Dynamical non-Abelian two-form: BRST quantization

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When an antisymmetric tensor potential is coupled to the field strength of a gauge field via a $B\wedge F$ coupling and a kinetic term for B is included, the gauge field develops an effective mass. The theory can be made invariant under a non-Abelian vector gauge symmetry by introducing an auxiliary vector field. The covariant quantization of this theory requires ghosts for ghosts. The resultant theory including gauge fixing and ghost terms is BRST invariant by construction, and therefore unitary. The construction of the BRST-invariant action is given for both Abelian and non-Abelian models of mass generation. [S0556-2821(97)05106-0]

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I. INTRODUCTION

The free antisymmetric tensor potential has one degree of freedom, a scalar [1]. This scalar can be coupled to an Abelian gauge field via a ‘‘topological’’ $B\wedge F$ term with a dimensionful coupling constant m of mass dimension one. The resulting theory, which is classically dual to the Goldstone model (the Abelian Stückelberg model), has three degrees of freedom which can be identified, both classically and quantum mechanically, with the propagating degrees of a massive gauge field of mass m [2–5]. This theory, as well as its vacuum, is invariant under both U(1) and the vector gauge symmetry $B_{\mu\nu}\rightarrow B_{\mu\nu}+\partial_{[\mu}\Lambda_{\nu]}$ with an arbitrary vector field Λ_μ. In other words, this model generates vector boson masses without symmetry breaking and without a residual Higgs field. The symmetries of the theory ensure that when fermions are included in the theory, only the transverse components of the gauge field couple to the fermionic current. The generic coupling term of mass dimension four between the antisymmetric tensor and fermions is of the form $\bar{\psi}(a+b\gamma_5)\sigma^{\mu\nu}B_{\mu\nu}\psi$, which is not invariant under the vector gauge transformations, and therefore cannot be included in the action if this symmetry is to be maintained. This implies that there is no three-point coupling, and therefore no loop, directly involving $B_{\mu\nu}$. Consequently it is straightforward to renormalize QED in which photons acquire mass via this mechanism [4].

The possibility that a non-Abelian version of this theory may exist as a consistently quantizable theory is an interesting one. Although many aspects of the standard model have been experimentally verified, the symmetry-breaking sector is still mostly unexplored and the source of some unanswered questions. So far experiments have not turned up an elementary scalar in any system of interacting particles, nor is there any positive evidence of an electroweak Higgs particle, either elementary or composite, at currently available energies. On the other hand, various theoretical arguments set the upper bound of the Higgs boson mass only a little out of reach of the present generation of accelerators. This suggests that perhaps we should consider alternative descriptions of the symmetry-breaking sector of the electroweak theory and prepare ourselves for the situation that no Higgs particle is ever found.

The Higgs sector as it stands has three equally important roles. One is to break the global $\mathrm{SU}(2)_{\mathrm{isospin}}\times\mathrm{U}(1)_{\mathrm{hypercharge}}$ symmetry down to the U(1) symmetry of electromagnetism. In the standard model the mechanism of symmetry breaking generates masses for the vector bosons $W^\pm$ and Z. In addition, the Yukawa coupling of the Higgs scalar to fermions breaks chiral symmetry and contributes to fermion mass generation. But suppose we consider the possibility that the three questions may be resolved separately. Then it makes sense to consider a mechanism to generate masses for vector bosons via a $B\wedge F$ interaction with an antisymmetric tensor, and look for the possibility of symmetry breaking and fermion mass generation in some other interaction in the theory, possibly as dynamical mechanisms.

But first we have to have a theory that can be consistently quantized, i.e., one that is both unitary and renormalizable. Various Higgs-free theories of massive non-Abelian vector bosons, including the Proca model, the Stückelberg model, the gauged nonlinear sigma model, or the Higgs model with a heavy Higgs boson, are either nonrenormalizable or violate unitarity. Therefore any other proposed mechanism must pass these two tests. As far as the antisymmetric tensor is concerned, the renormalizability of the Abelian theory does not really provide a pointer, because even a gauge variant mass term for the photon does not affect the renormalizability of QED [6]. However, as was pointed out elsewhere [7], it is possible to construct a non-Abelian theory which is power-counting renormalizable, has unbroken gauge symmetries, and has propagators which fall off as $1/k^2$ at high momentum, so there are no obvious obstructions to renormalizability. (Unlike the Freedman-Townsend model [8] which does not have a kinetic term for $B_{\mu\nu}$, the model proposed in [7] is not dual to the nonlinear sigma model.) But unitarity is another story.

