Constraints on the W_R mass and CP violation in left-right models

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We update the constraints on the right-handed W_R gauge boson mass, mixing angle ζ with the left-handed W_L gauge boson, and other parameters in general left-right symmetric models with different mechanisms of CP violation. Constraints mostly independent of any assumption on the quark sector are obtained from a reanalysis of muon decay data. The best χ^2 fit of the data gives $g_R/g_L=0.94\pm0.09$ for the ratio of right to left gauge couplings, with $M_{W_R}\geqslant 485$ GeV and $|\zeta|\leqslant 0.0327$. Fixing $g_L=g_R$ (in particular for manifestly left-right symmetric models), we obtain $M_{W_R}\gtrsim 549$ GeV and $|\zeta|\gtrsim 0.0333$. Estimates of the left-right hadronic matrix elements in the neutral kaon system and their uncertainties are revised using large N_c and chiral perturbation theory arguments. With explicitly given assumptions on the long-distance $(\Delta S=1)^2$ contributions to the K_L-K_S mass difference, lower bounds on M_{W_R} are obtained. With the same assumptions, one also gets strong upper bounds from the CP-violating parameter ϵ_K, for most of the parameter space of left-right models where the right-handed third family does not contribute in CP-violating quantities. For manifestly left-right symmetric models the lower bound obtained is $M_{W_R}\gtrsim(1.6^{+1.2}_{-0.7})$ TeV. [S0556-2821(97)01407-0]

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I. INTRODUCTION

While the standard model (SM) has been successful in its predictions over the past decades it is still not fully satisfactory in many ways. The origin of the maximal parity violation in the weak interactions, the origin of CP violation and the smallness of the ratio of neutrino masses to the top-quark mass are among the open questions which motivate searches for new physics beyond the electroweak scale. All these puzzles find natural answers in extensions of the SM based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [1,2]. The most definite benchmark of this class of models would be the discovery of the right-handed currents predicted by them.

Manifestations of right-handed charged currents have been looked for, both in high- and low-energy experiments. Direct searches at the Fermilab Tevatron set 652 GeV [3] as the lower bound of the right-handed gauge boson mass. If the right-handed neutrino is assumed to be much lighter than the W_R boson then this bound is increased to 720 GeV [4]. The contribution of virtual W_R excitations in low-energy processes can be used to constrain its mass, coupling, and other parameters too. In fact, so far, the most sensitive probe to the additional right-handed interaction is provided by the system of neutral kaons [5]. The right-handed charged current can give substantial contributions to strangeness changing in two units effective Lagrangian which governs the K^0-$\bar{K}^0$ mixing. Thus, well-measured observables of the neutral kaon system as the K_L-K_S mass difference $\Delta m_K \equiv m_{K_L}-m_{K_S}$ and the CP-violating parameter ϵ_K, are most suited for indirect limits on the right-handed gauge boson couplings.

Constraints on the W_R mass and its mixing angle with the left-handed W_L gauge boson have been studied, in general left-right (LR) models, extensively in [6]. However, we think that some additional analyses might be of interest. Namely, the CP-breaking mechanism effects have not been fully exploited in these analyses. Because the number of CP-violating phases, $[(N-1)(N-2)+N(N+1)]/2$ in a N-family LR model,[1] the effects of CP violation can be expected to be more important than those in the SM. In particular, CP-violating phases can also modify CP-conserving observables. For example, the contributions to Δm_K in LR models are proportional to cosines of differences of CP-violating phases which, in the general case, can be arbitrary and, therefore, reduce the limits considerably.

Moreover, the experimental value of ϵ_K, which in some particular cases (e.g., manifestly LR models with spontaneous breaking of CP) has been shown to be very constraining [7], has not been used to constrain other left-right models. We will show that indeed ϵ_K sets very constraining bounds in a large class of left-right models. Another point we would like to reanalyze is the reliability of the LR hadronic matrix elements estimates in the literature. In particular, we would like to make a realistic estimate of its uncertainty at present. This is necessary in order to have a meaningful comparison between the constraints obtained from the neutral kaon system and other results, e.g., muon decay data or collider experiments.

In this paper we update bounds on M_{W_R} and its mixing angle ζ in general LR symmetric models with different discrete symmetries on the Lagrangian. Bounds independent of the quark sector and hadronic physics uncertainties are set using updated electroweak data on the muon decay. Here, we assume that the right-handed neutrino is light enough to be produced in this decay. In the kaon system, we estimate the LR hadronic matrix elements and their present uncertainty using large N_c and chiral symmetry arguments. We derive bounds on M_{W_R} from measurements of both Δm_K and ϵ_K in

[1]As is well known, CP violation can occur even in the two-family case.

models with different CP breaking mechanisms and compare them with results from other sources.

As it has been pointed out by several authors [6,8,9], bounds on particular W_R parameters such as its mass, depend strongly on theoretical assumptions about the size of the right-handed gauge coupling g_R and/or the right-handed Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Combining the bounds obtained from the two neutral kaon observables we can eliminate one of the unknowns.

In the standard model, all three families need to be involved in CP violation observables. In addition, there is no CP violation if the up-type quarks or the down-type quarks are degenerated in mass [10], so that one can neglect the light-quark contributions owing to the large top-quark mass in observables such as ϵ_K. Observables such as Δm_K, only sensitive to the real part of the Lagrangian can, however, get contributions from each family separately. The K_L-K_S mass difference has been estimated in the SM in [11] by matching short- and long-distance contributions in a $1/N_c$ expansion (N_c is the number of QCD colors). The result indicates that the long-distance contributions in Δm_K in the SM are of the order of 50%. One thus expects this same large long-distance QCD contribution to appear in CP-violating observables when only the two lightest families are involved, as happens, in general, in LR models. More comments on this issue are in Sec. IV.

