Decay constants of pseudoscalar mesons in a relativistic quark model

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The decay constants of pseudoscalar mesons are calculated in a relativistic quark model which assumes that mesons are made of a valence quark-antiquark pair and of an effective vacuumlike component. The results are given as functions of quark masses and of some free parameters entering the expression of the internal wave functions of the mesons. Using $F_{\pi^+}=130.7$ MeV, $F_{K^+}=159.8$ MeV to fix the parameters of the model, we predict 60 MeV$\leqslant F_{D^+}\leqslant$185 MeV, 95 MeV$\leqslant F_{D_s}\leqslant$230 MeV, 80 MeV$\leqslant F_{B^+}\leqslant$205 MeV, 90 MeV$\leqslant F_{B_s}\leqslant$235 MeV for the light quark masses $m_u=5.1$ MeV, $m_d=9.3$ MeV, $m_s=175$ MeV and the heavy quark masses in the range 1GeV$\leqslant m_c\leqslant$1.6 GeV, 4.1GeV$\leqslant m_b\leqslant$4.5 GeV. In the case of light neutral mesons one obtains with the same set of parameters $F_{\pi^0}\approx 138$ MeV, $F_\eta\approx 130$ MeV, $F_{\eta'}\approx 78$ MeV. The values are in agreement with the experimental data and other theoretical results. [S0556-2821(97)07005-7]

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I. INTRODUCTION

The decay constants of pseudoscalar mesons have been treated by current algebra and PCAC (partial conservation of axial vector current) like simple scale parameters relating the meson fields with the coresponding axial vector currents. In quark models they are expressed by means of the quark-antiquark annihilation amplitude [1], but, although simple in principle, the calculation of the decay constants and, in general, of the electroweak form factors, is a difficult task due to the binding effects which escape a relativistic treatment.

A solution is to bypass the binding problem and work with free quarks. This is the way followed by QCD sum rules [2], which rely on the assumption of quark-hadron duality and relate the hadronic matrix elements with some quark and gluon transition amplitudes which can be evaluated within the perturbative QCD scheme. This is a fruitful method which produced among many other results, the values of the decay constants of heavy mesons too.

Another solution is to start from the very QCD principles in order to have a consistent description of the confinement. This is the way followed in lattice calculations, which succeded to give some reliable results, in spite of the technical difficulties raised by the enormous computational effort [3].

The oldest solution, which is still at use, is to assume that, in view of the locality of the weak current, the annihilation of the quark and of the antiquark takes place at vanishing relative distance [1,4]. One gets in this way $F\propto\psi(0)M^{-1/2}$, where ψ is the solution of a wave equation with a ''QCD inspired'' confining potential. Assuming that $\psi(0)$ is constant in the infinite mass limit, this leads to the well known scaling rule $F^2M=\text{const}$ [2] for heavy mesons.

Potential models are just quantum mechanics, where the annihilation process makes no sense since the quarks do not exist as independent particles, but only in the form of their bound state. This is perhaps the fundamental reason why these models give rather poor results in the case of light mesons. In the case of heavy mesons the large mass difference allows to see the quarks as independent particles, the light one moving in the field of forces created by its heavy partner at rest. In their case the potential models give sensibly better results.

An important step forward in introducing the independent quarks while preserving the valuable features of the potential models has been done by Isgur, Scora, Grinstein, and Wise [5]. Their ''mock meson'' is a system made of two almost free, independent particles, whose total momentum is equal to the meson momentum. The distribution of the quark-antiquark relative momentum is given by the Fourier transform of the solution of a wave equation with a suitable confining potential. The wave function and its Fourier transform are then L^2 integrable and the single ''mock meson'' state can be normalized like a single particle state. The annoying point is that, as mentioned in Ref. [5], a ''mock meson'' made of almost free quarks with a continuous distribution of the relative momentum has a false mass width because the sum of free quark momenta does not belong to a certain representation of the Lorentz group. This reflects the absence of a real Lorentz covariance and is the cause of some ambiguities when dealing with a moving ''mock meson'' [6].

