Low-energy signatures of semiperturbative unification

Christopher Kolda* and John March-Russell†
School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540

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We consider the low-energy signatures of, and high-energy motivations for, scenarios of semiperturbative gauge coupling unification. Such scenarios can leave striking imprints on the low-energy sparticle spectrum, including novel gaugino mass ratios (including $M_2/M_1 \approx 1$), substantial compression of the intragenerational squark-to-slepton mass ratios, and an overall lifting of scalar masses relative to the gauginos. We also demonstrate that the unification scale can be raised to $M_X \approx 4\times 10^{17}$ GeV while still in the perturbative regime-close to the one-loop heterotic string scale. We employ a three-loop calculation of the running of the gauge couplings as a test of the perturbativity of the high-scale theory. [S0556-2821(97)06107-9]

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I. INTRODUCTION

Two of the most compelling theoretical constructions since the advent of the standard model (SM) have been the concepts of grand unification [1] and supersymmetry (SUSY). However, it is by now well known that unification, specifically gauge coupling unification, and supersymmetry are intimately connected in light of the precise electroweak data from the CERN e^+e^- collider LEP. That the separate gauge couplings of the SU(3)×SU(2)×U(1) SM gauge group unify around the scale 2×10^{16} GeV if the SM is embedded in its supersymmetric extension, the minimal supersymmetric standard model (MSSM), can be taken as the first, albeit indirect, evidence for SUSY. The initial excitement surrounding this result has been replaced with a realistic reappraisal of the details of the calculation, showing that simple unification with a light SUSY spectrum and without modest grand unified theory (GUT) scale corrections predicts $\alpha_3(M_Z)$ to be larger than indicated by either LEP or low-energy data [2]. Nonetheless, there remains a remarkable level of agreement given the large number of model-dependent uncertainties that arise in the calculation.

It is necessary, however, to distinguish which aspects of the MSSM are fundamental to the observation of gauge coupling unification and which are coincidental. For example, consider the addition of extra matter, beyond that of the MSSM, at some arbitrary scale $M_n > M_Z$. Taking $\alpha_1 = \frac{5}{3}\alpha_Y$ and α_2 as measured inputs at the weak scale, one can use the one-loop renormalization group equations (RGE's) for the gauge couplings to yield a prediction of, e.g., the unification scale M_X:

$$\ln\left(\frac{M_X}{M_Z}\right) = \frac{2\pi}{b_2'-b_1'}\left(\frac{1}{\alpha_2}-\frac{1}{\alpha_1}\right) + \frac{\delta b_2-\delta b_1}{b_2'-b_1'}\ln\left(\frac{M_n}{M_Z}\right), \quad (1)$$

where $b_i' = b_i^0 + \delta b_i$ with b_i^0 the MSSM β-function coefficients and δb_i the contributions of the extra matter. A similar form applies for the modified prediction of $\alpha_3(M_Z)$. Thus we recover the very well-known result that, at one loop, new states which shift all three β functions identically (i.e., $\delta b_1 = \delta b_2 = \delta b_3 \equiv \delta b$) leave unchanged the predictions for the strong coupling and for the unification scale. Only the value of the experimentally inaccessible number α_X changes.

The requirement $\delta b_i = \delta b$ is met by adding states with quantum numbers such that they can be thought of as fitting into complete representations of some simple group containing the SM, presumably in conjugate pairs to allow vector-like SU(3)×SU(2)×U(1)-preserving mass terms and for anomaly cancellation. Thus the apparent unification already present in the MSSM is not simply an accident if there exist only complete ''GUT'' representations above the weak scale. From the point of view of gauge coupling unification, the MSSM and these extended variants are on an equivalent footing—there is no *current* experiment that favors one over the other.

That said, in this paper we will show that there can exist potentially dramatic and experimentally observable differences (at the next generation of colliders) between these models, given access to the sparticle spectrum. These predictions will provide a channel for detecting this extra matter through its virtual effects even if it is too heavy to be observed directly. Specifically, we will study intragenerational sparticle mass ratios and gaugino mass ratios as discriminants, under the assumption that they exhibit mass unification at the unification scale typical of supergravity-mediated models of SUSY breaking. That masses in the scalar sector are universal is a strong assumption; however, low-energy flavor-changing neutral current constraints require intergenerational universality, and so intragenerational universality seems motivated. Gaugino mass unification, on the other hand, is very well motivated, in both GUT's and string theory, as we will discuss in greater detail below.

Each of these mass ratios possesses various advantages and disadvantages. The sparticle masses have a quite sensitive dependence on the existence of even relatively small amounts of extra matter, but there are many other contributions to sparticle masses of a fairly generic nature—*D* terms from broken symmetries, Planck-scale corrections to the Kähler potential, etc.—that may make it difficult to disentangle the contributions of the extra matter in a unique way. On the other hand, the gaugino mass ratios, as we will show, are largely immune from corrections arising from unknown

*Electronic address: kolda@sns.ias.edu
†Electronic address: jmr@sns.ias.edu

high-scale physics. But we will also see that gaugino mass ratios differ from their canonical values only slightly in the presence of small amounts of extra matter; however, the ratios can begin to differ significantly once $\alpha_X > \alpha_3(M_Z)$.

It is an inevitable consequence of adding additional matter to the MSSM that α_X increases. Specifically, we will concern ourselves with scenarios of *semiperturbative unification* (SPU) in which matter in complete SU(5) multiplets is added at some intermediate scale $M_n < M_X$ such that $\alpha_X > \alpha_3(M_Z)$; in order to trust our results, however, *we require α_X to remain perturbative in the sense of quantum field theory*. A reasonable test of perturbativity is that the contributions from the $(n+1)$-loop RGE's are small compared to those at n loops. Because we will be working in a regime in which the $(n=1)$-loop contributions can be anomalously small, a test comparing two-loop to one-loop contributions can be misleading. Instead, we will test for perturbativity by comparing the three-loop contributions to those at two loops. This will force $\alpha_X \lesssim 1/2$. Note that we do *not* impose that the gauge coupling remain perturbative all the way up to the Planck (or reduced Planck) scale, since unification into a string theory can occur well below that scale, as we will discuss in the next subsection.

Our primary results are twofold: First, we generalize some recent results [3–5] to show that SPU pushes up the unification scale M_X, sometimes significantly, towards our expectation from string theory. Second, we examine the imprint left on the light SUSY spectrum by SPU and extra matter in general. We will finally consider threshold corrections, particularly those at the high scale.

Throughout, we will present our results both numerically and analytically. Because of the difficulty in finding simple and general analytic expressions when the two-loop contributions compete with the one-loop contributions, we will usually confine our analytic results to the interesting reference case of $b_3 = 0$, where the running of α_3 is entirely two loop in origin; for this case (in which $\alpha_X \simeq 0.22$), it is obvious that α_X is perturbative.

This paper is organized as follows. In the rest of this section, we will discuss SPU scenarios within the context of string theory and the relationship of SPU to the older idea of nonperturbative unification [6]. Those allergic to high-scale handwaving are encouraged to jump to Sec. II in which we briefly address the question of the amount and mass scale of the extra matter needed to achieve SPU of the gauge couplings and discuss the raising of the unification scale at two loops. Our primary results are contained within Sec. III in which we examine the low-energy consequences of SPU in the form of novel gaugino mass ratios and squark and slepton spectra. After our conclusions we include a brief appendix in which the three-loop RGE's are presented for the scenarios considered here, based on the recent work of Jack and co-workers [7,8].

