Relativistic corrections to the polarized structure functions in the resonance region

Dong Yu-bing
Faculty of Human Development, Kobe University, Kobe 657, Japan
and Institute of High Energy Physics, Academia Sinica, Beijing 100039, People's Republic of China

T. Morii
Faculty of Human Development, Kobe University, Kobe 657, Japan

T. Yamanishi
Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567, Japan

(Received 13 August 1996; revised manuscript received 2 December 1996)

Based on the relativistic harmonic oscillator model, the polarized structure functions of $\Gamma_1(Q^2)$ and $\Gamma_2(Q^2)$ are calculated in the resonance region. The relativistic effects on the transition amplitudes and the structure functions are addressed. [S0556-2821(97)04107-6]

PACS number(s): 12.39.Ki, 13.88.+e, 14.20.Gk

I. INTRODUCTION

The investigation of the spin-dependent structure functions of the proton and neutron and the study of the spin crisis have always been of great interest [1–3]. There are two important spin-dependent structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$. Here $g_1(x,Q^2)$ is given as [4]

$$g_1(x,Q^2)=\frac{MK}{8\pi^2\alpha(1+Q^2/\omega^2)}\left(\sigma_{1/2}(\omega,Q^2)-\sigma_{3/2}(\omega,Q^2)+\frac{2\sqrt{Q^2}}{\omega}\sigma_{TS}(\omega,Q^2)\right), \quad (1)$$

where M denotes the target nucleon mass, $K(\omega)$ is the photon flux (energy), and $\sigma_{1/2,3/2}$ and σ_{TS} represent the transverse photoabsorption cross sections on a polarized nucleon target and the interference cross section between transverse and longitudinal currents, respectively. For $g_1(x,Q^2)$, there are two major sum rules: One is the Drell-Hearn-Gerasimov (DHG) sum rule [5] in the real photon limit and the other is the sum rule in deep inelastic scattering (DIS) as follows:

$$\Gamma_1(Q^2)=\int_0^1 g_1(x,Q^2)dx=\begin{cases}-\omega_{\rm th}\kappa^2/4M, & Q^2=0,\\ z, & Q^2\to\infty.\end{cases} \quad (2)$$

Here κ is the anomalous magnetic moment of the nucleon, z is a positive quantity which is determined for deep inelastic scattering, and $\omega_{\rm th}=(Q^2+2m_\pi M+m_\pi^2)/2M$ is the threshold energy of pion photoproduction. Another polarized structure function is $g_2(x,Q^2)$ and is given as [4]

$$g_2(x,Q^2)=\frac{MK}{8\pi^2\alpha(1+Q^2/\omega^2)}\left[-\sigma_{1/2}(\omega,Q^2)+\sigma_{3/2}(\omega,Q^2)+\frac{2\omega}{\sqrt{Q^2}}\sigma_{TS}(\omega,Q^2)\right]. \quad (3)$$

For $g_2(x,Q^2)$, there are another two sum rules which are the Schwinger sum rule [6] in the real photon limit and the Burkhardt-Cottingham (BC) sum rule [7] in the deep inelastic scattering region as follows:

$$\Gamma_2(Q^2)=\int_0^1 g_2(x,Q^2)dx=\begin{cases}\omega_{\rm th}\kappa(\kappa+e_T)/4M, & Q^2=0,\\ 0, & Q^2\to\infty.\end{cases} \quad (4)$$

The DHG and Schwinger sum rules connect the helicity structure of the cross sections in the inelastic region with the ground-state properties. Since these sum rules are based on the general principle of physics such as Lorentz invariance, gauge invariance, crossing symmetry, causality, and unitarity, they give important grounds for our understanding of hadronic structure [8]. It should be emphasized that in the decomposition of the antisymmetric hadronic tensor, the scalar coefficients were chosen by Feynman [9] so that g_2 cancels in the scaling limit. However, g_2, which describes the difference between longitudinal and transverse polarized nucleons, should manifest itself in the DHG sum rule and the strong Q^2 dependence of the DHG sum rule can be simply attributed to g_2 [10].

Because of the sign change of the polarized structure function of the proton, $\Gamma_1^p(Q^2)$, from the real photon limit to the DIS in the large Q^2 limit, it has been speculated [3] that the DHG sum rule might play an important role in explaining the experimental data by the European Muon Collaboration (EMC) [1], which suggest that the spin of the proton is not carried by its valence quarks. To understand how this transition occurs will provide us insights into the Q^2 dependence of the spin structure function, in particular, for small Q^2 regions and, thus, the higher twist corrections to the EMC data. The existing data show that the function $\Gamma_1^p(Q^2)$ changes its sign at $Q^2=0.5$–1.0 GeV^2, where the baryon resonances play a dominant role [11,12]. Thus, the investigation of the spin-structure function in the resonance region becomes increasingly important both theoretically and experimentally. So far several experimental proposals in this region have been approved at continuous Electron Beam Accelerator Facility (CEBAF) [12]. In addition, the experiments for testing the DHG sum rule have been already accepted as an important research project both at MAMI in Mainz and at SPring-8 in Japan. At MAMI, the 50% polarized photon beam with energy from 0.14 to 0.8 GeV will be available for a test of the DHG sum rule in 1997. Moreover, by using almost 100% polarized photon beams produced by a laser technique, new experimental tests of the DHG sum rule will be expected in 1998 at SPring-8.

