Testing the equivalence principle through freely falling quantum objects

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Free fall in a uniform gravitational field is reexamined in the case of quantum states with and without a classical analogue. The interplay between kinematics and dynamics in the evolution of a falling quantum test particle is discussed allowing for a better understanding of the equivalence principle at the operational level. [S0556-2821(97)04302-6]

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I. INTRODUCTION

Gravity appears to be distinguished from all the other fundamental interactions by the remarkable feature of affecting all bodies in a universal way, regardless of their internal composition and mass. This fact, which requires in the Newtonian picture the gravitational force to be exactly proportional to the inertial mass, represents the physical cornerstone of Einstein's weak equivalence principle, establishing the local identification of gravity and acceleration as far as mechanical effects are concerned. In the famous gedanken experiment conceived by Galileo, the universality of the ratio between gravitational and inertial masses has been studied by imagining test bodies in free fall from the tower of Pisa [1]. Since that time several actual tests of the weak equivalence principle have been performed with very sensitive schemes, such as the ones exploiting pendula or torsion balances [2], but only classical test bodies have been involved. On the other hand, any attempt to merge quantum mechanics and gravity on an operational basis should start from the possibility of establishing the properties of the latter by using a generic body as a probe, regardless of its macroscopicity and hopefully without reference to classical physics. This path of reasoning leads to conceptual difficulties clearly focused, among the others, by Penrose when he writes: ''We see that this view of reality is very different from the one that we have become accustomed to from classical physics, where particles can be only in one place at a time, where the physics is local (except for action at a distance) and where each particle is a separate individual object which, when it is in free flight, can be considered in isolation from any other particle. All these classical conceptions must be overturned once we accept the reality of the state-vector. It is perhaps little wonder that most people are reluctant to do this'' [3]. As a first step along this direction, it is therefore natural to ask what happens if the Pisa gedanken experiment is repeated using properly prepared quantum test particles. This question, besides its above-mentioned conceptual importance, is also relevant in view of recent attempts and proposals to investigate gravitation using microscopic and mesoscopic systems, such as antimatter in free fall [4–6], cooled atoms in optical molasses [7] and opto-gravitational cavities [8,9], and interfering matter waves in both curved space-time and accelerated reference frames [10,11]. The content of the paper is organized as follows. In Sec. II, after some remarks on the preparation of the initial state for a Galileo experiment in the classical case, a similar prescription is discussed for all the quantum states mapped, in the macroscopic limit, into classical states. In Sec. III the peculiar case of quantum states without a classical analogue is dealt with, with a detailed analysis of the simplest class of Schrödinger cat states in the configurational space. In Sec. IV the two lowest momenta of the time-of-flight distributions for quantum states in free fall in a uniform gravitational field are evaluated. The dynamical effect of a continuous quantum measurement of position on the geodesic motion is dealt with in Sec. V. In Sec. VI some phenomenological consequences of the previous considerations are presented, with particular emphasis on the possibility of testing gravity with mesoscopic objects. Some general comments on the definitions of the weak equivalence principle which still hold in the quantum case are finally discussed.

II. GALILEIAN PREPARATION FOR CLASSICAL-LIKE STATES

In order to perform an ideal free fall experiment for two quantum particles having inertial masses $m_i^{(1)}$ and $m_i^{(2)}$, $m_i^{(1)} \neq m_i^{(2)}$, we have first of all to specify a proper initial preparation in such a way that any difference in the motion during the free fall must be ascribed to the effect of gravity. By recalling that within the classical Hamilton picture the Galileian prescription for initial positions and velocities fixes the ratio between the initial momenta in a well-defined way, $p_0^{(1)}/p_0^{(2)} = m_i^{(1)}/m_i^{(2)}$ it is natural to extend such a prescription to the quantum case, which can be also represented through a Hamiltonian scheme. Of course, the Heisenberg uncertainty principle prevents us from simultaneously defining, for each particle, initial position and momentum. If $|\psi_1\rangle$ and $|\psi_2\rangle$ denote the initial state vectors for particles 1 and 2 in the Schrödinger picture, the classical recipe can be reasonably rephrased by imposing the conditions

$$\langle \hat{z} \rangle_{\psi_1} = \langle \hat{z} \rangle_{\psi_2}, \qquad \frac{\langle \hat{p}_z \rangle_{\psi_1}}{m_i^{(1)}} = \frac{\langle \hat{p}_z \rangle_{\psi_2}}{m_i^{(2)}}, \tag{1}$$