A. SPU from a string perspective

That larger values of the unified coupling α_X may be preferred can be seen if we view the unification of couplings from the perspective of string theory. Probably the most serious phenomenological problem that faces string theory is that of dilaton runaway [9], which seems to be generic to string theories. In short, to all orders of perturbation theory (for a sypersymmetric string theory) the dilaton, whose expectation value sets the size of the gauge and other couplings, has no potential. When ''small'' nonperturbative effects are included (such as gaugino condensation in a hidden gauge group), a potential can be generated, but this potential *must vanish* as the dilaton vacuum expectation value goes to infinity and the theory becomes free. Thus, unless there is a local minimum at some intermediate value of the dilaton expectation value, the dilaton either runs away to a free theory or to a solution with a nonzero cosmological constant, presumably large. In order to generate a local minimum to stabilize the dilaton, one must almost certainly be outside the region of ''small'' nonperturbative effects, so that one can have competing terms. If all nonperturbative effects are of field-theoretic origin [i.e., instantonlike with behavior $\exp(-8\pi^2/g^2)$], then this seems to require very large couplings.

It seems at first to be a disaster that string theory must be strongly coupled in order to describe our universe. Not only does the observed unification within the MSSM predict a small, perturbative value for $g_{\rm string} \sim g_X$, but the recent results on the strong-weak coupling duality suggest that the dilaton runaway problem just reappears in a new guise if we move into the strongly coupled region $g_{\rm string} \gg 1$. Thus, from the duality argument, it appears that at best the value of $g_{\rm string}$ is in the region of intermediate coupling (probably near the electric-magnetic self-dual point $g_{\rm string} \sim \sqrt{2\pi}$), where *field-theoretic* nonperturbative effects are still negligible and cannot stabilize the dilaton.

A possible solution to this conundrum may lie in the observation that coupling strengths which within the context of field theory are perturbatively small can within the context of string theory be nonperturbatively large. In string theory there are expected to be corrections, specifically to the Kähler potential, which grow as fast as $\exp(-a/g)$, where $a \sim 1$ [10]. Thus we might hope that nature has chosen $g_{\rm string}$ such that $\exp(-a/g_{\rm string}) \sim 1$, while $\exp(-8\pi^2/g^2_{\rm string}) \ll 1$, allowing us to approach unification in perturbation theory while still understanding the stabilization of the dilaton. Values of $g_{\rm string}$ within the SPU range $g_3(M_Z) < g_{\rm string} \lesssim \sqrt{2\pi}$ are certainly within this domain.

In string theory there is also the well-known problem of the scale of coupling unification. One expects, for string theory, unification not only among the field-theoretic couplings, but also with gravity [11]. A one-loop calculation within weakly coupled heterotic string theory yields a prediction for the scale at which such unification occurs, the string scale, as a function of the unified coupling, $g_{\rm string}$ [12]:

$$M_{\rm string} = 5.3 g_{\rm string} \times 10^{17}\ \ {\rm GeV}, \tag{2}$$

only about one decade from the MSSM unification scale 2×10^{16} GeV with $\alpha_X \approx 1/25$. But converted to a prediction for $\alpha_3(M_Z)$ within the MSSM, the string result is many standard deviations away from the experimentally observed value. There have been many suggested resolutions to this disagreement [13], including the addition of matter in incomplete SU(5) multiplets, the inclusion of (hopefully large) string-scale threshold corrections, and even nonstandard affine levels for the affine algebras (Kac-Moody algebras), giving rise to the SM gauge interactions. Most recently, the question of the unification scale has been investigated within the context of strongly coupled $E_8 \times E_8$ string theory [14]. The low-energy limit of this theory is 11-dimensional supergravity with the 11th dimension being an interval. The length of this interval is essentially a free parameter which can be fit using α_X, M_X, and Newton's constant (in units of the 11-dimensional Planck length l_{11}), so that the unification scales in the string and field theories correspond. If we take α_X and M_X to be those of the MSSM, then the length of this interval is about $70 l_{11}$, quite large. This in turn has potentially interesting consequences for cosmology, axion dynamics, etc. [15]. However, larger values of α_X and M_X are in no way disfavored by this result; they simply lead to different values for the length of the interval and therefore different phenomenology.

In Sec. II, we will show that in SPU the unification scale is automatically raised by the two-loop effects, approaching in some cases the one-loop prediction of the string scale quite closely.

B. Relation to nonperturbative unification

Finally, we wish to mention the connections and differences of our SPU scenarios with an earlier scheme, nonperturbative unification (NPU), first proposed by Maiani, Parisi, and Petronzio [6] in 1979. The basic idea of NPU is that as more and more states are added to the particle spectrum, the β functions for the gauge couplings increase until a Landau pole at scale Λ develops. If unification occurs in the MSSM, then the amount and mass scale of extra matter can be chosen such that unification occurs right at $M_X = \Lambda$. The value of $g_X = g(\Lambda)$ then becomes irrelevant, all low-scale observables depending only on the scale Λ itself. The weak-scale values of the gauge couplings appear as infrared pseudofixed points of the renormalization group equations (RGE's).

A large number of analyses have been performed of the NPU scheme in the extended MSSM, most very similar in nature. These analyses have three rather generic problems: (i) They assume that the gauge couplings become nonperturbative at the unification scale, which is no more pleasant for string theory than very weak couplings, (ii) their only tool for analyzing the unification is perturbative RGE's, used despite the fact that the unification is supposedly nonperturbative, and (iii) thanks to the nonperturbative nature of the couplings, other observables such as scalar and gaugino masses are assumed to have uncontrollable corrections at the unification scale which prevent any prediction of their values at the weak scale. Thus the only discriminating signal of NPU is to actually find the extra matter through on-shell production. Within SPU, we will see that the coupling strengths necessary in order to render interesting effects at the weak scale observable are of intermediate strength. Moreover, we will have control of the perturbative expansion by checking against the three-loop contributions.

II. SEMIPERTURBATIVE UNIFICATION

In this work, we assume that nature chooses to unify semiperturbatively. Therefore the low-energy values of the gauge couplings which are measured experimentally are by definition close to their infrared pseudofixed point values and have their measured values thanks to some combination of extra matter at some unknown scales and possibly new large Yukawa couplings involving that extra matter. We have no knowledge *a priori* of these dynamics, but hope to study those effects which are independent of the details. Therefore we will be interested in increasing the β functions until the unification scale is pushed close to, but not above, the Landau scale and examining the resulting phenomenology.

All of our methods for analyzing physics within this domain will be perturbative. Each result derived perturbatively must then be checked against some test of perturbativity to ensure its validity. As already mentioned, a good test for a result derived at n loops would be a calculation of the $(n+1)$-loop corrections. This test does not work for $n=1$ for two reasons: The one-loop β function for α_3 is anomalously small in the region of interest, and many of the effects in which we are interested only arise at two loops. Therefore we will use as our test of perturbativity the ratio $|\beta_i^{(3)}/\beta_i^{(2)}|$ for each gauge group i, where $\beta^{(n)}$ is the n-loop gauge β function. For the purposes of this study, in calculating $\beta^{(3)}$ we will set all Yukawa couplings to zero; see the Appendix for a full discussion of the relevant RGE's. We will make the somewhat arbitrary, but reasonable, choice that the perturbative expansion is valid if $|\beta_i^{(3)}(M_X)/\beta_i^{(2)}(M_X)| \lesssim 1/2$ for all i. All of our results are derived under this constraint.

