Gluino contribution to the three-loop β function in the minimal supersymmetric standard model

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We deduce the gluino contribution to the three-loop QCD β function within the minimal supersymmetric standard model (MSSM) from its standard QCD expression. The result is a first step in the computation of the full MSSM three-loop β function. In addition, in the case of a light gluino it provides the strong three-loop SUSY correction to the extrapolation of the strong coupling constant from the low energy regime to the Z region and up to the squark threshold. [S0556-2821(97)01607-X]

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Although experimental measurements at the highest available energy are consistent with the standard model [1], the observed relationship of the strong coupling constant at the Z and the weak angle as well as the value of the b/τ mass ratio vis-à-vis the top quark mass remain strong indications of a supersymmetric (SUSY) grand unification above 10^{16} GeV and a SUSY threshold for squarks and sleptons in the 0.1 to 1 TeV region. In this unification picture the value of the SUSY threshold is very sensitive to the highest known (two-loop) contribution to the minimal supersymmetric standard model (MSSM) β function. At the one-loop order a SUSY threshold far below 100 GeV would be needed to fit the coupling constant measurements and such a low threshold is directly ruled out by the nonobservation of squarks and sleptons in Z decay. This can be seen by inserting the current experimental values for the couplings into the semianalytic expressions for the SUSY scale given in [2]. This suggests that the three-loop results could also be important especially as the precision of the measurements at the Z and beyond improves. As a first step in the calculation of the full three-loop β functions of the MSSM, we provide here the gluino contribution to the renormalization of the strong coupling constant. This gives the complete result in the region between the gluino mass and the squark mass which, in the light gluino scenario, extends from the low energy regime up to the Z and beyond up to the squark threshold. The standard Lagrangian density of the MSSM is summarized in [3]. The gluons interact with quarks and squarks in the fundamental representation of the gauge group SU(N), and with gluons, gluinos, and ghosts in the adjoint representation. In each representation the generators satisfy the commutation relations

$$[R^a,R^b]=if^{abc}R^c, \tag{1}$$

with the adjoint representation matrices being defined in terms of the structure constants $(F^a)_{bc}=if^{bac}$. The running of the strong coupling constant as a function of the scale μ is determined by the QCD β function

$$d\alpha_s/d(\ln\mu^2)=\alpha_s\beta(\alpha_s/4\pi), \tag{2}$$

where β has the perturbative expansion

$$\beta(x)=\beta_1x+\beta_2x^2+\beta_3x^3+\cdots. \tag{3}$$

Ignoring squark contributions, the one- and two-loop results in the minimally extended SUSY QCD [4] are

$$\beta_1=-\frac{11}{3}C_A+\frac{4}{3}\left(n_fT+\frac{n_{\tilde g}}{2}C_A\right), \tag{4}$$

$$\beta_2=-\frac{34}{3}C_A^2+\frac{20}{3}\left(n_fTC_A+\frac{n_{\tilde g}}{2}C_A^2\right)+4\left(n_fTC_F+\frac{n_{\tilde g}}{2}C_A^2\right), \tag{5}$$

where n_f is the number of quark flavors and $n_{\tilde g}$ is the number of gluino multiplets, C_A and C_F are the eigenvalues of the quadratic Casimir operators in the adjoint and fundamental representation, respectively, and T is the Dynkin index for the fundamental representation. Note that throughout this paper we use the framework of the dimensional regularization [5] and a minimal-subtraction- (MS-) or modified-MS ($\overline{\rm MS}$-)type prescriptions [6]. In the standard model ($n_{\tilde g}=0$), the three-loop coefficient is [7]

$$\beta_3^{(\rm SM)}=-\tfrac{2857}{54}C_A^3-n_fT(2C_F^2-\tfrac{205}{9}C_FC_A-\tfrac{1415}{27}C_A^2)-(n_fT)^2(\tfrac{44}{9}C_F+\tfrac{158}{27}C_A). \tag{6}$$