The biggest argument faced by any theory with massive vector bosons but without a Higgs-like excitation involves unitarity. Any theory with a Hermitian Hamiltonian operator is necessarily unitary. However, a gauge theory has several redundant degrees of freedom which have to be eliminated by gauge fixing. An explicitly Lorentz-covariant gauge-fixing term introduces states of negative norm in the theory which have to be eliminated in turn by introducing ghost fields. At this point the theory contains non-Hermitian fields

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and states of negative norm, so the unitary of the theory needs to be checked explicitly. One way of checking whether a theory unitary is to see if the action including the gauge fixing and ghost terms is invariant under Becchi-Rouet-Stora-Tyutin (BRST) transformations [9,10]. If it is invariant, it is possible to define the conserved Noether charge Q of the symmetry. This charge is nilpotent, $Q^2=0$, and defines a cohomology on the Fock space of the theory. The space of states $|\psi\rangle$ such that $Q|\psi\rangle=0$ but $|\psi\rangle\neq Q|\chi\rangle$ for any $|\chi\rangle$ can be identified with the physical subspace of the Fock space, and it can be shown that the S matrix of the theory is unitary in this physical subspace [11].

For the antisymmetric tensor potential, the Faddeev-Popov construction runs into problems because of the need for ghosts for ghosts [12]. It is well known that the constraints of the free antisymmetric tensor form a reducible system [13], as do the constraints of the pure $B\wedge F$ action. What is not so obvious (or well known) is that the constraints form a reducible system, both in the Abelian and the non-Abelian models, even when both the kinetic term and the $B\wedge F$ coupling term are present in the action [14,15]. (This is just a restatement of the fact that it is possible to introduce a kinetic term for $B_{\mu\nu}$ without breaking the vector gauge symmetry, and without introducing extra degrees of freedom.) As a result, ghost-for-ghosts are still a necessity, which causes problems for the Faddeev-Popov construction. A long time ago a geometric construction was proposed [12] for the construction of the BRST–anti-BRST–invariant quantum action for the Freedman-Townsend model. More recently, a geometric construction was proposed using a similar ''horizontality condition'' [16] for the model of vector boson mass generation with a non-Abelian antisymmetric tensor. A BRST–anti-BRST–invariant action was found this way. Therefore it is known that a covariant gauge fixed quantum action exists for the mass generation mechanism.

In this paper I demonstrate that it is possible to construct a BRST-invariant tree-level action in a covariant gauge starting from the classical action proposed in [7] and proceeding in a similar fashion to the textbook construction [6] for the free Yang-Mills theory. In Sec. II, the BRST-invariant action for the Abelian model is constructed, both for the sake of completeness and as a test case. The BRST transformations of the various fields and their ghosts in the non-Abelian model can be intuited from the Abelian case. In Sec. III, the BRST transformations of the non-Abelian fields are given, following as closely as possible the constructions for the Abelian model and the free Yang-Mills theory. Section IV contains a summary and discussion of results and some speculations.

II. THE ABELIAN MODEL

Let me begin by discussing the construction of a BRST-invariant quantum effective action for the dynamical Abelian two-form coupled to a gauge field. The theory under consideration is described by the classical action

$$S_0=\int d^4x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}+\frac{m}{4}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}B_{\rho\lambda}\right). \tag{2.1}$$

where $F_{\mu\nu}$ and $H_{\mu\nu\lambda}$ are the respective field strengths of A and B, $F_{\mu\nu}=\partial_{[\mu}A_{\nu]}=\partial_\mu A_\nu-\partial_\nu A_\mu$, and $H_{\mu\nu\lambda}=\partial_{[\mu}B_{\nu\lambda]}=\partial_\mu B_{\nu\lambda}+\partial_\nu B_{\lambda\mu}+\partial_\lambda B_{\mu\nu}$. This action remains invariant under the independent gauge transformations

$$A_\mu\to A_\mu+\partial_\mu\chi,\quad B_{\mu\nu}\to B_{\mu\nu}, \tag{2.2}$$

$$A_\mu\to A_\mu,\quad B_{\mu\nu}\to B_{\mu\nu}+\partial_{[\mu}\Lambda_{\nu]}. \tag{2.3}$$

