

## Reheating and causal thermodynamics

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The reheating process in inflationary universe models is considered as an out-of-equilibrium mixture of two interacting and reacting fluids, and studied within the framework of causal, irreversible thermodynamics. The evolution of the temperature and the decay rate as determined by causal thermodynamics are estimated at different stages of the process. A simple model is also used to find the perturbations of the expansion rate, including the possibility of damped oscillations. [S0556-2821(97)00208-7]

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### I. INTRODUCTION

The reheating period is a key ingredient in many inflationary models of the very early Universe. During reheating, most of the matter and radiation of the Universe is created via the decay of the inflaton field, while the temperature grows by many orders of magnitude. Traditionally, the idealized picture of a very quick decay in which the products reach equilibrium immediately is assumed [1]. However, recent quantum field theoretical investigations [2–10] indicate that the reheating period is characterized by complicated nonequilibrium processes, the main characteristics of which are initial, violent particle production via parametric resonance (“preheating”), with a highly nonequilibrium distribution of the produced particles, subsequently relaxing to an equilibrium state.

In this paper we aim at obtaining a phenomenological understanding of the reheating process within a model of two interacting and reacting fluids. We regard this approach as complementary to the above-mentioned quantum field theoretical studies. The model involves a fluid with the equations of state for matter (modeling scalar field oscillations about the true ground state in the reheating) that decays into a relativistic fluid. The implications of an intermediate decay into other massive (bosonic) particles that does not explicitly occur in this model, are assumed to be describable with the help of a large, effective decay rate of the initial into the final component of the entire process, leaving the detailed microphysical study of this epoch to quantum field theoretical investigations which are beyond the scope of the present paper.

While these simplifications may appear drastic, they open the possibility of studying the back reaction of the decay

process as a whole on the entire cosmological dynamics, including the behavior of the scale factor. We believe this to be a main advantage of our approach since most of the quantum field theoretical calculations do not even take into account the expansion of the Universe.

A two-fluid description of the expanding Universe is necessarily dissipative, even if the components are assumed to be intrinsically perfect fluids. This is also true for nonreacting fluids (conserved particle numbers), where the different cooling rates of the subsystems produce an effective, entropy-generating bulk viscous pressure of the system as a whole [11,12]. In fact, conservation of the particle number is only a very special case, particularly at high energies. The processes we are interested in are characterized by particle decay and production.

As has been shown recently, any deviation from detailed balance in the decay and inverse decay reactions in the expanding Universe gives rise to additional bulk pressures [13]. In the present case of strong perturbations of the detailed balance (the reactions are predominantly one directional with the inverse processes largely suppressed), one expects considerable bulk viscous pressures that characterize the deviations from equilibrium of the cosmological fluid as a whole. To describe this nonequilibrium process, we shall resort to the well-known Israel-Stewart theory of transport processes [14] which, because of its causality and stability properties [15], has been repeatedly applied in the cosmological context, see [16–18] and references therein.

In this theory, generalized “fluxes,” such as a bulk pressure, become dynamical degrees of freedom on their own and have associated relaxation times. Our point of view here is to regard the cosmic substratum during the reheating period as a causal, dissipative fluid, relaxing to equilibrium. (By “causal” we mean that dissipative signals propagate only at subluminal speeds.)

Bearing in mind that the Israel-Stewart theory was derived for small deviations from equilibrium (for attempts to apply it to far-from-equilibrium situations see [17,18]), we

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point out that in the present context it is the high “preheating” particle production rate that provides a “creation” contribution to the “effective” bulk pressure which may be much larger than the conventional, thermodynamic, dissipative bulk pressure. While the latter is considered to be small in the applications of this paper, the “effective” bulk pressure, that determines the reheating temperature, is not.

Section II establishes the basic relationships concerning both particle and energy-momentum nonconservation in our two-fluid system, and presents an expression for the evolution of the temperature of the overall fluid. Section III introduces the causal transport relation for the thermodynamic, dissipative bulk stress, and deduces the corresponding equation for the evolution of the Hubble parameter. Section IV A solves the latter equation at three different stages of the reheating by assuming a very short relaxation time, and determines the temperature evolution in each of them. The qualitative description of the dynamics is determined essentially by a single thermodynamical quantity, the dissipative contribution to the speed of sound. If this is larger than the adiabatic contribution to the sound speed at the beginning of reheating, we show that the temperature rises rapidly to a maximum (reheating) temperature, which we estimate. This condition for rising temperature is equivalent to a growth in the total particle number density, which is reasonable in the initial stage of reheating, when coherent oscillations of the scalar field lead to a huge production of particles. Thereafter, the temperature falls, but less rapidly than in the nondissipative case. In Sec. IV B, we construct a simple model to calculate the perturbations of the expansion rate due to causal viscous and reactive effects. The model includes the possibility of damped oscillations in the beginning of reheating. Finally, Sec. V summarizes our conclusions.

Units have been chosen so that  $c = k_B = \hbar = 1$ .