Problems of this kind are quite general in quark models and could not be solved without a relativistic theory of the binding. Unfortunately, the best relativistic theory we have at hand, the field theory in the perturbative approach, is unable to give an easy answer to the binding problem. In our opinion its failure in describing the bound states is due to the lack of a relativistic equivalent of the binding energy. We thus suggest to renounce representing the binding by a series of some quantum exchanges, since binding is not a perturbative effect and look instead for a relativistic generalization of the binding energy to be included in a ''mock meson'' with free quarks. In this way we hope to combine the valuable features of the potential models, which are suitable for describing the binding, but are improper for introducing the boost of the bound state, with those of the relativistic models which can boost the free states, but cannot describe the binding.

The model we propose has been recently applied to the

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weak radiative decays of pseudoscalar mesons [7] and to the decay $Z^0 \rightarrow \pi^0 \gamma$ [8]. In this paper we intend to exploit further the properties of the model and to perform some predictions on the values of the decay constants of heavy mesons.

The specific assumption of the model is that mesons are made of a valence $q\overline{q}$ pair bound together by some collective oscillation modes of the quark gluonic field. The last ones are described by an effective vacuumlike component Φ. Like the valence quarks, the effective field contributes with its own four-momentum to the meson momentum, but it has no mass shell constraint since it is far from being elementary.

Some ideas similar to the above ones may be found in the flux-tube model proposed in Ref. [9]. The effective field Φ may be considered as a kind of Fourier transform of the flux tube, but the formal frames are quite different in the two cases.

Turning now to the potential models, we notice that the essential consequence of a binding potential is the existence of some L^2 integrable solutions of the wave equation. Conversely, the existence of an L^2 integrable wave function may be considered as an evidence of a bound state and of a binding potential. In terms of Fourier transform this means that an L^2 integrable distribution of the relative momentum is a characteristic feature of a bound system. Looking from this point of view, we observe that in our model it is the effective field Φ which adjusts the continuous distribution of the relative momentum to the free particle behavior required by relativistic covariance. It is Φ which allows to have the quarks on their mass shell and a continuous distribution of the quark relative momentum while ensuring a definite mass for the ''mock meson.'' This argument gives a substantial support to the assumption that $\Phi(Q)$ represents the excitations responsible for the confinement and allows us to consider Q_μ the relativistic generalization of the potential energy. In fact, Q_0 only is the analogue of the potential energy of the $q\overline{q}$ system at rest. The spatial components **Q** introduced for relativistic consistency could be rather related with the fluctuations of the confining oscillation modes of the quark gluonic field, not with the magnitude of the binding forces.

An important ingredient of the model is the internal function of the compound system representing the hadron. In the lack of a dynamical equation for it, we shall use some trial functions allowing to ensure the integrability of the matrix elements of interest and to fulfill some consistency requirements.

Finally we wish to stress once again that a real relativistic treatment of a system made of independent constituents requires the use of the momentum space. If the states were defined in the configuration space, it would be necessary to introduce an independent time coordinate for each component, which is nonsense.

In the next section we discuss shortly the dynamical assumptions of the model and give the expressions of the decay constants as functions of the quark and meson masses.

The numerical results obtained with exponential and Gaussian internal functions are given in the third section. The fit of the pion and kaon decay constants with the experimental values is used to fix the parameters of the model.

We analyze the results in the fourth section and draw some general conclusions concerning the reliability of the model.

II. CALCULATION OF THE DECAY CONSTANTS

The fundamental dynamical assumption of the model is that the interaction inside the quark system representing a hadron can be treated independently from the external interaction. The first one is a mean field effect and is taken into account by means of the internal wave function, while the external interaction is the effect of some specific quantum fluctuations. We recall that this is also the main assumption underlying the Furry representation in field theory [10].

