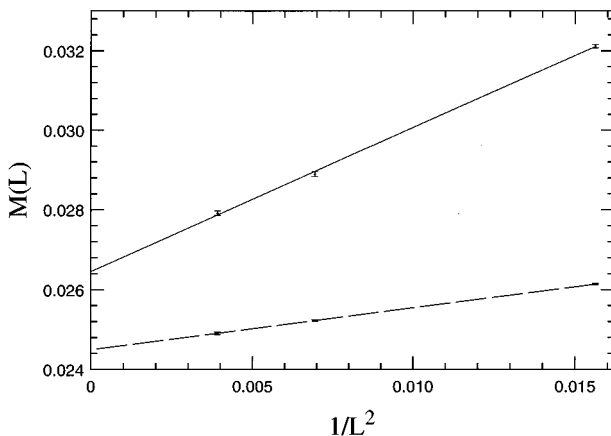


FIG. 1. Asymptotic values of M (straight line) and $M_s/10$ (dashed line) from their values for $L = 8, 12$, and 16 at $\beta = -1.5$.

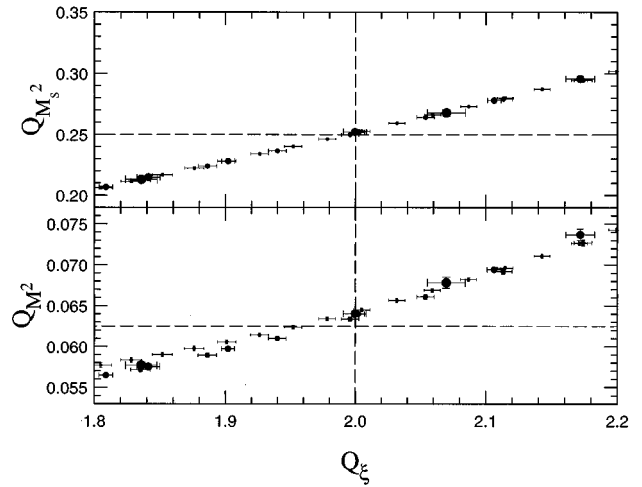


FIG. 2. Quotients of M_s^2 and M^2 as a function of the quotient of $\xi(L)$. The horizontal straight lines correspond to mean-field behavior. The symbol sizes are proportional to the lattice sizes.

for the susceptibility, and $x_M = -\beta$ for the magnetization. To calculate ν , we use $x_{d\xi/d\beta} = \nu + 1$. All along this paper we shall take $r = 2$.

We obtain the anomalous dimension η through the scaling relations

$$(2 - \eta)\nu = \gamma, \quad 2\beta = \nu(d - 2 + \eta). \quad (16)$$

The resulting values for the η exponent from these two relations will be denoted by η_χ and η_M , respectively.

As far as the exponent ν is concerned, we expect the same critical exponent for both correlation lengths, the ferromagnetic ξ^{FM} and the staggered ξ . We have found that the measures for ξ are more accurate so we have used this variable as correlation length.

We plot in Fig. 2 an example of how this method works in both channels. Notice that $Q_{M^2(s)}$ takes the value $2^{\gamma(s)/\nu-d}$ when $Q_\xi = 2$.

The resulting exponents are shown in Table II. After the name of the exponents their mean-field values are shown in square brackets [6]. The high accuracy reached on the measures of the η exponents is due, in part, to the strong statistical correlation between Q_ξ and Q_{M^2} .

We obtain a value for ν compatible with the mean-field prediction as well as for the magnetic exponents of the staggered sector. However, in the ferromagnetic channel, our exponents are close but not compatible with those given by mean-field theory.

Another possible interpretation of these values is that they could be *effective* critical exponents because of the presence of strong logarithmic corrections in the FM sector. To check this, let us suppose that there are logarithmic corrections only in the susceptibility. We do not take into account here the fact of possible logarithmic corrections to ξ as we use the values of the quotients measured at $r = 2$ value. This is an approximation that holds for large L . We address to Sec. IV for a more complete treatment of the logarithmic corrections. So that

$$\chi \sim L^m (\ln L)^{\bar{m}}, \quad (17)$$

TABLE II. Estimations for the critical exponents of the AFM RP² model.