Arriving at this result required the calculation of several hundred Feynman graphs, and a similar number is required to extend the result to the MSSM. However, the first step in this program, the incorporation of gluino loops, can be obtained by purely group theoretical methods. This leaves a sharply reduced number of graphs that must be treated in detail to determine the full three-loop β function. The main result of this paper, proven in the appendix, is that, in the MSSM above the gluino mass scale but below that of the squarks, there are only six linearly independent group factors, C_i, $i=0,\ldots,5$, among all the relevant Feynman graphs. In terms of these the three-loop β function of the MSSM takes the form

$$\beta_3=b_0C_0+b_1C_1+b_2C_2+b_3C_3+b_4C_4+b_5C_5. \tag{7}$$

The graphs involving solely gluons and ghosts have each a group factor proportional to $C_0=C_A^3$ as can be trivially deduced on dimensional grounds from the case where there are no quarks in the theory. Graphs involving a single fermion loop have group weights that are linear combinations of the three factors

$$C_1 = n_f T C_F^2 + n_{\tilde g} C_A^3/2, \tag{8}$$

$$C_2 = n_f T C_F C_A + n_{\tilde g} C_A^3/2, \tag{9}$$

$$C_3 = n_f T C_A^2 + n_{\tilde g} C_A^3/2. \tag{10}$$

Finally, all graphs involving two fermion loops have group weights that are linear combinations of the two factors

$$C_4 = (n_f T C_F + n_{\tilde g} C_A^2/2)(n_f T + n_{\tilde g} C_A/2), \tag{11}$$

$$C_5 = (n_f T C_A + n_{\tilde g} C_A^2/2)(n_f T + n_{\tilde g} C_A/2). \tag{12}$$

In the standard model case where $n_{\tilde g} = 0$, these results are trivial and not useful. The importance of Eqs. (8)–(12) is that they constrain the ways contributions from gluino loops follow from those of quark loops. Substituting C_0 through C_5 into Eq. (7) and comparing with Eq. (6) in the $n_{\tilde g} = 0$ limit suffices to determine the coefficients b_0 through b_5. The final result for the MSSM including gluinos but excluding squark contributions is, therefore,

$$\begin{aligned}\beta_3 = &-\tfrac{2857}{54} C_A^3 - n_f T\left(2C_F^2 - \tfrac{205}{9} C_F C_A - \tfrac{1415}{27} C_A^2\right)\\ &-(n_f T)^2\left(\tfrac{44}{9} C_F + \tfrac{158}{27} C_A\right) + \tfrac{988}{27} n_{\tilde g} C_A^3\\ &-n_{\tilde g} n_f T\left(\tfrac{224}{27} C_A^2 + \tfrac{22}{9} C_A C_F\right) - \tfrac{145}{54} n_{\tilde g}^2 C_A^3 .\end{aligned} \tag{13}$$

Assuming the gluino lies below the Z and the squarks above the Z, then at the Z scale, β_3 is $-9769/54$ in the standard model ($n_{\tilde g} = 0$) and $+14134/27$ in the MSSM ($n_{\tilde g} = 1$). If the gluino lies below the b quark, the value of α_s at the Z for a given α_s at m_b is increased by a non-negligible amount compared to the experimental error. A complete analysis of the precise effect is left for a later complete phenomenological analysis which should include the light gluino effect on the Z and τ decay widths [8,9]. The results found here for the MSSM using the dimensional regularization [5] framework, as expected, do not match those found recently [10,11] within the dimensional reduction framework and they do not have to. A similar scheme dependence has been found in [12].