The outline of the paper is the following. In Sec. II we briefly present the general structure of the left-right models we are interested in. In Sec. III we carry out the analysis of constraints from the muon decay data and in Section IV we discuss the effective Lagrangian with $\Delta S=2$ in LR models, obtaining constraints on the W_R mass from Δm_K and ϵ_K measurements. In these two sections, we put some emphasis in giving explicitly which have been, in each case, the assumptions and/or the range of applicability of our results. Our conclusions are given in Sec. V.

II. LEFT-RIGHT SYMMETRIC MODELS AND CP VIOLATION

Here, we present the basic structure of the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ left-right symmetric model we use in our analyses. Since we are interested in the charged current processes and CP-violation effects, we concentrate on the gauge and Higgs sectors of the model. In left-right models, each familiy of fermions

$$\Psi(x)\equiv\begin{pmatrix} \nu & u \\ l & d \end{pmatrix}^a \tag{1}$$

is assigned to doublets of the gauge groups $SU(2)_L$ and $SU(2)_R$ according to the chirality. The Latin index $a=1,\cdots,N$ is for the family. Here and in the rest of the paper, $\Psi_{L(R)}\equiv\{[1-(+)\gamma_5/2]\}\Psi$. The field $\Psi_L(x)$ transforms under $SU(2)_L\times SU(2)_R$ gauge rotations as (2,1) and the field $\Psi_R(x)$ as (1,2), where the representations are identified by their dimension. Quarks have $B-L=1/3$ and leptons $B-L=-1$. Their interactions with the corresponding charged gauge bosons are determined by the charged current Lagrangian

$$\mathcal{L}_{CC}=\sum_{a=1}^{N}\left[\frac{g_L}{\sqrt{2}}W_L^\mu(\overline{l_L^a}\gamma_\mu\nu_L^a+\overline{d_L^a}\gamma_\mu u_L^a)\right.$$
$$\left.+\frac{g_R}{\sqrt{2}}W_R^\mu(\overline{l_R^a}\gamma_\mu\nu_R^a+\overline{d_R^a}\gamma_\mu u_R^a)\right]+\text{H.c.} \tag{2}$$

The minimal set of fundamental scalars consists of a bidoublet $\phi(x)$, and a left-handed and a right-handed multiplets of Higgs boson fields. If one wants to realize the seesaw mechanism [2,12], the latter ones should be chosen to be triplets Δ_L and Δ_R. In this case the model contains the following set of Higgs fields:

$$\phi\equiv\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \quad \Delta_{L(R)}\equiv\begin{pmatrix} \Delta^+ & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^0 & -\Delta^+ \end{pmatrix}_{L(R)}. \tag{3}$$

The field matrix $\phi(x)$ transforms under $SU(2)_L\times SU(2)_R$ gauge rotations as (2,2), the field matrix Δ_L as (3,1), and the field matrix Δ_R as (1,3). The $B-L$ charge is zero for the bidoublet and two for the triplets. With this field content, the most general form of the scalar potential $V(\phi,\Delta)$ can be found in the literature [13]. Sometimes, the full Lagrangian of the theory is required to be invariant under the transformations

$$\Psi_L\leftrightarrow\Psi_R,\Delta_L\leftrightarrow\Delta_R,\phi\leftrightarrow\phi^\dagger. \tag{4}$$

These are the so-called manifestly LR symmetric models; in this case one also has $g_L=g_R$.

Spontaneous symmetry breaking is parametrized by the following vacuum expectation values (VEV's) of the scalar fields:

$$\langle\phi\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \quad \langle\Delta_{L,R}\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}. \tag{5}$$

The VEV's of the bidoublet $\phi(x)$ parametrize the spontaneous symmetry breaking of the SM gauge group $SU(2)_L \times U(1)_Y$, and generate Dirac mass terms to fermions through the Yukawa Lagrangian

$$\mathcal{L}_Y=\overline{\Psi}_L f\phi\Psi_R+\overline{\Psi}_L h\tilde{\phi}\Psi_R+\text{H.c.}, \tag{6}$$

where f and h are matrices in family space collecting the Yukawa couplings for both quarks and leptons, and $\tilde{\phi}\equiv\tau_2\phi^*\tau_2$, with τ_2 the second Pauli matrix. Summation over families is understood. Diagonalization of the up- and down-quark mass matrices in Eq. (6) provides us with the CKM matrices K_L and K_R which, in general, are different. Analogously, we get the CKM-like matrices for the leptons, U_L and U_R. The VEV's κ_1 and κ_2 give also mass to the left-handed W_L gauge boson. The left-handed triplet VEV v_L does not play any dynamical role in the symmetry breaking and is forced to be small because of its contribution to the ρ parameter [13,14]. The right-handed triplet Δ_R breaks the $SU(2)_R\times U(1)_{B-L}$ gauge group to $U(1)_Y$ and its VEV v_R gives mass to W_R. In general, the charged gauge boson mass eigenstates are mixings of the flavor eigenstates. The mixing angle is

$$\zeta \equiv \left(\frac{2r}{1+r^2}\right)\beta, \tag{7}$$

where $r \equiv \kappa_1/\kappa_2$ and $\beta \equiv M_{W_L}^2/M_{W_R}^2$. Just for notation purposes, we will continue to use W_L and W_R for the charged gauge boson eigenstates, assigning W_L to the eigenstate that reduces to the left-handed gauge boson when $\zeta=0$ and analogously for W_R.

There are two natural ways to obtain breaking of the CP symmetry in left-right models. First, CP is violated if the Yukawa coupling matrices f and h are complex. This is called hard CP violation and is the analogue of the CKM CP violation in the SM. Second, in LR models one can naturally extend the idea of spontaneous breaking of parity [15] to the spontaneous violation of CP [16,17]. This is parametrized by the VEV's v_L, v_R and κ_1, κ_2, which can, in principle, be complex and break CP. In general left-right models, CP violation can occur either due to just one of the mechanisms or to the combination of both.

III. UPDATED CONSTRAINTS FROM MUON DECAY DATA

Pure leptonic processes are free of both the assumptions on the unknown quark mixings and the uncertainty induced by our present limited knowledge on the low-energy QCD dynamics. They are thus, in principle, better suited for obtaining more model independent constraints. With such aim, we use the updated electroweak data [18] to reanalyze the muon decay data in the case of interest here.

