PHYSICAL REVIEW D VOLUME 55, NUMBER 7 1 APRIL 1997

Unified description of kaon electroweak form factors

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(Received 13 June 1996)

A calculation of the semileptonic decays of the kaon (K_{l3}) is presented. The results are direct predictions of a covariant model of the pion and kaon introduced earlier by Ito, Buck, and Gross. The weak form factors for K_{l3} are predicted with absolutely no parameter adjustments of the model. We obtained, for the form factor parameters, $f_-(q^2=m_l^2)/f_+(q^2=m_l^2)=-0.28$ and $\lambda_+=0.028$, both within experimental error bars. The connections of this approach to heavy quark symmetry are discussed. [S0556-2821(97)03007-5]

PACS number(s): 14.40.Aq, 11.10.St, 12.39.Ki, 13.10.+q

I. INTRODUCTION

The success of the Ito-Buck-Gross (IBG) [1] model in the description of many properties of both the π and K mesons motivated the calculation of the K_{l3} decays reported here. It is the K_{l3} decays that combine both the pion and kaon wave functions generated previously [1–3]. A successful K_{l3} calculation that is coupled to other observables constrains further the physics described by the model.

The work reported here is predictive and employs no new parameters and no parameter adjustment. We believe that our model, a covariant quark model, fills a gap between the low energy domain consistently described by chiral perturbation theory (CPT) and high energies where the operator product expansion (OPE) in QCD is applicable. It is a description of soft nonperturbative effects that we focus on in this work. The results, found below, are in good agreement with the data and are as good as the CPT approach [4,5] and the effective chiral Lagrangian (CL) approach [6], even though a low energy expansion is involved, and better, at least in the light-quark sector, than the quark potential model [Isgur-Scora-Grinstein-Wise (ISGW2)], with a hyperfine interaction, predictions [7]. It is noted that an older version of the quark potential model without a hyperfine interaction (ISGW) gives results similar to ours [8]. These comparisons and the details of our calculation are presented below. Nonetheless, with the success of CPT and this work, the question still remains for nuclear physics as to how to single out quark from hadronic structure. That is, where do hadrons leave off and quarks begin? The quark model has been very successful at reproducing hadronic static properties such as the mass spectrum and moments. But it is the dynamic properties, we feel, that will delineate the differences between hadronic physics and quark physics. For this reason, we take the position that not only is the $q^2=0$ or near $q^2=0$ physics important but the $q^2\neq 0$ domain will delineate theoretical approaches. Thus, our predictions for the nonperturbative weak transition form factors as a function of q^2 are also presented here in an attempt to attract both theoretical and empirical interest.

A detailed review of the theoretical and experimental status of semileptonic kaon decays is given in Refs. [5] and [9].

II. MODEL

The theoretical model employed is an extension of the Nambu–Jona-Lasinio (NJL) model but with a definite momentum distribution generated by a Lorentz-invariant separable interaction

$$V(p,k)=gf(p^2)f(k^2)[I\otimes I-(\gamma^5\lambda)\otimes(\gamma^5\lambda)], \qquad (1)$$

where $f(p^2)=(\Lambda^2-p^2)^{-1}$, with Λ being the interaction cutoff parameter for a given meson state, λ are the Gell-Mann flavor matrices, and p^2 and k^2 the relative four-momenta squared. With this choice of $q\overline{q}$ interaction, one can integrate all momentum integrals to infinity; there is no need for an integral cutoff as employed in the original NJL model. In our model, Eq. (1) enters in the interaction kernel of the Bethe-Salpeter equation. Since the interaction employed is nonlocal, gauge invariance is preserved by not only coupling the external vector field to the quarks but also directly to the vertices, resulting in an ''interaction current.'' The formalism that includes these ''interaction currents'' is found in Refs. [1,2]

The IBG model requires that the Bethe-Salpeter equation be solved for the vertex function Γ for each meson considered, where p is the four-momentum of the meson, as shown in Fig. 1.