The form we proposed for the meson state is [7]

$$\begin{aligned}|M_i(P)\rangle = \frac{i}{(2\pi)^3}\int \frac{d^3p}{e/m}\,\frac{d^3q}{\epsilon/m'}\,d^4Q\ \varphi(p,q;Q)\\ \times\overline{u(p)}\Gamma_M v(q)\ \chi^\dagger\lambda_i\psi\ \delta^{(4)}(p+q+Q-P)\\ \times\Phi^\dagger(Q)a^\dagger(p)b^\dagger(q)|0\rangle,\end{aligned} \tag{1}$$

where $a^\dagger, b^\dagger$ are the creation operators of the valence $q\overline{q}$ pair; u,v are Dirac spinors and Γ_M is a Dirac matrix ensuring the relativistic coupling of the quark spins. The quarks are supposed to be free; their creation and annihilation operators satisfy canonical commutation relations and commute with $\Phi^\dagger(Q)$, which describes the creation of a nonelementary excitation carrying the momentum Q_μ. The mass spectrum of the nonelementary excitations denoted by $\Phi\dagger$ and the internal distribution of momenta are described by the Lorentz invariant function $\varphi(p,q;Q)$. A natural assumption is that φ is a time independent, equilibrium distribution since the hadrons are long living. This means that as long as a quark system like that described by Eq. (1) is the single one in the external state and as long as it does not emit and absorb any electroweak quanta, the distribution of momenta is given by $\varphi(p,q;Q)$ and does not change. A straightforward consequence is that any time translation operator $U_s(t,t')$ describing the evolution of a quark system under the action of strong forces only can be replaced by unity when acting on a state like Eq. (1). This fact will allow us to perform some simplifications in the calculation of the matrix elements of interest.

For a better understanding of the present model, a comparison of the expression (1) with the ''mock meson'' in Ref. [5] is most useful. A first remark is that the continuous distribution of the relative momentum in a meson made of a quark and of an antiquark only, introduced by hand more than 20 years ago [5,11], follows naturally in our model from the existence of a third component, the field Φ, which contributes to the meson momentum. A second remark is that, unlike the ''mock meson,'' the expression (1) can be safely boosted due to the function $\delta^4(p+q+Q-P)$ which guarantees that the sum of the internal momenta belongs to the representation of the Lorentz group having the meson mass as invariant.

As concerns the concrete form of the internal function $\varphi(p,q;Q)$ some additional comments are necesary. It must be said that we have no *a priori* arguments for a particular form. However, we expect φ be such as to ensure the convergence of the integrals over the internal momenta in the expressions of the physical amplitudes.

Related to this fact, we remark that if the zeroth component of the momentum carried by the effective field Φ were positive in the rest frame of the meson, the function $\delta^{(3)}(\mathbf{p}+\mathbf{q}+\mathbf{Q})\delta(e+\epsilon+Q_0-M)$ would provide some natural upper bound for the quark energies. The integrals in the expression (1) of a meson state would then extend over a finite range and their convergence would be easy to ensure.

This is however not the case since, as shown by the electromagnetic form factors, there is no upper bound for the quark energy in a hadron. One must then allow for negative values of Q_0 and introduce some definite cutoff functions to ensure the convergence of the integrals. It is worth noticing that $Q_0 \leqslant 0$ means also a stability condition for the ''mock meson.''

The trial functions we shall use in the next as internal functions cut off the large values of Q_0 and $\mathbf{Q}$ only, but, due to the presence of the function $\delta^4(p+q+Q-P)$ we expect for them to provide the necessary cutoff for the quark momenta too. Indeed, it is easy to see that $(p-q)^2=2m^2+2m'^2-(P-Q)^2$ and hence, cutting off Q_0 and Q^2 means also to cut off the quark relative momentum.