## II. THE TWO-FLUID MODEL

Let us assume the energy-momentum tensor  $T^{ik}$  of the cosmic medium splits into two perfect fluid parts:

$$T^{ik} = T_1^{ik} + T_2^{ik}, \quad (1)$$

with  $(A = 1, 2)$

$$T_A^{ik} = \rho_A u^i u^k + p_A h^{ik}. \quad (2)$$

$p_A$  is the equilibrium pressure of species  $A$ . For simplicity, we assume that both components share the same four-velocity  $u^i$ , with projection tensor  $h^{ik} = g^{ik} + u^i u^k$ . The particle flow vector  $N_A^i$  of species  $A$  is

$$N_A^i = n_A u^i, \quad (3)$$

where  $n_A$  is the particle number density. We are interested in situations where neither the particle numbers nor the energy momentum of the components is separately conserved, i.e., particle interconversion and exchange of energy and momentum between the components are admitted.

The balance laws for the particle numbers are

$$N_{A;i}^i = \dot{n}_A + \Theta n_A = n_A \Gamma_A, \quad (4)$$

where  $\Theta \equiv u^i_{;i}$  is the fluid expansion and  $\Gamma_A$  is the rate of change of the number of particles of species  $A$ . There is particle production for  $\Gamma_A > 0$  and particle decay for  $\Gamma_A < 0$ . For  $\Gamma_A = 0$  we have separate particle number conservation.

Interactions between the fluid components amount to the mutual exchange of energy and momentum. Consequently, there will be no local energy-momentum conservation for the subsystems separately. Only the energy-momentum tensor of the system as a whole is conserved. Denoting the loss and source terms in the separate balances by  $t_A^i$  we may write

$$T_{A;k}^{ik} = -t_A^i, \quad (5)$$

implying

$$\dot{\rho}_A + \Theta(\rho_A + p_A) = u_i t_A^i, \quad (6)$$

and

$$(\rho_A + p_A) \dot{u}^i + p_{A,k} h^{ik} = -h_k^i t_A^k. \quad (7)$$

All the considerations to follow will be independent of the specific structure of the  $t_A^i$ . In general, there are no limitations on the strength or the structure of the interaction.

Each component is governed by a separate Gibbs equation:

$$T_A ds_A = d\left(\frac{\rho_A}{n_A}\right) + p_A d\left(\frac{1}{n_A}\right). \quad (8)$$

Using Eqs. (4) and (6), one finds for the time evolution of the entropy per particle

$$n_A T_A \dot{s}_A = u_i t_A^i - (\rho_A + p_A) \Gamma_A. \quad (9)$$

With nonvanishing source terms in the balances for  $n_A$  and  $\rho_A$ , the rate of change of entropy per particle is nonzero in general. Below, we shall deal with the special case that the terms on the right-hand side of Eq. (9) just cancel.

According to Eq. (5) the condition of energy-momentum conservation for the system as a whole,

$$(T_1^{ik} + T_2^{ik})_{;k} = 0, \quad (10)$$

implies

$$t_1^i = -t_2^i. \quad (11)$$

There is no corresponding condition, however, for the particle number balance as a whole. Defining the integral particle number density  $n$  as

$$n = n_1 + n_2, \quad (12)$$

we have

$$\dot{n} + \Theta n = n \Gamma, \quad (13)$$

with

$$n \Gamma = n_1 \Gamma_1 + n_2 \Gamma_2. \quad (14)$$

$\Gamma$  is the rate by which the total particle number  $N$  changes. We do *not* require  $\Gamma$  to be zero, since total particle number conservation is only a very special case, especially at high energies.

From now on we assume that the source terms on the right-hand side of Eq. (9) cancel among themselves, i.e., the entropy per particle of each of the components is preserved. The particles decay or come into being with a *fixed* entropy  $s_A$ . This adiabaticity condition amounts to the assumption that the particles at any stage are amenable to a perfect fluid description. With  $\dot{s}_A=0$  in Eq. (9), one has

$$u_i t_A^i = (\rho_A + p_A) \Gamma_A. \quad (15)$$

This relationship establishes a link between the source terms in Eqs. (4) and (6) which originally are independent quantities. The simplifying assumption  $\dot{s}_A=0$  takes into account the circumstance that the production process itself is the main source of entropy production, while dissipative processes within each of the separate components are less important. The cosmic fluid as a whole will be considered, however, as dissipative (see below).

Combining Eqs. (11) and (15) one has

$$u_i t_1^i = (\rho_1 + p_1) \Gamma_1 = -u_i t_2^i = -(\rho_2 + p_2) \Gamma_2, \quad (16)$$

which provides us with a relation between the rates  $\Gamma_1$  and  $\Gamma_2$ :

$$\Gamma_2 = - \left[ \frac{\rho_1 + p_1}{\rho_2 + p_2} \right] \Gamma_1. \quad (17)$$

Use of this relation in Eq. (14) yields

$$n\Gamma = n_1 \Gamma_1 h_1 \left[ \frac{1}{h_1} - \frac{1}{h_2} \right], \quad (18)$$

where  $h_A \equiv (\rho_A + p_A)/n_A$  are the enthalpies per particle. Total particle number conservation, i.e.,  $\Gamma=0$ , is only possible if  $h_1 = h_2$ .