As in any model in which light fermions have heavy boson-mediated interactions, the low-energy effective action of LR symmetric models contains the usual standard model bilinear terms plus four-fermion interactions. The latter are the result of integrating out the heavier LR degrees of freedom. Hence, precise low-energy tests of the light fermions constitute a window into the high-energy behavior of the model underlying the SM (in this case LR symmetric models). As was stated before, this procedure is cleaner in the leptonic sector where hadronization does not obscure it.

The effective Lagrangian which describes the contact four-fermion lepton-lepton interaction in LR models is [6]

$$\mathcal{L}_{\rm eff} = g_{LL} J_L^+ J_L^- + g_{LR} J_R^+ J_L^- + g_{RL} J_L^+ J_R^- + g_{RR} J_R^+ J_R^-, \tag{8}$$

with

$$J_{L(R)}^{-\mu} \equiv \overline{N_{L(R)}}\gamma^\mu U_{L(R)} E_{L(R)}, \tag{9}$$

where the three neutrino and charged lepton families are collected in the N and E vectors, respectively, and

$$J_{L(R)}^{+\mu} = (J_{L(R)}^{-\mu})^\dagger. \tag{10}$$

We assume that right-handed neutrinos are light enough to be produced in muon decays. With the present bounds on the left-handed neutrino masses [18], the left-handed U_L mixing matrix can be chosen to be diagonal. The couplings in Eq. (8) satisfy $g_{ij} = g_{ji}^*$ with

$$g_{LL} = \frac{g_L^2}{2M_{W_L}^2}(\cos^2\zeta + \beta\sin^2\zeta),$$

$$g_{LR} = \frac{g_L^2}{2M_{W_L}^2}\alpha(1-\beta)\sin\zeta\cos\zeta e^{i\varphi},$$

$$g_{RR} = \frac{g_L^2}{2M_{W_L}^2}\alpha^2(\sin^2\zeta + \beta\cos^2\zeta), \tag{11}$$

the angle φ is the relative phase of the bidoublet VEV's, and $\alpha \equiv g_R/g_L$. From W_L gauge boson and β decays, we know that the leptonic W_L vertices give the dominant contribution to the muon and τ meson leptonic decays. The SM predicts that this is indeed the unique tree-level contribution, then in the SM, to a very good approximation,

$$g_{LL} \simeq \frac{g_L^2}{2M_{W_L}^2}, \tag{12}$$

and $g_{LR} = g_{RR} \simeq 0$. In general LR models with the low-energy effective Lagrangian structure in Eq. (8), the measured muon decay width puts the following constraint:

$$8G_F^\mu = g_{LL}^2 + 2|g_{LR}|^2 + g_{RR}^2. \tag{13}$$

From here we can find the relation between the Fermi constant G_F^μ, α, β, and the mixing ζ,

$$\frac{G_F^\mu}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W(1-\Delta r)M_{W_L}^2}A, \tag{14}$$

with $|e|$ the electron electric charge and

$$A = (1+\alpha^4\beta^2)\cos^4\zeta + (\alpha^4+\beta^2)\sin^4\zeta + 2[\beta(1-\alpha^2)^2 + \alpha^2(1+\beta^2)]\sin^2\zeta\cos^2\zeta, \tag{15}$$

where we applied the one-loop standard model radiative correction Δr to the fine structure constant. The radiative correction Δr is evaluated numerically for $m_t = (175 \pm 6)$ GeV pole top-quark mass value [19] and $m_H = 300$ GeV, yielding $\Delta r = 0.053 \pm 0.003$. Since we assume that the SM provides the dominant contribution, any additional non-SM higher order correction is a subleading effect. Therefore, only SM radiative corrections are included.

To determine constraints on the parameters of our LR symmetric model, we have expressed the muon decay parameters in terms of α, ζ, and β as

$$\rho = \frac{3}{4A}[(1+\alpha^4\beta^2)\cos^4\zeta + (\alpha^4+\beta^2)\sin^4\zeta + 2\beta(1+\alpha^2)\sin^2\zeta\cos^2\zeta,]$$

$$\xi = -\frac{1}{A}[(\alpha^4\beta^2-1)\cos^4\zeta + (\alpha^4-\beta^2)\sin^4\zeta + 2\beta(\alpha^2-1)\sin^2\zeta\cos^2\zeta],$$

$$\xi' = \xi,$$

TABLE I. Constraints on the right-handed W_R gauge boson mass and mixing angle ζ for different values of $\alpha = g_R/g_L$. The second and third columns are the corresponding lower limits on M_{W_R} and upper limits on $|\zeta|$, respectively.

$\alpha = g_R/g_L$	M_{W_R} (GeV)	$\lvert\zeta\rvert$
0.50	$\geqslant$ 286	$\leqslant$ 0.0324
0.75	$\geqslant$ 379	$\leqslant$ 0.0321
1.00	$\geqslant$ 549	$\leqslant$ 0.0333
1.50	$\geqslant$ 825	$\leqslant$ 0.0330
2.00	$\geqslant$ 1015	$\leqslant$ 0.0327

$$\xi'' = \frac{1}{A}[(1+\alpha^4\beta^2)\cos^4\zeta + (\alpha^4+\beta^2)\sin^4\zeta + 2(\beta + 3\alpha^2 + 3\alpha^2\beta^2 - 5\alpha^2\beta)\sin^2\zeta\cos^2\zeta],$$

$$\delta = \frac{3}{4},$$

$$\overline{\eta} = \frac{2}{A}\alpha^2(1-\beta)^2\sin^2\zeta\cos^2\zeta, \tag{16}$$

where the overall normalization A is given in Eq. (15).

We have performed a best χ^2 fit of these parameters with the experimental data given by the Particle Data Group [18]. This gives lower limits on M_{W_R} and upper limits on the mixing angle ζ for different values of α as shown in Table I.