L_1, L_2	$\nu[0.5]$	Staggered		Ferromagnetic	
		$\eta_\chi[0]$	$\eta_M[0]$	$\eta_\chi[2]$	$\eta_M[2]$
4,8	0.527(8)	0.009(4)	0.008(4)	1.959(8)	1.963(9)
6,12	0.524(6)	0.006(3)	0.008(3)	1.976(5)	1.981(6)
8,16	0.512(4)	0.008(2)	0.005(2)	1.968(3)	1.973(3)
10,20	0.491(7)	0.013(4)	0.001(4)	1.964(6)	1.969(6)
12,24	0.496(9)	0.007(5)	0.009(5)	1.960(8)	1.965(7)

$$Q_\chi = \frac{\chi_{2L}}{\chi_L} = 2^m \left(\frac{\ln 2L}{\ln L} \right)^{\bar{m}} \equiv 2^m h_L^{\bar{m}}, \quad (18)$$

where $m = \gamma/\nu$. The effective exponents obtained with the standard FSS $m'_L = \ln Q_\chi / \ln 2$ can be written as

$$m'_L = m + \bar{m} \frac{\ln h_L}{\ln 2}. \quad (19)$$

The fit discarding the 4,8 pair for the ferromagnetic susceptibility gives

$$m = 0.09(9), \quad \bar{m} = -0.13(7), \quad \chi^2/N_{\text{DF}} = 0.04/2. \quad (20)$$

The fit (20) indicates that the exponents of the ferromagnetic channel in Table II are compatible with a mean-field situation with logarithmic corrections in the susceptibility.

C. Critical temperature

The thermodynamical critical temperature of our system can be estimated from the crossing points of the Binder cumulants for the different lattice sizes [12]. To obtain $\beta_c(\infty)$, we can extrapolate according to the formula

$$\beta_c(\infty) - \beta_c(L) \approx L^{-1/\nu}. \quad (21)$$

It should be noted that Eq. (21) is only a first approximation, because it does not include any logarithmic corrections. In Sec. IV C we will be able to measure $\beta_c(\infty)$ taking into account logarithmic corrections. However, it hardly modifies the value of the critical coupling obtained with this method.

We show in Fig. 3 the Binder cumulants for the staggered and ferromagnetic channels.

We have fitted the crossing points of the Binder cumulants of the $L=8$ lattice with lattices $L=12, 16, 20$, and 24 . The results of the fit are

<i>Staggered</i>	<i>Ferromagnetic</i>
$\beta_c(\infty) = -1.3426(3)$	$\beta_c(\infty) = -1.3421(6)$
$\chi^2/N_{\text{DF}} = 1.2/2$	$\chi^2/N_{\text{DF}} = 1.0/2$

Both values are compatible and, as we expect one transition point, we take the value of $\beta_c(\infty)$ with lower error, that is

$$\beta_c(\infty) = -1.3426(3). \quad (22)$$

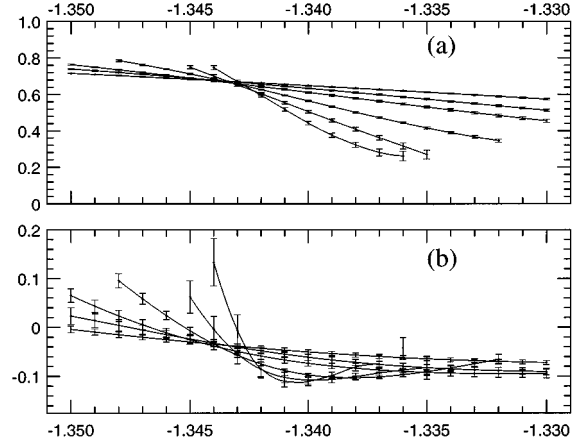


FIG. 3. Binder cumulants for both sectors, (a) staggered, (b) ferromagnetic.

Considering the crossing of the Binder cumulants of the $L=10$ lattice with the larger lattices scarcely changes the numbers.

IV. LOGARITHMIC CORRECTIONS OF THE RP² MODEL

As we have previously shown, the values obtained for the critical exponents for the four-dimensional AFM RP² model are compatible with those predicted by mean-field plus logarithmic corrections. They appear more clearly in the FM channel because in this sector there is no power-law divergence of the susceptibility, while in the staggered sector the logarithmic divergence is added to a power-law one. We need a modification of the standard FSS to include these corrections.