As is shown elsewhere [12,13], a system of two fluids, each of them perfect on its own, is dissipative in general and may be characterized by an energy-momentum tensor

$$T^{ik} = \rho u^i u^k + (p + \pi) h^{ik}. \quad (19)$$

The equilibrium pressure  $p$  of the total system and the energy density  $\rho$  are assumed to obey equations of state

$$p = p(n, n_1, T) \quad (20)$$

and

$$\rho = \rho(n, n_1, T), \quad (21)$$

where  $T$  is the equilibrium temperature of the system as a whole, *defined* by (cf. [11,12])

$$\rho_1(n_1, T_1) + \rho_2(n_2, T_2) = \rho(n, n_1, T). \quad (22)$$

It is worth mentioning that there does not exist a corresponding relation for the pressures. The partial pressures of the components do not add up to the equilibrium pressure in

general. The difference between the sum  $p_1(n_1, T_1) + p_2(n_2, T_2)$  and the equilibrium pressure  $p(n, T)$  contributes to the thermodynamical, dissipative bulk pressure  $\pi$ . A further source of  $\pi$  is deviations from detailed balance, i.e., contributions due to  $\Gamma_A \neq 0$  [13].

The behavior of the equilibrium temperature  $T$  of the system as a whole is governed by [13]

$$\frac{\dot{T}}{T} = -(\Theta - \Gamma) \frac{\partial_T p}{\partial_T \rho} - \frac{\Theta \pi}{T \partial_T \rho} + \left( \frac{\dot{T}}{T} \right)_*, \quad (23)$$

where the abbreviations  $\partial_T f = \partial f / \partial T$  and

$$\begin{aligned} \left( \frac{\dot{T}}{T} \right)_* &\equiv - \frac{p\Gamma - p_1\Gamma_1 - p_2\Gamma_2}{T \partial_T \rho} \\ &= \frac{\rho_1 + p_1}{T \partial_T \rho} \left( \frac{n_2}{n} p_1 - \frac{n_1}{n} p_2 \right) \left[ \frac{1}{n_1 h_1} + \frac{1}{n_2 h_2} \right] \Gamma_1 \end{aligned} \quad (24)$$

were used. The term (24) takes into account deviations from classical gas behavior. It vanishes for  $p_A = n_A T$ , i.e., for a mixture of classical gases. For a mixture of a classical and a quantum gas, or for a mixture of fermions and bosons, it will be nonzero in general. We shall restrict ourselves to the case of two classical fluids, i.e., to the case  $(\dot{T}/T)_* = 0$ . This simplifying assumption is in line with the general restriction of our approach concerning the detailed microphysics, pointed out in the introduction.

The temperature evolution equation (23) suggests that we define an ‘‘effective’’ bulk pressure

$$\pi_{\text{eff}} \equiv \pi - \frac{\Gamma}{\Theta} T \partial_T p = \pi - \frac{\Gamma}{\Theta} p, \quad (25)$$

so that Eq. (23) in the classical case may be written as

$$\frac{\dot{T}}{T} = -\Theta \frac{\partial_T p}{\partial_T \rho} - \frac{\Theta \pi_{\text{eff}}}{T \partial_T \rho}. \quad (26)$$

Our main concern in the following sections will be to study the influence of the different parts of the entropy producing ‘‘effective’’ bulk pressure  $\pi_{\text{eff}}$  on the cosmological evolution.

The  $\pi_{\text{eff}}$  term in Eq. (26) describes the deviation from the adiabatic temperature behavior. It is obvious that any  $\pi_{\text{eff}} < 0$  on the right-hand side of Eq. (26) yields a positive, i.e., ‘‘reheating’’ contribution to  $\dot{T}/T$ , counteracting the first term on the right-hand side of Eq. (26) that simply describes the adiabatic cooling due to the expansion.

As we shall discuss below, the nonequilibrium term on the right-hand side of Eq. (26) may overcompensate the adiabatic term during the initial ‘‘preheating’’ stage.

The effective bulk pressure (25) consists of the conventional, thermodynamic, dissipative bulk pressure  $\pi$  and a creation part  $-p\Gamma/\Theta$ . For  $\Gamma > \Theta$ , a condition that is expected to be satisfied during the initial stage of reheating, the creation part gives the dominant contribution to the effective bulk pressure. Even for a small or vanishing thermodynamic

viscous pressure  $\pi$ , the entropy-producing nonequilibrium parts on the right-hand side of the temperature law (26) may overcompensate the adiabatic part, provided only that the production rate  $\Gamma$  is sufficiently high. In other words, even the production of particles with an equilibrium distribution, equivalent to the possibility of a perfect fluid description, gives rise to “reheating” and entropy production. We shall consider this to be the dominant part of the entropy production during preheating. It is the particle production process itself which is connected with entropy production, simply through the enlargement of the phase space. There are additional entropy-producing contributions due to the fact that in reality the particles will deviate from equilibrium. These contributions will be subject to a causal transport equation in the following section.