According to the above considerations, a meson at rest is supposed to have an internal function of the following kind:

$$\varphi(p,q;Q)=D_M\sigma(Q_0,\mathbf{Q}), \tag{2}$$

$$\sigma(Q_0,\mathbf{Q})=\exp\left[\frac{Q_0}{\alpha}-\frac{|\mathbf{Q}|}{\beta}\right]\theta(Q^2)\,\theta(-Q_0), \tag{2a}$$

$$\sigma(Q_0,\mathbf{Q})=\exp\left[\frac{Q_0}{\alpha}-\frac{\mathbf{Q}^2}{\beta^2}\right]\theta(Q^2)\,\theta(-Q_0), \tag{2b}$$

$$\sigma(Q_0,\mathbf{Q})=\exp\left[-\frac{Q_0^2}{\alpha^2}-\frac{\mathbf{Q}^2}{\beta^2}\right]\theta(Q^2)\,\theta(-Q_0), \tag{2c}$$

where M is the meson mass, α,β are the free parameters of the model ensuring the desired convergence of the integrals. Lorentz covariance of the internal function becomes obvious if one writes Q_0 and $|\mathbf{Q}|$ as $Q_0=(P\cdot Q)/M, |\mathbf{Q}| =\sqrt{(P\cdot Q)^2/M^2-Q^2}$, where P is the meson momentum. The functions θ in Eqs. (2a), (2b), (2c) express the fact that Q is timelike and Q_0 negative, in agreement with our assumption that Q_μ is the relativistic generalization of the potential energy in the bound system.

Before proceeding to the evaluation of the decay constants, we have to make an explicit statement on the vacuum expectation value of the effective field. As mentioned above, the momentum carried by $\Phi^\dagger$ is not subject of a mass-shell constraint, since it represents the creation of a collective excitation, not of an elementary one. Accordingly, we assume that:

$$\Phi(Q_1)\Phi(Q_2)\cdots\Phi^+(Q_n)=\Phi(Q_1+Q_2\cdots-Q_n)$$

and that the vacuum expectation value of the effective field $\Phi(Q)$ is

$$\langle 0|\Phi(Q)|0\rangle=\mu^4\int d^4X\exp(-iQ\cdot X)=(2\pi)^4\mu^4\delta^{(4)}(Q). \tag{3}$$

We emphasize that the appearance of the function $\delta^{(4)}$ in Eq. (3) is essential for ensuring the overall energy-momentum conservation and for preserving the Lorentz covariance of the model.

The constant μ^4 in Eq. (3), introduced for dimensional reasons, is related to the volume of a large four-dimensional box of interest for our problem by $\mu^4=(VT)^{-1}$. A short comment on the box size will be given in the last section.

Then, using the relations (1) and (3) we get the following expression for the norm of the single meson state:

$$\begin{aligned}\langle M_i(P')|M_j(P)\rangle=&(2\pi)^3\delta_{ij}\delta^{(3)}(P-P')\delta(E-E')\\&\times(2\pi)^4\mu^4\int\frac{d^3p}{e/m}\frac{d^3q}{\epsilon/m'}d^4Q\\&\times\delta^{(4)}(p+q+Q-P)\varphi(p,q;Q)^2\\&\times\mathrm{Tr}\left(\frac{\hat{p}+m}{2m}\Gamma_M\frac{\hat{q}-m'}{2m'}\Gamma_{M'}\right).\end{aligned} \tag{4}$$

The function $\delta^{(3)}(P-P')\delta(E-E')$ in Eq. (4) originating from $\delta^{(4)}(p+q+Q-P)$ in the definition of a single meson state can also be written as $(E/M)\delta^{(3)}(P-P')\delta(M-M')$. It is a signal for the continuous mass spectrum of the complex system representing the meson and cannot be modified without renouncing the real Lorentz invariance of the model. This forces us to treat the physical meson as a mixed state whose probability density is the mass distribution function $\rho(M,M_0)$ with M_0 as central value and the normalization condition