where for simplicity we have restricted ourselves to a one-dimensional representation along the vertical z direction and $\langle \hat{z} \rangle_\psi$ and $\langle \hat{p}_z \rangle_\psi$ denote, as usual, the expectation values for position and momentum operators, respectively. Furthermore, our description will be confined to the motion of nonrelativistic quantum particles, implying values of the initial velocities in Eqs. (1) to be small compared to the speed of light. From the mathematical viewpoint, Eqs. (1) impose a relative constraint on the average values of the position and momentum probability distributions associated with the states $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. Some other remarks are in order. First, leaving aside the special choice of starting from a given position at rest ($\langle \hat{p}_z\rangle_{\psi_1}=\langle \hat{p}_z\rangle_{\psi_2}=0$), we are forced in general to deal with two different initial states $|\psi_1\rangle$ and $|\psi_2\rangle$ in the single particle Hilbert space; this is analogous to the classical situation, where the representative point of the classical initial state in the phase space is different for the two masses. Second, deep differences between the concepts of classical and quantum state exist, which also produce differences in the corresponding prepared systems. The probabilistic interpretation underlying quantum mechanics will allow us to speak of probability distributions, for instance, characterized by *mean* initial conditions such as Eqs. (1), as opposed to the well-defined values for the relevant classical observables. Moreover, as a striking difference with respect to the classical case, conditions (1) are far from univocally determining the initial state of the two particles. On the contrary, the Galileian prescription gives rise to equivalence classes of states in the Hilbert space, each class possessing a defined mean position and velocity and collapsing in the classical limit into a state having both quantities sharply defined. It is also rewarding to point out that this line of reasoning, being ultimately motivated by the correspondence principle, applies in a strict sense to all quantum states for which a classical interpretation is obtainable. Such a class of states does not exhaust the totality of the admissible ones in the Hilbert space. For all the other states, namely, the family of genuinely quantum states for which no classical counterpart exists and the correspondence principle cannot be invoked, the Galileian prescriptions (1) have to be *postulated* and deserve therefore a more careful discussion.

III. CASE OF STATES HAVING NO CLASSICAL ANALOGUE

Because of the superposition principle, states of an intrinsically quantum nature arise even starting from states which in the classical limit corresponds to macroscopically distinguishable ones. Let $|\psi_n\rangle$, $n=1,\ldots,N$, denote a set of states in the Hilbert space of a given quantum system, and let us suppose these states to be macroscopically distinct. Any superposition state $|\psi_0\rangle=\Sigma_n c_n|\psi_n\rangle$ is also permitted, the complex coefficients c_n ensuring the overall normalization. The nonclassical nature of this superposition state can be made explicit by considering the density matrix representation

$$\hat{\rho}=|\psi_0\rangle\langle\psi_0|=\sum_n |c_n|^2|\psi_n\rangle\langle\psi_n|+\sum_{n,m\neq n} c_n^* c_m|\psi_n\rangle\langle\psi_m|, \tag{2}$$

the off-diagonal terms being responsible for correlations of a purely quantum-mechanical origin. In the classical limit, due to the action of decoherence mechanisms, interference effects are lost and the pure state description (2) becomes identical to the statistical mixture characterized by the diagonal probability weights $|c_n|^2$ alone. Given the superposition state $|\psi_0\rangle$, the separation indicated in Eq. (2) between diagonal and off-diagonal contributions also reflects on observable properties of the system, such as average values of Hermitian operators. For a generic observable $\hat{\mathcal{O}}$, one can think its overall mean value in the state $|\psi_0\rangle$ as formed, according to Eq. (2), by two distinct terms:

$$\langle\hat{\mathcal{O}}\rangle_{\psi_0}=\langle\hat{\mathcal{O}}\rangle_{\text{classical}}+\langle\hat{\mathcal{O}}\rangle_{\text{purely quantum}}. \tag{3}$$

If only nondiagonal entries are different from zero in Eq. (3) ($\langle\psi_n|\hat{\mathcal{O}}|\psi_n\rangle=0$ for all n), the mean value of the observable $\hat{\mathcal{O}}$ has only contributions of intrinsically quantum-mechanical origin.

Coming back to the free fall problem, the previous considerations apply if a nonclassical superposition state is selected as initial state for one or both test particles $m_i^{(1)}$ and $m_i^{(2)}$, the decomposition (3) holding in this case for the position and momentum operators $\hat{z}$ and $\hat{p}_z$ involved in Eq. (1). Despite the fact that purely quantum expectation values may emerge, compatibility with the classical limit is again maintained provided that the Galileian conditions (1) are satisfied, in the sense that both states are mapped into initial configurations having the same (classical) position and the same (classical) velocity.