The self-energy of each flavor quark is treated by solving the Schwinger-Dyson equation

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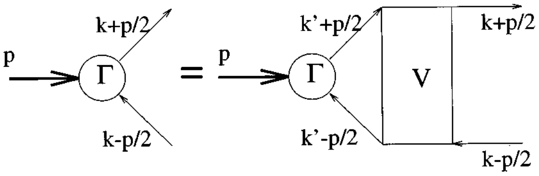

FIG. 1. Bethe-Salpeter equation.

$$\Sigma(k^2)=4in_f g f(k^2)\int \frac{d^4p}{(2\pi)^4} f(p^2)\frac{m_0+\Sigma(p^2)}{p^2-[m_0+\Sigma(p^2)]^2}, \tag{2}$$

where n_f is the number of quark flavors (equal to 3 in our $q\bar{q}$ system) in a coupled sense (coupled via quark masses and interaction strength) to the Bethe-Salpeter equation, though in the case of the strange quark mass the self-energy is assumed to be the (constituent) quark mass and is treated as a parameter to be fixed [3].

Electomagnetic gauge invariance is imposed on the electromagnetic current of a pseudoscalar meson:

$$J^\mu=F(q^2)(p+p')^\mu, \tag{3}$$

where $F(q^2)$ is the meson charge form factor and $p(p')$ is the four-momentum of initial (final) meson. The work of Buck, Williams, and Ito (BWI) [3] has shown that both the pion and kaon (charged and neutral) charge form factors can be predicted and it is the pion and kaon vertices from this work that are employed in the calculation of the weak form factors.

III. WEAK FORM FACTORS

In the standard model, the weak current for K_{l3} decays has the structure

$$J^\mu=\frac{G_F}{\sqrt{2}}V_{us}[f_+(q^2)(P_K+P_\pi)^\mu+f_-(q^2)(P_K-P_\pi)^\mu], \tag{4}$$

where P_K and P_π are the kaon and pion four-momenta, $q=P_K-P_\pi$, and $f_\pm$ are dimensionless form factors, G_F is a Fermi constant, and V_{us} is a CKM matrix element.

The semileptonic decays studied are

$$\begin{aligned}(K_{e3})K^\pm&=\pi^0e^\pm\nu_e,\\ K_L^0&=\pi^\pm e^\mp\nu_e,\\ (K_{\mu3})K^\pm&=\pi^0\mu^\pm\nu_\mu,\\ K_L^0&=\pi^\pm\mu^\mp\nu_\mu.\end{aligned} \tag{5}$$

In the limit of exact isospin symmetry, $m_u=m_d$, form factors of charged and neutral kaon decays are related,

$$f_\pm^\pm/f_\pm^0=1/\sqrt{2},$$

and in the limit of exact SU(3) symmetry, the form factor f_- is zero. For the decay channel, the transferred four-momentum q is timelike, and the physical region is limited to $m_l^2\leqslant q^2\leqslant(m_K-m_\pi)^2$. The vertices appearing in this weak current and the ones employed in this work are the kaon and pion vertices (wave functions) previously obtained by BWI: namely,

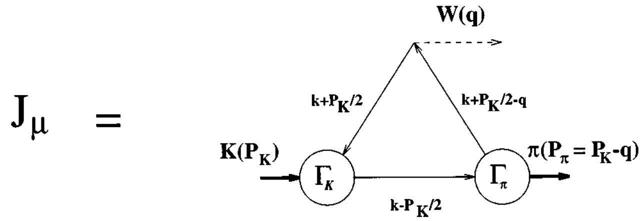

FIG. 2. Triangle diagram for the charged weak current of K_{l3} decay.

$$\Gamma_{K,\pi}(k)=\frac{N_{K,\pi}\gamma^5}{\Lambda_{K,\pi}^2-k^2}, \tag{6}$$

with $N_{K,\pi}$ being the normalization.