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APPENDIX

The group factors occuring in the β function are calculated in terms of traces of representation matrices. The fundamental representation matrices T^a, and the adjoint representation matrices F^a satisfy

$$T^a T^a = C_F \hat{\mathbf{1}}, \quad Tr(T^a T^b) = T\delta^{ab}, \tag{A1}$$

$$F^a F^a = C_A \hat{\mathbf{1}}, \quad Tr(F^a F^b) = C_A \delta^{ab}, \tag{A2}$$

$$F^a F^b F^a = C_A F^b/2, \quad T^a T^b T^a = (C_F - C_A/2) T^b, \tag{A3}$$

$$Tr(T^a T^b T^c) = T(d^{abc} + i f^{abc})/2, \quad Tr(F^a F^b F^c) = i f^{abc} C_A/2, \tag{A4}$$

$$f^{abc} f^{abc} = C_A (N^2 - 1), \tag{A5}$$

where

$$C_A = 2TN, \quad C_F = T(N^2 - 1)/N. \tag{A6}$$

The unit matrices appearing in Eqs. (A1),(A2) are N and $N^2 - 1$ dimensional for the fundamental and adjoint representations of SU(N), respectively. The arbitrary normalization of the generators is usually chosen so that $T = 1/2$. The gauge coupling constant usually quoted in experimental analyses uses this normalization. The result of this work, that the three-loop β function including gluino but not squark contributions is a linear combination of the six quoted group factors, is proven in this appendix.

We begin by noting that the MS scheme strong coupling renormalization constant Z_{α_s} can be determined via the renormalization constants $\tilde Z_1$ for the ghost-ghost-gluon vertex, $Z_3^{1/2}$ for the gluon propagator, and $\tilde Z_3^{1/2}$ for the ghost propagator (see, e.g., [12] for more details)

$$Z_{\alpha_s} = \tilde Z_1^{\,2} Z_3^{-1} \tilde Z_3^{-2}, \tag{A7}$$

where by the definition of the MS-type schemes [6] each renormalization constant Z_i is a polynomial

$$Z_i = 1 + \sum_{n \geq 1} Z_i^{(n)}(\alpha_s)\varepsilon^{-n}, \tag{A8}$$

with $\varepsilon = (4 - D)/2$. Now, the QCD β function can be obtained from the following relation (see, e.g., [12])

$$\beta = \frac{\partial Z_{\alpha_s}^{(1)}}{\partial \ln \alpha_s}. \tag{A9}$$

Contributions from graphs with no fermion loops each have the group factor $C_A^3 = C_0$ as can be seen from the standard model result equation (6). The relevant Feynman graphs involving at least one fermion loop are those in Fig. 1 with external gluons being attached at all possible points. We may average over the $N^2 - 1$ color gluons. Thus, the group factors that occur are $1/(N^2 - 1)$ times the group factors of the graphs obtained by adding an internal gluon line in all possible ways to the graphs of Fig. 1, thus transforming them into four-loop graphs. In each of these four-loop graphs there are six gluon vertices. We may classify them by the resulting number of gluons attached to the outer fermion loop in Figs. 1(a)–1(d). We ignore for the present quartic couplings. Each resulting graph will then have n connections on the outer fermion line and $6 - n$ connections in the interior and we treat the graphs in order of decreasing n. Equation (1) tells us that the group factor for a graph with any order of gluon attachments on the outer loop is a linear combination of the group factor from the graph with the gluons attached in a standard order and group factors from graphs with fewer gluon attachments on the outer loop. This relation may be

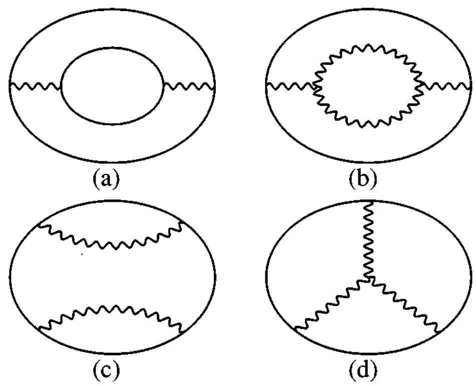

FIG. 1. Basic topologies for the calculation of the gluon propagator renormalization. Solid lines represent fermions, either quarks or gluinos, and wavy lines represent gluons. The pair of external gluons are attached at all possible positions.