A. FSS with logarithmic corrections

Let us consider an observable $O(t)$ whose behavior near the critical point would have a logarithmic contribution

$$\langle O(t) \rangle \sim t^{-x} |\ln t|^{\bar{x}}. \quad (23)$$

We will follow [13] to take into account the logarithmic corrections: the scaling variable $s(L, t)$ of Eq. (9) is now substituted by $\xi(L, 0)/\xi(t)$:

$$\langle O(L, t) \rangle = \langle O(t) \rangle \mathcal{F}_O \left(\frac{\xi(L, 0)}{\xi(t)} \right). \quad (24)$$

Formula (24) coincides with Eq. (9) below the upper critical dimension, where $\xi(L, 0) \sim L$, otherwise it can also take into account the logarithmic corrections to the finite volume correlation length.

Let us suppose that, to leading order,

$$\xi(L, 0) \sim L^{\hat{\alpha}} (\ln L)^{\hat{\beta}}, \quad (25)$$

where $\hat{\alpha}$ and $\hat{\beta}$ are two exponents that depend on the theory. For the $O(N)$ models, $\hat{\alpha} = 1$, $\hat{\beta} = \frac{1}{4}$ [11]. The transition at finite L takes place when t is such that $\xi(L, 0) \sim \xi(t)$. If $\xi(t) \sim t^{-\nu} |\ln t|^{\bar{\nu}}$, then, employing Eq. (25),

$$t \sim L^{-\hat{\alpha}/\nu} (\ln L)^{(\bar{\nu}-\hat{\beta})/\nu}. \quad (26)$$

Below four dimensions $\hat{\alpha}=1$ and $\hat{\beta}=\bar{\nu}=0$. Now, we have

$$s(L,t) \equiv \frac{\xi(L,0)}{\xi(t)} \sim \frac{L^{\hat{\alpha}} (\ln L)^{\hat{\beta}}}{t^{-\nu} |\ln t|^{\bar{\nu}}}, \quad (27)$$

so that making use of Eq. (26), and with a change of variable similar to Eq. (11), we obtain

$$\langle O(L,t) \rangle = L^{\hat{\alpha}x/\nu} (\ln L)^{x/\nu(\hat{\beta}-\bar{\nu})+\bar{x}} f_o \left(\frac{\xi(L,t)}{L^{\hat{\alpha}} (\ln L)^{\hat{\beta}}} \right), \quad (28)$$

which is the equation analogue to Eq. (13). We follow now the same method as in the standard FSS case: we compute the quotient

$$Q_o = \frac{\langle O(2L,t) \rangle}{\langle O(L,t) \rangle} = 2^{\hat{\alpha}x/\nu} \left(1 + \frac{\ln 2}{\ln L} \right)^{x/\nu(\hat{\beta}-\bar{\nu})+\bar{x}} \times \frac{f_o \left(\frac{\xi(2L,t)}{(2L)^{\hat{\alpha}} (\ln 2L)^{\hat{\beta}}} \right)}{f_o \left(\frac{\xi(L,t)}{L^{\hat{\alpha}} (\ln L)^{\hat{\beta}}} \right)}. \quad (29)$$

Measuring Q_o at the point t_L where

$$\frac{\xi(2L, t_L)}{\xi(L, t_L)} = 2^{\hat{\alpha}} h_L^{\hat{\beta}}, \quad (30)$$

with $h_L \equiv 1 + \ln 2 / \ln L$, we find

$$Q_o(t_L) = 2^{\hat{\alpha}x/\nu} h_L^{x/\nu(\hat{\beta}-\bar{\nu})+\bar{x}}. \quad (31)$$

This is the new expression that substitutes Eq. (15) when there are logarithmic corrections, from it we can extract the exponent x/ν . We no longer have to measure Q_o where the quotient of correlation lengths is 2, but instead where it equals $2^{\hat{\alpha}} h_L^{\hat{\beta}}$.

B. Staggered channel: Renormalized four-point coupling

We proceed now to calculate the renormalized four-point coupling of our theory. The limit we are interested in is

$$g_R = \lim_{L \rightarrow \infty} g_R[L, \beta_c(\infty)]. \quad (32)$$

The evolution of g_R at the critical temperature with L is shown in Table III, where we have used the value for $\beta_c(\infty)$ of Eq. (22).

When hyperscaling is violated by logarithms, $g_R \sim |\ln t|^{-\bar{\rho}}$ [14]. Let us apply the FSS formula (24):

$$g_R(L,t) = g_R(t) \mathcal{F}_{g_R} \left(\frac{L^{\hat{\alpha}} (\ln L)^{\hat{\beta}}}{t^{-1/2} |\ln t|^{\bar{\nu}}} \right). \quad (33)$$

The scaling behavior with L at the critical point is

TABLE III. The renormalized four-point coupling $g_R[L, \beta_c(\infty)]$.