From Eq. (4), one can uncouple $f_\pm$:

$$f_\pm=\frac{(P_K\mp P_\pi)^2J^\mu(P_K\pm P_\pi)^\mu-(m_K^2-m_\pi^2)J^\mu(P_K\mp P_\pi)^\mu}{(P_K-P_\pi)^2(P_K+P_\pi)^2-(m_K^2-m_\pi^2)^2}. \tag{7}$$

To compare to available experimental data, the following low-q^2 expansion is used for the form factors:

$$f_\pm(q^2)=f_\pm(q^2=m_l^2)\left(1+\lambda_\pm\frac{q^2}{m_{\pi^+}^2}\right), \tag{8}$$

where $\lambda_\pm$ is the slope of $f_\pm$ evaluated at $q^2=m_l^2$ and $f_\pm(q^2=m_l^2)$ corresponds to the normalization. Note that it is the charged, not the neutral, pion mass that enters the above expansion.

Another set of the form factor parameters commonly used in the literature is λ_+, λ_0, arising as coefficients of linear expansions of the form factors f_+ and f_0, with f_0 defined as

$$f_0=f_++\frac{q^2}{m_K^2-m_\pi^2}f_-. \tag{9}$$

The form factors f_+ and f_0 describe, respectively, P-wave and S-wave projections of weak current matrix elements in the crossed channel.

To obtain the values of $\lambda_\pm$, a calculation of J^μ must be performed. In this work, J^μ is the direct result of a triangle diagram (Fig. 2) with a flavor-changing operator having V-A spin structure $\gamma^\mu(1-\gamma^5)$. In the standard model, the K_{l3} decay form factors are determined only by the vector part of the charged weak current operator.

Integrals with respect to loop momentum were evaluated in the following way. In the expression for the weak current given by a Feynman diagram, Fig. 2, the spin trace was calculated and the terms dependent on loop momentum in the numerator were divided out by corresponding terms in the denominator. This procedure reduces the expression for ‘‘impulse’’ current, Fig. 2, to the sum of scalar integrals of products of three to five denominator factors (three of them

TABLE I. Model predictions for the parameters of K_{l3} decay form factors.

	CPT [5], CL [6]	VMD [9]	ISGW2 [7]	This work	Experiment [11]
λ_+	0.031, 0.0328	0.0245	0.019	0.028	0.0286 ± 0.0022 (K_{e3})
$\xi_A(0)$	-0.164 ± 0.047,[a] -0.235	-0.28	-0.28	-0.28	-0.35 ± 0.15 ($K_{\mu3}$)
λ_0	0.017 ± 0.004, 0.0128	0.0	-0.005[b]	0.0026	0.004 ± 0.007 ($K^+_{\mu3}$)
					0.025 ± 0.006 ($K^0_{\mu3}$)

[a]From the corresponding values of λ_+ and λ_0 [5].
[b]From the corresponding values of λ_+ and $\xi_A(0)$.

coming from quark propagators and two from meson-$q\bar{q}$ vertex form factors). Each denominator factor is a polynomial quadratic in the loop momentum. The terms involving three denominators are, in fact, scalar three-point functions which may be expressed analytically in terms of Spence functions [10]. In this work, to calculate the three-point functions, we parametrized them in terms of two Feynman parameters. Integration with respect to one Feynman parameter was done analytically, and the other was a numerical Gauss integration. We did not use a low-q^2 expansion to evaluate loop integrals but we do extract from our results the low-q^2 behavior and find it is consistent with the low-q^2 expansion of Eq. (8) above, employed by all researchers.