$$\int\rho(M,M_0)dM=1. \tag{5}$$

Accordingly, the normalization condition for the meson wave function writes

$$\begin{aligned}&(2\pi)^4\mu^4\frac{1}{2M_0^2}D_M^2\int\frac{d^3p}{e/m}\frac{d^3q}{\epsilon/m'}d^4Q\,\delta^{(4)}(p+q+Q-P)\\&\times\sigma^2(Q_0,\mathbf{Q})\left(\frac{\hat{p}+m}{2m}\Gamma_M\frac{\hat{q}-m'}{2m'}\right)=1\end{aligned} \tag{6}$$

and the density of states in phase space modifies by replacing $1/(2\pi)^3(d^3P/2E)$ with $[1/(2\pi)^3](d^3P/2E)\rho(M,M_0)dM$.

It is important to notice that the continuous distribution of masses is not at all unusual, but it is a natural consequence of the uncertainties produced by quantum fluctuations in any system. Since all the hadrons, with the exception of the proton, are unstable, the mass distribution function ρ may be taken of Breit-Wigner form $\rho(M,M_0)=(\pi)^{-1}\Gamma[(M-M_0)^2+\Gamma^2]^{-1}$ where Γ is the particle width. However, if the dependence of the matrix elements on the meson mass M is rather smooth and if the width of the mass distribution function is small, one can replace M by M_0 in the expressions of the matrix elements and perform the integral over meson masses in the new expression of the density of states by using the normalization condition (5). The calculation can then proceed like in the old case, using wave functions which satisfy the normalization condition (6).

The matrix element of interest for the leptonic decay of a meson, written in the lowest order of perturbation with respect to the weak interaction, is

$$\langle 0|U_s(+\infty,0)A_\mu(0)U_s(0,-\infty|M(P)\rangle = iF_M P_\mu, \quad (7)$$

where the operator $U_s(t,t')$ describes the evolution of a system under the action of strong interaction among the constituents, and A_μ is the free-field weak current of interest in the process. It is important to notice that in soft processes, like, for instance, the present one, the perturbative expansion of U_s is inappropriate. For this reason we shall not consider the virtual states generated by the evolution operator in the perturbative approach, but merely look at the real modifications which could appear in the distribution of flavors and momenta during the time translation. In the above case no such changes could appear, since the real vacuum and the single meson state are stable states whose content does not change under the action of strong interaction and consequently both time translation operators in Eq. (7) can be replaced by unity.

By using the relation (3), the canonical anticommutation relations of the fermionic operators and integrating over the internal momenta, we obtain from the matrix element (7) the following expression for the decay constants:

$$F_M = (2\pi)^4\mu^4 D_M 2\pi\sqrt{3}\frac{p(m+m')}{M}\left[1-\frac{(m-m')^2}{M^2}\right], \quad (8)$$

where $p=(M/2)\sqrt{[1-(m+m')^2/M^2][1-(m-m')^2/M^2]}$ and the factor $\sqrt{3}$ comes from the colors.

It is worthwhile noticing that the leptonic decay constant in Eq. (8) is proportional to the internal wave function at $Q_\mu=0$, which means the absence of any other excitations beside the valence quarks. Expressing this result in more general terms, one may say that the leptonic decay constants are proportional to the value of the internal function at vanishing contribution from the binding effects. This is in remarkable agreement with the old assumption that F_P is proportional to the internal wave function at vanishing distance between the quarks [1,4], since, according to the asymptotic freedom, this is the point in the configuration space where the confining forces vanish. It is a strong argument for considering the present model as a real relativistic generalization of the potential models.