We also, of course, assume that the near unification of the three gauge couplings in the MSSM is not an accident. For the purposes of our calculations, we will denote as M_X the scale at which $\alpha_1(Q) = \alpha_2(Q)$ and determine the corresponding value of $\alpha_3(M_Z)$ as a prediction by running back down to the weak scale. Since we only allow complete GUT multiplets to be added to the MSSM, we know that we cannot disrupt the full unification that occurs in the MSSM by much.[1]

As we do not have control over the specifics of the dynamics which are occurring between the weak and unification scales, we need an appropriate parametrization for describing the unknown effects. If the effective theory at the weak scale is the MSSM, then there are essentially only two degrees of freedom for exploring SPU: the representations of the extra matter and the mass scale at which they couple. Consider the toy case where the new matter is degenerate at the weak scale, $M_n = M_Z$. We can derive bounds on δb by requiring that $1/\alpha_X$ approach zero from above. Thus, at one loop,

$$\delta b \lesssim \left(b_1^0 - \frac{\alpha_2}{\alpha_1} b_2^0 \right) \left(\frac{\alpha_2}{\alpha_1} - 1 \right)^{-1} \simeq 4.6. \tag{3}$$

A **5** and **10** of SU(5) contribute 1/2 and 3/2 to δb, respectively, while a **16** of SO(10) contributes 2. Equation (3) then

[1]The precise value of $\alpha_3(M_Z)$ is not a particularly useful prediction of SPU (or the MSSM for that matter) without considering the corrections at the weak scale, logarithmic and nonlogarithmic, which are known to be large [2]. In this sense, we are not requiring precise unification of α_3 with α_1 and α_2. Furthermore, note that shifts in $\alpha_3(M_Z)$ which arise due to the extra matter at two loops are typically canceled against one-loop threshold contributions from the splittings in the masses of the new matter generated by their anomalous dimensions [3,16].

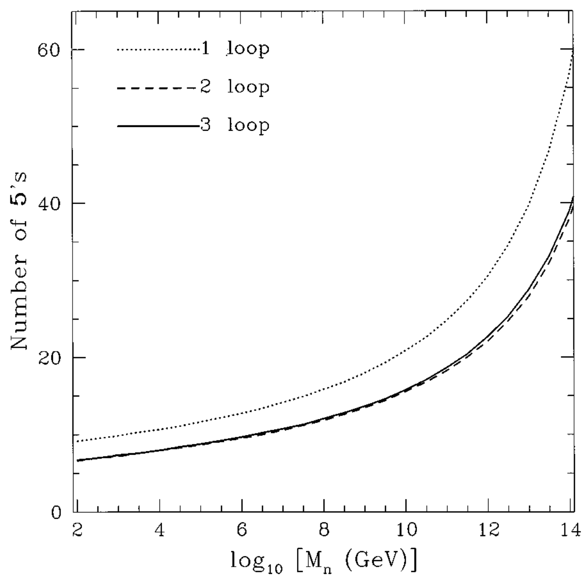

FIG. 1. Dependence of the maximal number of **5**'s on their mass scale, calculated at one, two, and three loops, and with all Yukawa couplings set to zero.

sets the maximum number of additional **5**'s, **10**'s, and **16**'s at 9, 3, and 2, respectively. There are no other representations which can be added at the weak scale. If $M_n \gg M_Z$, then there can be correspondingly more states added or, alternatively, larger GUT representations.

It seems then that an effective number of **5**'s, **10**'s, or **16**'s added to the model at the weak scale may be a good parametrization for studying the new effects of SPU. In particular, we will study how the phenomenology changes as the effective number of representations is increased to the SPU point. We will choose the *effective* number of **5**'s, **10**'s, or **16**'s as the degree of freedom in most cases and differentiate it from the *actual* number n_i by denoting it $n_{i,\mathrm{eff}}$ for $i=5$, 10, or 16. Since we are absorbing not only the number of extra representations into $n_{i,\mathrm{eff}}$ but also their mass scale and other effects (see below), *it is not necessary that $n_{i,\mathrm{eff}}$ be an integer*.

There are two dominant effects which change the number of representations that can or must be added to the MSSM at a given scale M_n to achieve SPU. Two-loop contributions to the RGE's tend to increase the gauge β functions. In the MSSM (with or without extra matter), this decreases the amount of matter that can or must be added compared to the one-loop case. In Fig. 1 we plot the extreme upper bound on n_5 as a function of M_n. (By ''extreme upper bound'' we mean the value beyond which the Landau scale occurs below the unification scale, calculated to the stated order in perturbation theory. This is not to be confused with our usual definition of an upper bound, which requires $|\beta_i^{(3)}/\beta_i^{(2)}|<1/2$ at the unification scale.) Results are shown at one, two, and three loops, where for the purposes of the figure all Yukawa couplings are set to zero. In the one-loop case, one can rescale the y axis to n_{10} or n_{16} using the relation $n_5=3n_{10}=4n_{16}$; the two-loop corrections do not have any such simple scaling. Note that $\delta n \simeq 3$ or 4 in going from one loop to two loops over most of the range of M_n.

Yukawa couplings enter the two-loop β functions with opposite signs from the gauge contributions and therefore slow the running of the gauge couplings, an effect which might either increase or decrease the amount of extra matter which is allowed. One can parametrize this effect as a shift in $n_{i,\mathrm{eff}}$ away from n_i; that is, the effects of the Yukawa couplings mimic extra matter:

$$\beta_i=\alpha_i^2\left(\frac{b_i}{2\pi}-\sum_j \frac{a_{ib}}{32\pi^3}\, y_b^2\right)+O(\alpha^3), \tag{4}$$

for some Yukawa coupling y_b. Ignoring the running of the Yukawas themselves and generalizing Eq. (3), one finds[2]

$$\delta n_{5,\mathrm{eff}}=\sum_b \frac{y_b^2}{8\pi^2}\left(a_{1b}-\frac{\alpha_2}{\alpha_1}\, a_{2b}\right)\left(\frac{\alpha_2}{\alpha_1}-1\right)^{-1}. \tag{5}$$

For Yukawa couplings like those of the SM particles (including the top Yukawa coupling), $n_{5,\mathrm{eff}}<0$, so that more matter can be added to the spectrum and still maintain perturbativity. This is not true for generic Yukawa couplings; in particular, most R-parity-violating couplings lead to $n_{5,\mathrm{eff}}>0$, though their coefficients are usually assumed to be very small.