### III. THE CAUSAL EVOLUTION EQUATION

In the Israel-Stewart second order theory of irreversible processes, the viscous pressure  $\pi$  is a dynamical degree of freedom on its own [14]. Instead of applying the much more involved full Israel-Stewart theory, we shall restrict ourselves to the so-called *truncated* version [17–19] of this theory, since the latter already captures the essence of non-instantaneous relaxations. While the full and the truncated theories may disagree significantly if applied to far-from-equilibrium situations [17–19], they are expected to provide similar results near equilibrium [18]. Now, reheating is anything but close to equilibrium. But as already mentioned, the main contribution to the entropy production during the decay process may be traced back to the second term in the expression (25) for the effective bulk pressure. The thermodynamic, dissipative bulk pressure  $\pi$  may be regarded as a small perturbation under these circumstances and is supposed to obey the truncated causal transport equation

$$\pi + \tau \dot{\pi} = -\zeta \Theta. \tag{27}$$

The quantity  $\tau$  denotes the relaxation time associated with the thermodynamic, dissipative bulk pressure, i.e., the time the system would take to come to equilibrium (perfect fluid behavior) if the generalized “force,” in this case the expansion  $\Theta$ , were suddenly turned off. It may be related to the bulk viscosity  $\zeta$  by [19] (see also [18])

$$\frac{\zeta}{\tau} = c_b^2(\rho + p), \quad c_b^2 \leq 1 - c_s^2, \tag{28}$$

where  $c_s$  is the adiabatic sound speed and  $c_b$  is the bulk viscous contribution to the speed of sound  $v$ , given by  $v^2 = c_s^2 + c_b^2$ . Thus,

$$\pi + \tau \dot{\pi} = -\Theta c_b^2 \rho \gamma \tau. \tag{29}$$

Here and throughout, we will use the abbreviation

$$\gamma \equiv 1 + \frac{p}{\rho},$$

where  $\gamma$  is *not* assumed constant.

The Einstein field equations for a spatially flat Robertson-Walker spacetime are

$$3H^2 = \kappa \rho, \tag{30}$$

where  $\kappa$  is Einstein’s gravitational constant, and

$$\dot{H} = -\frac{1}{2} \kappa (\rho + p + \pi), \tag{31}$$

where  $H = \frac{1}{3} \Theta = \dot{R}/R$  and  $R$  is the cosmic scale factor. By Eqs. (30) and (31), the thermodynamic bulk pressure may be written as

$$\kappa \pi = -3 \gamma H^2 - 2 \dot{H}. \tag{32}$$

To calculate  $\dot{\pi}$  one first has to determine  $\dot{p}$  via

$$\dot{p} = \frac{\partial p}{\partial n} \dot{n} + \frac{\partial p}{\partial T} \dot{T}. \tag{33}$$

Using the balance law (13) for  $n$ , and the evolution equation (23) for  $T$ , we find

$$\dot{p} = -(3H - \Gamma)(\rho + p)c_s^2 - 3H\pi \left( \frac{\partial_T p}{\partial_T \rho} \right), \tag{34}$$

with the adiabatic sound velocity  $c_s$  given by

$$c_s^2 = \left( \frac{\partial p}{\partial \rho} \right)_{\text{isentropic}} = \frac{n}{(\rho + p)} \frac{\partial p}{\partial n} + \frac{T}{(\rho + p)} \frac{(\partial_T p)^2}{\partial_T \rho}. \tag{35}$$

By differentiating Eq. (31), we get

$$\begin{aligned} \dot{\pi} = & -\frac{2}{3} \frac{\ddot{H}}{H^2} \rho - 2 \frac{\dot{H}}{H} \rho \left( 1 + \frac{\partial_T p}{\partial_T \rho} \right) - 3H(\rho + p) \frac{\partial_T p}{\partial_T \rho} \\ & + (3H - \Gamma)(\rho + p)c_s^2. \end{aligned} \tag{36}$$

Using Eqs. (32) and (36) in Eq. (29), the evolution equation for  $H$  becomes

$$\begin{aligned} \tau \left[ \frac{\ddot{H}}{H^2} + 3 \frac{\dot{H}}{H} \left( 1 + \frac{\partial_T p}{\partial_T \rho} \right) + \frac{9}{2} H \gamma \left( \frac{\partial_T p}{\partial_T \rho} - c_s^2 - c_b^2 \right) + \frac{3}{2} c_s^2 \gamma \Gamma \right] \\ + \frac{\dot{H}}{H^2} + \frac{3}{2} \gamma = 0. \end{aligned} \tag{37}$$