Recently, Soffer and Teryaev [13] and Li and Dong [14] have pointed out that the investigation of the Q^2 evolution of the polarized structure functions $\Gamma_1(Q^2)$ and $\Gamma_2(Q^2)$ in the resonance region sheds light on the important role of the interference cross section σ_{TS} between transverse and longitudinal currents, which is presented in the third term in Eqs. (1) and (3). In Ref. [14] the evolution of the structure functions $\Gamma_1(Q^2)$ and $\Gamma_2(Q^2)$ in the finite Q^2 region is given based on a nonrelativistic model. Moreover, the two sum rules for the polarized structure function of Γ_2 in Eq. (4) are also obtained based on the same model. Much of what we know and what we have done about the excited baryon states has been based on a simple nonrelativistic quark models [15–18]. However, the success of the nonrelativistic quark model in describing the baryon-photon couplings cannot be naturally extended to electroproduction amplitudes. In the nonrelativistic quark model, the charge radius of the proton is too small [18] and the form factors fall off faster than the dipole form for a large Q^2 value. Similar problems also exist in the calculation of the Q^2 dependence of the form factors for higher excited baryons [19]. There are still discrepancies between the predictions of the nonrelativistic quark model and the extracted helicity amplitudes [20], though we do not have much experimental data on excited baryons. Further experimental study of the properties of resonances is necessary for the analysis of electroproduction and will be done at CEBAF, MAMI, and SPring-8.

To avoid the above problem, in most of the nonrelativistic calculations of the helicity amplitudes and polarized structure functions the equal velocity reference frame (EVF) [21] is often used inconsistently to calculate the transition helicity amplitudes because in the EVF the proton form factor prefers the dipole form [21], while the polarized structure function is calculated in the lab frame. It is expected that a relativistic description of baryon states is necessary if one wants to get a good understanding of the Q^2 dependence of the helicity amplitudes $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ and, accordingly, the absorption cross sections $\sigma_{1/2}$, $\sigma_{3/2}$, and σ_{TS}. More exact calculations of the photoabsorption cross section and predictions both for the sum rules and the Q^2-dependent evolution of the polarized structure functions $\Gamma_1(Q^2)$ and $\Gamma_2(Q^2)$ are urgently required for the forthcoming experiments at CEBAF, MAMI, and SPring-8.

The purpose of this work is to give a relativistic description for the polarized structure functions and their Q^2-dependent evolution in the resonance region based on a relativistic harmonic oscillator model (RHOM). We know that the relativistic harmonic oscillator model, as first proposed in Refs. [22–24] for the study of the proton form factor, enables us to take into account the Lorentz contraction effect of the composite particle wave function. The effect is very essential, since at finite Q^2 range, it provides a dipolelike Q^2 dependence of the nucleon elastic form factor. Because of this advantage, it has been widely used for the description of the hadron properties [23–29].

This paper is organized as follows. In Sec. II, a brief introduction of the RHOM is given. The calculation of transition amplitudes and photoabsorption cross sections will be given in Sec. III. Section IV is devoted to concluding remarks.

II. RELATIVISTIC HARMONIC OSCILLATOR MODEL

As is well known, the relativistic harmonic oscillator model was first introduced for the calculation of the proton form factors. In the frame of the RHOM, the three-body system satisfies the equation

$$\left(\sum_{i=1}^{3}\Box_i-\overline{\kappa}\sum_{i\neq j=1}^{3}(x_i-x_j)^2+V_0\right)\Psi(x_1,x_2,x_3)=0, \quad (5)$$

where $\overline{\kappa}$ and V_0 are the parameter of the harmonic oscillator model and the depth of the potential, respectively. The squared four-vectors are defined as $x^2=x_0^2-\vec{x}^2$ and $\Box=\partial_0^2-\vec{\nabla}^2$. Introducing the relativistic Jacobian coordinates

$$R=\frac{1}{\sqrt{3}}(x_1+x_2+x_3), \quad (6)$$

$$\rho=\frac{1}{\sqrt{2}}(x_1-x_2), \quad \lambda=\frac{1}{\sqrt{6}}(x_1+x_2-2x_3),$$

Eq. (5) can be diagonalized as

$$[\Box_R+\Box_\rho+\Box_\lambda-\omega^2\rho^2-\omega^2\lambda^2+V_0]\Psi(R,\rho,\lambda)=0, \quad (7)$$

where $\omega^2=3\overline{\kappa}$. In the center-of-mass frame, the spatial wave function of the ground state is described by

$$\Psi_0=\exp(-iMR/\sqrt{3})\Psi_{\rm int}(\rho,\lambda), \quad (8)$$

$$\Psi_{\rm int}(\rho,\lambda)=\Pi_{v=\rho,\lambda}\Psi(v),$$

$$\Psi(v)=\left(\frac{\omega}{\pi}\right)\exp\{-\tfrac{1}{2}\omega(v_0^2+\vec{v}^2)\},$$

where $v=(\rho,\lambda)$ stands for the two independent variables of the system. As stressed by Lipes [25], in the center-of-mass frame, the excited state with the internal degree of freedom can be expressed as

$$\Psi_{n_xn_yn_z}=N_{n_xn_yn_z,l_xl_yl_z}[\Pi_{i,j=x,y,z}H_{n_i}(\rho)H_{n_j}(\lambda)]\Psi_0(R,\rho,\lambda), \quad (9)$$

where $N_{n_xn_yn_z,l_xl_yl_z}$ is the normalization constant and H is the Hermite polynomial in ρ and λ. Since the timelike excitation implies the state of imaginary mass, to avoid this catastrophe, we, as in Ref. [25], restrict ourselves to spacelike excitations only in the center-of-mass frame.