As is well known, a nice class of quantum states without classical counterpart is offered by the so-called Schrödinger cat states, first introduced in [12]. Let us consider the coherent superposition of two macroscopically distinguishable states in the configurational space, represented by a wave function of the form

$$\psi_0(z)=N\left\{c_+\exp\left(-\frac{(z-z_0+\Delta)^2}{2\Delta_0^2}\right)+c_-\exp\left(-\frac{(z-z_0-\Delta)^2}{2\Delta_0^2}\right)\right\}, \tag{4}$$

consisting of the sum of two Gaussian peaks at $z=z_0\pm\Delta$, $\Delta>0$, each of width Δ_0. Here $c_\pm$ are two complex coefficients, for which we denote by ϑ the relative phase, and N is a normalization constant determined (up to an irrelevant phase factor) by

$$|N|^2=(\pi\Delta_0^2)^{-1/2}\{|c_+|^2+|c_-|^2+2\,\mathrm{Re}(c_+c_-^*)\times\exp(-\Delta^2/\Delta_0^2)\}^{-1}. \tag{5}$$

For null separation ($\Delta=0$) the special case of a normalized Gaussian wave packet with vanishing average momentum is recovered. The expectation values of position and momentum in the state (4) are calculated, obtaining

$$\langle\hat{z}\rangle_{\psi_0}=z_0-\Delta\frac{|c_+|^2-|c_-|^2}{|c_+|^2+|c_-|^2+2\,\mathrm{Re}(c_+c_-^*)\exp(-\Delta^2/\Delta_0^2)}, \tag{6}$$

$$\langle\hat{p}_z\rangle_{\psi_0}=-2\hbar\frac{\Delta}{\Delta_0^2}\frac{\mathrm{Im}(c_+c_-^*)\exp(-\Delta^2/\Delta_0^2)}{|c_+|^2+|c_-|^2+2\,\mathrm{Re}(c_+c_-^*)\exp(-\Delta^2/\Delta_0^2)}. \tag{7}$$

The average position is simply the center of mass of the cat, weighted by the asymmetry between the coefficients $|c_+|^2$ and $|c_-|^2$. By Fourier transforming Eq. (4), it is seen that no diagonal momentum contributions are present, leading to a purely quantum momentum in Eq. (7) with the form of a typical interference factor, $\mathrm{Im}(c_+ c_-^*) = |c_+||c_-|\sin\vartheta$. A vanishing value of $\langle \hat{p}_z \rangle_{\psi_0}$ is found if the relative phase $\vartheta = k\pi$, $k \in Z$, corresponding to the even (male) and odd (female) combinations of definite parity cat states [13]; a maximum contribution is instead achieved at $\vartheta = (2k+1)\pi/2$, $k \in Z$, corresponding to cat wave functions already introduced by Yurke and Stoler [14]. In order to study the free fall motion of two catlike quantum particles, one has to match prescriptions (1) by suitably gauging the parameters of the wave function (4), which is, at least in principle, possible. Let us suppose for definiteness to maintain the width Δ_0 fixed and $|c_+| = |c_-|$ in such a way that the initial average position (6) is constrained to be z_0. The simplest choice is clearly represented by parity eigenstates, i.e., by even or odd cat states necessarily starting at rest. If, however, nonvanishing average momenta are present, one can exploit the possibility to tune the separation $\Delta_{1,2}$ between the peaks, with ϑ fixed, i.e.,

$$\frac{\Delta_2}{\Delta_1} \exp\left(- \frac{\Delta_2^2 - \Delta_1^2}{\Delta_0^2} \right) \frac{1 + \cos\vartheta \exp(-\Delta_1^2/\Delta_0^2)}{1 + \cos\vartheta \exp(-\Delta_2^2/\Delta_0^2)} = \frac{m_i^{(2)}}{m_i^{(1)}}, \quad (8)$$

or the relative quantum phase $\vartheta_{1,2}$, with Δ fixed, i.e.,

$$\frac{\sin\vartheta_2 [1 + \exp(-\Delta^2/\Delta_0^2)\cos\vartheta_2]}{\sin\vartheta_1 [1 + \exp(-\Delta^2/\Delta_0^2)\cos\vartheta_1]} = \frac{m_i^{(2)}}{m_i^{(1)}}. \quad (9)$$

Having assigned the initial preparation of each particle, we will analyze in the following two sections the evolution of the system during the free fall as, respectively, predicted by ordinary quantum mechanics for closed systems and by a more general description also including the effect of a measuring apparatus.

IV. RUNNING THE GEDANKEN EXPERIMENT: THE UNMEASURED EVOLUTION

In the nonrelativistic picture we are considering, the time evolution is generated by the Hamiltonian $\hat{H} = \hat{p}_z^2/2m_i + m_g g \hat{z}$, m_g denoting the gravitational mass. In analogy with Galileo's procedure let us focus, for instance, on the time of flight of the quantum particles. Because of the already remarked fuzziness of quantum states, even if the measurements are treated as ideal, allowing us to estimate all the measured quantities with infinite precision, these times can be only predicted in a probabilistic way. Given our particles with inertial masses $m_i^{(1)}$ and $m_i^{(2)}$, the experiment consists in recording two time-of-flight distributions corresponding to the time of arrival from the initial height $z_0 = \langle \hat{z} \rangle_{\psi_0}$, at variance with the classical case where two single time values are sampled. Therefore, the full knowledge of such time-of-flight distributions can in principle be demanded to gain a complete information on the free fall dynamics. To the best of our knowledge, no general consensus has been reached so far on a consistent definition of the time-of-flight probability density. In the probabilistic language, the question is related to the so-called first passage time problem, which has been solved within the natural framework offered by stochastic mechanics [15] in the special case of stationary states [16]. Various attempts in different directions have been made, as indicated in a recent work on the subject where an operatorial solution is proposed [17]. Leaving aside a rigorous derivation which is not essential to sketch our main line of reasoning, we will now limit ourselves to simple arguments. The average time of flight for the test mass $m_i^{(k)}$ can by straightforwardly calculated by means of the Ehrenfest theorem which, owing to the linearity of the gravitational potential, allows us to obtain the average position at time t in the classical form