Our task of computing four-dimensional loop integrals of products of more than three denominators is greatly simplified when taking advantage of the fact that only two external momenta in the integrand are linearly independent. As a result, products of four and five denominators are reduced to the sum of products of three denominators with redefined masses M_i. In one representation, this procedure is described by the following identity, assuming a four-momentum integration is performed:

$$\frac{1}{(k^2-m_0^2)[(k+q_1)^2-m_1^2][(k+q_2)^2-m_2^2][(k+q_3)^2-m_3^2][(k+q_4)^2-m_4^2]} = \sum_{i,j,n} \frac{a_i}{(k^2-M_i^2)[(k+q_j)^2-M_j^2][(k+q_n)^2-M_n^2]}, \tag{10}$$

if

$$\begin{pmatrix} q_3 \\ q_4 \end{pmatrix} = \mathbf{A} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \quad \det\mathbf{A} \neq 0,$$

where the sum is taken over different combinations of the external four-momenta q_i involved in the reaction, m_i are quark masses and mass parameters in meson-$q\bar{q}$ vertex form factors, a_i are coefficients independent of loop momentum k, and $\mathbf{A}$ is a 2×2 matrix setting relations between external momenta in the integrand. After this reduction, scalar three-point integrals are computed within the technique described above.

IV. RESULTS

In the physical region of K_{e3} decays, q^2 may be as low as the square of the electron mass, 25×10^{-8} GeV$^2/c^2$, and as high as the square of the mass difference between the kaon and the pion, 0.123 GeV$^2/c^2$. The form factors $f_\pm$ in this region with a good precision appear to be linear functions of q^2, thereby justifying a linear parametrization of Eq. (8) usually employed in analyses of experimental data [11]. To compare our results with experiment, we extracted the slopes and ratios of the form factors $f_\pm$ at $q^2=m_l^2$ via numerical differentiation. Numerical values for the parameters in this calculation were taken to be the same as in Ref. [3], viz. $m_{u,d}=$ 250 MeV, $m_s=$ 430 MeV, $\Lambda_\pi=$ 600 MeV, and $\Lambda_K=$ 690 MeV.

The direct predictions of our approach for λ_+ and $f_-/f_+|_{q^2=m_l^2}$ are 0.028 and -0.28, respectively. Our result for the form factor f_+ at zero momentum transfer, $f_+(0)=0.952$, is consistent with the Ademollo-Gatto theorem [13]. These results are to be compared to the experimental values of

$$\lambda_+=0.0286\pm0.0022, \quad \xi_A=f_-/f_+=-0.35\pm0.15.$$

We obtain $\lambda_-=$ 0.029; i.e., in our model both f_- and f_+ have approximately the same slopes, in agreement with early quark model results [14]. Our calculation for K_{e3} and $K_{\mu3}$ yields equal results, within the quoted precision, since the $\lambda_\pm$ are almost constant in the range $m_e^2\leq q^2\leq m_\mu^2$. Naturally, the decay rates should be different due to phase space factors; they can be calculated by known formulas in terms for form factor slopes (see, e.g. Ref. [5]); however, we have yet to perform the calculation of these rates.

Table I illustrates the comparison between our work, that of CPT, vector meson dominance (VMD), and the ISGW2 model. One sees that the work reported on here compares very favorably to experiment and to CPT, except for the prediction for λ_0.

A prediction for the slope parameter λ_0 obtained within our model is 0.0026, which is consistent with experiments on charged kaon decays ($\lambda_0^+ = 0.004 \pm 0.007$) and inconsistent with neutral kaon decay measurements ($\lambda_0^0 = 0.025 \pm 0.006$). Since the experimental results for this slope parameter are not firm, it is hard to draw any positive conclusions about the agreement or disagreement of our result for λ_0 with experiment. However, we can compare it with predictions from other models. It can be seen from the Table I that the quark model in general gives much smaller numbers for λ_0 than CPT.

To test the sensitivity of our results, an arbitrary change in the Λ_π cutoff from 600 MeV to 450 MeV (a 25% change) results in $\lambda_+ = 0.028$–0.031 (an almost 10% change), $\xi_A(0)$ from -0.28 to -0.27 ($\approx 5\%$ change), and λ_0 from 0.0026 to 0.0058 ($\approx 100\%$ change), respectively. One is reminded that changing Λ_π changes the pion charge radius as well as the pion decay constant. In fact, the value $\Lambda_\pi =$ 450 MeV was used in Refs. [1,2] as the best fit to the pion decay constant and charge radius alone, and this parameter was adjusted to 600 MeV in Ref. [3] to be able to treat *both* the pion *and* kaon in a coupled approach.