Moreover, in any arbitrary frame, for a variable we covariantly have

$$-(v_0^2+\vec{v}^2)=v^2-2\left(\frac{p\cdot v}{M}\right)^2, \quad (10)$$

TABLE I. The spatial matrix elements $\langle [N_6,L^p]_{Nms}|e^{ik\cdot r_3}|[56,0^+]_{00s}\rangle$ of the nonrelativistic (NR) quark model and RHOM in the q frame. The full matrix elements are obtained by multiplying the entries in this table by a factor of $e^{-(k^2/6\alpha)}$ and $e^{-(k^2+\omega^2)/6\alpha f}$ for the NR and RHOM cases, respectively.

$[N_6,L^p]_{Nms}$	NR	RHOM
$[56,0^+]_{00s}$	1	$\frac{1}{f^2}$
$[70,1^-]_{10\rho}$	0	0
$[70,1^-]_{10\lambda}$	$-\frac{ik}{\sqrt{3\alpha}}$	$-\frac{iE_f}{\sqrt{3\alpha}f^3M_f}\left(k-\frac{p_f\omega}{E_f}\right)$
$[56,0^+]_{20s}$	$\frac{k^2}{6\sqrt{3}\alpha}$	$\frac{1}{\sqrt{3}f^2}\left[1-\frac{1}{f}\left(1-\frac{k^2}{6\alpha f}\right)\right]+\frac{p_f}{\sqrt{3}M_f^2f^3}\left[2p_f-\frac{p_f(k^2+\omega^2)}{6\alpha f}+\frac{E_fk\omega}{3\alpha f}\right]$
$[56,2^+]_{20s}$	$-\frac{k^2}{3\sqrt{6}\alpha}$	$\frac{1}{\sqrt{6}f^2}\left[\frac{1}{f}-1-\frac{k^2}{3\alpha f^2}\right]+\frac{2p_f}{\sqrt{6}M_f^2f^3}\left[2p_f\left(1-\frac{(k^2+\omega^2)}{12\alpha f}\right)+\frac{E_f\omega k}{3\alpha f}\right]$
$[70,0^+]_{20\rho}$	0	0
$[70,0^+]_{20\lambda}$	$\frac{k^2}{6\sqrt{3}\alpha}$	$\frac{k^2}{6\sqrt{3}f^4\alpha}+\frac{p_f}{6\sqrt{3}M_f^2f^4\alpha}[p_f(k^2+\omega^2)-2E_f\omega k]$
$[70,2^+]_{20\rho}$	0	0
$[70,2^+]_{20\lambda}$	$\frac{k^2}{3\sqrt{6}\alpha}$	$-\frac{1}{\sqrt{6}f^2}\left[\frac{1}{f}-1-\frac{k^2}{3f^2\alpha}\right]+\sqrt{\frac{2}{3}}\frac{p_f}{6M_f^2\alpha f^4}[p_f(k^2+\omega^2)-2E_f\omega k]$

and note that in the covariant wave function of a state with momentum p_μ, the argument of H becomes the components of the spacelike four-vectors $\rho_\mu-(p\cdot\rho)/M^2p_\mu$ and $\lambda_\mu-(p\cdot\lambda)/M^2p_\mu$ for excited states.

III. CALCULATION OF HELICITY AMPLITUDES AND STRUCTURE FUNCTIONS

As shown in Eqs. (1) and (3), to calculate the polarized structure functions Γ_1 and Γ_2 in the resonance region, we first compute the helicity amplitudes as in Ref. [14]. In this paper, we give priority to studying the relativistic effect both in the baryon wave function and in the photon wave function. As for the transverse and longitudinal transition operators, we still use them as given by Close and Li [30] and in Ref. [14], since these transition operators could generate the sum rules for the polarized structure functions Γ_1 and Γ_2 both in the real photon limit and in deep inelastic scattering regions, though it originates from the nonrelativistic reduc tion of the electromagnetic interaction between photon and

TABLE II. The spatial matrix elements $\langle [N_6,L^p]_{Nms}|e^{ik\cdot r_3}p_3^{\pm}|[56,0^+]_{00s}\rangle$ of the nonrelativistic (NR) quark model and RHOM in the q frame. Keys as in Table I.

$[N_6,L^p]_{Nms}$	NR	RHOM
$[56,0^+]_{00s}$	0	0
$[70,1^-]_{1\pm1\rho}$	0	0
$[70,1^-]_{1\pm1\lambda}$	$\pm i\sqrt{2\alpha/3}$	$\pm i\sqrt{2\alpha/3}\,1/f^2$
$[56,0^+]_{20s}$	0	0
$[56,2^+]_{2\pm1s}$	$\pm\frac{k}{3}$	$\pm E_f/3f^3M_f[k-(p_f\omega/E_f)]$
$[70,0^+]_{20\rho}$	0	0
$[70,0^+]_{20\lambda}$	0	0
$[70,2^+]_{2\pm1\rho}$	0	0
$[70,2^+]_{2\pm1\lambda}$	$\mp k/3$	$\mp E_f/3f^3M_f[k-(p_f\omega/E_f)]$

TABLE III. The spatial matrix elements $\langle[N_6,L^p]_{Nms}|e^{ik\cdot r_3}p_3^z|[56,0^+]_{00s}\rangle$ of the nonrelativistic (NR) quark model and RHOM in the q frame. Keys as in Table I.

