Application of sum rules to heavy baryon masses

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Model-independent sum rules are applied to recent measurements of heavy c-baryon and b-baryon masses. The sum rules are generally satisfied to the same degree as for the light (u,d,s) baryons. [S0556-2821(97)03601-1]

Recent measurements of the masses of a number of heavy baryons (with c or b quarks) now make it possible to test model-independent sum rules that were derived some time ago using fairly minimal assumptions within the quark model. [1,2] The sum rules depend on standard quark model assumptions and the additional assumption that the interaction energy of a pair of quarks in a particular spin state does not depend on which baryon the pair of quarks is in ("baryon independence"). This is a weaker assumption than full SU(3) symmetry of the wave function, which would require each individual wave function to be SU(3) symmetrized. Instead, we use wave functions with no SU(3) symmetry as described in Ref. [3]. No assumptions are made about the type of potential, and no internal symmetry beyond baryon independence is assumed. The sum rules allow any amount of symmetry breaking in the interactions and individual wave functions, but do rest on baryon independence for each quark-quark interaction energy. A more detailed discussion of the derivation of the sum rules is given in Ref. [1]. In a previous paper, we applied an isospin breaking sum rule to the Σ_c charge states [2]. In the present paper we test sum rules that connect baryon states of different isospin, which we characterize as medium strong energy difference sum rules.

Before looking at the heavy baryon masses, we review the application of the model independent sum rules to the light baryons. There, the following sum rules have been derived [3]

$$\tfrac{1}{3}(\Omega^- - \Delta^{++}) \;=\; \Xi^{*0} - \Sigma^{*+} = \;\; \Xi^0 - \Sigma^+, \quad (1)$$

$$(147 \pm 1) \qquad (149 \pm 1) \qquad (125)$$

$$2\bar{N} + 2\bar{\Xi} - 3\Lambda - \bar{\Sigma} \;=\; \Omega^- + \bar{\Delta} - \overline{\Sigma^*} - \overline{\Xi^*}. \quad (2)$$

$$(-26) \qquad\qquad (-14 \pm 2)$$

The baryon symbol has been used for its mass. A star indicates spin 3/2, and a bar over the symbol represents an average over the particular isospin multiplet. Where specific charges are indicated, these could be changed using the isospin breaking sum rules in Ref. [1]. The experimental value in MeV for each sum is given below each equation.

The Gell-Mann–Okubo formula and equal spacing in the decuplet, which would follow if SU(3) symmetry were broken only by an octet component, correspond to each side of Eq. (2) being zero. The deviations of each side in Eq. (2)

from zero indicate that the light baryon interactions break that assumption (which is not required for the sum rules) on the order of about 10 MeV. The deviations between the two sides in Eq. (2), and by the $\Xi^0 - \Sigma^+$ term compared to the other expressions in Eq. (1), indicate that the light baryon wave functions also violate our baryon independence assumption to the extent of about 10–20 MeV in the masses. We note that this breaking of baryon independence occurs for the two cases where baryons of different spin are compared. Similar deviation should be expected in the heavy baryon sum rules, even if the symmetry breaking heavy quark interactions are suppressed by the heavy quark mass. This is because the light quark two-body interactions remain the same when one light quark is replaced by a charmed (or bottom) quark.

For extension to heavy baryons, it is convenient to combine Eqs. (1) and (2) in the form

$$(\Delta^+ - p) \;=\; (\Sigma^{*0} - \Sigma^0) + \tfrac{3}{2}(\Sigma^0 - \Lambda^0)$$

$$(297) \qquad\qquad (307)$$

$$= (\overline{\Xi^*} - \bar{\Xi}) + \tfrac{3}{2}(\Sigma^0 - \Lambda^0). \quad (3)$$

$$(330)$$

Equation (3) shows a spread of ~ 30 MeV corresponding to breaking of that order of baryon independence in the light baryons. This demonstrates the ambiguity that arises when the sum rules are extended to heavy baryons.