We eliminate now the constant D_M in Eq. (8) with the aid of the normalizaton condition (6) and write the decay constants in terms of μ and of the integral over the internal distribution of momenta which is in fact a function of meson and quark masses as well as of the model parameters α and β:

$$F_M = (2\pi)^2\mu^2(48\pi^3)^{1/2}p(m+m')\left(1-\frac{(m-m')^2}{M^2}\right)\Bigg\{\int dQ_0\mathbf{Q}^2 d|\mathbf{Q}|\sigma^2(Q_0,\mathbf{Q}) \times\left[\frac{(M-Q_0)^2-\mathbf{Q}^2-(m-m')^2}{M^2[(M-Q_0)^2-\mathbf{Q}^2]}\right]\sqrt{[(M-Q_0)^2-\mathbf{Q}^2-m^2-m'^2]^2-4m^2m'^2}\Bigg\}^{-1/2}. \quad (9)$$

Similar expressions can be written for the decay constants of neutral mesons. Defining them as in Ref. [12] one has

$$F_{M^0} = (2\pi)^2\mu^2(24\pi^3)^{1/2}\sum_{i=u,d,s}\kappa_i^2 m_i\left(1-\frac{4m_i^2}{M^2}\right)^{1/2} \times\Bigg\{\sum_{i=u,d,s}\kappa_i^2\int dQ_0|\mathbf{Q}|^2 d|\mathbf{Q}|\sigma^2(Q_0,\mathbf{Q})\left(\left[\left(1-\frac{Q_0}{M}\right)^2-\frac{\mathbf{Q}^2}{M^2}-\frac{2m_i^2}{M^2}\right]^2-\frac{4m_i^4}{M^4}\right)^{1/2}\Bigg\}^{-1/2}, \quad (10)$$

where m_i are the quark masses, $\kappa_i = a(\lambda_3)_{ii}+b(\lambda_8)_{ii}+c(\lambda_0)_{ii}$ with λ_j the Gell-Mann matrices [12], $a=1$; $b=c=0$ for π^0, $a=0$; $b=\cos\theta_P$; $c=-\sin\theta_P$ for η, $a=0$; $b=\sin\theta_P$; $c=\cos\theta_P$ for η' and $\theta_P=-10^0$ or $\theta_P=-23^0$ [12].

III. NUMERICAL RESULTS

Before proceeding to the numerical calculations we have to analyze the relation of the model parameters α, β, μ, with the general features of the bound $q\bar{q}$ system.

First of all we remind that Q_0 is the analogue of the potential energy in the nonrelativistic models. In the present approach it is the energy of the oscillation modes of the quark-gluonic field confining the valence quarks inside the meson. Its cutoff parameter, α, must be chosen such as to ensure a relative stability of F_P with the increase of M_P. [See Eq. (9)]. Our tests with $\alpha=\kappa\sqrt{mm'M/(m+m')}$, $\alpha=\kappa\sqrt{\sqrt{mm'}M}$, $\alpha=\kappa(m+m')$, and $\alpha=\kappa M$ where M is the meson mass and κ a universal parameter, proved that the last choice is the best. All the others either lead to very small values for the decay constants of the heavy mesons or do not allow to fit pion and kaon decay constants with the same set of parameters, as required from the beginning.

The same stability argument forces us to introduce an additional cutoff for $|\mathbf{Q}|$, the momentum carried by the effective component, since the simple requirement for Q^2 to be positive would lead to a too strong increase of F_P with the meson mass. We recall that $|\mathbf{Q}|$ has been introduced for relativistic consistency; we did not relate it to the potential en-

TABLE I. Decay constants of heavy mesons. The indices (a), (b), (c) correspond to the trial functions (2a), (2b), (2c).