Equation (37) is the causal dynamical equation in general form. We now specify the equations of state implicit in  $\rho$  and  $p$ . We assume that fluid one is described by the equations of state for nonrelativistic matter, i.e.,

$$\rho_1 = n_1 m + \frac{3}{2} n_1 T, \quad p_1 = n_1 T, \quad m \gg T, \tag{38}$$

while fluid two is relativistic:

$$\rho_2 = 3n_2 T, \quad p_2 = n_2 T. \tag{39}$$

Consequently, Eq. (37) reduces to

$$\begin{aligned} & \tau \left[ \frac{\ddot{H}}{H^2} + \left( \frac{5n_1 + 8n_2}{n_1 + 2n_2} \right) \frac{\dot{H}}{H} + 3 \left\{ \frac{(n_1 + n_2)n_1 m}{(n_1 + 2n_2)(n_1 m + 3n_2 T)} \right. \right. \\ & \quad \left. \left. - \frac{3}{2} \gamma c_b^2 \right\} H + \frac{(5n_1 + 8n_2)n_1 m}{8(n_1 + 2n_2)(n_1 m + 3n_2 T)} Q \right] \\ & + \frac{\dot{H}}{H^2} + \frac{3(n_1 m + 4n_2 T)}{2(n_1 m + 3n_2 T)} = 0, \end{aligned} \quad (40)$$

where  $Q \equiv |\Gamma_1| > 0$  was assumed, i.e., the nonrelativistic component decays. This is the causal evolution equation to be solved.

#### IV. CAUSAL DYNAMICS OF REHEATING

The Eq. (40) is a complicated nonlinear second order equation. To solve it, one could resort to numerical integration. However, this is strongly dependent on initial conditions and on the form of the decay rate  $Q$ , and it cannot readily give an idea of the overall dynamical features implied by the equation. Furthermore, part of the purpose of our thermodynamical approach is to avoid detailed complexities and to aim for an overall qualitative understanding arising from the constraint of causality.

In Sec. IV A, we use Eq. (40) to estimate the temperature and decay rate during reheating, neglecting the small perturbation of the expansion rate due to thermodynamic viscous effects. (Notice that the gravitational field equations “feel” only the thermodynamic viscous pressure.) It turns out that under this condition the reheating dynamics may be discussed in terms of the ratio  $c_b/c_s$  of the dissipative to the adiabatic contributions to the speed of sound. Then, in Sec. IV B, we calculate the perturbation of the expansion rate in a simple case.

##### A. Temperature and decay rate

It is reasonable to assume that the expansion is approximately governed by nonviscous effects, and that the latter can be treated as a back reaction. In this approximation, we neglect the terms multiplying  $\tau$  in Eq. (40), and arrive at the equation governing the expansion rate:

$$\frac{\dot{H}}{H^2} + \frac{3(n_1 m + 4n_2 T)}{2(n_1 m + 3n_2 T)} \approx 0. \quad (41)$$

We solve this for three different stages of the decay. Then we use Eq. (40) to determine the corresponding decay rate  $Q$ . The evolution of the temperature is given by

$$\frac{\dot{T}}{T} = \frac{1}{6(n_1 + 2n_2)T} [-12H(p_1 + p_2 + \pi) + n_1 m Q] \quad (42)$$

as follows from Eq. (26) and from

$$\Gamma = \frac{n_1 m}{4n T} Q,$$

which is a consequence of Eq. (18) and the equations of state (38) and (39). In the perfect fluid limit  $\pi = Q = 0$ , one recovers  $\dot{T}/T = -2H$  for  $n_1 \gg n_2$ , i.e., the nonrelativistic case,

while for radiation ( $n_1 \ll n_2$ ), the well-known behavior  $\dot{T}/T = -H$  is reproduced. Assuming that  $|\pi| \ll p_1 + p_2$ , in agreement with the applicability conditions of the Israel-Stewart theory, Eq. (42) may be written as

$$\frac{\dot{T}}{T} \approx -2H \left( \frac{n_1 + n_2}{n_1 + 2n_2} \right) \left[ 1 - \frac{\Gamma}{3H} \right]. \quad (43)$$

It follows that the temperature rises when  $\Gamma > 3H$ . By Eq. (13), this is equivalent to  $\dot{n} > 0$ , i.e., to a growth in the total particle number density (cf. [20]). As discussed in Sec. I, such abundant net particle creation is expected to occur in the initial nonrelativistic stage of reheating.

##### 1. Nonrelativistic regime

In this regime the fluid is dominated by massive particles, so that  $n_1 \gg n_2$ ,  $\gamma \approx 1$ . Then, Eq. (41) reduces to

$$\dot{H} + \frac{3}{2} H^2 \approx 0 \Rightarrow H \approx \frac{2}{3t}. \quad (44)$$

Inserting this solution back into Eq. (40), we find that the decay rate at the beginning of reheating is

$$Q \approx \frac{36}{5} c_b^2 H \Rightarrow \Gamma \approx 3 \left( \frac{c_b}{c_s} \right)^2 H, \quad \text{where } c_s^2 \approx \frac{5}{3} \frac{T}{m}. \quad (45)$$