L	Staggered	Ferromagnetic
8	3.16 (3)	-0.19 (3)
10	2.84 (4)	-0.19 (5)
12	2.61 (5)	-0.21 (5)
16	2.34 (4)	-0.23 (5)
20	2.08 (9)	-0.23 (11)
24	1.95 (13)	-0.15 (17)

$$g_R(L,0) = |\ln t|^{-\bar{\rho}} \lim_{t \rightarrow 0} \mathcal{F}_{g_R} \left(\frac{L^{\hat{\alpha}} (\ln L)^{\hat{\beta}}}{t^{-1/2} |\ln t|^{\bar{\nu}}} \right). \quad (34)$$

We can eliminate the t dependence in Eq. (34) if $\lim_{t \rightarrow 0} \mathcal{F}_{g_R}(z) \sim z^{-\bar{\rho}/\bar{\nu}}$, which makes the $|\ln t|^{-\bar{\rho}}$ factor disappear. After that, we employ relation (26) and get finally

$$g_R(L,0) \sim (\ln L)^{-\bar{\rho}}, \quad (35)$$

which is also directly obtained from Eq. (28). We can obtain $\bar{\rho}$ fitting the values of Table III to the functional form (35). The fit for all lattice sizes, $L \geq 8$, yields (Fig. 4)

$$\bar{\rho} = 1.07(6), \quad \chi^2/N_{DF} = 0.8/4. \quad (36)$$

The result (36) implies triviality for the staggered sector of the AFM RP^2 model in four dimensions. The renormalized coupling goes to zero because of logarithmic corrections, exactly in the same way as in the ferromagnetic $O(N)$ models, for which $\bar{\rho} = 1$ [15].

The behavior of the ferromagnetic channel is, however, rather different from that of the staggered sector. From the data of Table III we cannot conclude an asymptotic value for the renormalized coupling. We have seen above that the logarithmic corrections are very strong in this channel, and maybe for that reason, the renormalized coupling is very hard to measure. To conclude about the triviality of this sector, we will try to study in full detail the logarithmic corrections of this channel, following the FSS analysis derived in Sec. IV A.

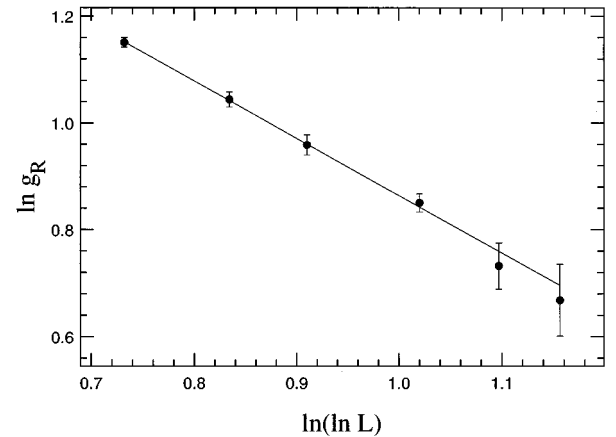


FIG. 4. Fit of $g_R(L)$ (staggered channel) at $\beta_c(\infty)$.

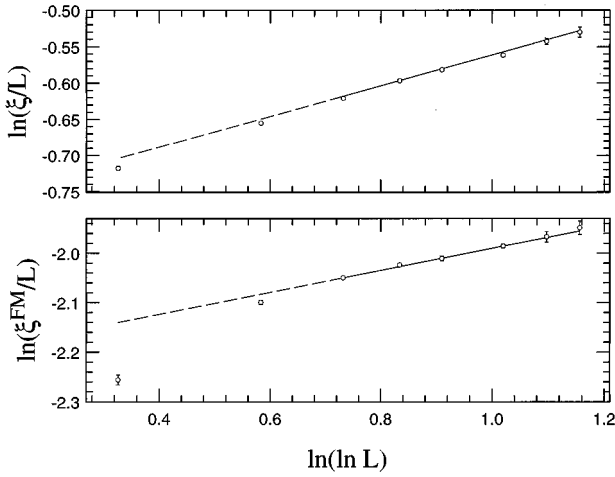


FIG. 5. Determination of the exponent $\hat{\beta}$ of the FSS formulas from the behavior of ξ^{FM} and ξ measured at the mean values of β_c from the fits with $L \geq 8$.