$[N_6,L^p]_{Nms}$	NR	RHOM
$[56,0^+]_{00s}$	$-\frac{k}{3}$	$-\frac{E_i}{3f^3M_i}\left(k-\frac{p_i\omega}{E_i}\right)$
$[70,1^-]_{10\rho}$	0	0
$[70,1^-]_{10\lambda}$	$-\frac{i\sqrt{3\alpha}}{3}\left(1-\frac{k^2}{3\alpha}\right)$	$-\frac{i\sqrt{\alpha}E_fE_i}{\sqrt{3}f^3M_iM_f}\left[1-\frac{k^2}{3\alpha f}+\frac{p_ip_f}{E_iE_f}\left(1-\frac{\omega^2}{3\alpha f}\right)+\frac{\omega k}{6\alpha}\left(\frac{p_i}{E_i}+\frac{p_f}{E_f}\right)\right]$
$[56,0^+]_{20s}$	$\frac{k}{3\sqrt{3}}\left(1-\frac{k^2}{6\alpha}\right)$	$\frac{E_i}{3\sqrt{3}f^3M_i}\left\{\left(\frac{2}{f}-1-\frac{k^2}{6\alpha f^2}\right)k-\left(\frac{1}{f}-1-\frac{k^2}{6\alpha f^2}\right)\frac{p_i\omega}{E_i}-\frac{6p_f}{M_f^2f}\left[\left(k-\frac{p_i\omega}{E_i}\right)\left(-\frac{p_f}{2}+\frac{p_f(\omega^2+k^2)}{36\alpha f}-\frac{E_f\omega k}{18\alpha f}\right)+\frac{E_f}{6}\left(\omega-\frac{p_ik}{E_i}\right)\right]\right\}$
$[56,2^+]_{20s}$	$-\frac{2k}{3\sqrt{6}}\left(1-\frac{k^2}{6\alpha}\right)$	$-\frac{\sqrt{2}E_i}{3\sqrt{3}f^3M_i}\left\{\left(\frac{3}{2f}-\frac{1}{2}-\frac{k^2}{6\alpha f^2}\right)k-\left(\frac{1}{f}-1-\frac{k^2}{3\alpha f^2}\right)\frac{p_i\omega}{2E_i}-\frac{6p_f}{M_f^2f}\left[\left(k-\frac{p_i\omega}{E_i}\right)\times\left(-\frac{p_f}{2}+\frac{p_f(\omega^2+k^2)}{36\alpha f}-\frac{E_f\omega k}{18\alpha f}\right)+\frac{E_f}{6}\left(\omega-\frac{p_ik}{E_i}\right)\right]\right\}$
$[70,0^+]_{20\rho}$	0	0
$[70,0^+]_{20\lambda}$	$\frac{k}{3\sqrt{3}}\left(1-\frac{k^2}{6\alpha}\right)$	$\frac{E_i}{3\sqrt{3}f^4M_i}\left\{\left(1-\frac{k^2}{6\alpha f}\right)k+\frac{p_i\omega k^2}{6\alpha E_if}-\frac{p_f}{M_f^2}\left[\left(k-\frac{p_i\omega}{E_i}\right)\times\left(p_f-\frac{p_f(K^2+\omega^2)}{6\alpha f}+\frac{E_f\omega k}{3\alpha f}\right)-E_f\left(\omega-\frac{p_ik}{E_i}\right)\right]\right\}$
$[70,2^+]_{20\rho}$	0	0
$[70,2^+]_{20\lambda}$	$\frac{2k}{3\sqrt{6}}\left(1-\frac{k^2}{6\alpha}\right)$	$-\frac{E_i}{3\sqrt{6}f^3M_i}\left\{\left(1-\frac{3}{f}+\frac{k^2}{6\alpha f^2}\right)k-\frac{p_i\omega}{E_if}\left[\frac{1}{f}\left(1-\frac{k^2}{3\alpha f}\right)-1\right]-\frac{2p_f}{M_f^2f}\left[\left(k-\frac{p_i\omega}{E_i}\right)\times\left(p_f-\frac{p_f(k^2+\omega^2)}{6\alpha f}+\frac{E_f\omega k}{3\alpha f}\right)-E_f\left(\omega-\frac{p_ik}{E_i}\right)\right]\right\}$

quarks [11,30].

The transverse operator for the electromagnetic interaction between quark and photon in the nonrelativistic approximation is [30]

$$H_t=\sum_j\left\{e_j\vec{r}_j\cdot\vec{E}_j-\frac{e_j}{2m_j}\vec{\sigma}_j\cdot\vec{B}_j-\frac{e_j}{4m_j}\vec{\sigma}_j\cdot\left[\vec{E}_j\times\frac{\vec{p}_j}{2m_j}-\frac{\vec{p}_j}{2m_j}\times\vec{E}_j\right]\right\}+\sum_{j<l}\frac{1}{4M}\left[\frac{\vec{\sigma}_j}{m_j}-\frac{\vec{\sigma}_l}{m_l}\right]\cdot(e_l\vec{E}_l\times\vec{p}_j-e_j\vec{E}_j\times\vec{p}_l), \quad (11)$$

and the longitudinal current J_0 is [14]

$$J_0=\sqrt{\frac{1}{2\omega}}\left\{\sum_j\left(e_j+\frac{ie_j}{4m_j^2}\vec{k}\cdot(\vec{\sigma}_j\times\vec{p}_j)\right)e^{i\vec{k}\cdot\vec{r}_j}-\sum_{j<l}\frac{i}{4M}\left(\frac{\vec{\sigma}_j}{m_j}-\frac{\vec{\sigma}_l}{m_l}\right)\cdot(e_j\vec{k}\times\vec{p}_le^{i\vec{k}\cdot\vec{r}_j}-e_l\vec{k}\times\vec{p}_je^{i\vec{k}\cdot\vec{r}_l})\right\}, \quad (12)$$

where the electric and magnetic fields are defined as

$$\vec{E}=i\omega\sqrt{4\pi}\sqrt{\frac{1}{2\omega}}\vec{\epsilon}\exp(-i\vec{k}\cdot\vec{r}),$$

$$\vec{B}=i\sqrt{4\pi}\sqrt{\frac{1}{2\omega}}\vec{\epsilon}\times\vec{k}\exp(-i\vec{k}\cdot\vec{r}). \quad (13)$$