It is a specific feature of the present first order approximation that it fixes the rates  $Q$  and  $\Gamma$  which, in our general setting, are input parameters.  $Q$  and  $\Gamma$  are positive and proportional to  $H$ . Moreover, our approximation relates  $c_b$ , the bulk viscous contribution to the sound speed, to  $Q$  and  $\Gamma$ . The exact value of  $\Gamma$  is fixed by the ratio  $c_b^2/c_s^2$ . Since, according to Eq. (45),  $c_s^2 \ll 1$ , a value  $c_b^2 \approx 1$  is admitted by Eq. (28). This amounts to  $\Gamma \gg H$ , i.e., violent particle production as required for “preheating.” A sufficiently high value of  $\Gamma$  also determines the entropy production. Neglecting second order terms in  $\pi$ , the entropy flow vector is given by  $S^a \approx n s u^a$ , where  $s$  is the entropy per particle. For large  $\Gamma$ , neglecting the change in the entropy per particle, the entropy production density is approximately

$$S_{;a}^a \approx 3 \left( \frac{c_b}{c_s} \right)^2 s H. \quad (46)$$

Equation (42) implies that at the beginning of the reheating, the rate of temperature change is given by

$$\frac{\dot{T}}{T} \approx -2 \left[ 1 - \left( \frac{c_b}{c_s} \right)^2 \right] H. \quad (47)$$

In the initial stage, a very large rate of creation of particles can lead to a growth in the net number density and thus in the temperature. By Eq. (47), this is again equivalent to the effective dissipative contribution  $c_b$  to the sound speed exceeding the adiabatic contribution  $c_s$ :

$$\dot{n} > 0 \Leftrightarrow \dot{T} > 0 \Leftrightarrow c_b > c_s. \quad (48)$$

Equation (47) then implies that initially the temperature rises extremely rapidly (see also [20]). It reaches a maximum,

which we could call the reheating temperature  $T_{\text{reh}}$ , following standard terminology, and then decreases as  $c_b$  falls below  $c_s$ . By Eq. (45), the reheating temperature is given by

$$T_{\text{reh}} \approx \frac{3}{5} c_b^2 m. \quad (49)$$

It follows from Eq. (46) that there is a high entropy production in this regime. Alternatively, this regime may be characterized in terms of the effective bulk pressure (25). For  $\Gamma \geq 3H$ , the condition  $|\pi| \ll p_1 + p_2 = p$  leads generally to

$$\pi_{\text{eff}} \approx -\frac{\Gamma}{3H} p. \quad (50)$$

Using the relations (45), one gets

$$\pi_{\text{eff}} \approx -\left(\frac{c_b}{c_s}\right)^2 p \approx -\frac{3}{5} n m c_b^2 \approx -n T_{\text{reh}}, \quad (51)$$

equivalent to

$$\frac{|\pi_{\text{eff}}|}{p} \approx \frac{T_{\text{reh}}}{T}. \quad (52)$$

For large particle production the condition  $|\pi_{\text{eff}}| \gg p$  is satisfied. The reheating temperature (49) may be understood as the temperature for which  $|\pi_{\text{eff}}| \approx p \approx p_1$ .

### 2. Intermediate regime

Here, the energy densities of both components are comparable, i.e.,  $n_1 m \approx 3n_2 T$  so that  $\gamma \approx \frac{7}{6}$  and consequently, Eq. (41) simplifies to

$$\dot{H} + \frac{7}{4} H^2 \approx 0 \Rightarrow H \approx \frac{4}{7t}. \quad (53)$$

Then, Eq. (40) implies that the decay rate is

$$Q \approx \frac{1}{2} (1 + 42c_b^2) H \Rightarrow \Gamma \approx \frac{3}{8} \left[ 1 + 8 \left( \frac{c_b}{c_s} \right)^2 \right] H, \quad \text{where } c_s^2 \approx \frac{4}{21}. \quad (54)$$

As can be seen from Eq. (42), the temperature behavior is

$$\frac{\dot{T}}{T} \approx -\frac{7}{8} \left[ 1 - \frac{8}{7} \left( \frac{c_b}{c_s} \right)^2 \right] H. \quad (55)$$

After the initial stage in the nonrelativistic regime, the creation of ultrarelativistic particles slows down, while the nonrelativistic particles continue to decay, and we no longer expect that  $\dot{n}$  is positive. Thus,  $\dot{T} < 0$ , although by Eq. (55) the cooling rate is less than the nondissipative case ( $c_b = 0$ ). We expect that the temperature should decrease monotonically after reaching its maximum  $T_{\text{reh}}$ . By Eq. (55), this is the case provided that

$$c_b^2 < \frac{7}{8} c_s^2 \Rightarrow c_b^2 < \frac{1}{6}. \quad (56)$$

We expect that Eq. (56) is easily satisfied in the intermediate regime, after the initial violent rate of creation of particles has passed, and the two-fluid mixture evolves increasingly towards ‘‘normal’’ two-fluid behavior, for which  $c_b \ll 1$  [see Eq. (53) of [12]].