C. Computation of the logarithmic corrections

1. Correlation lengths

To parametrize the logarithmic corrections, we need a lot of, in principle, unknown exponents: $\hat{\alpha}, \hat{\beta}, \bar{\nu}, \dots$. Therefore, it will be necessary to make a few assumptions about these exponents.

The most important exponents are $\hat{\alpha}$ and $\hat{\beta}$, Eq. (25), because we have to measure at the points which satisfy Eq. (30). Assuming a mean-field plus logarithmic corrections scenario, we expect $\hat{\alpha} = 1$. To find out the value of exponent $\hat{\beta}$, we use the prediction (25) for the correlation length in a finite lattice at $\beta_c(\infty)$:

$$\ln \frac{\xi_L}{L} = C + \hat{\beta} \ln(\ln L). \quad (37)$$

We have performed the fit for both correlation lengths, the ferromagnetic (ξ^{FM}), and the staggered (ξ) one (Fig. 5). In order to monitorize subleading effects, we have compared the fits with $L \geq 8$, and $L \geq 10$. We have found that the fit for ξ^{FM} is more stable with growing lattice sizes. In Table IV, we show the fit parameters. The infinite volume critical coupling, and the fit-parameter errors have been estimated from the increment in one unit of the χ^2 function. Comparing with the previous determination of the critical coupling (22), both determinations are consistent and of similar accuracy, although logarithmic corrections to scaling were not considered previously. Our value for the exponent $\hat{\beta}$ is consistent

TABLE IV. Fits for the logarithmic corrections to the correlation lengths at the critical point.

	Fit	χ^2/N_{DF}	$\hat{\beta}$	$\beta_c(\infty)$
ξ	$L_{\min}=8$	2.1/3	0.21(2)	-1.3423(3)
	$L_{\min}=10$	0.3/2	0.16(3)	-1.3424(6)
ξ^{FM}	$L_{\min}=8$	0.8/3	0.22(4)	-1.3425(3)
	$L_{\min}=10$	0.2/2	0.17(8)	-1.3424(3)

with the predicted value for the $O(N)$ models, $\hat{\beta} = 0.25$, especially the one obtained from the ferromagnetic correlation length.

2. Magnetic operators

We shall try to control other logarithmic corrections by making use of the results of Sec. IV A. From Eq. (29), we know that

$$Q_\chi = 2^{\hat{\alpha}\gamma/\nu} h_L^{\gamma/\nu(\hat{\beta}-\bar{\nu})+\bar{\gamma}}, \quad (38)$$

when measuring the quotient at the point which verifies the condition (30). The logarithmic corrections are given in terms of the unknown exponents $\bar{\nu}$ and $\bar{\gamma}$ but, as we assume mean-field exponents, $\gamma = 0$, $\nu = 0.5$, we can reduce Eq. (38) to

$$\ln Q_\chi = \bar{\gamma} \ln h_L. \quad (39)$$

In a similar way, if we take the magnetization,

$$Q_M = 2^{-\hat{\alpha}\beta/\nu} h_L^{-\beta/\nu(\hat{\beta}-\hat{\nu})+\bar{\kappa}}, \quad (40)$$

or (mean field: $\beta = 1$)

$$\ln Q_M = -2 \ln 2 + \bar{\kappa} \ln h_L, \quad (41)$$

where $\bar{\kappa} = -2(\hat{\beta} - \bar{\nu}) + \bar{\beta}$. Therefore, from every pair of lattices $L, 2L$, we can obtain the exponents $\bar{\gamma}, \bar{\kappa}$. This is shown in Table V.

Notice that the $\bar{\gamma}$ and $\bar{\kappa}$ values are very close to their corresponding values for the magnetization in $O(3)$ and $O(4)$ models. It should be understood that these values ($\bar{\gamma} \sim 0.5$ and $\bar{\kappa} \sim 0.25$) are calculated for the order parameter on the fundamental (vectorial) representation of the $O(N)$ group. However, the critical exponents of the FM magnetization in the RP^2 model and those of the order parameter in the tensorial representation for the $O(N)$ models are the same at the mean-field level. Let us remark that the $\bar{\gamma} \sim 2\bar{\kappa}$ just means that $\langle M^2 \rangle \sim \langle M \rangle^2$.

This final result from the FM sector completes the conclusion that we obtained after examining the renormalized coupling of the staggered sector in Sec. IV B: the RP^2 model is trivial because of the logarithmic corrections to the mean-field behavior.

The question of the SSB pattern remains unsolved as the ferromagnetic susceptibility is only logarithmically divergent. We recall that the power-law behavior was crucial to check the symmetry breaking in three dimensions [6].