This theory has three degrees of freedom [14], one of which couples to A_μ in a fashion similar to the Goldstone mode in the Higgs mechanism. The interaction between the gauge field and the antisymmetric tensor has a two-point vertex operator proportional to the momentum. The ''physical'' propagator—so called because it couples to external fermion currents—can be calculated by summing over all gauge propagators with insertions of antisymmetric tensor propagators [4]. The physical propagator has a pole at $k^2=m^2$, i.e., this theory can be thought of as a (gauge-invariant) theory of a massive Abelian gauge field, with no other degree of freedom.

In this section I shall give a straightforward construction of the BRST-invariant action for the Abelian model (2.1). Starting with the free action S_0, the gauge-fixing terms in the covariant Lorentz gauge are added, and the Faddeev-Popov ghost terms are computed so as to cancel exactly the variation of the gauge fixing terms. The notation used in this section and the next one follows that of [6]. The BRST transformations of A_μ and $B_{\mu\nu}$ are given by their gauge transformations with Grassmann-valued gauge parameters ω and ω_μ, respectively:

$$\delta A_\mu=\partial_\mu\omega\,\delta\lambda,\quad \delta B_{\mu\nu}=(\partial_\mu\omega_\nu-\partial_\nu\omega_\mu)\,\delta\lambda. \tag{2.4}$$

As is obvious, there is a further symmetry under which ω_μ is shifted by the gradient of a scalar. This implies that the effective action needs to be gauge-fixed for ω_μ as well, otherwise the ghost propagator does not exist. This introduces a commuting ghost β for ω_μ. I can now choose the gauge-fixing part of the effective action to be

$$\mathcal{L}_{\rm GF}=-\frac{1}{2\xi}(F_1)^2-\frac{1}{2\eta}F_2^\mu F_{2\mu}-\frac{1}{2\zeta}(F_3), \tag{2.5}$$

where the fields are fixed in covariant gauges,

$$F_1=\partial_\mu A^\mu,\quad F_2^\mu=\partial_\nu B^{\mu\nu},\quad F_3=(\partial_\mu\overline{\omega^\mu})(\partial_\nu\omega^\nu). \tag{2.6}$$

The BRST transformations of the ghost fields can now be written down along the lines of the standard procedure for gauge theories:

$$\delta\omega=0,\quad \delta\overline{\omega}=\frac{1}{\xi}\partial_\mu A^\mu\delta\lambda,$$

$$\delta\omega_\mu=\partial_\mu\beta\,\delta\lambda,\quad \delta\overline{\omega_\mu}=\frac{1}{\eta}\partial^\nu B_{\mu\nu}\delta\lambda,$$

$$\delta\beta=0,\quad \delta\overline{\beta}=-\frac{1}{\zeta}(\partial_\mu\overline{\omega^\mu})\,\delta\lambda. \tag{2.7}$$

The ghost terms in the action are chosen to compensate for the variation in the gauge-fixing terms, and are, therefore,

$$\mathcal{L}_{\rm FP}=\partial_\mu\overline{\omega}\partial^\mu\omega-\partial_\mu\overline{\omega_\nu}(\partial^\mu\omega^\nu-\partial^\nu\omega^\mu)+\partial_\mu\overline{\beta}\partial^\mu\beta. \tag{2.8}$$

The total action

$$S=S_0+\int d^4x\mathcal{L}_{\rm GF}+\int d^4x\mathcal{L}_{\rm FP} \tag{2.9}$$

is now fully gauge fixed but is invariant under the BRST transformations as given in Eq. (2.7).