These bounds are stronger than the ones obtained in previous analyses due to the improvement in experimental data precision. Letting α to vary freely, the best χ^2 is obtained for $\alpha = 0.94 \pm 0.09$, with

$$M_{W_R} \geqslant 485 \quad \text{GeV}, \quad |\zeta| \leqslant 0.0327. \tag{17}$$

IV. BOUNDS ON M_{W_R} IN CP-VIOLATING LEFT-RIGHT MODELS

As in the SM, in left-right models there are two types of contributions to strangeness changing in two units processes, namely, $\Delta S = 2$ transitions induced by box diagrams [20,21] or short-distance contributions, and $(\Delta S = 1)^2$ transitions or long-distance contributions [22]. As noticed first in [22] for the real part and in [23] for the imaginary one, the long-distance $(\Delta S = 1)^2$ contributions can be large. Its possible importance in LR models was already pointed out in [16,17]. Their calculation involves a good mastering of the QCD long-distance part and in particular of the so-called $\Delta I = 1/2$ rule for $K \to \pi\pi$ decays. As mentioned in Introduction, its relative importance in CP-violating observables in general LR models can be larger than that in the SM and of the same order as for CP-conserving quantities such as Δm_K. This is because, in general LR models, the presence of all the SM families is not anymore required in CP-violating observables and the top-quark contribution will not dominate in general. One thus expects long- and short-distance processes to contribute with the same weight in CP-conserving and CP-violating observables. To have a hint, we can compare with what happens in the SM. The short- and long-distance contributions to the CP-conserving observable Δm_K were computed in [11] within the SM. The result was that both contributions are of the same size when the scale separating both regions is around 1 GeV.

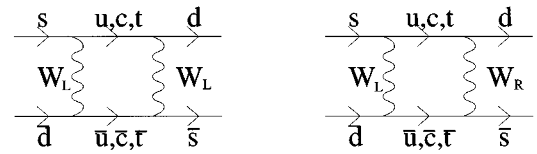

FIG. 1. Diagrams showing the dominant box-diagram contributions to the $\Delta S = 2$ effective Lagrangian in left-right models.

Therefore, there could be large cancellations in general LR models between long- and short-distance contributions. The precise analysis requires a careful study of the relative phases and of the LR hadronic matrix elements of $\Delta S = 1$ transitions. This is outside the scope of this paper and will be presented elsewhere.

However, with some more or less strong assumptions on the long-distance contributions, which will be given explicitly in each case, one can still obtain relevant bounds from the short-distance $\Delta S = 2$ contributions.

There already exists an extensive literature on the box-diagram contributions in LR symmetric models [5,16,17,24] (for the dominant contributions see, Fig. 1).

Namely, the exchange of two left-handed gauge bosons (LL diagram) and the exchange of one left- and one right-handed gauge bosons (LR diagrams). Contributions coming from right-right gauge boson or physical Higgs boson exchanges are suppressed by boson masses as well as by small Yukawa couplings in the latter case.

For the CKM matrices K_L and K_R, we use the following parametrizations. In the left-handed sector, we use a typical SM parametrization of the CKM matrices, i.e., three angles and one phase δ. This can be done because of our convention in which all additional phases are shifted to the right-handed sector. For the right-handed CKM matrices, the most general matrix has six phases and three angles. As mentioned in Introduction, in general, the right-handed third-family contribution is not needed for CP-conserving or CP-violating observables: this leaves us with three observable phases; therefore, we can take the following Wolfenstein-like parametrization for the lightest two right-handed families of quark submatrix,

$$e^{i\gamma}\begin{bmatrix} e^{-i\delta_2}\left(1-\frac{\lambda_R^2}{2}\right) & e^{-i\delta_1}\lambda_R \\ -e^{i\delta_1}\lambda_R & e^{i\delta_2}\left(1-\frac{\lambda_R^2}{2}\right) \end{bmatrix}, \tag{18}$$

which violates unitarity by terms proportional to λ_R^4 This is naturally small in left-right models if the same hierarchy in the angles as in the left-handed sector holds. The parameter λ_R is the right sector analogous to the Cabibbo angle in the Wolfenstein parametrization of the left-handed CKM matrix $\lambda \simeq |V_{us}| \simeq 0.22$.

As pointed out in [17], the charm-charm contribution dominates over the top-top and top-charm contributions in the right-handed sector, unless a fine-tuning of the parameters is made. Assuming the same hierarchy in the right-handed CKM angles as the one in the left-handed ones (notice that they are equal in some LR models) and the same order of magnitude for the contributing right-handed phases in each type of box diagram, this dominance can be quantified in terms of CKM matrix elements and quark masses by the factor $\lambda^8(m_t^2/m_c^2)|\ln(m_t/M_{W_L})/\ln(m_c/M_{W_L})|\simeq 0.02$ for the top-top contributions over the charm-charm contributions and by $\lambda^4 m_c/m_t\simeq 10^{-5}$ for the top-charm contributions over the charm-charm contributions. Diagrams with unphysical scalars are suppressed by a $m_c^2/M_{W_L}^2$ factor. This dominance is also true for the contributions of the left-handed sector to the real part of the effective $\Delta S=2$ Lagrangian. However, the contribution of the left-handed sector is dominated by the top quark instead; therefore, we keep the full contribution from the left-handed sector to the $\Delta S=2$ effective Lagrangian (see, for instance, [25]). The effective $\Delta S=2$ Lagrangian in LR models is thus well approximated by