Although SPU has no effect on the scale of unification at one loop, at two loops this no longer holds. For small values of α_X the two-loop effects are calculable and negligible, but for SPU, the shifts in the unification scale can be substantial. Consider, for example, our reference case of $b_3=0$. It is necessary to solve the α_3 equation

$$\frac{d\alpha_3}{dt}=\frac{6}{\pi^2}\,\alpha_3^3+\cdots\ , \tag{6}$$

where the ellipsis represents subleading terms which go as $\alpha_3^2\alpha_{1,2}$ and are also down by small coefficients. Making the good approximation of dropping these terms, we get the solution

$$\frac{1}{\alpha_3^2(\mu)}\simeq\frac{1}{\alpha_3^2(M_Z)}-\frac{12}{\pi^2}\ln\left(\frac{\mu}{M_Z}\right), \tag{7}$$

and so $\alpha_X>\alpha_3(M_Z)$, as expected. With $\alpha_3(M_Z)=0.12$, this leads to $\alpha_X\simeq 0.22$. In this case the expression for the unification scale becomes

$$M_X\simeq M_X^{(1)}\left[\prod_{i=1,2}\left(\frac{\alpha_X}{\alpha_i}\right)^{(b_{2i}-b_{1i})/2b_i(b_1-b_2)}\right]\times\exp\left[\frac{\pi}{24}\,\frac{b_{23}-b_{13}}{b_1-b_2}\left(\frac{1}{\alpha_3(M_Z)}-\frac{1}{\alpha_X}\right)\right], \tag{8}$$

where $M_X^{(1)}$ is the one-loop unification scale, about $(2\text{–}3)\times 10^{16}$ GeV. Note that, for SPU, $[1/\alpha_3(M_Z)-1/\alpha_X]>0$ and $b_1-b_2=28/5$, but the b_{ij} depend on the type of matter added to set $b_3=0$, i.e., either two **10**'s or six **5**'s. For either case the first factor coming from the U(1) and SU(2) contri-

[2]For each observable the precise definition of $n_{i,\mathrm{eff}}$ in terms of n_i, the new mass scale, and any other effects such as Yukawa couplings differs slightly. This particular definition is appropriate for parametrizing effects of Yukawa couplings on α_X and thus on the amount of extra matter needed for SPU.

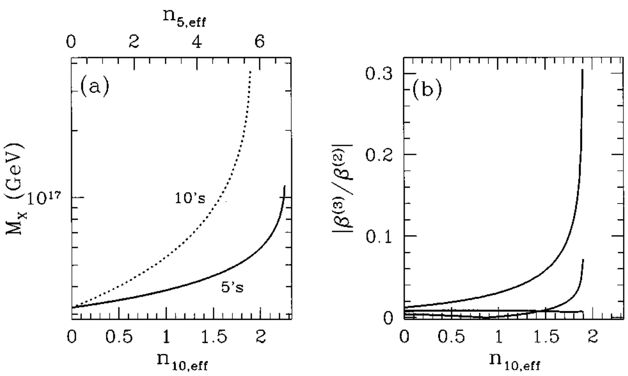

FIG. 2. (a) Dependence of the unification scale on the type and amount of extra matter; (b) ratio of the three-loop to two-loop contributions to the three β functions at the unification scale with extra **10**'s. The ratio for the U(1) coupling provides the strongest constraint, followed by that of the SU(2).

butions raises the unification scale by a factor ~ 4. On the other hand, the exponential [SU(3) contribution] depends strongly on type of additional matter since

$$b_{23}-b_{13}=\begin{cases} 64/5 & \text{if } n_{10}=2, \\ 0 & \text{if } n_5=6, \end{cases} \tag{9}$$

which results in an additional enhancement in Eq. (8) of ~ 3 in the case of **10**'s, but none in the case of the **5**'s. This is due to the presence of (**3**,**2**) states in the decomposition of the **10** which lead to enhanced b_{23} entries in the two-loop β-function coefficients. Thus there is a quantitative difference at two loops between adding (one-loop) equivalent amounts of **5**'s and **10**'s.

In Fig. 2(a) we show a full three-loop numerical calculation of the unification scale as function of $n_{5,\mathrm{eff}}$ and $n_{10,\mathrm{eff}}$. This clearly shows the increase in the unification scale for both **5**'s and **10**'s, and that the increase is more marked in the second case. In line with the analytic estimates in Eq. (8), the unification scale for **10**'s is about a factor of 3 higher than that for **5**'s. It is quite remarkable that the unification scale in these models, especially in the case of **10**'s, approaches quite closely the one-loop heterotic string prediction ($\sim 1\times 10^{18}$ GeV for the appropriate value of g_{string}). Note that this occurs without the introduction of split multiplets or large weak or string scale threshold corrections.

In Fig. 2(b) we plot the ratio of the three-loop term to the two-loop term evaluated at the unification scale for the three SM gauge couplings. Notice that the perturbative expansion breaks down first in β_1, with $|\beta_1^{(3)}/\beta_1^{(2)}|$ reaching values near/below 1/2 as we approach the cutoff in the amount of extra matter. We take this as a strong indication that our perturbative calculations are under control.

As the scale of extra matter increases, more matter is needed to reach SPU. However, the prediction of M_X remains roughly constant, as we have checked numerically. We also note that the case of the **16**'s is intermediate to those of the **5**'s and **10**'s, as one might expect. But there are questions which cannot be addressed within the context of this parametrization which assumes all the extra matter sits at the weak scale. Two such issues are the question of b-τ unification in simple GUT's and the existence of infrared pseudofixed points for the soft masses and couplings which arise in running between the string-Planck scale and a true GUT scale. It is well known that even within the MSSM b-τ unification only occurs near the infrared pseudofixed point of one of the third generation Yukawa couplings and, preferably, for smaller values of $\alpha_3(M_Z)$. Since the Yukawa unification depends on the values of α_3 at all scales between M_Z and M_X, it is clear that the scale of the extra matter is of primary concern. Lanzagorta and Ross [4] have also considered a case similar to SPU in which the extra matter sits near or above the unification scale, so that the running from the GUT scale to the string scale is semiperturbative. There one finds interesting fixed point structures in the soft masses and couplings which then set the boundary condition for further running down to the weak scale. Once again, the scale of the extra matter is of primary importance and so our effective parametrization is not applicable.

III. LOW-ENERGY SIGNATURES

The low-energy signatures of SPU on which we focus all involve changes to the spectra of sparticle masses at the weak scale. Further, all the statements that we make in this regard will be in the context of supergravity- (SUGRA-) mediated supersymmetry-breaking scenarios [17]. The reason for this restriction is that the main effect of the additional matter on sparticle masses will be radiative, through the modified RGE running of the soft SUSY-breaking parameters, and this only occurs if the soft masses are induced above the scale of the additional matter. Therefore we will, for example, have nothing to say about the case of gauge-mediated SUSY breaking where the scale at which the soft terms are induced in the observable sector is very close to the scale of the additional messenger matter.

In line with the usual assumptions, we will take the soft terms induced at the unification scale to be *universal* in form. This is certainly a strong assumption for the soft scalar masses, but one of our main points will be that the usual low-energy predictions of such a scenario can be greatly altered even without violations of universality at the high scale. Furthermore, for the *gaugino* masses the universality assumption is relatively mild, as we will review below.