$$\langle z^{(k)}(t) \rangle = -\frac{1}{2} \frac{m_g^{(k)}}{m_i^{(k)}} g t^2 + \frac{\langle \hat{p}_z \rangle_{\psi_0}}{m_i^{(k)}} t + \langle \hat{z} \rangle_{\psi_0}, \quad k = 1,2. \quad (10)$$

By setting $\langle z^{(k)}(t) \rangle = 0$, the mean time of flight at ground level is obtained; in the particular case $\langle \hat{p}_z \rangle_{\psi_0} = 0$, the corresponding expression is

$$T_{\mathrm{of}}^{(k)} = \sqrt{2 \left(\frac{m_i^{(k)}}{m_g^{(k)}} \right) \frac{z_0}{g}}, \quad k = 1,2. \quad (11)$$

A rough estimate of the fluctuations around this mean value, taking into account the spreading of the state during the motion, can be given by evaluating $\sigma_{T_{\mathrm{of}}} \approx \sigma_z(T_{\mathrm{of}})/v_z(T_{\mathrm{of}})$, where $\sigma_z^2(T_{\mathrm{of}})$ and $v_z(T_{\mathrm{of}})$ are the position variance of the state and the average velocity at time $t = T_{\mathrm{of}}$, respectively. By exploiting Eq. (10), this may be shown to be equivalent to another possible definition of $\sigma_{T_{\mathrm{of}}}$, resulting from the semidifference between the times t_+ and t_- for which $\langle z(t_\pm) \rangle \pm \sigma_z(t_\pm) = 0$. Let us consider in detail the behavior of our quantum probes $m_i^{(1)}$ and $m_i^{(2)}$, when each of them is allowed to be initially prepared with average position z_0 and vanishing average momentum in the form of a Gaussian or a catlike state (this latter necessarily possessing a well-defined parity). The general expression of the position variance as a function of time, $\sigma_z^2(t) = \langle \hat{z}^2 \rangle_{\psi_t} - \langle \hat{z} \rangle_{\psi_t}^2$, has been calculated by following the Schrödinger evolution having the wave function (4) as initial condition (see Appendix A for details). The result of the calculation can be written as

$$\sigma_z(t) = \sqrt{\frac{\Delta_0^2}{2} + \frac{\Delta^2}{1 \pm e^{-\Delta^2/\Delta_0^2}} + \frac{\hbar^2}{2 m_i^2 \Delta_0^2} \left(1 \mp 2 \frac{\Delta^2}{\Delta_0^2} \frac{e^{-\Delta^2/\Delta_0^2}}{1 \pm e^{-\Delta^2/\Delta_0^2}} \right) t^2}, \quad (12)$$

the upper and lower signs referring to the even (male, $c_+ = c_- = 1$) and odd (female, $c_+ = -c_- = 1$) cat states, respectively. As we will discuss in more detail elsewhere [18], the rate of spreading for a cat state may become less than the Gaussian one. Equation (12) reduces to the familiar formula for the spreading of Gaussian wave packets if $\Delta = 0$, while slight modifications to the Gaussian case are obtained in the limit $\Delta/\Delta_0 \gg 1$. Let us then consider an intermediate regime with $\Delta \approx \Delta_0$, moreover assuming Δ_0 small enough to become negligible with respect to the evolution induced contribution in the total spreading $\sigma_z(t)$. One can thus approximate Eq. (12) by the leading time-dependent term and collect the available information concerning the time-of-flight distribution in the form of average time and standard deviation as

$$T_{\rm of}\pm\sigma_{T_{\rm of}}=\sqrt{2\left(\frac{m_i^{(k)}}{m_g^{(k)}}\right)\frac{z_0}{g}}\pm\sqrt{\frac{2}{2}}\epsilon\frac{\hbar}{\Delta_0 m_g^{(k)}g},\qquad k=1,2, \tag{13}$$

where $k=1,2$ distinguishes, as usual, the two particles and ϵ is a numerical factor given by

$$\epsilon=\begin{cases}1 & \text{Gaussian state,}\\ \left(\dfrac{e-1}{e+1}\right)^{\pm 1/2} & \text{male}(+)/\text{female}(-)\text{cat state.}\end{cases} \tag{14}$$

Equation (13) can in fact be used with a general validity, provided the final time in Eq. (12) is long enough, which can in turn be obtained by properly adjusting the initial height. It is manifest from Eq. (13) that, despite the semiclassical preparation formulas (1), the time-of-flight distributions corresponding to different quantum objects are different, due either to different masses or different initial states. As an interesting feature, the ratio between inertial and gravitational mass contributes to the average time of flight, whereas the fluctuations around this value are affected, at this stage, by the *gravitational* mass alone.