It should be noted that our model gives a stable, with respect to the variation of the model parameter, prediction for $\xi_A(0)$, and appears to give a highly parameter-dependent result for λ_0. This model dependence is due to the cancellation between two large terms on the right-hand side (RHS) of Eq. (9). The situation is different in CPT, where uncertainties due to higher-order loop corrections give rise to an about 30% uncertainty for $\xi_A(0)$, -0.164 ± 0.047, and about a 25% uncertainty for λ_0 [5].

Such a difference for the slope of the scalar form factor λ_0 predicted by constituent quark models (present work and [7]) and CPT is mainly due to chiral loop corrections in CPT. While these corrections are important features of CPT, the present work is not that of CPT and does not include chiral logarithms or loop corrections. It is very important, therefore, to pursue future work to clarify any qualitative and quantitative theoretical differences between the two approaches.

Finally, the CL approach suggests that once the ratio of the weak form factors is known, then an estimate of the mass of the strange σ (m_{σ_K}), a meson with $J^P = 0^+$, can be made. The relationship referred to is [6]

$$\xi_A(0) = (M_K^2 - M_\pi^2)(M_\rho^{-2} - M_{\sigma_K}^{-2}). \tag{11}$$

Taking our result for $\xi_A(0)$ and assuming duality between our model predictions and the model with effective exchanges of vector and scalar mesons at low q^2, we have $m_{\sigma_K} =$ 1.5 GeV, which compares favorably to the mass of $K_0^*(1430)$. A test of this value could be made through the hypernuclear spectroscopy measurements (CEBAF E89-009, CEBAF PR–95–002) [15], inferring the interaction that contains this strange σ.

Another feature of our approach is revealed in the limit as Λ_K, m_K, and m_s become infinitely large. The ratio $\xi_A(0)$ is calculated in this limit and its asymptotic value is -1; Fig. 3 illustrates the mass dependence. In the limit of heavy quark symmetry (HQS), the q^2 dependence of the semileptonic decay form factors is factorized out in the form of an Isgur-Wise function [12], and ξ_A is given by the combination of the initial (M) and final (M') meson masses,

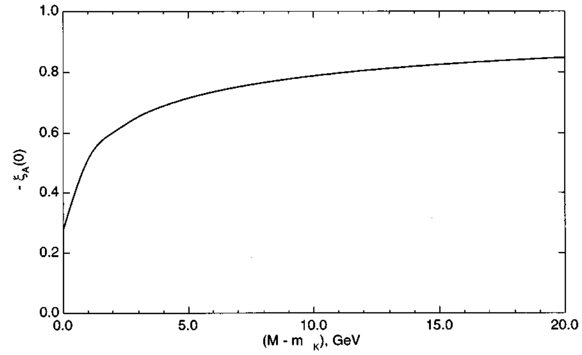

FIG. 3. Ratio $\xi_A(0)$ as a function of the initial meson mass.

$$\xi_A = -\frac{M - M'}{M + M'}. \tag{12}$$

As a result, $\xi_A|_{\mathrm{HQS}} = -1$ if the initial meson is much heavier than the final. Note that for HQS to be applied, both initial and final mesons should be heavy, whereas assuming m_s to be large in our model, we keep the final meson light. This implies that this particular result of HQS appears to be more generally applicable. Of course, the ratio is zero for mesons of equal mass.

Though it is tempting to make exuberant statements with regard to identical results, one is cautioned by the manner in which the limits are taken and the nature of the physics examined, respectively.