### 3. Ultrarelativistic regime

In this last stage of reheating, the energy density becomes dominated by the radiation fluid, i.e.,  $n_2 T \gg n_1 m$  and  $\gamma \approx \frac{4}{3}$ . Therefore, from Eq. (41) we have

$$\dot{H} + 2H^2 \approx 0 \Rightarrow H \approx \frac{1}{2t}. \quad (57)$$

Then, Eq. (40) implies the decay rate:

$$Q \approx 36c_b^2 \left( \frac{n_2 T}{n_1 m} \right) H \Rightarrow \Gamma \approx 3 \left( \frac{c_b}{c_s} \right)^2 H, \quad \text{where } c_s^2 \approx \frac{1}{3}. \quad (58)$$

The temperature change follows from Eq. (42) as

$$\frac{\dot{T}}{T} \approx -\left[ 1 - \left( \frac{c_b}{c_s} \right)^2 \right] H. \quad (59)$$

As argued above, we expect that  $c_b < c_s$  is easily satisfied, so that the temperature continues to fall, although still at a reduced rate relative to the nondissipative case. Towards the end of reheating the cosmic medium approaches a perfect, relativistic fluid with vanishing  $\zeta$ , so that  $c_b^2$  must tend to zero. For any nonzero  $c_b^2$  the decay rate  $Q$  diverges in the limit  $n_1 m \ll n_2 T$ .

### B. Perturbations of the expansion rate

It is possible, given *a priori* the forms of the relaxation time  $\tau$  and the decay rate  $Q$ , to calculate the perturbations of the expansion rate  $H$  due to causal viscous and reaction effects, via Eq. (40). We illustrate this with a simple model which, in particular, can accommodate oscillations in  $H$  during the initial stage of reheating (compare [21], p. 241).

The simple model is based on the ansatz that  $\tau^{-1}$  and  $Q$  are proportional to the expansion rate in the initial stage of reheating, i.e.,

$$\tau^{-1} = \nu H, \quad Q = \beta H, \quad (60)$$

where  $\nu$  and  $\beta$  are positive constants (with  $\nu > 1$  for a consistent hydrodynamic description). The ansatz for  $Q$  is consistent with Eq. (45) if  $c_b$  is constant. Using Eqs. (60) in Eq. (40) in the nonrelativistic regime, we find that the evolution of the expansion rate is governed by

$$\ddot{H} + (5 + \nu) H \dot{H} + \left[ 3 + \frac{3}{2} \nu + \frac{5}{8} \beta - \frac{9}{2} c_b^2 \right] H^3 \approx 0. \quad (61)$$

Now, we know that

$$H = \frac{2}{3t} + h, \quad \text{where } |h| \ll \frac{2}{3t}. \quad (62)$$

Substituting Eq. (62) into Eq. (61) and linearizing, we find that the perturbation  $h$  is governed by

$$\begin{aligned} \ddot{h} + \left[ \frac{2}{3}(5 + \nu) \right] t^{-1} \dot{h} + \left[ \frac{1}{6}(4 + 8\nu + 5\beta - 36c_b^2) \right] t^{-2} h \\ \approx \left[ \frac{1}{27}(36c_b^2 - 5\beta) \right] t^{-3}. \end{aligned}$$

This is the equation of a forced damped oscillator, as is readily seen after the change of variable to  $s = \ln t$ :

$$\begin{aligned} h'' + \left[ \frac{1}{3}(7 + 2\nu) \right] h' + \left[ \frac{1}{6}(4 + 8\nu + 5\beta - 36c_b^2) \right] h \\ \approx \left[ \frac{1}{27}(36c_b^2 - 5\beta) \right] e^{-s}. \end{aligned} \quad (63)$$

The simplest case is Eq. (45), i.e.,  $5\beta = 36c_b^2$ , for which Eq. (63) leads to the overdamped perturbation

$$h(t) \approx \epsilon_1 t^{-2} + \epsilon_2 t^{-(1+2\nu)/3}. \quad (64)$$

Damped oscillations of  $h$  about the zero order solution  $2/(3t)$  occur when

$$30\beta > 216c_b^2 + (2\nu - 5)^2, \quad (65)$$

which shows that  $c_b$  is a damping factor, while the decay coefficient  $\beta$  contributes to oscillation. A further constraint on the thermodynamic parameters arises from the requirement that the particular integral of Eq. (63) must be small compared to  $2/(3t)$ :

$$|B| \equiv \left| \frac{36c_b^2 - 5\beta}{5\beta - 36c_b^2 + 4\nu - 4} \right| \ll 1. \quad (66)$$

If Eqs. (65) and (66) are satisfied, then the damped oscillatory perturbation is given by

$$h(t) \approx \frac{2}{9} B t^{-1} + t^{-(7+2\nu)/6} [\epsilon_1 \cos(\omega \ln t) + \epsilon_2 \sin(\omega \ln t)], \quad (67)$$

where

$$\omega = \frac{1}{6} [30\beta - 216c_b^2 - (2\nu - 5)^2]^{1/2}$$

is the frequency of oscillation. A simple choice that satisfies Eqs. (65) and (66) is

$$\nu = \frac{5}{2}, \quad \beta = \frac{2}{165} + \frac{36}{5} c_b^2,$$

in which case

$$H \approx \frac{2}{3t} - \frac{1}{450t} + \frac{1}{t^2} \epsilon \exp\left(i \frac{\sqrt{11}}{33} \ln t\right).$$