Under a BRST transformation the variation in the action can be written as a total divergence:

$$\delta S=\int \partial_\mu Y^\mu=0,$$

$$Y^\mu=\frac{m}{2}\epsilon^{\mu\nu\lambda\rho}\omega_\nu F_{\lambda\rho}-\frac{1}{\xi}(\partial_\nu A^\nu)\partial^\mu\omega+\frac{1}{\eta}(\partial^\lambda B_{\nu\lambda})(\partial^\mu\omega^\nu-\partial^\nu\omega^\mu)-\frac{1}{\zeta}(\partial_\nu\overline{\omega^\nu})\partial^\mu\beta. \tag{2.10}$$

The conserved Noether current for the BRST symmetry is thus

$$\begin{aligned}j^\mu&=\sum\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\frac{\delta\phi}{\delta\lambda}-Y^\mu\\&=-F^{\mu\nu}\partial_\nu\omega+\frac{m}{2}\epsilon^{\mu\nu\lambda\rho}\partial_\nu\omega B_{\lambda\rho}-\frac{1}{\xi}(\partial_\nu A^\nu)\partial^\mu\omega\\&\quad-(\partial^\mu\overline{\omega^\nu}-\partial^\nu\overline{\omega^\mu})\partial_\nu\beta-\frac{1}{2}H^{\mu\nu\lambda}(\partial_\nu\omega_\lambda-\partial_\lambda\omega_\nu)\\&\quad+\frac{1}{\eta}(\partial^\sigma B_{\nu\sigma})(\partial^\mu\omega^\nu-\partial^\nu\omega^\mu)+\frac{1}{\zeta\eta}(\partial_\nu\omega^\nu)(\partial_\lambda B^{\mu\lambda})\\&\quad-\frac{1}{\zeta}(\partial_\nu\overline{\omega^\nu})\partial^\mu\beta-\frac{m}{2}\epsilon^{\mu\nu\lambda\rho}\omega_\nu F_{\lambda\rho}.\end{aligned} \tag{2.11}$$

The BRST charge constructed from this current, $Q_{\rm BRST}=\int j^0d^3x$, is nilpotent, $Q^2_{\rm BRST}=0$. More explicitly,

$$\frac{\delta^2}{\delta\lambda^2}\{A_\mu,B_{\mu\nu},\omega,\omega_\mu,\beta,\overline{\omega,\omega_\mu},\overline{\beta}\}=0, \tag{2.12}$$

where the last three fields satisfy the equality on shell, as is the case with $\overline{\omega}$ in free Maxwell theory. Off shell their third variations vanish:

$$\frac{\delta^3}{\delta\lambda^3}\{\overline{\omega,\omega_\mu},\overline{\beta}\}=0. \tag{2.13}$$

III. THE NON-ABELIAN MODEL

The non-Abelian model [7] starts with a naïve non-Abelianization of the action (2.1) to a compact gauge group, which I shall choose to be SU(N) for convenience. To begin with, the field strength $F_{\mu\nu}$ is now defined as the curvature of an SU(N) gauge connection:

$$F^a_{\mu\nu}=\left(-\frac{i}{g}[D_\mu,D_\nu]\right)^a=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu-gf^{abc}A^b_\mu A^c_\nu. \tag{3.1}$$

In order to keep the $B\wedge F$ term invariant under SU(N) gauge transformations, $B_{\mu\nu}$ has to transform in the adjoint representation of the gauge group. This implies that in the kinetic term for $B_{\mu\nu}$ the derivative operator ∂_μ should be replaced by the gauge covariant derivative operator D_μ, and the field strength $H_{\mu\nu\lambda}$ should be defined as $H_{\mu\nu\lambda}=D_{[\mu}B_{\nu\lambda]}$. The resulting action

$$S=\int d^4x\left(-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{1}{12}H^a_{\mu\nu\lambda}H^{a\mu\nu\lambda}+\frac{m}{4}\epsilon^{\mu\nu\rho\lambda}B^a_{\mu\nu}F^a_{\lambda\rho}\right), \tag{3.2}$$