Probably the single most interesting and distinctive signature of SPU, at least near the upper limit of the allowed range of unified coupling, is the change in the low-energy gaugino mass ratios. Recall the usual situation within SUGRA-mediated SUSY breaking, where at the unification scale we expect universal gaugino masses

$$M_1(M_x)=M_2(M_x)=M_3(M_x)\equiv M_{1/2}. \tag{10}$$

Given this boundary condition, the low-energy ratios are determined by the running from M_X down to the weak scale. The two-loop RGE's for the gaugino masses are very close in form to those of the gauge couplings:

$$\frac{dM_i}{dt}=\frac{b_i}{2\pi}\alpha_i M_i+\frac{b_{ij}}{8\pi^2}\alpha_i\alpha_j(M_i+M_j)$$
$$+\frac{\alpha_{ic}}{32\pi^3}\alpha_i y_c^2 M_i+\cdots , \tag{11}$$

where the ellipsis represents A-term contributions which can be shown to be small both in the MSSM and with extra matter (see the work of Yamada in Ref. [18]). The β-function coefficients b_i, b_{ij}, and a_{ic} are equal to those for the gauge couplings given in the Appendix [18]. Such a form for the gaugino RGE's implies that the ratio

$$R_i \equiv M_i/\alpha_i \tag{12}$$

satisfies the equation

$$\frac{dR_i}{dt} = \frac{b_{ij}}{8\pi^2}\,\alpha_j^2 R_j + \cdots , \tag{13}$$

where again the ellipsis represents the small A-term contributions. In other words, the ratio is constant at one loop, but runs at two loops. In the case of the unextended MSSM, the change in the ratio due to the two-loop term is quite small, and we get the standard result that, at the weak scale,

$$M_i/M_j = \alpha_i/\alpha_j , \tag{14}$$

up to relatively small weak-scale threshold corrections and conversions from dimensional reduction with modified minimal subtraction ($\overline{\mathrm{DR}}$) masses to pole masses. Thus $M_3:M_2:M_1 \sim 7.4:2.0:1.0$.

In the SPU scenario these ratios can change dramatically. In our reference case with $b_3=0$, the equation for R_3 has an $\alpha_3^2 R_3$ term with large coefficient and $\alpha_i^2 R_i$ terms ($i=1,2$) that are suppressed both by relatively small coefficients and the fact that below M_X the U(1) and SU(2) couplings decrease quite quickly. Keeping just the dominant $\alpha_3^2 R_3$ term leads to the expression (valid to roughly 10%)

$$R_3(M_Z) \simeq \frac{M_{1/2}}{\alpha_X}\left(\frac{\alpha_3(M_Z)}{\alpha_X}\right). \tag{15}$$

The term in parentheses is the modification to the usual result and amounts to a 40% change in the predicted value of R_3 as compared to the MSSM.

For the other ratios $R_{1,2}$, a similar approximation scheme is applicable. The $\alpha_3^2 R_3$ term again dominates for most of the running, although the $\alpha_{1,2}^2 R_{1,2}$ terms can provide a numerically significant correction near the unification scale. These can be simply dealt with by substituting in the one-loop expressions and integrating. The general form of the prediction for the ratios R_1 and R_2 is then

$$R_i(M_Z) \simeq \frac{M_{1/2}}{\alpha_X}\left\{1 - B_i - \sum_{j=1,2}\frac{b_{ij}}{4\pi b_j}\,\alpha_X\right\} + B_i R_3(M_Z), \tag{16}$$

where the constants $B_i = b_{i3}/b_{33}$ are ratios of 2 two-loop β-function coefficients that depend upon the type of extra matter. We have also dropped small correction terms of order $M_{1/2}\alpha_i(M_Z)/4\pi\alpha_X$. The form of Eq. (16) is actually *valid beyond the particular case of* $b_3=0$—it is the generalization of the usual one-loop relation $R_i = M_{1/2}/\alpha_X$ to two loops.

For R_2 in the specific case of $b_3=0$, this leads to the prediction

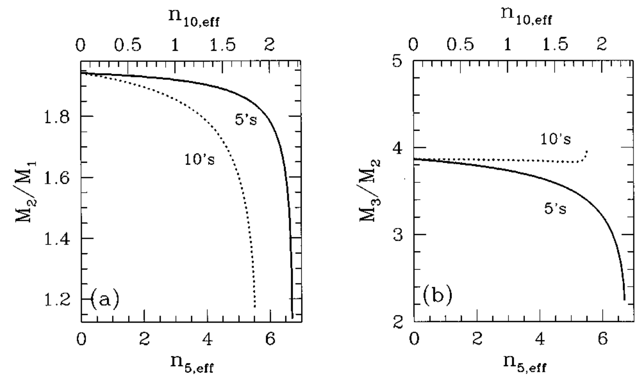

FIG. 3. Ratios of the gaugino masses (a) M_2/M_1 and (b) M_3/M_2 as functions of the amount and type of extra matter. Solid lines are for the case of additional **5**'s, dotted lines for **10**'s.

$$R_2(M_Z) \simeq \frac{M_{1/2}}{\alpha_X}\{1 - B_2 - A_2\alpha_X\} + B_2 R_3(M_Z), \tag{17}$$

where the constants A_2 and B_2 take on the values $281/96\pi$ and 5/6 ($95/32\pi$ and 1/2) in the **10** (**5**) case, respectively.

The ratio R_1 may be handled in an identical way, leading to the relation

$$R_1(M_Z) = \frac{M_{1/2}}{\alpha_X}\{1 - B_1 - A_1\alpha_X\} + B_1 R_3(M_Z), \tag{18}$$

in the $b_3=0$ case, with $A_1 = 337/480\pi$ ($147/160\pi$) and $B_1 = b_{13}/b_{33} = 17/30$ (1/2), in the **10** (**5**) case.

In Fig. 3 we show the results of the numerical evaluation of the low-energy gaugino mass ratios (a) M_2/M_1 and (b) M_3/M_2 as a function of $n_{5,\mathrm{eff}}$ and $n_{10,\mathrm{eff}}$. The most remarkable feature of the figure is that with SPU one can have large changes in the gaugino mass ratios away from the canonical values of α_i/α_j. In particular, M_2/M_1 can approach unity, depending on the type of extra matter. This has strong phenomenological consequences. One of the neutralinos of the MSSM can be essentially photino like, rather than B-ino like as is usually assumed. If that neutralino is the lightest SUSY particle, then it can be the dark matter in the universe with properties markedly different than one might expect from B-ino-like dark matter [19]. For example, because it lacks a coupling to the Higgs boson, it does not self-annihilate as efficiently, resulting in a higher relic density than for a similarly massive B-ino.

As a way to differentiate the cases of the **5**'s and **10**'s, it would be useful to have access to the M_3/M_2 mass ratio. Figure 3(b) reveals that as one approaches SPU, M_3/M_2 remains constant or slightly increases for **10**'s, while it decreases for **5**'s; the behavior for the **5**'s is well described by Eqs. (15)–(18), but that of the **10**'s requires a much more detailed analytic analysis because of cancellations among competing terms. [Note that Fig. 3 was made assuming that $\alpha_3(M_Z)$ is brought back to the experimentally measured values using threshold effects.]