Finally, we note that the perturbations of the Hubble rate induce perturbations of the temperature via the temperature evolution equation (42). Using Eq. (60) and  $n_1 \gg n_2$ ,  $p_1 + p_2 + \pi \approx n_1 T$ , we find

$$\dot{T} + 2HT \approx \frac{1}{6} \beta m H.$$

Denoting by  $\bar{T}$  the dominant zero order part of the temperature, which is determined by  $\bar{H} \equiv 2/(3t)$ , and by  $\delta T$  the perturbation induced by  $\delta H \equiv h$ , this equation leads to

$$(\delta T)' + 2\bar{H} \delta T \approx \left( \frac{1}{6} \beta m - 2\bar{T} \right) h. \quad (68)$$

Equation (68) determines the temperature perturbation explicitly in terms of the Hubble rate perturbation. Clearly, if  $h$  has an oscillatory component, then so will  $\delta T$ .

## V. CONCLUDING REMARKS

A qualitative analysis based on a thermodynamic two-fluid model subject to causality has produced overall features which are consistent with expectations. The decay rate of nonrelativistic particles implied by causality is positive throughout the reheating process (thus providing a consistency check on our approach), and proportional to the expansion. The overall behavior of the temperature and decay rate are essentially determined by  $c_b$ , the dissipative contribution to the sound speed. Provided that  $c_b$  exceeds the adiabatic contribution  $c_s$  in the beginning of reheating, equivalently, provided that the total number density grows by virtue of superabundant particle creation, the temperature rises very rapidly at the start of reheating. It quickly reaches a maximum value (49), whereafter it falls with expansion, at a reduced rate (determined by  $c_b$ ) relative to the nondissipative case. Large amounts of entropy are generated in the early stage, as shown by Eq. (46). A simplified model (60) allows us to calculate explicitly the decaying perturbations of the Hubble rate, including a case of oscillatory perturbations (67) around matter-dominated behavior.

We have thus shown the capacity of a phenomenological model based on causal relativistic thermodynamics to predict the expected basic features of reheating with economy and simplicity, and independent of detailed knowledge of the interaction. This should be counted as another success of the Israel–Stewart theory (as adapted to deal with interacting fluids), and another argument in favor of the theory with its causality and stability properties.

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- [1] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990).
- [2] L. Kofman, A. Linde, and A. A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994).
- [3] D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee, and A. Singh, *Phys. Rev. D* **51**, 4419 (1995).
- [4] Y. Shtanov, J. Traschen, and R. Brandenberger, *Phys. Rev. D* **51**, 5438 (1995).
- [5] M. Yoshimura, *Prog. Theor. Phys.* **94**, 873 (1995).
- [6] E. Calzetta and B. L. Hu, *Phys. Rev. D* **52**, 6770 (1995).
- [7] L. Kofman, A. Linde, and A. A. Starobinsky, *Phys. Rev. Lett.* **76**, 1011 (1996).
- [8] D. Boyanovsky, M. D’Attanasio, H. J. de Vega, and R. Holman, *Phys. Rev. D* **54**, 1748 (1996).
- [9] D. I. Kaiser, *Phys. Rev. D* **53**, 1776 (1996).
- [10] H. Fujisaki, K. Kumekawa, M. Yamaguchi, and M. Yoshimura, *Phys. Rev. D* **53**, 6805 (1996).
- [11] N. Udey and W. Israel, *Mon. Not. R. Astron. Soc.* **199**, 1137 (1982).
- [12] W. Zimdahl, *Mon. Not. R. Astron. Soc.* **280**, 1239 (1996).
- [13] W. Zimdahl, *Mon. Not. R. Astron. Soc.* (to be published).
- [14] W. Israel, *Ann. Phys. (N.Y.)* **100**, 310 (1976); W. Israel and J. M. Stewart, *ibid.* **118**, 341 (1979); D. Pavón, D. Jou, and J. Casas-Vázquez, *Ann. Inst. Henri Poincaré, A* **36**, 79 (1982).
- [15] W. Hiscock and L. Lindblom, *Ann. Phys. (N.Y.)* **151**, 466 (1983); *Phys. Rev. D* **35**, 3723 (1987).
- [16] V. Romano and D. Pavón, *Phys. Rev. D* **50**, 2572 (1994).
- [17] R. Maartens, *Class. Quantum Grav.* **12**, 1455 (1995).
- [18] W. Zimdahl, *Phys. Rev. D* **53**, 5483 (1996).
- [19] R. Maartens, “Causal Thermodynamics in Relativity, Hanno Rund Workshop Proceedings,” Report No. astro-ph/9609119 (unpublished).
- [20] W. Zimdahl and D. Pavón, *Mon. Not. R. Astron. Soc.* **266**, 872 (1994).
- [21] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, *Phys. Rep.* **215**, 203 (1992).