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\Delta S=2}\simeq -\frac{G_F^2}{4\pi^2}M_{W_L}^2\Bigg[&\{(\lambda_c^*)_{LL}^2\eta_1(\mu)S(x_c)\\ &+(\lambda_t^*)_{LL}^2\eta_2(\mu)S(x_t)\\ &+2(\lambda_c^*)_{LL}(\lambda_t^*)_{LL}\eta_3(\mu)S(x_c,x_t)\}O_{LL}(x)\\ &+2x_c\ln(x_c)\frac{M_{W_L}^2}{M_{W_R}^2}\frac{g_R^2}{g_L^2}(\lambda_c^*)_{LR}(\lambda_c^*)_{RL}\Bigg(\eta_4(\mu)O_S(x)\\ &+\eta_5(\mu)\left\{O_{LR}(x)+\frac{2}{N_c}O_S(x)\right\}\Bigg)\Bigg],\end{aligned}\tag{19}$$

where

$$S(x)=x\left[\frac{1}{4}+\frac{9}{4}\frac{1}{(1-x)}-\frac{3}{2}\frac{1}{(1-x)^2}\right]-\frac{3}{2}\left[\frac{x}{1-x}\right]^3\ln x;$$

and

$$S(x,y)=x\left[\ln\left(\frac{y}{x}\right)-\frac{3y}{4(1-y)}\left(1+\frac{y\ln y}{1-y}\right)\right],\tag{20}$$

with $x_i=m_i^2/M_{W_L}^2$ for m_i the i-flavor modified minimal subtraction scheme ($\overline{\rm MS}$) pole quark mass. In Eq. (19), $(\lambda_i)_{AB}\equiv(K_A)_{id}(K_B^*)_{is}$ and

$$\begin{aligned}O_{LL}(x)&\equiv(\overline{d_L}\gamma^\mu s_L)(x)(\overline{d_L}\gamma_\mu s_L)(x),\\ O_{LR}(x)&\equiv(\overline{d_L}\gamma^\mu s_L)(x)(\overline{d_R}\gamma_\mu s_R)(x),\\ O_S(x)&\equiv(\overline{d_R}s_L)(x)(\overline{d_L}s_R)(x).\end{aligned}\tag{21}$$

Color indices are summed inside brackets. The factors $\eta_i(\mu)$ include the short-distance QCD corrections from integrating out degrees of freedom between the W_R scale until some scale μ lower than the charm quark mass. They were calculated for the LR diagrams in [16] to one loop and in [21,26] at two loops for the LL part; we, however, only keep the one-loop value for consistency with the LR part and functions of the heavy-quark masses, gauge boson masses, and $\alpha_S(\mu)$.

Matrix elements of the Lagrangian in Eq. (19) are, of course, scale independent. The scale μ dependence of the $\eta_i(\mu)$ factors cancels the one of the matrix elements of the operators which are multiplying. The dependence of the hadronic matrix elements on the renormalization scale μ just appears at next-to-leading order in $1/N_c$ which can at present be only estimated using models. Below, we shall see that the leading order in $1/N_c$ estimate of the numerically dominant LR hadronic matrix element, which is model independent, has already a sizable uncertainty. We expect the matching scale μ to be a typical hadronic scale around the rho meson mass. In the case of the operators $O_S(x)$ and $O_{LR}(x)+(2/N_c)O_S(x)$, the present usage and uncertainties of the leading $1/N_c$ estimate (see below) make varying the renormalization scale μ in $\eta_{4,5}(\mu)$ between 0.7 GeV and 1.2 GeV good enough for our purposes.

Using $\overline{m_c}=(1.23\pm0.05)$ GeV [27] for the $\overline{\rm MS}$ pole charm-quark mass, $\Lambda_{\rm QCD}^{(3)}=(350\pm100)$ MeV [18], and the three-flavor one-loop running of $\alpha_S(\mu)$, we get from [16] that $\eta_4(\mu)$ varies between 3.5 and 6.5 and $\eta_5(\mu)$ between 0.3 and 0.4. These values depend on the right-handed scale very weakly: varying M_{W_R} from 1 to 10 TeV, they change a couple of percent only. For the LL short-distance factors we take [26] $\eta_1(\mu)\simeq(1.0\pm0.2)\alpha_S(\mu)^{-2/9}$, $\eta_2(\mu)\simeq(0.6\pm0.1)\alpha_S(\mu)^{-2/9}$, and $\eta_3(\mu)\simeq(0.4\pm0.1)\alpha_S(\mu)^{-2/9}$.

Let us now study the hadronic matrix elements needed. For the LL operator we use the standard parametrization in terms of the $\hat{B}_K$ parameter

$$\langle K^0|O_{LL}(x)|\overline{K^0}\rangle=\tfrac{4}{3}\,\alpha_S(\mu)^{2/9}\hat{B}_K f_K^2 m_K^2.\tag{22}$$

The kaon decay coupling constant f_K is 113 MeV [18] in this normalization. The hadronic matrix element $\langle K^0|O_{LL}(x)|\overline{K^0}\rangle$ has been subject of much more work and its present knowledge is summarized in [28]. We use the present-favored range of values $\hat{B}_K=0.70\pm0.10$ [28].

The vacuum insertion approximation (VIA) has been used generally in the literature to estimate LR hadronic matrix elements. The same procedure gives $B_K=1$ at any scale. Unfortunately, the VIA is not a systematic expansion in any parameter so that an estimate of the reliability of its predictions and/or improving them is not possible. We use instead the large N_c expansion [29], together with chiral perturbation theory (CHPT) (see [30] for a recent introductory review and references), counting as organizative schemes. The combination of both techniques allows for a more systematic expansion and estimate of the uncertainties as we see below. In the case of the LL operator, the leading $1/N_c$ model-independent result gives $\hat{B}_K=3/4$ with 0.25 as the estimated uncertainty. Using the same expansion for the LR hadronic matrix elements, we get

$$\langle K^0|O_S(x)|\overline{K^0}\rangle=\frac{\langle\overline{s}s+\overline{d}d\rangle^2}{4f_K^2}+O\left(\frac{m_K^2 f_K^2}{N_c}\right).\tag{23}$$

$$\left\langle K^0 \left| O_{LR}(x) + \frac{2}{N_c} O_S(x) \right| \overline{K^0} \right\rangle = -f_K^2 m_K^2 + O\left(\frac{\alpha_S}{\pi} \frac{\langle \overline{q}q \rangle^2}{f_K^2} \right), \tag{24}$$

where the quark condensates $\langle 0|\overline{s}s + \overline{d}d|0\rangle$ can be obtained from

$$\langle 0|\overline{s}s + \overline{d}d|0\rangle = -2 f_K^2 \frac{m_K^2}{m_s + m_d}(1-\delta_K). \tag{25}$$