$\alpha_{(i)}, \beta_{(i)}$ $(2\pi)^4\mu^4_{(i)}$			Decay constants			
	m_c	m_b	F_D	F_{D_s}	F_B	F_{B_s}
$\alpha_{(a)}=0.075M$	1.0	4.1	126	152	107	108
$\beta_{(a)}=0.096$ GeV	1.3	4.3	109	141	92	95
$(2\pi)^4\mu^4_{(a)}=310$ MeV^4	1.6	4.5	56	91	64	80
$\alpha_{(a)}=0.05M$	1.0	4.1	125	151	111	109
$\beta_{(a)}=0.065$ GeV	1.3	4.3	111	142	95	96
$(2\pi)^4\mu^4_{(a)}=57$ MeV^4	1.6	4.1	60	95	77	82
$\alpha_{(a)}=0.025M$	1.0	4.1	121	144	98	113
$\beta_{(a)}=0.032$ GeV	1.3	4.3	111	139	87	102
$(2\pi)^4\mu^4_{(a)}=3.3$ MeV^4	1.6	4.5	65	98	72	88
$\alpha_{(b)}=0.075M$	1.0	4.1	155	188	148	172
$\beta_{(b)}=0.082$ GeV	1.3	4.3	137	177	126	151
$(2\pi)^4\mu^4_{(b)}=397$ MeV^4	1.6	4.5	73	118	102	127
$\alpha_{(b)}=0.05M$	1.0	4.1	156	190	153	176
$\beta_{(b)}=0.054$ GeV	1.3	4.5	140	180	132	157
$(2\pi)^4\mu^4_{(b)}=71.7$ MeV^4	1.6	4.5	79	124	108	133
$\alpha_{(b)}=0.025M$	1.0	4.1	155	188	156	178
$\beta_{(b)}=0.027$ GeV	1.3	4.5	143	181	136	160
$(2\pi)^4\mu^4_{(b)}=4.1$ MeV^4	1.6	4.5	85	129	114	138
$\alpha_{(c)}=0.075M$	1.0	4.1	185	227	203	235
$\beta_{(c)}=0.04$ GeV	1.3	4.3	168	216	177	209
$(2\pi)^4\mu^4_{(c)}=117$ MeV^4	1.6	4.5	95	149	144	178
$\alpha_{(c)}=0.05M$	1.0	4.1	183	223	204	235
$\beta_{(c)}=0.027$ GeV	1.3	4.3	168	214	178	210
$(2\pi)^4\mu^4_{(c)}=22.3$ MeV^4	1.6	4.5	99	152	148	181
$\alpha_{(c)}=0.025M$	1.0	4.1	176	213	198	229
$\beta_{(c)}=0.014$ GeV	1.3	4.3	163	207	175	207
$(2\pi)^4\mu^4_{(c)}=1.3$ MeV^4	1.6	4.5	99	151	147	180

ergy, but rather to some fluctuations in the momentum carried by the collective excitations denoted by Φ. The parameter β in Eqs. (2a), (2b), and (2c) is hence a measure of the fluctuation amplitude and we shall assume that it does not depend on the quark or meson masses because the vacuumlike excitations are not sensitive to the flavors. However, we expect β be smaller than the cutoff parameter of Q_0 in the case of heavy mesons, because the fluctuation effect must be negligible in their cases.

The parameter $(2\pi)^4\mu^4$ is assumed to be an universal constant, related to the volume of the four-dimensional box relevant for the process. It has the same meaning like the four-dimensional volume of the lattice, in lattice calculations [3]. Its independence on the masses will be used as a consistency condition in fixing the parameters α and β of the model.

Specifically, our procedure is to reverse the equation (9) and to express μ in terms of F_M in the case of the π and K mesons, whose decay constants $F_\pi=130.7$ MeV, $F_K=159.8$ MeV [12] are taken as input. The next step is to search for the values of the parameters α and β yielding the same value for μ in these cases. The parameters α and β satisfying this consistency requirement will then be introduced in Eq. (8) in order to obtain the decay constants of the D and B mesons.

A last comment concerns the quark masses. We recall that in our model the quarks are assumed to be free and the weak current entering the matrix element (7) is expressed in terms of free quark fields. Then, as already emphasized in the comments to Eq. (7), it is natural to assume that the quarks are of the current type and that their masses are rather small, not far from the chiral symmetry limit. The calculations have been done using the values of the light quark masses $m_u=5.1$ MeV, $m_d=9.3$ MeV, $m_s=175$ MeV resulting from the chiral perturbation theory [13]. These values are also in agreement with the quark masses recently obtained in lattice calculations [3]. Heavy quark masses have been taken in the range quoted by Particle Data [12]. The results obtained using the trial functions (2a), (2b), (2c) are listed in Table I. The heavy quark masses are given in GeV and the decay constants are given in MeV.