Since Eq. (11) holds, a necessary and sufficient condition in order to conclude

$$\frac{m_i^{(1)}}{m_g^{(1)}}=\frac{m_i^{(2)}}{m_g^{(2)}}=\text{const} \tag{15}$$

is to observe, in close analogy with the classical experiment, identical values for the average times of flight. Let us suppose that the equality (15) has been established for two particles initially prepared in the same type of state, for instance, a Gaussian one. At first sight one may wonder that, at variance with the classical behavior, a dependence upon the mass survives in time-of-flight spreading, allowing one to distinguish again the resulting distribution patterns. At closer inspection, the role and nature of the mass dependence can be understood on the basis of kinematical arguments. Whatever such a dependence will be, the crucial point is that it can be recovered from an equivalent problem where it has a completely kinematical origin. Let us imagine a laboratory in which a test particle of mass m_i is initially placed on the top, at a height z_0 above the floor, with the laboratory being then propelled at uniform acceleration $\vec{a}$ with respect to an inertial reference frame, $\vec{a}$ directed upward. The motion of the particle, which is supposed to freely evolve in the noninertial frame, can be derived from the inertial free motion by a standard procedure, as outlined in Appendix B. Provided that equality (15) is satisfied and $a=g$, formally identical evolution equations are thus found either for the motion of a quantum particle subjected to a uniform gravitational field with strength g in a inertial reference or for the motion of the same particle freely evolving in a noninertial reference accelerated with $a=g$. In particular, an observer inside the accelerating laboratory should see the particle hitting the ground level after the same average time as in Eq. (13) but with a variance proportional to the inertial mass, the only property of the body available in this case. More generally, once Eq. (15) is fulfilled, the theory shows a complete identification between the effects of gravitation and acceleration, predicting in particular that time-of-flight probability distributions of identical form, with identical mass dependence in *every* momentum, not just in the second one as here considered, are detected if the motion is performed in the gravitational or accelerated laboratory.

By summarizing, the widespread quoted sentence according to which all bodies equivalently prepared fall precisely the same way in a gravitational field has to be carefully interpreted when quantum objects are considered. Unlike the classical case, this does not imply that only mass-independent observables are found. On the contrary, the time-of-flight probability distributions of quantum particles remain mass dependent, but such a dependence is *expected in order that the weak equivalence principle be preserved at the quantum level*.

V. MEASURED EVOLUTION

Until now, the discussion has been carried out without considering the effect of the measurement apparatus. Indeed, detection schemes can be designed in which each falling atom impulsively interacts with the meter just before stopping its evolution, that is, at the arrival time. However, experimental situations involving a continuous monitoring of the atom throughout the whole free fall can be investigated (see [19] for recent proposals) and the perturbation introduced by the meter cannot be reduced without a parallel limitation on the extracted information also resulting. Such an influence has then in principle to be taken into account, as we are going to discuss in this section. Various models have been designed to include the effect of a continuous measurement process into the dynamics, the so-called measurement quantum mechanics, an account of which can be found, for instance, in [20]. In the so-called nonselective approach, i.e., when no particular history of measurement is selected, the evolution of the system under a continuous measurement of position $\hat{z}$ is described through a master equation for the reduced density matrix operator:

$$\frac{d}{dt}\hat{\rho}(t)=-\frac{i}{\hbar}[\hat{H},\hat{\rho}]-\frac{\kappa_z}{2}[\hat{z},[\hat{z},\hat{\rho}]], \tag{16}$$

$\hat{H}$ being the Hamiltonian of the system, $[,]$ the commutator, and κ_z expressing [in $(m^2/\text{Hz})^{-1}$] the coupling of the position meter to the test particle. Hereafter, κ_z is assumed to be time dependent. Equation (16) for the system under consideration can be rewritten as

$$\frac{\partial}{\partial t}\rho(z,z',t)=\left\{\frac{i\hbar}{2m}\left(\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial z'^2}\right)-\frac{i m_g g}{\hbar}(z-z')-\frac{\kappa_z}{2}(z-z')^2\right\}\rho(z,z',t), \tag{17}$$

with $\rho(z,z',t)=\langle z|\hat{\rho}(t)|z'\rangle$ denoting the coordinate representation of the density operator. Besides some numerical

TABLE I. Predicted standard deviations for the times of flight (in msec) of freely falling Gaussian states and Schrödinger cat (male, $c_+ = c_- = 1$ and female, $c_+ = -c_- = 1$) states, starting at rest from an height $z_0 = 3$ mm. The corresponding average time of flight is $T_{\mathrm{of}} = 24.74$ ms and the values $\Delta = \Delta_0 = 100$ nm have been chosen.