TABLE IV. The spatial matrix elements $\langle[N_6,L^p]_{Nms}|e^{ik\cdot r_2}p_1^{\pm}|[56,0^+]_{00s}\rangle$ of the nonrelativistic (NR) quark model and RHOM in the q frame. Keys as in Table I.

$[N_6,L^p]_{Nms}$	NR	RHOM
$[56,0^+]_{00s}$	0	0
$[70,1^-]_{1\pm1\rho}$	$\mp i\sqrt{\alpha/2}$	$\mp i\sqrt{\alpha/2}\,1/f^2$
$[70,1^-]_{1\pm1\lambda}$	$\mp i\sqrt{\alpha/6}$	$\mp i\sqrt{\alpha/6}\,1/f^2$
$[56,0^+]_{20s}$	0	0
$[56,2^+]_{2\pm1s}$	$\mp k/6$	$\mp E_f/6f^2M_f[k-(p_f\omega/E_f)]$
$[70,0^+]_{20\rho}$	0	0
$[70,0^+]_{20\lambda}$	0	0
$[70,2^+]_{2\pm1\rho}$	0	0
$[70,2^+]_{2\pm1\lambda}$	$\mp k/3$	$\mp E_f/3f^2M_f[k-(p_f\omega/E_f)]$

To consider the relativistic effect in the photon wave function, we simply replace $\exp(-i\vec{k}\cdot\vec{r})$ by $\exp(ik\cdot r)$, where $k\cdot r=\omega t-\vec{k}\cdot\vec{r}$.

In order to calculate the helicity amplitudes $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$, we have to compute the matrix elements between the initial ground state and final excited state for transverse and longitudinal transition operators. First of all, we covariantly boost the wave functions for the ground initial state and excited final state to a definite frame (we call it the q frame which is the same as the equal velocity reference frame (EVF) used in the literature [21]), as in Refs. [23,25,26]. In this frame, the momenta and energies of the initial and final states are

$$\vec{p}_f=-\frac{M_f}{M_i}\vec{p}_i=\frac{\vec{q}}{2},\quad E_f^2=\frac{M_f^2}{M_i^2}E_i^2. \tag{14}$$

Since the wave functions in Eqs. (8) and (9) for the ground and excited states represent the wave functions in their center-of-mass frame for each state, Lorentz transformation for the variables $(\vec{\rho},\vec{\lambda})$ is necessary to boost the wave functions to the definite q frame. The advantage of introducing the q frame is that the matrix elements can be expressed analytically which are displayed in Tables I–V. In these tables, we list the calculated spatial matrix elements between the ground state and the excited baryon states in the q frame based on the RHOM. In addition, the conventional nonrelativistic results are also displayed in the tables for comparison. In the tables, $[N_6,L^p]_{Nms}$ represents the spatial wave function of the baryon: N_6, L, p, N, m, and s stand for the SU(6) representation, angular momentum, parity, harmonic oscillator shell, the third component of the angular momentum, and the symmetry of the state, respectively. In the tables, $E_{i,f}$, $p_{i,f}$, and $M_{i,f}$ are the energy, momentum, and mass for initial and final states, respectively. They are defined as

$$p_i=-M_i\sqrt{\frac{E_f^L-M_f}{2M_f}},\quad p_f=\sqrt{M_f\frac{(E_f^L-M_f)}{2}},$$

$$E_{i,f}=\sqrt{M_{i,f}^2+p_{i,f}^2}, \tag{15}$$

where E_f^L is the energy of the final state in the lab frame. In the tables, $f=E_f^L/M_f$ and α is the harmonic oscillator constant parameter. In what follows we take $\alpha=0.16\ \mathrm{GeV}^2$. This value is the one used in the work by Koniuk and Isgur [18] which is in agreement with the calculations of photon production and baryon spectrum [18]. The photon momentum k and energy ω which are defined in the q frame can also be expressed in the lab frame by Lorentz transformation:

$$k=\sqrt{\frac{E_f^L+M_f}{2M_f}}\left(k^L-\sqrt{\frac{E_f^L-M_f}{E_f^l+M_f}}\omega^L\right),$$

$$\omega=\sqrt{\frac{E_f^L+M_f}{2M_f}}\left(\omega^L-\sqrt{\frac{E_f^L-M_f}{E_f^L+M_f}}k^L\right), \tag{16}$$

where k^L and ω^L stand for the photon momentum and energy in the lab frame, respectively.

From Tables I–V, one can find that for elastic proton-photon scattering, the form factor $F(Q^2)$ of the proton in the RHOM is given by

$$F(Q^2)=\frac{1}{[1+Q^2/2M^2]^2}\exp\left(-\frac{Q^2}{6\alpha[1+Q^2/2M^2]}\right). \tag{17}$$

Comparing them to the results of matrix elements in the nonrelativistic quark model, we find in the tables that in the nonrelativistic approximation, the matrix element in the RHOM will reduce to the one in the nonrelativistic case.