described by a graphical equivalence in color space illustrated in Fig. 2. Since Eq. (1) holds for all representations, the same equivalence is valid on a gluon or ghost loop also. Thus, the linearly independent group factors may be found by considering planar graphs only. When gluinos are incorporated into the theory, each graph with a quark loop has a corresponding graph with one or more quark loops replaced by gluino loops. The gluino contribution to the β function is given by the quark contribution except for the replacement of the fundamental representation matrices by the adjoint representation matrices and by an extra factor of 1/2 for each gluino loop due to the Majorana nature of the gluino. This factor of 1/2 has its source in the fact that, unlike the case for Dirac fermions, one must ignore the direction of the gluino line in a Feynman graph in calculating statistical and symmetry factors. The only linearly independent group factor coming from graphs with $n=6$ may be taken as the planar graph obtained by attaching another gluon line at adjacent points on the outer loop of Fig. 1(c). Using the identities of Eqs. (A1) and (A2), the corresponding group factor is seen to be the C_1 of Eq. (8):

$$C_1=\frac{1}{N^2-1}\left[n_f\mathrm{Tr}(T^aT^aT^bT^bT^cT^c)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^aF^bF^bF^cF^c)\right]. \tag{A10}$$

The traces here are easily evaluated using Eqs. (A1)–(A5). Because of Fig. 2, the group factor from the various nonplanar graphs with $n=6$ are linear combinations of C_1 and group factors coming from graphs with $n<6$. Thus, Eq. (A10) can be taken to be the only linearly independent group factor coming from the $n=6$ graphs. Graphs with $n=5$ can be obtained by adding a gluon line to Fig. 1(d) or Fig. 1(c). Using Fig. 2, one sees that all such graphs lead to group factors that are proportional to the C_2 of Eq. (9):

$$C_2=\frac{-i}{N^2-1}\left[n_f\mathrm{Tr}(T^aT^aT^bT^cT^d)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^aF^bF^cF^d)\right]f^{bcd} \tag{A11}$$

plus possibly a linear combination of group factors from graphs with lower n. It is clear, using the tracelessness of the representation matrices and the equivalence of Fig. 2, that this is the only linearly independent group factor one can write with five attachments on the outer loop and one internal attachment. We proceed, therefore, to $n=4$. A linearly independent group factor with $n=4$ is the C_3 of Eq. (10),

$$C_3=\frac{4}{N^2-1}\left[n_f\mathrm{Tr}(T^aT^bT^cT^d)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^bF^cF^d)\right]f^{abe}f^{cde}. \tag{A12}$$

This appears when the extra gluon line is added connecting the gluon lines of Fig. 1(c). A second group invariant corresponding to $n=4$ is

$$\frac{1}{N^2-1}\left[n_f\mathrm{Tr}(T^aT^aT^bT^c)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^aF^bF^c)\right]\mathrm{Tr}(F^bF^c). \tag{A13}$$

However, by explicit calculation this is seen to be proportional to C_2. At this point one should investigate the graph where to Fig. 1(d) one adds a gluon line with one leg on the outer loop and the other attaching to make a quartic coupling. However, the group factor at a quartic coupling is a sum of three terms each of which is a product of two f^{abc}. Thus, in group space the quartic coupling is equivalent to a sum of products of triple vertices as expressed in Fig. 3. Again, this figure has the meaning that the group factor of a graph including the quartic coupling is a linear combination of group factors where the four legs are linked in the three possible ways by triple couplings. The consequence of this equivalence is that we may neglect all graphs with quartic couplings in determining the number and form of linearly

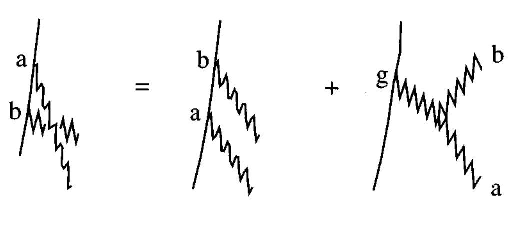

FIG. 2. Diagrammatic equivalence in group space. The group factor from a graph containing the left-hand side is a linear combination of the group factors from graphs in which each of the terms on the right-hand side is inserted in its place.