In Table I, we list the measured heavy baryon masses that will be used in the sum rules. We indicate the expected baryon assignments in Table I. The $\Xi_c'^+$ is the spin 1/2 usc

TABLE I.   Heavy baryon masses used in the sum rules.

Baryon	Mass (MeV)	Reference
Σ_c^{++}	$\Lambda_c^+ + 168.0 \pm 0.3$	[4]
Σ_c^+	$\Lambda_c^+ + 168.7 \pm 0.4$	[4]
Σ_c^{*++}	$\Lambda_c^+ + 245 \pm 7$	[5]
$\Xi_c'^+$	2563 ± 15	[6]
Ξ_c^{*0}	2643.3 ± 2.2	[7]
Ω_c^0	2700 ± 3	[8]
Σ_b^-	$\Lambda_b^0 + 173 \pm 9$	[9]
Σ_b^{*-}	$\Lambda_b^0 + 229 \pm 9$	[9]

baryon having the *u*-*s* quarks in a spin 1 state. We extend Eq. (3) to charmed baryons by changing the *s* quark into a *c* quark. This leads to [10]

$$(\Sigma^{*0}-\Lambda^0)+\tfrac{1}{2}(\Sigma^0-\Lambda^0)=(\Sigma_c^{*+}-\Lambda_c^+)+\tfrac{1}{2}(\Sigma_c^+-\Lambda_c^+).$$
(4)

$$(307) \qquad\qquad (330\pm7)$$

We have used the light Σ baryons for the left hand side of Eq. (4). Use of other combinations could change the left hand side to 297 or 330 MeV, as indicated in Eq. (3). Equation (4) is written in terms of differences from the Λ_c mass which is how the Σ_c and Σ_c^* masses are measured. We have had to use the measured Σ_c^{*++} mass for the Σ_c^{*+} mass in Eq. (4), but that difference is probably small.

Changing the *c* quark in any *c*-baryon sum rule to a *b* quark leads to the corresponding sum rule for *b* baryons. Applying this to Eq. (4) leads to

$$(\Sigma^{*0}-\Lambda^0)+\tfrac{1}{2}(\Sigma^0-\Lambda^0)=(\Sigma_b^{*0}-\Lambda_b^0)+\tfrac{1}{2}(\Sigma_b^0-\Lambda_b^0).$$
(5)

$$(307) \qquad\qquad (316\pm10)$$

The sum rules in Eqs. (4) and (5) are satisfied to about the same extent as the light baryon sum rules.

In Ref. [2] we used a sum rule to predict the Ξ_c' mass which has now been measured. This permits a test of the sum rule, which we write here as

$$\Sigma^++\Omega^--\Xi^0-\Xi^{*0}=\Sigma_c^{++}+\Omega_c^0-2\Xi_c'^{+}.$$
(6)

$$(15) \qquad\qquad (27\pm30)$$

The left hand side of Eq. (6) could be made to vary between -3 and $+27$ MeV, by using Eq. (1) to substitute other light baryon combinations.

The charmed spin 3/2 counterpart of Eq. (6) can be used to predict the as yet unmeasured Ω_c^{*0} mass

$$\Omega_c^{*0}=\Omega_c^0+2(\Xi_c^{*+}-\Xi_c'^{+})-(\Sigma_c^{*++}-\Sigma_c^{++})=2783\pm30,$$
(7)

where we have used the most similar *c*-baryon combination rather than using any light baryons. This increases the error on the prediction, but keeps the entire equation within the charmed quark sector.

We see that the medium strong energy difference sum rules are satisfied at least as well for the heavy baryons as for the light quark baryons. However the situation is not as nice for the isospin breaking mass differences in the case of the Σ_c. In Ref. [2] we showed that the Σ_c sum rule is violated by three standard deviations, while the corresponding light baryon sum rule is satisfied. Since sum rules in disagreement are of more concern than those which are satisfied, resolving the Σ_c mass differences is of prime importance.

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[10] Equation (4) is a combination of several equations in Ref. [1].