TABLE V. Exponents of the logarithmic corrections of the FM sector of the RP^2 model.

L_1, L_2	$\bar{\gamma}$	$\bar{\kappa}$
4,8	0.45 (2)	0.23 (6)
6,12	0.45 (1)	0.22 (6)
8,16	0.49 (1)	0.24 (3)
10,20	0.53 (2)	0.26 (1)
12,24	0.52 (3)	0.26 (2)

V. CONCLUSIONS

We have examined the triviality question of the four-dimensional AFM RP^2 model, which presents a second order transition. A very interesting feature of this model is that it presents two different order parameters. A detailed study of these two sectors reveals that the model has a trivial continuum limit. We have been able to calculate explicitly the logarithmic corrections to the mean-field behavior by means of a FSS analysis also valid at the critical dimension of the model.

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APPENDIX: THE OVERRELAXED ALGORITHM

Overrelaxation is a local microcanonical update algorithm. It makes the maximum change in the variable at a given point without modifying the energy. We will now describe how this method works in our model. It is easy to see that

$$(\mathbf{v}_i \cdot \mathbf{v}_j)^2 = \text{tr} \mathbf{T}_i \mathbf{T}_j + \frac{1}{3}, \quad (\text{A1})$$

so that the change $\mathbf{T}_i \rightarrow \mathbf{T}'_i$ must be such that the $\text{tr} \mathbf{T}_i \mathbf{N}$ is conserved, where \mathbf{N} is the sum of the tensors at the neighbor points of i . If we set $\mathbf{T}' = R \mathbf{T} R^{-1}$, the conservation of the energy is then expressed by

$$[R, \mathbf{N}] = 0. \quad (\text{A2})$$

In addition, we must ensure that the new tensor belongs to RP^2 , so that the change in \mathbf{T} is associated with a change in \mathbf{v} : $\mathbf{v}' = R \mathbf{v}$. As the vectors are normalized, this puts also the condition of unitarity on the matrix R .

In order to satisfy these two conditions, let us write \mathbf{N} as $\mathbf{N} = U \mathbf{\Lambda} U^{-1}$, where $\mathbf{\Lambda}$ is the matrix of eigenvalues of \mathbf{N} , and define $C = U^{-1} R U$. Then, the updating conditions, written in terms of the matrix C , are

$$[C, \mathbf{\Lambda}] = 0, \quad C^{-1} = C^+. \quad (\text{A3})$$

As $\mathbf{\Lambda}$ is a diagonal matrix (A3) implies that C has to be also diagonal, and $C^2 = \mathbf{1}$, which means that its three eigenvalues will be ± 1 . We have reduced our updating process to a choice of the matrix C . Here enters the second characteristic of the overrelaxed algorithm: the change in the vector \mathbf{v} should be maximum, which can be achieved by minimizing the value of the squared scalar product

$$\mathcal{A} = (\mathbf{v} \cdot \mathbf{v}')^2 = (\mathbf{v} \cdot R \mathbf{v})^2 = (\tilde{\mathbf{v}} \cdot C \tilde{\mathbf{v}})^2, \quad (\text{A4})$$

where

$$\tilde{\mathbf{v}} = U^{-1} \mathbf{v} \equiv (x_1, x_2, x_3). \quad (\text{A5})$$

To do the update, we then have to take the three numbers $c_i = \pm 1$ that minimize the quantity

$$\mathcal{A} = (c_1 x_1^2 + c_2 x_2^2 + c_3 x_3^2)^2. \quad (\text{A6})$$

To sum up, the overrelaxed algorithm consists of calculating the matrix \mathbf{N} of nearest neighbors, its eigenvectors to obtain U , and then looking for the minimum of the combinations in Eq. (A6).

It is easy to see that this algorithm verifies detailed balance: if we make $\mathbf{v} \rightarrow \mathbf{v}' \rightarrow \mathbf{v}''$, so that $\mathbf{v}'' = U C' \tilde{\mathbf{v}}'$ and $\tilde{\mathbf{v}}' = C \tilde{\mathbf{v}}$, we have to minimize

$$(\tilde{\mathbf{v}}' \cdot C' \tilde{\mathbf{v}}')^2 = (\tilde{\mathbf{v}} \cdot C C' C \tilde{\mathbf{v}})^2, \quad (\text{A7})$$

and the last expression was minimized by C , so that $C = C C' C$ or $C' = C$. Therefore, $\mathbf{v}'' = \mathbf{v}$.

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