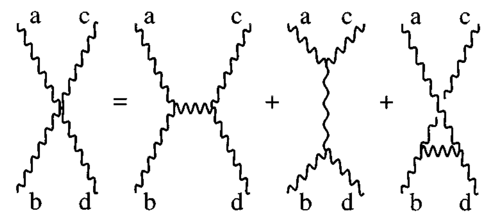

FIG. 3. Graphical equality in group space between a quartic coupling and three pairs of triple couplings. As in Fig. 2, the equality has the meaning that the group factor of a graph containing the quartic coupling is a linear combination of the group factors obtained by replacing that coupling in turn by each of the subgraphs shown on the right-hand side.

independent group factors. With $n=4$ we also have a contribution from the two-fermion loop topology of Fig. 1(a) where a gluon line is attached at adjacent points on the outer loop and each of the fermion lines can represent either a quark or a gluino. The corresponding group factor is C_4:

$$C_4=\frac{1}{N^2-1}\left[n_f\mathrm{Tr}(T^aT^aT^bT^c)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^aF^bF^c)\right]\times\left[n_f\mathrm{Tr}(T^bT^c)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^bF^c)\right]. \quad \text{(A14)}$$

All other graphs with four attachments on the outer loop can be seen, using Fig. 2, to be linear combinations of C_1 through C_4 and possible group factors appearing at lower n. At $n=3$ we have the group factor from Fig. 1(a) with the extra gluon connecting the outer and inner loops. The corresponding group factor is

$$\frac{-2i}{N^2-1}\left[n_f\mathrm{Tr}(T^a[T^b,T^c])+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^a[F^b,F^c])\right]\times\left[n_f\mathrm{Tr}(T^aT^bT^c)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^bF^c)\right]=C_5. \quad \text{(A15)}$$

Here, we have used the fact that the divergent part of Fig. 1(a) when gluon-crossed graphs are included is totally antisymmetric in (a),(b),(c). This can be seen from the fact that the divergent subgraph of Fig. 1(a) with an extra gluon joining the two fermion loops is a renormalization of the triple gluon vertex. The other two-fermion-loop possibility at $n=3$ is obtained by connecting the extra gluon line to Fig. 1(a) with one end on a triple gluon vertex. The corresponding group factor is proportional to C_5 as can be seen by using Fig. 2. The only remaining possibility with $n=3$ is

$$\frac{1}{N^2-1}\left[n_f\mathrm{Tr}(T^a[T^b,T^c])+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^a[F^b,F^c])\right]\mathrm{Tr}(F^aF^bF^c), \quad \text{(A16)}$$

coming for example from adding a gluon line to Fig. 1(b) or Fig. 1(d). However, this is easily seen to be proportional to C_3. Finally, we consider graphs with $n=2$. One such graph coming from Fig. 1(a) is equivalent to a graph treated earlier under interchange of the outer and inner loops. The only other group invariants one can construct with $n=2$ are of the form

$$C=\frac{1}{N^2-1}\left[n_f\mathrm{Tr}(T^aT^b)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^b)\right][A\mathrm{Tr}(F^aF^bF^cF^c)+B\mathrm{Tr}(F^aF^e)\mathrm{Tr}(F^eF^b)], \quad \text{(A17)}$$

all of which are proportional to C_3. The B term, in fact, never appears in the three-loop β function since after cutting a gluon line in the four-loop graph to make the external gluons, the graph must remain one-particle-irreducible. This exhausts the graphs contributing to the gluon propagator renormalization.