Furthermore, Fig. 4 illuminates our predictions for $f_+(q^2)$ at space like momentum transfers describing the neutrinoproduction processes $\nu\pi \to lK$ and $\nu\pi^0 \to l\pi^+$ and corresponding weak lepton capture. We stress that no low-q^2 expansion was assumed in our calculations, so that present results have the same validity range in terms of q^2 as results of Refs. [1–3] for electromagnetic form factors of pions and kaons. The form factor f_+ at large q^2 behaves as $1/q^4$ (up to a logarithmic correction), indicating that our model effectively describes a soft, nonperturbative reaction mechanism, and does not include perturbative QCD contributions.

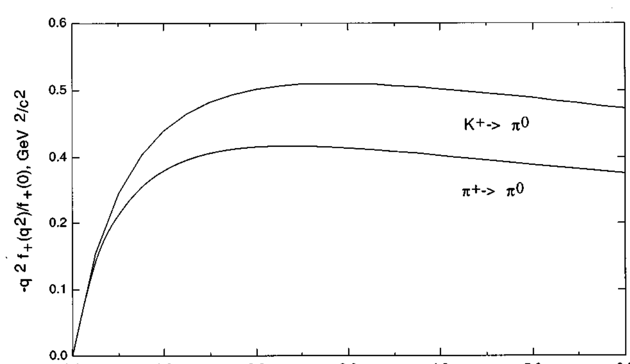

FIG. 4. Form factors of weak transitions $K^+ \to \pi^0$ and $\pi^+ \to \pi^0$ at space like transferred momenta.

The weak $K \rightarrow \pi$ transition form factors in the spacelike region could be possibly accessed experimentally in the production of kaons on a hadronic target induced by neutrinos or lepton weak capture. The latter possibility is being studied for a CEBAF experiment [16].

It would be instructive to see if the earlier success of the IBG model, which includes pion and kaon observables as well as the results of this present work, can be reproduced with other interactions and/or with other wave equations; by this is meant the predictive characteristics associated with the low-energy axial anomaly, such as the pion transition and elastic charge form factors, kaon charge form factors, and K_{l3} decays.

In brief, weak form factors and slope parameters have been calculated for K_{l3} decays. The results compare very favorably to available experimental data. The model employed was that of IBG [1-3] and there were no parameter adjustments, thus rendering this calculation predictive.

ACKNOWLEDGMENTS

The work of A.A. was supported by the U.S. Department of Energy under Contract No. DE-AC05-84ER40150; the work of W.W.B. was supported by the National Science Foundation Grant No. HRD-9154080. We would like to acknowledge useful discussions with J. Goity, N. Isgur, A. Radyushkin, and R. Williams in the course of this work.

[1] H. Ito, W. W. Buck, and F. Gross, Phys. Lett. B **287**, 23 (1992).
[2] H. Ito, W. W. Buck, and F. Gross, Phys. Rev. C **45**, 1918 (1992).
[3] W. W. Buck, R. A. Williams, and H. Ito, Phys. Lett. B **351**, 24 (1995).
[4] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 517 (1985).
[5] J. Bijnens, G. Colangelo, G. Ecker, and J. Gasser, in *The Second DAΦNE Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (INFN, Rome, Italy, 1995), p. 315.
[6] E. P. Shabalin, Sov. J. Nucl. Phys. **51**, 296 (1990).
[7] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1992).
[8] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989): D. Scora, Ph.D. thesis, Toronto University, 1993.
[9] L.-M. Chounet, J.-M. Gaillard, and M. K. Gaillard, Phys. Rep. **4**, 199 (1974).
[10] G. 't Hooft and M. Veltman, Nucl. Phys. **B153**, 365 (1979).
[11] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[12] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[13] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).
[14] N. Isgur, Phys. Rev. D **12**, 3666 (1975).
[15] R.E. Chrien, L.G. Tang, and E.V. Hungerford, CEBAF Proposal No. PR-89-009 (unpublished); A. Margaryan and L. Tang, spokespersons, CEBAF Proposal No. PR-95-002 (unpublished).
[16] J. M. Finn (private communication).