The parameter $\delta_K = 0.35 \pm 0.10$ has been calculated using finite energy QCD sum rules [31]. Using the $\overline{\mathrm{MS}}$ running masses at 1 GeV, $m_s + m_d = (185 \pm 30)$ MeV [32], we obtain

$$\langle K^0|O_S(x)|\overline{K^0}\rangle(\mu) = (0.013 \pm 0.006)\mathrm{GeV}^4 \tag{26}$$

and

$$\left\langle K^0 \left| O_{LR}(x) + \frac{2 O_S(x)}{N_c} \right| \overline{K^0} \right\rangle(\mu) = -(3 \pm 3) \times 10^{-3} \mathrm{GeV}^4, \tag{27}$$

where the scale μ varies between 0.7 GeV and 1.2 GeV. Because of the chiral structure of these operators, we observe that the $1/N_c$ corrections to the matrix element $\langle K^0|O_S(x)|\overline{K^0}\rangle$ are suppressed by a $m_K^2 f_K^4/\langle \overline{q}q\rangle^2$ factor. This makes the main uncertainty in this matrix element operator to be the one in the determination of the quark condensates, i.e., δ_K. This translates into a 50% uncertainty in this matrix element, which can only be reduced with a more accurate determination of the quark condensates. The matrix element of the operator $O_{LR}(x) + 2O_S(x)/N_c$ has even larger relative uncertainties due again to its chiral structure. In this case, there are nonfactorizable $1/N_c$ corrections which are not chirally suppressed, instead there is an additional α_S/π suppressing factor. Fortunately, its leading order in $1/N_c$ contribution is chirally suppressed. In addition, as we saw before, the short-distance coefficient $\eta_5(\mu)$ is very small. The discussion above makes clear that a large N_c estimate of the matrix elements of the operators in the Lagrangian in Eq. (19) is enough at present for this case.

Let us study the imaginary part of the $\Delta S = 2$ efffective Lagrangian. For the left-handed sector we need the concurrence of the three families so that the dominance of the charm-charm box diagrams is not true anymore; in fact, the top-top box diagram contribution dominates the CP-violating part of the $\Delta S = 2$ effective Lagrangian. The dominance of the charm-charm contributions is still true for the imaginary part in the right-handed sector unless $\delta_1 = \delta_2 + n\pi$. Therefore, we can take the same effective Lagrangian in Eq. (19) unless $\delta_1 = \delta_2 + n\pi$. In that case, the right-handed sector behaves as the left-handed one in this respect, and no interesting bounds can be obtained.

In the approximations [33], where

$$\Delta m_K \simeq 2\mathrm{Re}M_{12}, \quad \text{with } M_{12} \simeq -\langle K^0|\mathcal{L}_{\mathrm{eff}}^{\Delta S=2}|\overline{K^0}\rangle/2m_K \tag{28}$$

and

$$\epsilon_K \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{\mathrm{Im}M_{12}}{\Delta m_K^{\mathrm{exp}}}, \tag{29}$$

where we have included in ϵ_K the long-distance contributions to $\mathrm{Re}M_{12}$ in the experimental value of Δm_K, and using the effective Lagrangian in Eq. (19), we obtain

$$\Delta m_K \simeq \left[(0.40 \pm 0.20) - (4.5 \pm 2.5) \frac{g_R^2}{g_L^2} \lambda_R \times \left(1 - \frac{\lambda_R^2}{2}\right) \cos(\delta_2 - \delta_1) \left(\frac{1\,\mathrm{TeV}}{M_{W_R}} \right)^2 \right] \Delta m_K^{\mathrm{expt}} \tag{30}$$

and

$$\epsilon_K \simeq e^{i\pi/4} \left[(2.7 \pm 1.0) \times 10^{-3} \sin(\delta) - (1.6 \pm 0.9) \frac{g_R^2}{g_L^2} \times \lambda_R \left(1 - \frac{\lambda_R^2}{2}\right) \sin(\delta_2 - \delta_1) \left(\frac{1\,\mathrm{TeV}}{M_{W_R}} \right)^2 \right], \tag{31}$$

where the first contribution inside the squared brackets is the LL contribution in each case, $\Delta m_K^{\mathrm{expt}} = (3.491 \pm 0.014) \times 10^{-15}$ GeV [18], and $|\epsilon_K^{\mathrm{expt}}| = (2.26 \pm 0.02) \times 10^{-3}$ [18]. For the top-quark mass we have used the $\overline{\mathrm{MS}}$ pole top-quark mass value $\overline{m_t} = (167 \pm 6)$ GeV [19]. Notice that these two observables only constrain two, δ and $\delta_1 - \delta_2$, out of the seven CP-violating phases we can have in the most general left-right model. The left-handed long-distance contributions to ϵ_K (in our parametrization of CP phases) are expected to be negligible since in this case the physics is dominated by the large top-quark mass contributions. Therefore, for the left-handed part, the box diagram is a good approximation in this case. There are though, in principle, right-handed long-distance contributions to ϵ_K which are expected to be, as said before, of the same order as the right-handed short-distance ones.