One may worry that these predictions for the gaugino mass ratios suffer from large uncertainties due to threshold corrections, either at the low or, especially, at the high scale, due to the large amount of matter present. However, this is not the case. Part of the reason is obvious: Because the one-loop runnings of the gaugino masses and the gauge couplings are identical, all logarithmically enhanced threshold terms such as

$$\frac{\alpha_X}{4\pi}\ln\left(\frac{M_V^2}{M_X^2}\right) \quad \text{or} \quad \frac{\alpha_X}{4\pi}\ln\left(\frac{M_c^2}{M_X^2}\right), \tag{19}$$

where M_V (M_c) is the mass of some superheavy vector (chiral) multiplet, cancel in the ratio M_i/α_i. Actually, the situation is even better than this. The one-loop nonlogarithmically enhanced threshold corrections to the gaugino masses have been calculated in Ref. [20], resulting in the expression

$$\frac{M_i(\mu)}{\alpha_i(\mu)} = \frac{M_{1/2}}{\alpha_X} + \frac{1}{4\pi}\left\{2T_i^{(V)}[M_{1/2}(\mu)-\delta m] + \sum_{c=\text{chiral}} T_i(R_c)B_c\right\}, \tag{20}$$

where δm is the mass of the fermion component of the Nambu-Goldstone multiplet induced by SUSY breaking [and is $O(M_Z)$], the B_c are the standard B terms for the chiral multiplets, and the T_i group factors are defined in the Appendix. In the case of universal scalar mass terms (and B parameters), the contributions of complete GUT multiplets to $T_i(R_c)B_c$ add equally to each ratio M_i/α_i and are thus harmless in the M_i/M_j ratios. The only nonvanishing contribution from chiral multiplets in the universal case arises from heavy Higgs triplets and is, thus, independent of the amount of extra matter. Similarly, there is a small correction arising from the heavy *vector* multiplets, whose contribution only depends on the gauge structure of the underlying theory. The final result is a total threshold correction to the gaugino mass ratios of only a few percent, independent of any unsplit chiral multiplets at the high scale. Thus high-scale field-theoretic corrections to our expressions are generically under control.

Gaugino mass unification is also a generic prediction of string theory. One-loop perturbative string threshold corrections to universality have been considered and argued to be small except in the limit where the moduli F terms are much larger than those of the dilaton. There is also the question of nonperturbative corrections to gaugino masses in string theory. Banks and Dine [21] have argued that through a combination of the holomorphy of the gauge kinetic functions f_i and discrete gauged subgroups of Peccei-Quinn symmetries, one can show that string nonperturbative corrections to gaugino masses behave as $\exp(-8\pi^2/g^2)$, of the same order as field-theoretic nonperturbative effects, which we are by the definition of SPU taken to be small. Therefore string-induced corrections to our expressions are also generically under control.

We now turn to the other major low-energy signal for a larger value of the unified coupling, the squark and slepton spectrum. One interesting feature of the modified spectrum is that as a function of the amount of additional matter, the shifts in the squark and slepton masses occur well before the maximal SPU point is reached. It is therefore sufficient to consider only the one-loop equations for the running of these parameters to gain a good understanding of the changes. In our analytic results we will also concentrate on the first two generations of squarks and sleptons so as to avoid complications due to the large top Yukawa coupling; however, our numerical results include all MSSM Yukawa contributions. The general form for the one-loop squark and slepton RGE's is

$$\frac{dm_c^2}{dt} = -\frac{1}{2\pi}\sum_i \gamma_i^{(c)}\alpha_i M_i^2, \tag{21}$$

where the anomalous dimension coefficients $\gamma_i^{(c)}$ are not modified by the addition of extra matter. They take the values $\gamma^{(Q)}=(1/15,3,16/3)$ for the $\widetilde{Q}$ squark doublets, $\gamma^{(u)}=(16/15,0,16/3)$ for the $\widetilde{u}_R$ squarks, $\gamma^{(d)}=(4/15,0,16/3)$ for the $\widetilde{d}_R$ squarks, $\gamma^{(L)}=(3/5,3,0)$ for the $\widetilde{L}$ slepton doublets, and $\gamma^{(e)}=(12/5,0,0)$ for the $\widetilde{e}_R$ sleptons. The only dependence on n_5 and n_{10} comes through the running of α_i and M_i^2. In the case of the MSSM, we can solve these RGE's by using the one-loop relation $M_i/\alpha_i = M_{1/2}/\alpha_X$, leading to

$$m_c^2(M_Z) = m_0^2 + \sum_{i=1}^{3}\frac{\gamma_i^{(c)}}{2b_i}M_{1/2}^2\left\{1-\frac{\alpha_i^2(M_Z)}{\alpha_X^2}\right\}, \tag{22}$$

where we have assumed $b_i \neq 0$ and m_0 is the soft SUSY-breaking scalar mass communicated by supergravity at the high scale (assumed universal for simplicity). Explicitly,

$$m_{\widetilde{Q}}^2 = m_0^2 + M_{1/2}^2\{-\tfrac{8}{9}(1-\alpha_3^2/\alpha_X^2) + \tfrac{3}{2}(1-\alpha_2^2/\alpha_X^2) + \tfrac{1}{198}(1-\alpha_1^2/\alpha_X^2)\}, \tag{23}$$

$$m_{\widetilde{u}}^2 = m_0^2 + M_{1/2}^2\{-\tfrac{8}{9}(1-\alpha_3^2/\alpha_X^2) + \tfrac{8}{99}(1-\alpha_1^2/\alpha_X^2)\}, \tag{24}$$

$$m_{\widetilde{d}}^2 = m_0^2 + M_{1/2}^2\{-\tfrac{8}{9}(1-\alpha_3^2/\alpha_X^2) + \tfrac{2}{99}(1-\alpha_1^2/\alpha_X^2)\}, \tag{25}$$

for the first two generation squarks, and

$$m_{\widetilde{L}}^2 = m_0^2 + M_{1/2}^2\{\tfrac{3}{2}(1-\alpha_2^2/\alpha_X^2) + \tfrac{1}{22}(1-\alpha_1^2/\alpha_X^2)\}, \tag{26}$$

$$m_{\widetilde{e}}^2 = m_0^2 + M_{1/2}^2\{\tfrac{2}{11}(1-\alpha_1^2/\alpha_X^2)\}, \tag{27}$$

for the sleptons. An important qualitative feature of these solutions in the MSSM is that the SU(3) terms dominate because of the large ratio $\alpha_i^2(M_Z/)\alpha_X^2 \sim 9.8$. This enhancement of the SU(3) contributions relative to those arising from SU(2) and U(1) is due to the fact that with the standard MSSM spectrum the color coupling is still asymptotically free, while the SU(2) and U(1) couplings are not. Therefore, as we approach the point where α_3 is no longer asymptotically free, we expect a substantial compression of the squark and slepton spectrum. Specifically, the form of the contribution due the SU(3) quantum numbers of the states for our reference case ($b_3=0$) is modified to

$$\Delta_3 m_c^2(M_Z) = \frac{\gamma_3^{(c)}\pi}{36}\frac{M_{1/2}^2}{\alpha_X}\left\{1-\frac{\alpha_3^3(M_Z)}{\alpha_X^3}\right\}. \tag{28}$$

This leads to a prediction for the masses in the $b_3=0$ case (equations on the left) as compared to the MSSM (equations on the right) of

$$\begin{aligned}
m_{\tilde{Q}}^2&=m_0^2+(2.1)M_{1/2}^2 & & m_{\tilde{Q}}^2=m_0^2+(7.2)M_{1/2}^2,\\
m_{\tilde{u}}^2&=m_0^2+(1.8)M_{1/2}^2 & & m_{\tilde{u}}^2=m_0^2+(6.7)M_{1/2}^2,\\
m_{\tilde{d}}^2&=m_0^2+(1.75)M_{1/2}^2 & \text{vs} \quad & m_{\tilde{d}}^2=m_0^2+(6.7)M_{1/2}^2,\\
m_{\tilde{L}}^2&=m_0^2+(0.4)M_{1/2}^2 & & m_{\tilde{L}}^2=m_0^2+(0.5)M_{1/2}^2,\\
m_{\tilde{e}}^2&=m_0^2+(0.12)M_{1/2}^2 & & m_{\tilde{e}}^2=m_0^2+(0.15)M_{1/2}^2.
\end{aligned} \tag{29}$$

This compression of the squarks down towards the sleptons is a general signature of SPU that sets in well before the nonperturbative limit at the high scale is reached and does not depend on large two-loop contributions.