Using Eq. (10) and the same sets of parameters as above we calculated also the decay constants of the lightest pseudoscalar mesons. The results are quoted in Table II.

IV. COMMENTS AND CONCLUSIONS

Analyzing the numerical results in Table I, one notices that, for each of the tested internal functions, the decay constants do not change significantly when passing from one set of parameters α, β, μ to another set which fits the values of F_π and F_K. Indeed, for a change with 200% of α and β,

TABLE II. The decay constants of the lightest neutral mesons. The trial functions are indicated in the first column.

	$\pi^0(135)$	$\eta(547)$ $\theta_P=-10°$	$\eta(547)$ $\theta_P=-23°$	$\eta'(958)$ $\theta_P=-10°$	$\eta'(958)$ $\theta_P=-23°$
(2a)	137–139	128–139	77–78	67–69	94–97
(2b)	≈ 139	≈ 131	77–78	75–76	105–107
(2c)	135–139	129–132	76–78	77–81	108–114

$(4\pi)^4\mu^4$ changes with two orders of magnitude, while the theoretical values of the decay constants change with less than 10%.

The decay constants are more sensitive to the variation of the heavy quark masses and could be used in principle for a more precise determination of the last ones. The comparison with the experimental values [12] $F_D \leqslant 300, F_{D_s} = 232 \pm 45 \pm 20 \pm 48$ MeV, or $F_{D_s} = 344 \pm 37 \pm 52 \pm 42$ MeV and with the values yielded by QCD sum rules [4,14] and lattice calculations [3] $F_D \approx (1.35 \pm 0.04 \pm 0.06) F_\pi$, $F_{D_s} \approx (1.55 \pm 0.10) F_\pi$, $F_B \approx (1.49 \pm 0.06 \pm 0.05) F_\pi$, $F_B = 185 \pm 40$ MeV shows that the agreement is better at the lowest values of the heavy quark masses. Things look mainly the same for any of the trial functions, but the best fit of the data seems to be done with the internal function (2c). Of course, this is just a qualitative estimate. A more reliable test could be provided by the fit of the weak or electromagnetic form factors, which are very sensitive to the form of the internal function.

In the case of neutral mesons, we found $(F_\eta)_{\rm th}$ in the range 128–139 MeV for $\theta_P = -10°$, which is in agreement with $F_\eta = 133 \pm 10$ MeV quoted in Ref. [12], but $(F_{\pi^0})_{\rm th} \approx 138$ MeV, slightly larger than $F_{\pi^0} = 119 \pm 4$ MeV in Ref. [12]. One sees also that the calculated values of $F_{\eta'}$ both for $\theta_P = 10°$ and $\theta_P = -23°$ are smaller than $F_{\eta'} = 126 \pm 7$ MeV, quoted in Ref. [12]. The differences noticed above must not be taken too seriously because of the large uncertainties entering the values quoted in Ref. [12]. They come from the extrapolation on the meson mass shell when deriving F_{P^0} with the aid of the axial anomaly but, for η, η' they come also from the uncertainties in the mixing angle.

Resuming, one may say that the present model yields reasonable values for the decay constants of light *and* heavy mesons using the same set of parameters, which is quite remarkable.

A last comment concerns the parameter μ. As it can be seen from Table I, its values resulting from the fit of the pion and kaon decay constants are in the range 0.2–0.7 MeV. Recalling that μ^{-4} is equal to the four-dimensional volume VT of a very large box containing the meson, we get a box size of about 300–1000 fm, quite large in comparison with the meson size which is less than 1 fm. The large value found for the box size, sensible larger than the lattice size [3], as well as the relative independence of the results in Tables I and II on the value of μ, if μ is sufficiently small, are strong arguments for the consistency of the present model.

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