To see the relativistic effect on the polarized structure functions Γ_1 and Γ_2 numerically, we have calculated the polarized structure functions based on the RHOM, according to the method of Ref. [14]. The calculated results for the polarized structure functions $g_1^{p,n}$ and $g_2^{p,n}$ are shown in Figs. 1–4, where the nonrelativistic results in the lab frame are also plotted for comparison. In Figs. 5 and 6, the calculated helicity amplitudes for the resonances $P_{33}(1232)$ and $S_{11}(1535)$ are exhibited because the DHG sum rule is saturated dominantly by the resonance $P_{33}(1232)$ [3] and furthermore the resonance $S_{11}(1535)$ is an important state to test the longitudinal transition amplitude.

IV. CONCLUDING REMARKS

First of all, it should be stressed that the term σ_{TS} has an important role on the prediction of the Q^2 evolution of the polarized structure functions, especially on the structure

TABLE V. The spatial matrix elements $\langle [N_6,L^P]_{Nms}|e^{ik\cdot r_2}p_1^z|[56,0^+]_{00s}\rangle$ of the nonrelativistic (NR) quark model and RHOM in the q frame. Keys as in Table I.

$[N_6,L^P]_{Nms}$	NR	RHOM
$[56,0^+]_{00s}$	$\frac{k}{6}$	$\frac{E_i}{6M_if^3}\left[k-\frac{p_i\omega}{E_i}\right]$
$[70,1^-]_{10\rho}$	$i\sqrt{\frac{\alpha}{4}}\left(1-\frac{k^2}{6\alpha}\right)$	$\frac{i\sqrt{\alpha}E_iE_f}{2f^3M_iM_f}\left[\left(1-\frac{k^2}{6\alpha f}\right)\left(1+\frac{p_ip_f}{E_iE_f}\right)+\frac{k\omega}{12\alpha f}\left(\frac{p_i}{E_i}+\frac{p_f}{E_f}\right)\right]$
$[70,1^-]_{10\lambda}$	$i\sqrt{\frac{\alpha}{12}}\left(1+\frac{k^2}{6\alpha}\right)$	$\frac{i\sqrt{\alpha}E_iE_f}{\sqrt{12}f^3M_iM_f}\left[\left(1+\frac{k^2}{6\alpha f}\right)\left(1+\frac{p_ip_f}{E_iE_f}\right)-\frac{k\omega}{3\alpha f}\left(\frac{p_i}{E_i}+\frac{p_f}{E_f}\right)\right]$
$[56,0^+]_{20s}$	$-\frac{k}{6\sqrt{3}}\left(1-\frac{k^2}{6\alpha}\right)$	$-\frac{E_i}{6\sqrt{3}f^3M_i}\Big\{k\left(\frac{2}{f}-1-\frac{k^2}{6\alpha f^2}\right)+\frac{\omega p_i}{E_i}\left(1-\frac{1}{f}+\frac{k^2}{6\alpha f^2}\right)-\frac{6p_f}{M_f^2f}\Big[\left(k-\frac{p_i\omega}{E_i}\right)\left(-\frac{p_f}{2}+\frac{p_f(k^2+\omega^2)}{36\alpha f}-\frac{E_f\omega k}{18\alpha f}\right)+\frac{E_f}{6}\left(\omega-\frac{p_ik}{E_i}\right)\Big]\Big\}$
$[56,2^+]_{20s}$	$\frac{\sqrt{6}k}{18}\left(1-\frac{k^2}{6\alpha}\right)$	$\frac{E_i}{3\sqrt{6}f^3M_i}\Big\{k\left(\frac{2}{f}-1-\frac{k^2}{6\alpha f^2}\right)-\frac{\omega p_i}{2E_i}\left(\frac{2}{f}-1-\frac{k^2}{3\alpha f^2}\right)-\frac{12p_f}{M_f^2f}\Big[\left(k-\frac{p_i\omega}{E_i}\right)\left(-\frac{p_f}{4}+\frac{p_f(k^2+\omega^2)}{72\alpha f}-\frac{E_f\omega k}{36\alpha f}\right)+\frac{E_f}{12}\left(\omega-\frac{p_ik}{E_i}\right)\Big]\Big\}$
$[70,0^+]_{20\rho}$	$\frac{k^3}{72\alpha}$	$\frac{E_iE_f^2}{f^5M_iM_f^2}\Big\{\frac{k^3}{72\alpha}-\frac{\omega^3p_i}{72\alpha E_i}+\frac{\omega kp_f}{36E_f}\Big[\left(k-\frac{p_i\omega}{E_i}\right)-\frac{p_f}{2E_f}\left(\omega-\frac{p_ik}{E_i}\right)\Big]\Big\}$
$[70,0^+]_{20\lambda}$	$\frac{\sqrt{3}k}{9}\left(1-\frac{k^2}{24\alpha}\right)$	$\frac{E_i}{3\sqrt{3}f^4M_i}\Big\{k\left(1-\frac{k^2}{24\alpha f}\right)+\frac{\omega p_ik^2}{24E_i\alpha f}+\frac{p_f}{M_f^2}\Big[\left(k-\frac{p_i\omega}{E_i}\right)\times\left(p_f-\frac{p_f(k^2+\omega^2)}{24\alpha f}+\frac{E_f\omega k}{12\alpha f}\right)-E_f\left(\omega-\frac{p_ik}{E_i}\right)\Big]\Big\}$
$[70,2^+]_{20\rho}$	$\frac{\sqrt{2}k^3}{72\alpha}$	$\frac{\sqrt{2}E_iE_f^3}{f^5M_iM_f^2}\Big\{\frac{k^3}{72\alpha}-\frac{\omega^3p_i}{72\alpha E_i}+\frac{\omega kp_f}{36E_f}\Big[\left(k-\frac{p_i\omega}{E_i}\right)-\frac{p_f}{2E_f}\left(\omega-\frac{p_ik}{E_i}\right)\Big]\Big\}$
$[70,2^+]_{20\lambda}$	$\frac{\sqrt{6}k}{9}\left(1-\frac{k^2}{24\alpha}\right)$	$\frac{2E_i}{3\sqrt{6}f^4M_i}\Big\{k\left(1-\frac{k^2}{24\alpha f}\right)+\frac{\omega p_ik^2}{24E_i\alpha f}-\frac{3p_f}{M_f^2}\Big[\left(k-\frac{p_i\omega}{E_i}\right)\times\left(-\frac{p_f}{3}+\frac{p_f(k^2+\omega^2)}{72\alpha f}-\frac{E_f\omega k}{36\alpha f}\right)+\frac{E_f}{3}\left(\omega-\frac{p_ik}{E_i}\right)\Big]\Big\}$