is invariant under SU(N) gauge transformation, but does not contain a natural generalization of the vector gauge symmetry in Eq. (2.3) under which one expects to find $B_{\mu\nu}\to B_{\mu\nu}+D_{[\mu}\Lambda_{\nu]}$, with Λ_μ an arbitrary vector field transforming homogeneously under the gauge group. Even though this is a symmetry of the last term of the action, the second term is not invariant under this transformation. The absence of this symmetry shows up starkly when one tries to find the propagating degrees of freedom in this theory by restricting the fields to the constraint surface according to Dirac's prescription. The matrix of Poisson Brackets of the constraints turn out to be field dependent. As a result, it is not possible to find local coordinates of the reduced phase space, or a Hamiltonian that keeps the degrees of freedom on the constraint surface. A detailed analysis of constraints will be presented elsewhere [15], but it turns out that the simplest way to construct a reduced phase space is to introduce an auxiliary vector field C_μ, also transforming in the adjoint representation of the gauge group, so as to compensate for the variation of the action (3.2) under the non-Abelian vector gauge symmetry. This does not introduce any new propagating degrees of freedom, as C_μ turns out to be fully constrained. The need for this auxiliary field also shows up in the covariant quantization of the Freedman-Townsend model [12], but here its essential purpose [15] is to enforce the constraint $[F_{\nu\lambda},H^{\mu\nu\lambda}]=0$.

Let me therefore define the compensated field strength $\widetilde{H}_{\mu\nu\lambda}$:

$$\begin{aligned}\widetilde{H}^a_{\mu\nu\lambda}&=(D_{[\mu}B_{\nu\lambda]})^a-ig[F_{[\mu\nu},C_{\lambda]}]^a\\&=\partial_{[\mu}B^a_{\nu\lambda]}-gf^{abc}A^b_{[\mu}B^c_{\nu\lambda]}+gf^{abc}F^b_{[\mu\nu}C^c_{\lambda]}.\end{aligned} \tag{3.3}$$

As is obvious, this field strength is invariant under the combined transformations

$$B_{\mu\nu}\to B_{\mu\nu}+D_{[\mu}\Lambda_{\nu]},\quad C_\mu\to C_\mu+\Lambda_\mu, \tag{3.4}$$

where Λ^a_μ are real vector fields. It should also be noted that the last term in the definition of $\widetilde{H}_{\mu\nu\lambda}$ vanishes in the case where the gauge group is Abelian, so that $\widetilde{H}_{\mu\nu\lambda}$ is an allowed generalization of the Abelian field strength. Now I can write down an action which is invariant both under the gauge group and the vector transformations (3.4):

$$S_0=\int d^4x\left(-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{1}{12}\widetilde{H}^a_{\mu\nu\lambda}\widetilde{H}^{a\mu\nu\lambda}+\frac{m}{4}\epsilon^{\mu\nu\rho\lambda}B^a_{\mu\nu}F^a_{\lambda\rho}\right). \quad (3.5)$$

It should be noted that this action is invariant under the non-Abelian vector gauge symmetry given in Eq. (3.4) without any modification of the interaction term as long as the fields vanish sufficiently rapidly at infinity. Also, the auxiliary field C_μ is nondynamical—there is no quadratic term corresponding to it in the action, and the propagator is zero at tree level. From now on I shall work only with the compensated field strength $\widetilde{H}_{\mu\nu\lambda}$ and not refer to the naïve field strength $H_{\mu\nu\lambda}$, so I can drop the tilde and write $H_{\mu\nu\lambda}$ whenever I mean $\widetilde{H}_{\mu\nu\lambda}$.

It can be shown by an analysis of constraints that there are three degrees of freedom for each gauge index in this theory. The quadratic terms in this theory are identical, for each gauge index, to the Abelian action. As a result, the tree-level effective propagator for the gauge field can be computed exactly in the same fashion and leads to a pole at $k^2=m^2$. And there is no residual scalar.