However, to assess the overall scale of the squark and slepton spectrum, it is necessary to reexpress $M_{1/2}$ in terms of the physically observable gaugino masses. In particular, from Eq. (15) we find that (in the case $b_3=0$) $M_{1/2}\simeq 2.9M_3$ (versus $M_{1/2}\simeq 0.33M_3$ in the MSSM). Written in terms of the gluino mass parameter, Eqs. (29) become

$$\begin{aligned}
m_{\tilde{Q}}^2&=m_0^2+(17.6)M_3^2 & & m_{\tilde{Q}}^2=m_0^2+(2.4)M_3^2,\\
m_{\tilde{u}}^2&=m_0^2+(15.1)M_3^2 & & m_{\tilde{u}}^2=m_0^2+(2.2)M_3^2,\\
m_{\tilde{d}}^2&=m_0^2+(14.7)M_3^2 & \text{vs} \quad & m_{\tilde{d}}^2=m_0^2+(2.2)M_3^2,\\
m_{\tilde{L}}^2&=m_0^2+(3.4)M_3^2 & & m_{\tilde{L}}^2=m_0^2+(0.17)M_3^2,\\
m_{\tilde{e}}^2&=m_0^2+(1.0)M_3^2 & & m_{\tilde{e}}^2=m_0^2+(0.05)M_3^2.
\end{aligned} \tag{30}$$

Given the bound on the gluino mass of roughly 180 GeV from the Collider Detector at Fermilob (CDF) [22] (this is in the limit of heavy squarks which is appropriate here), we find that first two generations of squark doublets have a mass of at least 750 GeV in our reference case (see also Ref. [3]), while the sleptons range in mass from 180 to 330 GeV, always assuming $m_0=0$. (The right-handed top squark, because of the large top quark Yukawa coupling, has a reduced mass relative to the other squarks—numerically, we find its lower bound to be 500 GeV, ignoring left-right mixing.) Therefore the squark and slepton spectrum has to be heavier than is apparent in Eqs. (29).

The essential physics is demonstrated in Fig. 4, where we have taken a constant value of $M_3=200$ GeV and $\tan\beta=2$ and shown how the scalar masses change as a function of the amount of extra **5**'s. (The case of extra **10**'s is essentially identical.) In the figure are plotted the first- and second-generation squarks and sleptons and the right-handed top squark $\tilde{t}$, which falls significantly below the other squarks due to the large top Yukawa coupling. The figure clearly shows the overall lifting of the scalar masses with respect to the gauginos; the compression of the scalar mass ratios is also present, but more difficult to see.

It is well known that in the MSSM electroweak symmetry breaking (EWSB) is induced when the $(\text{mass})^2$ of one of the Higgs doublets is driven negative by radiative effects enhanced by the large top quark Yukawa coupling. We have studied this question numerically and found that this physics is qualitatively unaffected by the inclusion of extra matter. In particular, the $|m_{H_u}^2|$ scales with $n_{5,\text{eff}}$ in a similar fashion as the other scalars. Therefore we expect the value of μ in the MSSM superpotential to be much larger than the gaugino masses, so that the lightest SUSY state in this SPU scenario will be a neutralino, which is dominantly photinolike.

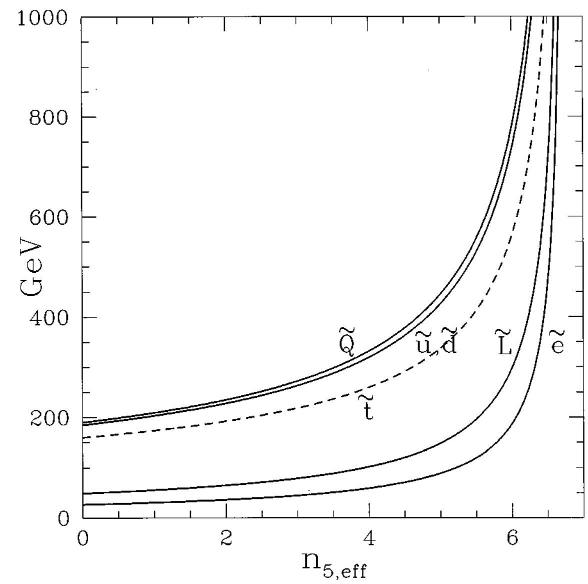

FIG. 4. Squark and slepton masses as a function of the number of additional **5**'s, for constant $M_3=200$ GeV and $m_0=0$. The $\tilde{t}$ is shown as a dashed line. The $\tilde{u}$ and $\tilde{d}$ contours are coincident.

IV. CONCLUSIONS

In this paper we have considered the possibility that the gauge couplings unify within the semiperturbative regime at high scales. Although such scenarios are from an experimental viewpoint currently on an equivalent footing to the MSSM, we showed that they can lead to striking experimental signatures. In contrast to previous studies of nonperturbative unification, we have been able to make reliable predictions in our scenario by utilizing the three-loop gauge coupling RGE's as a test of the sensitivity of our predictions to higher-loop effects.

The addition of extra matter changes the usual spectrum of scalar masses which one derives from minimal supergravity-mediated models of SUSY breaking and, more surprisingly, shifts significantly the relations among the gaugino masses at the weak scale. In particular, we find that $M_2/M_1\simeq 1$ can be achieved for values of the unified coupling for which the field theory is still perturbative. This can lead to a host of phenomologically interesting effects coming from the photinolike nature of the lightest SUSY state. Interestingly, it may be possible to use some observables (e.g., M_3/M_2) as discriminants among the various types of extra matter.

We have also demonstrated that a generic prediction of SPU is the raising of the unification scale well above the canonical value of the MSSM. For **10**'s, in particular, we find $M_X\approx 4\times 10^{17}$ GeV, remarkably close to the one-loop string unification scale.

Overall, we find that the idea of SPU is both motivated and potentially testable at the next generation of colliders through its novel effects on the sparticle spectrum.