	Gaussian state	Male cat state	Female cat state
He [a]	10.86	7.38	15.98
Be	4.82	3.28	7.09
Na	1.89	1.29	2.78
Rb	0.36	0.24	0.53
Cs	0.33	0.22	0.48

[a] ^3S metastable helium.

factor, the coupling constant κ_z is equal to the reciprocal of the position noise spectral density. If this last is decreased, i.e., the position sensitivity is increased, the last term in the right-hand side of Eq. (17) will dominate over the others. As remarked above, under this assumption the considerations made in the previous section can be affected by the measurement process and have to be reanalyzed in detail. A straightforward but tedious calculation (see Appendix A for more details) allows one to explicitly solve the master equation (17) when the initial wave function has a Gaussian or catlike form. In particular, it is possible to calculate the average position and its variance versus time. It turns out that, while the former is left unchanged with respect to the unmeasured case (10), the position variance is modified by the measurement coupling, the net effect being represented by an additive time-dependent contribution of the form

$$\sigma_z^2(t;\kappa_z) = \sigma_z^2(t;\kappa_z = 0) + \frac{\kappa_z}{3}\left(\frac{\hbar}{m_i}\right)^2 t^3, \tag{18}$$

with $\sigma_z^2(t;\kappa_z = 0)$ given in Eq. (12). It is worth observing that an additional mass dependence is obtained in this case. Despite the fact that such a dependence is not different from the unmeasured one, the ratio $\hbar/m_i$ still appearing, an explanation analogous to the one delineated at the end of the previous section requires care, due to the physical meaning of the coupling κ_z. In order to state the problem in an accelerated frame some additional assumptions on the behavior of such a parameter are required. Since in general the coupling between the meter and the test mass is not a purely mechanical one, the question can be properly addressed only by *postulating* the validity of the strong equivalence principle.

The previously noted lack of effect on the average position is interpreted as a manifestation of the Ehrenfest theorem for potentials written as polinomials up to the third power of distance [21]. However, for motions in more complicated gravitational fields, the last term in the right-hand side of Eq. (17) can create differences in the average position of particles having different masses, something like a *gravitational* quantum Zeno effect. This would be, on the other hand, the signal of a contrast between a body at the same time being in free fall and having its position continuously registered via a meter interacting with it. Tests of the equivalence principle, although still viable, become more complicated since disentanglement of the effect of the meter from the purely gravitational one is required. This problem is also present in the case of other tests of the equivalence principle such the ones exploiting rigid objects. For such configurations additional complications arise from the difficulty to achieve the quantum domain in macroscopic bodies [22].

VI. PHENOMENOLOGY

Recent progress in the manipulation of atomic states gives hope to make the considerations presented here less remote from experimental investigation than expected. Atomic mixed states with a Gaussian phase space distribution, although far from the minimum uncertainty value of the pure state configuration, have been already prepared and used to study the free fall of atoms in a semiclassical regime, as reported in [8]. A cloud of cesium atoms was trapped, cooled at a temperature of a few μK, and then released at an average height of 2.91 mm above an atomic mirror made by a dielectric surface and a repulsive evanescent field. Various bounces of the cloud were observed, and the time of flight was measured. In a successive experiment, a vibrating mirror was used to show phase modulation of atomic waves. In this case, an accurate study of the time of flight was reported, showing that resolutions of the order of 0.5 ms can be achieved [23]. On the other hand, preparation of even and odd superposition states has been proposed [24], and Schrödinger cats of a single trapped Be^+ ion with a separation $\Delta \approx 10^2$ nm have been recently generated and detected in laboratory [25]. Merging these accomplishments, an experiment in which a Gaussian or a Schrödinger cat state of matter at the single atom level is bouncing over an atomic mirror can be envisaged. Some numerical values, evaluated from Eq. (13) for different atomic species routinely manipulated in laboratory, are reported in Table I, allowing us to clarify the orders of magnitude involved in a possible experimental test. The already achieved time-of-flight resolution quoted in [23] allows one to observe different time-of-flight distributions either due to different masses (within the same column) or to different states (within the same row). Experiments of this kind should also stimulate further thoughts on consistent definitions of the time-of-flight distributions in the quantum domain. Moreover, similar experiments should be performed by actually observing the time-of-flight distributions in accelerated frames, for instance, exploiting centrifugal force fields, to verify if the weak equivalence principle, as supposed in our discussion, still holds in the quantum realm. In performing such experiments, one should be prepared to possible surprises, since no evidence is so far available that nature preserves the equivalence principle at the quantum level.