The construction of the BRST-invariant action will follow those for the Abelian model above and Yang-Mills theory, and also that for the pure $B\wedge F$ topological field theory. The gauge-fixing terms are easy to write down,

$$\mathcal{L}_{\rm GF}=-\frac{1}{2\xi}(\partial_\mu A^{a\mu})^2-\frac{1}{2\eta}(\partial_\nu B^{a\mu\nu})^2-\frac{1}{\zeta}(\partial_\mu\overline{\omega}^{a\mu})(\partial_\nu\omega^{a\nu}), \quad (3.6)$$

as are the Faddeev-Popov ghost terms,

$$\mathcal{L}_{\rm FP}=-\overline{\omega}^a\frac{\delta}{\delta\lambda}(\partial_\mu A^{a\mu})-\overline{\omega}^{a\mu}\frac{\delta}{\delta\lambda}(\partial^\nu B^a_{\mu\nu})-\overline{\beta}^a\frac{\delta}{\delta\lambda}(\partial_\nu\omega^{a\nu}). \quad (3.7)$$

These terms were written down simply by generalizing the Abelian case, and the ghost fields are also defined as generalizations of the Abelian model. Now, however, an interesting difference shows up. The fields $\beta,\overline{\beta}$ were needed in the Abelian case in order to compensate for the gauge fixing of the ghost ω_μ. In the non-Abelian model, ω^a_μ needs a gauge fixing term for the same reason, namely that the propagator cannot be defined until that has been done. In the Abelian model, this showed up as the symmetry of the action under $\omega_\mu\to\omega_\mu+\partial_\mu\theta$. Alternatively, the need for this ghost of ghost was a consequence of a symmetry $\Lambda_\mu\to\Lambda_\mu+\partial_\mu\chi$, with χ an arbitrary scalar, which is hidden in the vector gauge transformation (2.3). In the non-Abelian model, it is still not possible to define the ghost propagator and the ghosts need gauge fixing. One can try to implement a similar symmetry transformation, $\Lambda_\mu\to\Lambda_\mu+D_\mu\chi$, where Λ_μ and χ are now in the adjoint representation of the gauge group. However, this leads to the following set of transformations:

$$\delta B^a_{\mu\nu}=-gf^{abc}F^b_{\mu\nu}\chi^c, \quad \delta C^a_\mu=(D_\mu\chi)^a, \quad (3.8)$$

unlike in the Abelian case, where $\delta B_{\mu\nu}=0$ under such a transformation. This implies that there has to be a ghost field corresponding to this transformation, as was found by the authors of [12] in the context of the Freedman-Townsend model. The complete set of BRST transformations can now be written down, simply by generalizing the Abelian case, remembering that all the fields and the ghosts transform in the adjoint representation, and including this extra ghost:

$$\begin{aligned}
\delta A^a_\mu&=(D_\mu\omega)^a\,\delta\lambda\\
\delta\omega^a&=-\frac{1}{2}gf^{abc}\omega^b\omega^c\,\delta\lambda\\
\delta\overline{\omega^a}&=\frac{1}{\xi}(\partial_\mu A^{a\mu})\,\delta\lambda\\
\delta B^a_{\mu\nu}&=[-gf^{abc}B^b_{\mu\nu}\omega^c+(D_{[\mu}\omega_{\nu]})^a-gf^{abc}F^b_{\mu\nu}\theta^c]\,\delta\lambda\\
\delta C^a_\mu&=[-gf^{abc}C^b_\mu\omega^c+\omega^a_\mu+(D_\mu\theta)^a]\,\delta\lambda\\
\delta\omega^a_\mu&=[-gf^{abc}\omega^b_\mu\omega^c+(D_\mu\beta)^a]\,\delta\lambda\\
\delta\overline{\omega^a_\mu}&=\frac{1}{\eta}(\partial^\nu B_{\mu\nu})\,\delta\lambda\\
\delta\beta^a&=-gf^{abc}\beta^b\omega^c\,\delta\lambda\\
\delta\overline{\beta^a}&=-\frac{1}{\zeta}\partial_\mu\overline{\omega^{a\mu}}\,\delta\lambda\\
\delta\theta^a&=(-gf^{abc}\theta^b\omega^c-\beta^a)\,\delta\lambda\\
\delta\overline{\theta^a}&=0.
\end{aligned} \quad (3.9)$$

This set of transformations has the correct limits—if ω^a is the only nonvanishing ghost, these would be the transformations corresponding to an SU(N) symmetry, whereas if f^{abc} and C_μ are set to zero, the Abelian BRST transformations (2.7) are recovered. It is straightforward to check that this set of transformations is nilpotent in a manner similar to the Abelian case:

$$\frac{\delta^2}{\delta\lambda^2}\{A^a_\mu,B^a_{\mu\nu},C^a_{\mu\nu},\omega^a,\omega^a_\mu,\beta^a,\theta^a\}=0. \quad (3.10)$$