The ghost propagator renormalization contributes no new group factors. This can be seen by noting that the ghost-ghost-gluon vertex has the same group factor f^{abc} as the triple gluon vertex. The ghost propagator corrections can be obtained from the topologies of Fig. 1 by adding a gluon in all possible ways which lead to at least one closed gluon loop and then replacing each gluon loop in turn by a ghost loop. Cutting one of the ghost lines then leads to a ghost propagator renormalization graph. All of these clearly have the same group structure as the corresponding contribution to the gluon propagator renormalization. In fact, since the role of the ghosts is to cancel unphysical Lorentz modes of the gluons, one can anticipate that the ghost propagator and vertex renormalizations contribute no new linearly independent group factors. It is useful, however, to see this in detail.

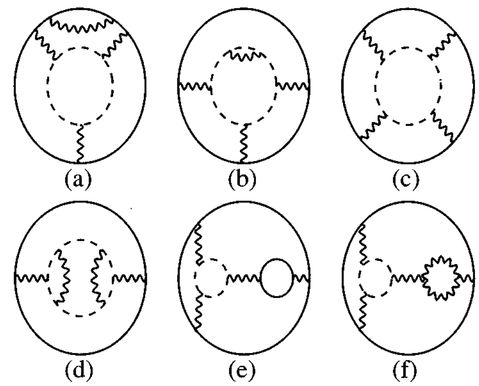

FIG. 4. Basic topologies with at least one fermion loop for the renormalization of the ghost-ghost-gluon vertex. Fermions, gluons, and ghosts are indicated by solid, wavy, and dashed lines, respectively.

We, therefore, consider finally contributions to the β function from renormalization of the ghost-ghost-gluon coupling. The corresponding graphs are given by attaching an external gluon in all possible ways to the graphs of Fig. 1 and then at all possible points joined by a continuous gluon line attaching a pair of external ghosts and changing the continuous gluon line between them into a ghost line. Since the ghost-ghost-gluon vertex, like the triple gluon vertex, is proportional to f^{abc}, the infinite corrections to the vertex must also have this group structure. Since $f^{abc}f^{abc}=C_A(N^2-1)$, all the group factors from the renormalization of this vertex can be obtained by considering all the vacuum to vacuum, five-loop graphs including a fermion loop and a ghost loop that remain one-particle irreducible after at least one particular gluon-ghost-ghost vertex is excised. The group factors that occur in the gluon-ghost-ghost vertex renormalization are then the group factors of these five-loop graphs divided by $C_A(N^2-1)$. Each of these graphs consists of a ghost loop and fermion loop with $3<n<6$ gluon vertices on the ghost loop. Four representative graphs are shown in Fig. 4. The three-loop, ghost-ghost-gluon vertex correction can be restored by excising a ghost-ghost-gluon vertex from these five-loop graphs in all possible ways that leave the graph one-particle irreducible. Since there are eight gluon vertices in a five-loop graph, all the graphs with six attachments to the ghost loop will have two attachments to the fermion loop as in Fig. 4(d). Each such graph, therefore, will have a factor

$$n_f\mathrm{Tr}(T^aT^b)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^b)=\left(n_fT+\frac{n_{\tilde{g}}}{2}C_A\right)\delta^{ab}. \quad \text{(A18)}$$

The group factor for such graphs will, therefore, be $(n_f T+n_{\tilde{g}}C_A/2)/C_A$ times the group factor (C_A^3) of the three-loop gluon propagator correction with no fermion loops. The resultant group factor is C_3. A graph such as Fig. 4(a) with three gluons attached to the ghost loop clearly gives no new group factors since

$$\frac{1}{C_A(N^2-1)}\mathrm{Tr}(F^gF^hF^i)=\frac{if^{ghi}}{2(N^2-1)}. \tag{A19}$$

That is, a graph with a ghost loop attached to the rest of the graph by three gluons has the same group factor as the graph where the ghost loop is shrunk to a point at a triple gluon vertex leading to one of the graphs treated in the gluon propagator renormalization. Graphs with five vertices on the ghost loop [e.g, Fig. 4(b)] have the group factors

$$\frac{1}{C_A(N^2-1)}\left[n_f\mathrm{Tr}(T^bT^cT^d)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^bF^cF^d)\right]\mathrm{Tr}(F^aF^aF^bF^cF^d) \tag{A20}$$

and

$$\frac{1}{C_A(N^2-1)}\left[n_f\mathrm{Tr}(T^bT^cT^d)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^bF^cF^d)\right]\mathrm{Tr}(F^aF^bF^aF^cF^d). \tag{A21}$$