function g_2. The same result has been already addressed in Ref. [14]. From Eq. (17), one can find that the inclusion of relativistic effects both in the wave functions of the photon and baryon and the correct treatment of boosting would lead us to the proper description for the nucleon form factor. Moreover, it should be emphasized that consideration of relativistic corrections also enables us to get different behaviors of the Q^2-dependent evolution of the helicity amplitudes. This conclusion can be seen from Figs. 5 and 6. The differences between the descriptions in the nonrelativistic calculation and the one in the RHOM for the Q^2-dependent evolution of the helicity amplitudes shown in Figs. 5 and 6 are in agreement with calculations in a light-front framework [31] given by Capstick and Keister qualitatively. It is expected that our calculations for the transition amplitudes with relativistic corrections could be tested in future experiments

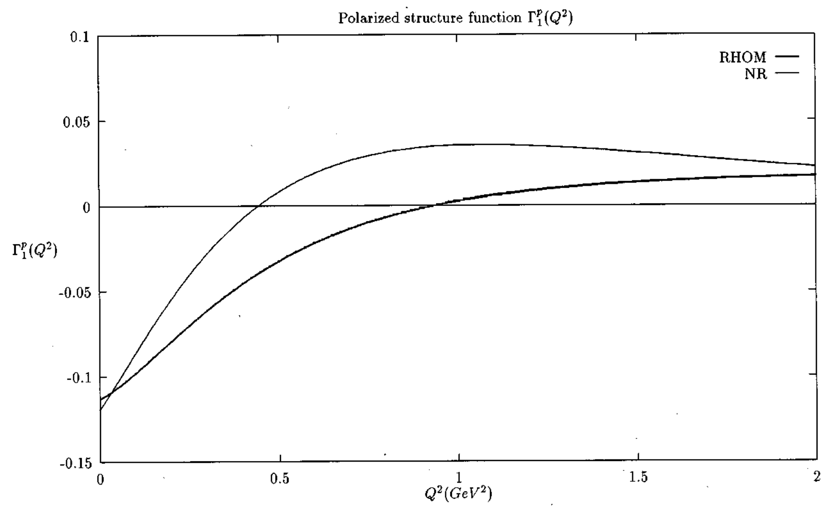

FIG. 1. The calculation for the polarized structure function $\Gamma_1^p(Q^2)$.

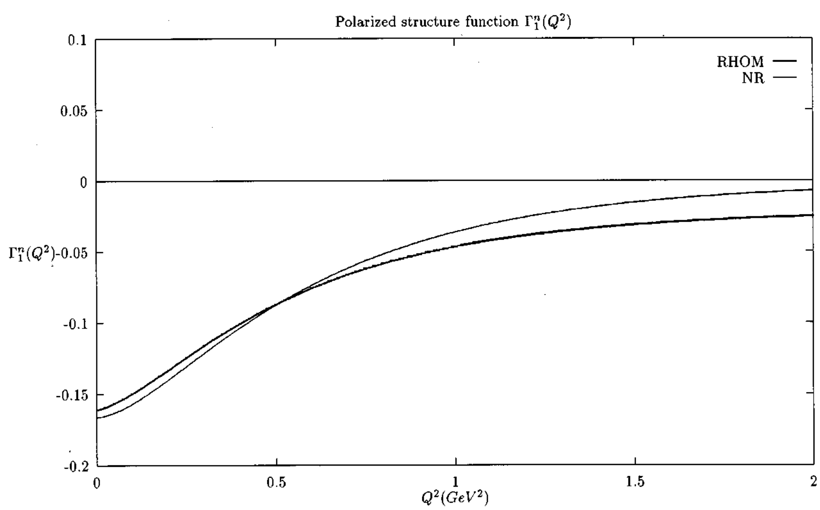

FIG. 2. The calculation for the polarized structure function $\Gamma_1^n(Q^2)$.