It is also straightforward to show that the set of the BRST transformations as posited above leaves the sum of the gauge-fixing and ghost Lagrangians invariant:

$$\frac{\delta}{\delta\lambda}(\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP})=0. \quad (3.11)$$

The total BRST-invariant action can now be written as a sum of three terms, the gauge term, the gauge fixing term, and the ghost contribution:

$$S=\int d^4x(\mathcal{L}_0+\mathcal{L}_{\rm GF}+\mathcal{L}_{\rm FP}), \tag{3.12}$$

with

$$\mathcal{L}_0=-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{1}{12}\widetilde{H}^a_{\mu\nu\lambda}\widetilde{H}^{a\mu\nu\lambda}+\frac{m}{4}\epsilon^{\mu\nu\rho\lambda}B^a_{\mu\nu}F^a_{\lambda\rho},$$

$$\mathcal{L}_{\rm GF}=-\frac{1}{2\xi}(\partial_\mu A^{a\mu})^2-\frac{1}{2\eta}(\partial_\nu B^{a\mu\nu})^2$$
$$-\frac{1}{\zeta}(\partial_\mu\overline{\omega^{a\mu}})(\partial_\nu\omega^{a\nu}),$$

$$\mathcal{L}_{\rm FP}=\partial^\mu\overline{\omega^a}(D_\mu\omega)^a-gf^{abc}\partial^\nu\overline{\omega^{a\mu}}B^b_{\mu\nu}\omega^c+\partial^\nu\overline{\omega^{a\mu}}(D_{[\mu}\omega_{\nu]})^a$$
$$-gf^{abc}\partial_\nu\overline{\beta^a}\omega^{b\nu}\omega^c+\partial^\mu\overline{\beta^a}(D_\mu\beta)^a. \tag{3.13}$$

This action is fully gauge fixed with respect to the SU(N) gauge transformations, as well as the vector gauge transformations (3.4), but it is invariant under the BRST transformations given in Eq. (3.9). This action also implies the nilpotence of the BRST transformation on $\overline{\omega}$, $\overline{\omega_\mu}$, β and θ,

$$\frac{\delta^2}{\delta\lambda^2}\{\overline{\omega^a},\overline{\omega^a_\mu},\overline{\beta^a},\overline{\theta^a}\}=0, \tag{3.14}$$

taking into account their equations of motion. Off shell, their third variations vanish,

$$\frac{\delta^3}{\delta\lambda^3}\{\overline{\omega^a},\overline{\omega^a_\mu},\overline{\beta^a},\overline{\theta^a}\}=0, \tag{3.15}$$

just as in the case of $\overline{\omega^a}$ in the case of pure Yang-Mills theory. It is now possible to construct the BRST-invariant Noether current for this action in the same manner as in the Abelian case. The variation of the action vanishes:

$$\frac{\delta}{\delta\lambda}S=\int d^4x\,\partial_\mu Y^\mu=0, \tag{3.16}$$

with

$$Y^\mu=\frac{m}{2}\epsilon^{\mu\nu\lambda}\omega^a_\nu F^a_{\lambda\rho}-\frac{1}{\xi}[\partial_\nu A^{a\nu}(D^\mu\omega)^a]-\frac{1}{\eta}(\partial^\lambda B^a_{\nu\lambda})$$
$$\times[gf^{abc}B^{b\mu\nu}\omega^c-(D^{[\mu}\omega^{\nu]})^a+gf^{abc}F^{b\mu\nu}\theta^c]$$
$$-\frac{1}{\zeta}(\partial_\lambda\overline{\omega^{a\lambda}})[-gf^{abc}\omega^{b\mu}\omega^c+(D^\mu\beta)^a]. \tag{3.17}$$