Let us now apply our results to left-right models with different symmetries in the Lagrangian. First, we consider a manifestly LR symmetric model invariant under the transformation (4) and with spontaneous breakdown of CP. Remember that $g_R = g_L$ in these models. Diagonalization of the quark mass matrices for this case has been studied in [16,17]. In this type of models, $\lambda_R = \lambda$ and all the relevant phases in the quark sector, namely, δ, δ_1, and δ_2 in our parametrization, are proportional to a single quantity, $r = |\kappa_1/\kappa_2|\sin\varphi$ [7,17]. Moreover, when solving for the quark masses one finds the requirement $r < |m_b/m_t|$ [7,17] which implies the suppression of the phases δ, δ_1, and δ_2 by this factor. This particular feature makes this type of models very predictive. The expressions of the phases δ, δ_1, and δ_2 in terms of r, CKM matrix elements, and quark masses can be found in [7]. The numerical analysis in [7] shows that the phases δ, δ_1, and δ_2 are actually very small and $\cos(\delta_2 - \delta_1) \simeq 1$, so that there is no suppression in Δm_K because of CP-violating phases. In this case and when the long-distance contributions are smaller than $\Delta m_K^{\rm expt}$ we have that $(\Delta m_K)_{\rm box}$ has to be positive. Using this positivity, one gets from Eq. (30) the following lower bound

$$M_{W_R} \gtrsim (1.6^{+1.2}_{-0.7})\,\text{TeV}. \tag{32}$$

This is quite general since it only requires the natural expectation that long-distance contributions to Δm_K are smaller than the experimental value. The assumption on the long-distance contributions done in [5] is similar to ours but the input values and the hadronic matrix elements used are quite different. The fact that the bound they get coincides with the central value in Eq. (32) is a numerical accident. (Notice that the left-handed part alone in [5] gives a 90% contribution to $\Delta m_K^{\rm expt}$.) The bound obtained in [17] is not valid for models with $\cos(\delta_1-\delta_2)\geqslant 0$ as, for instance, manifestly left-right models we are considering here. Another important difference of our analysis respect to the ones in [5,17] is the error bars. They reflect the uncertainties in the hadronic matrix elements, and some input parameters, mainly, $\Lambda_{\rm QCD}$ as discussed above. Taking into account realistic uncertainties, we still get a lower bound ($M_{W_R}\gtrsim 0.9$ TeV) from Δm_K (together with the assumption on the long-distance contributions explained above), which is larger than the muon decay constraint we got in Sec. III, $M_{W_R}\gtrsim 549$ GeV, and the Tevatron direct limits $M_{W_R}\gtrsim 652$ GeV [3], $M_{W_R}\gtrsim 720$ GeV [4]. This shows the powerfulness of this low-energy observable.

For manifestly LR models with no spontaneous CP breaking, where CP violation is parametrized by the complex Yukawa couplings, the transformation (4) requires $K_L=K_R$. Therefore, only one independent CP-violating phase remains, the one analogous to the SM CKM phase δ. The lower bound (32) holds in these models since we have there $\delta_1=\delta_2=0$.

Finally, we analyze more general left-right models where we do not impose any discrete symmetry in the Lagrangian. As said before, in this case there are seven independent CP-violating phases (one in the left-handed and six in the right-handed in our parametrization sector). They can have both complex Yukawa (hard) origin and/or spontaneous symmetry-breaking origin and, in general, $\lambda_R\neq\lambda$. In the case where $\cos(\delta_1-\delta_2)\geqslant 0$ and again long-distance contributions are smaller than the measured Δm_K^2 there is a lower bound on the W_R mass analogous to Eq. (32), which takes the form

$$M_{W_R} \gtrsim (3.4^{+2.5}_{-1.5})\frac{g_R}{g_L}\sqrt{\lambda_R\left(1-\frac{\lambda_R^2}{2}\right)\cos(\delta_2-\delta_1)}\;\text{TeV}. \tag{33}$$

If $\cos(\delta_1-\delta_2)\leqslant 0$, there is no cancellation in Eq. (30) and to get lower bounds we need a stronger assumption on the long-distance contributions, namely, that they are smaller than the experimental value of ϵ_K and add positively, so in this case we have the constraint $(\Delta m_K)_{\rm box}\leqslant \Delta m_K^{\rm expt}$. This case is actually the one treated in [17]. Using this inequality and Eq. (30), we get

$$M_{W_R} \gtrsim (2.7^{+2.1}_{-1.2})\frac{g_R}{g_L}\sqrt{-\lambda_R\left(1-\frac{\lambda_R^2}{2}\right)\cos(\delta_2-\delta_1)}\,\text{TeV}. \tag{34}$$

Putting the worse case, $\cos(\delta_2-\delta_1)=-1$, and $\lambda_R=\lambda$ (as done in [17]), we get $M_{W_R}\gtrsim(1.3^{+1.0}_{-0.6})$ TeV. This lower bound updates the one in [17]. Notice again that a realistic estimate of the present uncertainties allows, in this case, lower bounds $M_{W_R}\gtrsim 0.7$ TeV, just slightly larger than direct Tevatron searches.

Let us turn to the analysis of the CP-violating parameter ϵ_K. From Eq. (31) one notices that unless $(g_R/g_L)^2\lambda_R\sin(\delta_1-\delta_2)$ is close to zero, the experimental value for ϵ_K is saturated almost completely by the LL contribution, i.e., the LR contribution to ϵ_K for light W_R gauge bosons is naturally larger than the LL one in general left-right models. The reason was already given in Introduction, it is the need to have the three families involved with non-degeneracy of the up-quark masses in the left-handed sector, while only the two lightest are needed, in general, in the right-handed sector. This combined with the observed hierarchy of the left-handed CKM angles[2] suppresses, in general, the LL contribution to CP-violating parameters with respect to the LR one (in our parametrization).

Again, assuming long-distance contributions to ϵ_K to be smaller than $\epsilon_K^{\rm expt}$ as noticed in [7], allows one to get relevant upper bounds on M_{W_R} for most of the parameter space in general left-right models. For instance, if $\sin(\delta)\leqslant 0.6$ in Eq. (31), we need a contribution larger or of the order of $\pm 10^{-5}$ from the right-handed part. We can, from this observation, get a natural upper bound on the W_R mass in general left-right symmetric models, so that we obtain the following upper bound:

$$M_{W_R} \leqslant O\left[350\frac{g_R}{g_L}\sqrt{\lambda_R\left(1-\frac{\lambda_R^2}{2}\right)|\sin(\delta_2-\delta_1)|}\right]\text{TeV}. \tag{35}$$

This bound will be violated unless $\sin(\delta_1-\delta_2)$ is close to zero. In that case, the observed ϵ_K value has to be saturated by the left-handed contribution. How close to zero depends on the lower bound values for M_{W_R}. Combining the lower bound in Eq. (33) and the upper bound in Eq. (35), we get that they are self-consistent unless

$$|\sin(\delta_1-\delta_2)|\leqslant 10^{-4}, \tag{36}$$

i.e., $|\delta_1-\delta_2|$ very close to 0, π, or 2π. The upper bound in Eq. (35) can be reduced by reducing the upper bound to $\sin(\delta)$, of course.