VII. DISCUSSIONS AND CONCLUSIONS

Besides the experimental feasibility, some conceptual observations concerning the interplay between quantum mechanics and gravitation are in order. First of all, the presence of the mass in the time-of-flight distributions cannot be ascribed to the fact that the test objects possess, in some sense, an extended structure. It is indeed apparent from Eqs. (12) and (13) that, due to the uncertainty principle, a further increase in the spreading is found if more pointlike structures (i.e., smaller Δ_0 values) are allowed. Such a dependence can be instead deeply related to the impossibility of reproducing, for any quantum object, the classical concept of a *deterministic* trajectory. It may be helpful to recall that in Nelson's picture of quantum mechanics [15] the kinematics is modelized in terms of *stochastic* trajectories in the configuration space. Within this framework, it is not surprising that the combination $\hbar/m$ ultimately appears in Eq. (13), which is nothing but the Brownian diffusion coefficient accounting for the degree of stochasticity of quantum kinematics as opposite to the deterministic classical one. As a related question, which is preexisting to the introduction of the gravitational field itself, this unavoidable stochasticity introduces troubles if the same procedure of classical relativity is adopted to operatively define geodesics curves and associated inertial frames. In the quantum case, the possibility of a simple identification between the world lines of freely falling bodies and a set of preferred entities with a purely geometric nature clearly no longer holds. We refer to [26,27] for a detailed account on the definition of reference frames by means of material quantum objects as a preliminary step toward quantum gravity; see also [28] for an attempt to give a variational definition of a quantum geodesic within a relativistic stochastic scheme.

In summary, we have discussed a revival of the Galileo free fall experiment using quantum test objects. Both the initial preparation and the dynamical evolution have been analyzed with special care to states of intrinsically quantum nature. It turns out that, despite the possibility of *weighting* different quantum objects by looking at their free fall evolution, a complete identification between the effects of gravitation and acceleration is expected in agreement with the equivalence principle. Some troubles may instead emerge by including a continuous measurement process, which demands for a reformulation of the concept of *free* fall itself. It is not unlikely that an operative definition of the equivalence principle consistent with quantum measurement theory will require the emergence of new concepts in gravitation [3].

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APPENDIX A: SOLUTION OF THE MASTER EQUATION WITH MEASUREMENT COUPLING

The master equation (17) can be conveniently rewritten in terms of new independent variables u and v defined as

$$u=\frac{z+z'}{2}, \quad v=z-z', \tag{A1}$$

obtaining

$$\frac{\partial}{\partial t}\rho(u,v,t)=\left\{2i\mu\frac{\partial^2}{\partial u\partial v}-i\nu v-\frac{\kappa_z}{2}v^2\right\}\rho(u,v,t), \tag{A2}$$

with $\mu=\hbar/2m_i$ and $\nu=m_g g/\hbar$. By performing the Fourier transform with respect to the variable u,

$$\widetilde{\rho}(\alpha,v,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{-i\alpha u}\rho(u,v,t)du, \tag{A3}$$

we get from Eq. (A2) the equation

$$\frac{\partial}{\partial t}\widetilde{\rho}(\alpha,v,t)=\left\{-2\mu\alpha\frac{\partial}{\partial v}-i\nu v-\frac{\kappa_z}{2}v^2\right\}\widetilde{\rho}(\alpha,v,t). \tag{A4}$$

The solution of Eq. (A4) is

$$\widetilde{\rho}(\alpha,v,t)=\widetilde{\rho}_0(\alpha,v-2\mu\alpha t)\exp\{i\nu(\mu\alpha t^2-vt)-\kappa_z v^2 t+2\mu\alpha\kappa_z v t^2-\tfrac{4}{3}\kappa_z\mu^2\alpha^2 t^3\}, \tag{A5}$$

$\widetilde{\rho}_0(\alpha,v)$ denoting the Fourier transform of the initial density matrix $\rho_0(u,v)$. The average position and the average square position at time t are, respectively, given by

$$\langle\hat{z}(t)\rangle=\int dz z\rho(z,z,t), \quad \langle\hat{z}^2(t)\rangle=\int dz z^2\rho(z,z,t). \tag{A6}$$

In the case the initial density matrix $\rho_0(z,z')=\psi_0(z)\psi_0(z')^*$ arises from a Gaussian or a catlike wave function ψ_0 as considered in Sec. III, the Fourier transform (A5) can be explicitly inverted and the above quantities analytically evaluated. The Ehrenfest expression (10) for the average position is then recovered, whereas the complete expression for the position variance is

$$\begin{aligned}\sigma_z^2(t)=&2A(t)+\frac{\Delta^2}{B}\left[|c_+|^2+|c_-|^2-\frac{1}{B}(|c_+|^2-|c_-|^2)^2\right]\\&-\frac{8\hbar}{B^2m_i}\left[(|c_+|^2-|c_-|^2)\mathrm{Im}(c_+c_-^*)\frac{\Delta^2}{\Delta_0^2}e^{-\Delta^2/\Delta_0^2}\right]t\\&-\frac{1}{B}\left[\frac{2\hbar^2}{m_i^2}\mathrm{Re}(c_+c_-^*)\frac{\Delta^2}{\Delta_0^4}e^{-\Delta^2/\Delta_0^2}\right.\\&\left.+\frac{4\hbar^2}{Bm_i^2}\mathrm{Im}^2(c_+c_-^*)\frac{\Delta^2}{\Delta_0^4}e^{-2\Delta^2/\Delta_0^2}\right]t^2,\end{aligned} \tag{A7}$$