The Noether current is, therefore,

$$j^\mu=\sum\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\frac{\delta\phi}{\delta\lambda}-Y^\mu$$
$$=\left(-F^{a\mu\nu}+\frac{m}{2}\epsilon^{\mu\nu\lambda\rho}B^a_{\lambda\rho}-\frac{1}{\xi}g^{\mu\nu}(\partial_\lambda A^{a\lambda})-gf^{abc}C^b_\lambda H^{c\mu\nu\lambda}\right)(D_\nu\omega)^a$$
$$-\frac{1}{2}H^{a\mu\nu\lambda}[-gf^{abc}B^b_{\nu\lambda}\omega^c+(D_\nu\omega_\lambda-D_\lambda\omega_\nu)^a-gf^{abc}F^b_{\nu\lambda}\theta^c]+\frac{1}{\eta}(\partial^\lambda B^a_{\nu\lambda})[-gf^{abc}B^{b\mu\nu}\omega^c+(D^\mu\omega^\nu-D^\nu\omega^\mu)^a]$$
$$+\frac{1}{\zeta\eta}(\partial_\lambda\omega^{a\lambda})(\partial^\sigma B^{a\mu\sigma})-\frac{1}{2}gf^{abc}(\partial^\mu\overline{\omega^a})\omega^b\omega^c-(\partial^\mu\overline{\omega^{a\nu}}-\partial^\nu\overline{\omega^{a\mu}})[-gf^{abc}\omega^b_\nu\omega^c+(D_\nu\beta)^a]$$
$$+gf^{abc}\partial^\mu\overline{\beta^a}\beta^b\omega^c-\frac{1}{\zeta}(\partial_\lambda\overline{\omega^{a\lambda}})[-gf^{abc}\omega^{b\mu}\omega^c+(D^\mu\beta)^a]. \tag{3.18}$$

Just as in the Abelian case, the BRST charge constructed from this current,

$$Q_{\rm BRST}=\int j^0 d^3x \tag{3.19}$$

is nilpotent, $Q^2_{\rm BRST}=0$, and implements the BRST transformations on the fields, as can be explicitly checked by writing out the charge in terms of the canonically conjugate momenta to the fields and the ghosts.

IV. DISCUSSION

Let me first summarize what has been done so far. First I constructed a BRST-invariant gauge fixed action for the Abelian mass generation mechanism. The transformations in the Abelian case were then generalized to the non-Abelian mechanism. The non-Abelian BRST transformations reduce to those for the Abelian case or for the free Yang-Mills case in the appropriate limits. The gauge fixed effective Lagrangian was constructed by including the appropriate ghost terms which leave the total action invariant under the BRST transformations. This invariance leads to a conserved BRST charge which is nilpotent on the Fock space. The cohomology of the BRST charge can be identified with the physical subspace of the Hilbert space, and the unitarity of the S-matrix is guaranteed on the physical states.

It is possible to compute the Slavnov-Taylor identities for the non-Abelian theory starting from the BRST-invariant effective action of Eq. (3.12). It is outside the scope of this paper to do that, or to construct counterterms and prove perturbative renormalizability of the theory, which will be done elsewhere. It should be noted that no kinetic term (or any other quadratic term) for C_μ was required for the nilpotence of the BRST transformations, i.e., for the construction of a BRST-invariant quantum action for the theory. Thus C_μ remains a nondynamical auxiliary field at tree level even after quantization.

Does anything change when fermions are coupled to the theory? If the fermions are minimally coupled *only* to the gauge field A_μ, it is easy to see that the resulting theory can be made BRST invariant in the same way as before after adding in the usual BRST transformations of fermions in gauge theories. In the Abelian model, fermions cannot couple to the antisymmetric tensor because the minimal coupling breaks the vector gauge symmetry. In the non-Abelian model, the vector gauge symmetry is enforced by the introduction of the auxiliary C_μ. As a result it is possible to couple the non-Abelian antisymmetric tensor to fermions, the general term for minimal coupling being $\bar{\psi}(a+b\gamma_5)\sigma^{\mu\nu}(B_{\mu\nu}-D_{[\mu}C_{\nu]}\psi)$. This term is invariant under both the continuous symmetries, but breaks chiral symmetry. It is plausible that fermion mass is generated as a dynamical effect as a result of chiral symmetry breaking via this term.

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