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APPENDIX

The form of the three-loop β functions is

$$\frac{d\alpha_i}{dt}=\alpha_i^2\left\{\frac{b_i}{2\pi}+\frac{1}{8\pi^2}(b_{ij}\alpha_j+a_{ia}\tilde{y}_a)+\frac{1}{32\pi^3}(b_{ijk}\alpha_j\alpha_k+c_{ija}\alpha_j\tilde{y}_a+a_{iab}\tilde{y}_a\tilde{y}_b)\right\}, \tag{A1}$$

where $\tilde{y}_a=y_a^2/4\pi$. In the case of an additional n_5 **5**'s and n_{10} **10**'s, the one- and two-loop coefficients are well known:

$$b_i=[33/5,1,-3]+\tfrac{1}{2}(n_5+3n_{10})[1,1,1] \tag{A2}$$

and

$$b_{ij}=\begin{bmatrix}199/25 & 27/5 & 88/5\\ 9/5 & 25 & 24\\ 11/5 & 9 & 14\end{bmatrix}+n_5\begin{bmatrix}7/30 & 9/10 & 16/15\\ 3/10 & 7/2 & 0\\ 2/15 & 0 & 17/3\end{bmatrix}+n_{10}\begin{bmatrix}23/10 & 3/10 & 24/5\\ 1/10 & 21/2 & 8\\ 3/5 & 3 & 17\end{bmatrix}. \tag{A3}$$

The a_{ia} can be found in the literature and do not change as additional matter is added.

The three-loop coefficients in the dimensional reduction with modified minimal subtraction ($\overline{\mathrm{DR}}$) scheme for a simple gauge group have been recently calculated in Ref. [7]. This was extended to the MSSM in Ref. [8]; expressions for the a_{iab}, b_{ijk}, and c_{ija} in the MSSM (or in the MSSM with extra **16**'s) can be easily extracted from the explicit expressions given there. For the purposes of this study, we are setting the Yukawa contributions in the three-loop RGE's to zero, keeping only the pure gauge pieces.

That there is a scheme dependence to the coefficients of the three-loop β functions is well known. Our choice of $\overline{\mathrm{DR}}$, however, is the natural one, since it is within this scheme that gauge and gaugino unifications are expected to hold.

The three-loop gauge contributions to the RGE's for a product group can be written in the $\overline{\mathrm{DR}}$ scheme as

$$\left.\frac{d\alpha_i}{dt}\right|_{3\ \mathrm{loop}}=\frac{\alpha_i^2}{32\pi^3}\left\{\alpha_i^2 b_i C(G_i)[4C(G_i)-b_i]+8C(G_i)\sum_{a,j}\alpha_i\alpha_j T_i(R_a)C_j(R_a)-6\sum_{a,j}\alpha_j^2 b_j T_i(R_a)C_j(R_a)-8\sum_{a,j,k}\alpha_j\alpha_k T_i(R_a)C_j(R_a)C_k(R_a)\right\}, \tag{A4}$$

where i,j,k label gauge groups and a labels matter representations. The Casimir invariants have the usual definitions

$$C_i(R)\delta_m^n\equiv(\mathbf{t}_i^A\mathbf{t}_i^A)_m^n,\quad T_i(R)\delta^{AB}\equiv\mathrm{Tr}_R(\mathbf{t}_i^A\mathbf{t}_i^B), \tag{A5}$$

with $\mathbf{t}_i$ the generators of gauge group i. In our normalization, for SU(N), $C(G)=N$, $T(R)=\frac{1}{2}$ for a fundamental and $C(R)=\frac{3}{4}$ or $\frac{4}{3}$ for a fundamental of SU(2) or SU(3).

Plugging into the general form for the MSSM with additional **5**'s and **10**'s, one finds the b_{ijk} to be

$$\begin{aligned}b_{1jk}\alpha_j\alpha_k={}&\left(-\tfrac{32117}{375}-\tfrac{7507}{900}n_5-\tfrac{12859}{300}n_{10}-\tfrac{7}{40}n_5^2-\tfrac{207}{40}n_{10}^2-\tfrac{9}{4}n_5n_{10}\right)\alpha_1^2+\left(-\tfrac{81}{5}-\tfrac{27}{4}n_5-\tfrac{261}{20}n_{10}-\tfrac{27}{40}n_5^2-\tfrac{27}{40}n_{10}^2-\tfrac{9}{4}n_5n_{10}\right)\alpha_2^2\\&+\left(\tfrac{484}{15}-\tfrac{506}{45}n_5-\tfrac{154}{5}n_{10}-\tfrac{4}{5}n_5^2-\tfrac{54}{5}n_{10}^2-6n_5n_{10}\right)\alpha_3^2+\left(-\tfrac{69}{25}-\tfrac{27}{50}n_5-\tfrac{1}{50}n_{10}\right)\alpha_1\alpha_2+\left(-\tfrac{1096}{75}-\tfrac{64}{225}n_5-\tfrac{344}{75}n_{10}\right)\alpha_1\alpha_3\\&+\left(-\tfrac{24}{5}-\tfrac{8}{5}n_{10}\right)\alpha_2\alpha_3,\end{aligned}$$

$$\begin{aligned}b_{2jk}\alpha_j\alpha_k={}&\left(-\tfrac{457}{25}-\tfrac{441}{100}n_5-\tfrac{1513}{300}n_{10}-\tfrac{9}{40}n_5^2-\tfrac{9}{40}n_{10}^2-\tfrac{3}{4}n_5n_{10}\right)\alpha_1^2+\left(35-\tfrac{33}{4}n_5-\tfrac{99}{4}n_{10}-\tfrac{13}{8}n_5^2-\tfrac{117}{8}n_{10}^2-\tfrac{39}{4}n_5n_{10}\right)\alpha_2^2\\&+\left(44-18n_5-\tfrac{118}{3}n_{10}-18n_{10}^2-6n_5n_{10}\right)\alpha_3^2+\left(\tfrac{9}{5}+\tfrac{3}{10}n_5+\tfrac{1}{10}n_{10}\right)\alpha_1\alpha_2+\left(-\tfrac{8}{5}-\tfrac{8}{15}n_{10}\right)\alpha_1\alpha_3+(24+8n_{10})\alpha_2\alpha_3,\end{aligned}$$

$$\begin{aligned}b_{3jk}\alpha_j\alpha_k={}&\left(-\tfrac{1702}{75}-\tfrac{2689}{900}n_5-\tfrac{3353}{300}n_{10}-\tfrac{1}{10}n_5^2-\tfrac{27}{20}n_{10}^2-\tfrac{3}{4}n_5n_{10}\right)\alpha_1^2+\left(-27-\tfrac{27}{4}n_5-\tfrac{117}{4}n_{10}-\tfrac{27}{4}n_{10}^2-\tfrac{9}{4}n_5n_{10}\right)\alpha_2^2\\&+\left(\tfrac{347}{3}+\tfrac{215}{9}n_5+\tfrac{215}{3}n_{10}-\tfrac{11}{4}n_5^2-\tfrac{99}{4}n_{10}^2-\tfrac{33}{2}n_5n_{10}\right)\alpha_3^2+\left(-\tfrac{3}{5}-\tfrac{1}{5}n_{10}\right)\alpha_1\alpha_2\\&+\left(\tfrac{22}{15}+\tfrac{4}{45}n_5+\tfrac{2}{5}n_{10}\right)\alpha_1\alpha_3+(6+2n_{10})\alpha_2\alpha_3.\end{aligned} \tag{A6}$$

The MSSM is recovered for $n_5=n_{10}=0$.

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