Each of these has a group factor proportional to C_3 as can be seen by using Eqs. (A1)–(A3). Graphs where there is a triple gluon vertex can be reduced to a linear combination of Eqs. (A20) and (A21) using the equivalence of Fig. 2. The graph of Fig. 4(c) where the fermion and ghost loop are joined by four gluons has, when all the crossed gluon graphs are considered, the group factor

$$\frac{1}{C_A(N^2-1)}\left[n_f\mathrm{Tr}(T^aT^bT^cT^d)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^bF^cF^d)\right]\times\{A\mathrm{Tr}([F^a,F^b]F^cF^d)+B\mathrm{Tr}(F^a[F^b,F^c]F^d)+C\mathrm{Tr}(F^aF^bF^cF^d-F^cF^bF^aF^d)\}. \tag{A22}$$

The symmetry of the divergent part of these graphs is determined from the fact that the divergent subgraph is a renormalization of the quartic gluon vertex. Each of these three terms is proportional to C_3. The group factor for Fig. 4(f) is also proportional to C_3 as is that for the case of Fig. 4(f) with fermion and gluon loops interchanged. Finally, there is also a two-fermion-loop graph renormalizing the ghost-ghost-gluon vertex. This is shown in Fig. 4(e) where the vertex renormalization is obtained by excising and discarding a ghost-ghost-gluon vertex that leaves a connection from the ghost loop to each of the fermion loops. The group factor for Fig. 4(e) is

$$\frac{1}{C_A(N^2-1)}\left(n_f\mathrm{Tr}(T^aT^bT^c)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^aF^bF^c)\right)\left(n_f\mathrm{Tr}(T^cT^d)+\frac{n_{\tilde{g}}}{2}\mathrm{Tr}(F^cF^d)\right)\mathrm{Tr}(F^aF^bF^d)=-\frac{C_5}{4}. \tag{A23}$$

This completes the proof that, when gluinos are included, there are only six linearly independent group factors in the three-loop β function for the SU(N) strong coupling constant, namely, C_A^3 and the five group factors given in Eqs. (8)–(12). This leads to the result stated in Eqs. (7) and (13) of the text.

[1] M. Demarteau, in Proceedings of DPF 96 Meeting, Minneapolis, 1996 (unpublished); J. Hewett, *ibid.*
[2] L. Clavelli, Phys. Rev. D **45**, 3276 (1992).
[3] J. Rosiek, Phys. Rev. D **41**, 3464 (1990).
[4] D. R. T. Jones, Nucl. Phys. **B87**, 127 (1975).
[5] G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972).
[6] G. 't Hooft, Nucl. Phys. **B61**, 455 (1973); W. Bardeen, A. Buras, D. Duke, and T. Muta, Phys. Rev. D **18**, 3998 (1978).
[7] O. V. Tarasov, A. A. Vladimirov, and A. Yu. Zharkov, Phys. Lett. **93B**, 429 (1980).
[8] L. Clavelli and L. R. Surguladze, Phys. Rev. Lett. **78**, 1632 (1997).
[9] K. G. Chetyrkin, Report No. MPI/PHT/96-83, hep-ph/9608480 (unpublished); Report No. hep-ph/9610531 (unpublished).
[10] I. Jack, D. R. T. Jones, and C. G. North, Phys. Lett. B **386**, 138 (1996).
[11] P. M. Ferreira, I. Jack, and D. R. T. Jones, Phys. Lett. B **387**, 80 (1996).
[12] O. V. Tarasov and A. A. Vladimirov, Dubna JINR Report No. E2-80-483, 1980 (unpublished).