having denoted $A(t)=\Delta_0^2/4+\hbar^2/(4m_i^2\Delta_0^2)t^2+\hbar^2/(6m_i^2)\kappa_z t^3$ and $B=|c_+|^2+|c_-|^2+2\mathrm{Re}(c_+c_-^*)\exp(-\Delta^2/\Delta_0^2)$, respectively. The unmeasured evolution is obtained when $\kappa_z=0$, while the evolution of a Gaussian state corresponds to the choice $\Delta=0$.

APPENDIX B: NONRELATIVISTIC SCHRÖDINGER EVOLUTION IN ACCELERATED FRAMES

Let us consider a free nonrelativistic quantum particle, satisfying the one-dimensional Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t}(z,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial z^2}(z,t), \tag{B1}$$

and let us introduce an accelerate frame of reference by means of the coordinate transformation

$$z' = z - vt - \tfrac{1}{2}at^2, \qquad t' = t, \tag{B2}$$

v and a being constant. As a consequence of Eqs. (B2), a corresponding transformation on the space of states will be induced, mapping the wave function $\psi(z,t) \to \psi'(z',t')$. We represent such a transformation via the ansatz

$$\psi'(z',t') = e^{if(z',t')}\psi(z(z',t'),t(z',t')), \tag{B3}$$

where the real function $f(z',t')$ has been introduced to allow the possibility of a local phase factor and $(z(z',t'),t(z',t'))$ denotes the inverse transformation, obtainable from Eq. (B2) by letting $v \mapsto -v$, $a \mapsto -a$. The equation of motion satisfied by the transformed wave function (B3) with a generic f can be straightforwardly obtained from Eq. (B1):

$$i\hbar\frac{\partial\psi'}{\partial t'} - \hbar\frac{\partial f}{\partial t'}\psi' + (v+at')\left(i\hbar\frac{\partial\psi'}{\partial x'} - \hbar\frac{\partial f}{\partial x'}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi'}{\partial x'^2} + \frac{\hbar^2}{2m}\left(\frac{\partial f}{\partial x'}\right)^2 - \frac{i\hbar^2}{m}\frac{\partial f}{\partial x'}\frac{\partial\psi'}{\partial x'} - \frac{i\hbar^2}{2m}\frac{\partial^2 f}{\partial x'}\psi'. \tag{B4}$$

If the function $f(z't')$ is now chosen to be of the form

$$f(z',t') = -\frac{mv}{\hbar}\left(z' + \frac{vt'}{2}\right) - \frac{mat'}{\hbar}\left(z' + \frac{vt'}{2} + \frac{at'^2}{6}\right), \tag{B5}$$

Eq. (B4) simplifies as follows:

$$i\hbar\frac{\partial\psi'}{\partial t'}(z',t') = -\frac{\hbar^2}{2m}\frac{\partial^2\psi'}{\partial z'^2}(z',t') + maz'\psi'(z',t'). \tag{B6}$$

The second term in the right-hand side of Eq. (B6) represents the effect of an effective potential $V_{in}(z') = maz'$ which, in the classical limit, corresponds to the well-known inertial force for the case of constant acceleration. By putting $a=0$, we recover the Galileo transformation between two inertial frames of reference translating with relative velocity v. In this case the invariance of Eq. (B1) reflects, as expected, the validity of the Galilean relativity principle [29]. According to Eq. (B3), the wave functions are related in this case by the transformation

$$\psi'(z',t') = \exp\left\{-\frac{i}{\hbar}\left(mvz' + \frac{v^2t'}{2}\right)\right\}\psi(z'+vt',t'). \tag{B7}$$

One can easily check that the phase factor involved in Eq. (B7) is just the one needed to ensure the correct transformations of the average values of position and momentum, namely,

$$\langle \hat{z}'\rangle = \langle \hat{z}\rangle - vt, \quad \langle \hat{p}'\rangle = \langle \hat{p}\rangle - mv, \tag{B8}$$

in agreement with the Heisenberg equations of motion

$$\frac{d\hat{z}'}{dt} = \frac{d\hat{z}}{dt} - v = \frac{\hat{p}'}{m}, \quad \frac{d\hat{p}'}{dt} = \frac{d\hat{p}}{dt} = 0. \tag{B9}$$

Finally, we write the density matrix associated with the pure state (B3) with f given by Eq. (B5) as

$$\rho'(z_1',z_2',t') = \exp\left\{-\frac{i}{\hbar}(mv+mat')(z_1'-z_2')\right\} \times \rho(z_1'+vt'+\tfrac{1}{2}at'^2, z_2'+vt'+\tfrac{1}{2}at'^2, t'). \tag{B10}$$

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