[2]Left-handed CKM angles are measured in tree-level processes, so we expect non-SM physics effects there to be negligible.

Let us now see how this bound applies to the case of manifestly left-right symmetric models with spontaneous CP violation. As was shown in [7], in this type of models $|\sin(\theta)|$ with $\theta=\delta$, δ_1, or δ_2 is typically of the order of 10^{-2}. Therefore, the value of the phases in this type of models satisfies the requirements for the bound in Eq. (35) to hold in most of the parameter space. Using $\sin(\delta_1-\delta_2)\simeq 10^{-2}$ as a typical value of this region of parameter space, we get

$$M_{W_R}\lesssim 20 \text{ TeV}. \tag{37}$$

A counter example of the upper bound in Eq. (35) happens, for instance, in manifestly symmetric left-right models with only complex Yukawa coupling CP violation or in models with $\delta_2=\delta_1+n\pi$.

The bound in Eq. (35) is indicating that left-right symmetric models prefer (in general) ''light'' W_R gauge bosons. Only very particular models, those where CP violation requires also the third family in the right-handed sector, can naturally accommodate very heavy W_R gauges boson masses. This is a nontrivial bound which, for instance, constrains which type of CP violation can have a model with a left-right symmetric scale of the order of (10^7-10^9) TeV, such as some left-right symmetric models with a seesaw mechanism for neutrino masses favored by neutrino physics [2].

V. CONCLUSIONS

In this work we have studied and updated the bounds on the W_R gauge boson mass and its mixing angle ζ with its left-handed partner in general LR model.

Results independent of the quark sector assumptions and low-energy QCD uncertainties have been obtained by reanalyzing muon decay data from the CERN e^+e^- collider LEP. In the case the left-handed gauge coupling is equal to the right-handed one (for instance, in manifestly left-right symmetric models), we get $M_{W_R}\geqslant 549$ GeV and $|\zeta|\leqslant 0.0333$. Without that constraint, the best χ^2 fit to the muon decay data is obtained for $g_R/g_L=0.94\pm 0.09$ with $M_{W_R}\geqslant 485$ GeV and $|\zeta|\leqslant 0.0327$.

We have also considered bounds on the W_R mass imposed by the neutral kaon observables, namely, the CP-conserving Δm_K and the CP-violating ϵ_K. This has been done in left-right models with different mechanisms of CP violation. We have estimated the LR hadronic matrix elements and their present uncertainty using a combined large N_c expansion and CHPT analysis. The uncertainty of the LR boxes' contribution to $\mathcal{L}_{\text{eff}}^{\Delta S=2}$ is due both to short-distance QCD corrections and hadronic matrix element contributions and can be as large as 80%. We want to emphasize that the low-energy QCD dynamics is the main present uncertainty in using the K^0-$\bar{K}^0$ system to constrain left-right symmetric model parameters. We have included this uncertainty in our results.

With some explicitly given assumptions on the long-distance contributions to Δm_K and ϵ_K such as that the long-distance contributions to a given observable are smaller than its experimental value (see Sec. IV), we get updated lower bounds on M_{W_R} from the CP-conserving Δm_K for general left-right symmetric models and strong upper bounds from the CP-violating ϵ_K parameter for most of the parameter space (36) in general left-right models. The bounds obtained favor quite light W_R gauge bosons, namely, masses below a few hundreds of TeV's (tens in the case of manifestly left-right models with spontaneous CP violation). This upper bound does not hold for the class of left-right symmetric models where one needs the third family in the right-handed sector. This can give a hint, in model building, of the CP-violating sector of the left-right symmetric models where a very large left-right symmetric scale is required, as for instance models with a seesaw mechanism for the neutrino masses [2]. Upper bounds on the W_R mass were obtained previously in [17,35]. The hypothesis made in these references was, however, very strong, namely, that ϵ_K is saturated completely by the right-handed contribution. The parameter space scenario where our upper bound holds is much broader, only a small deviation from the saturation of ϵ_K by the left-handed contribution is enough. Our study also clarifies which type of CP violation do these upper bounds correspond to. Namely, the experimental value on the CP-violating parameter ϵ_K likes to have a quite light intermediate left-right scale when the right-handed third family is not required to contribute.

In particular, we have obtained the lower bound, $M_{W_R}\gtrsim(1.6^{+1.2}_{-0.7})$ TeV, in manifestly left-right symmetric models with either spontaneous and/or hard CP violation. This bound complements the one we got from the muon decay data in Sec. III and the direct Tevatron bounds [3,4]. For more general LR models (see Sec. IV for details), the lower bound we get is in Eq. (33).

This work shows the large potential and complementarity of low-energy physics to the direct searches in high-energy experiments; in particular, in constraining new physics and/or in giving hints on model building. CP-violating quantities are very interesting in that respect because of its large suppression in the SM. We have seen that only two CP-violating phases, out of the seven in the most general left-right model, are constrained from ϵ_K; probably, other low-energy CP-violating observables give complementary information. One should also keep in mind that large CP-violating phases are welcome for baryogenesis [34]. More work is needed, however, in the low-energy QCD mastering in order to improve the constraints we can get. The present hadronic uncertainties dominate by far the error bars in the constrains we have obtained. In particular, the inclusion of $(\Delta S=1)^2$ long-distance contributions, especially to Δm_K, will refine the constraints obtained here.

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