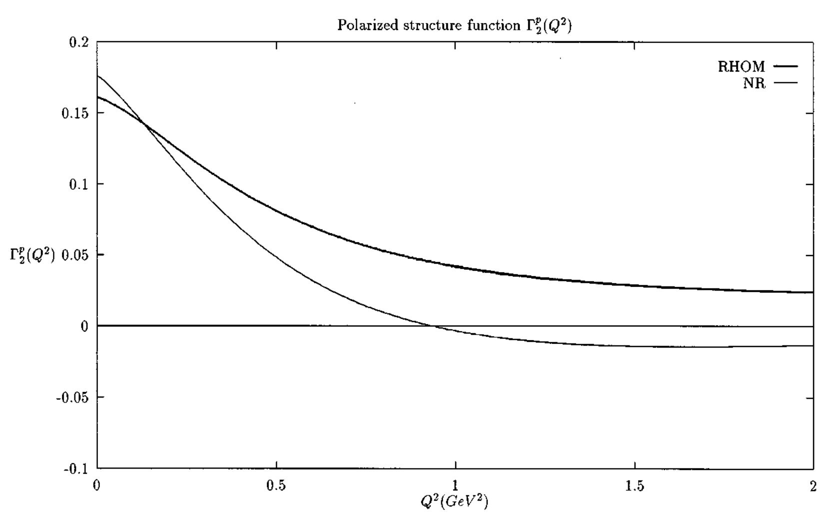

FIG. 3. The calculation for the polarized structure function $\Gamma_2^p(Q^2)$.

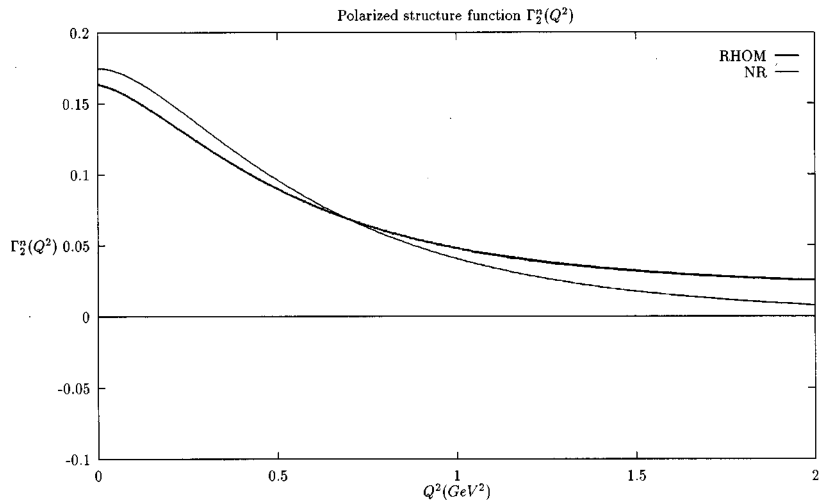

FIG. 4. The calculation for the polarized structure function $\Gamma_2^n(Q^2)$.

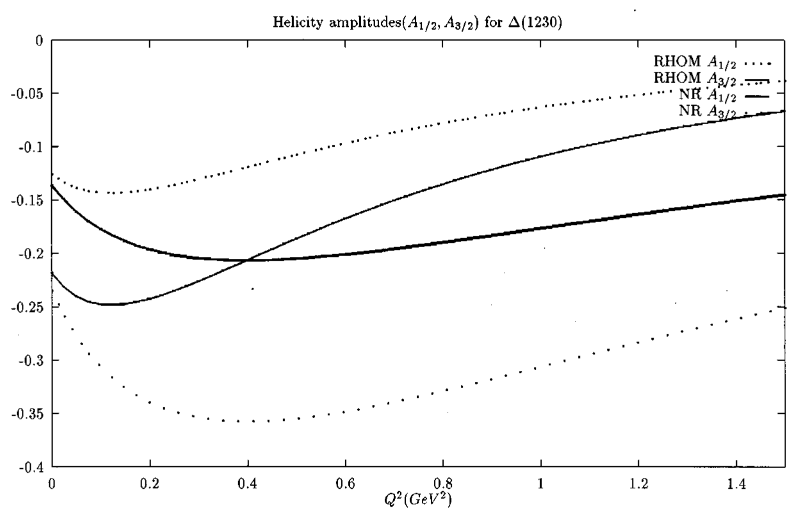

FIG. 5. The calculation for the helicity amplitudes $A_{1/2}(Q^2)$ and $A_{3/2}(Q^2)$ of $P_{33}(1232)$ in units of $\mathrm{GeV}^{-1/2}$.

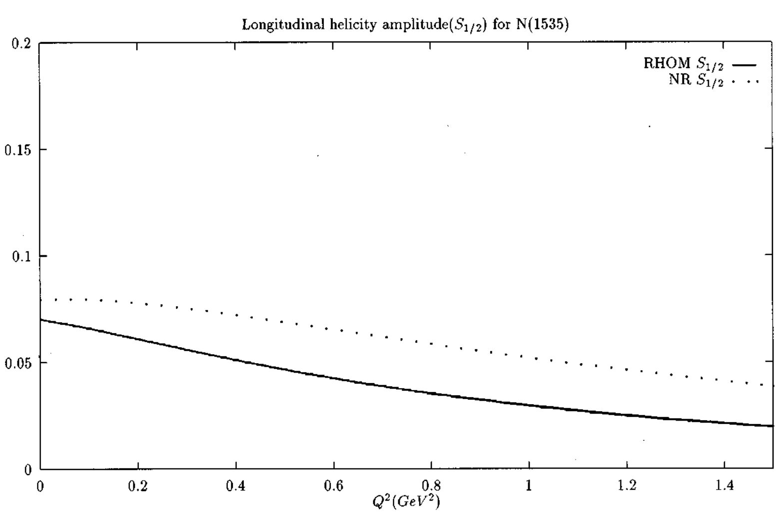

FIG. 6. The calculation for the helicity amplitude $S_{1/2}(Q^2)$ of $S_{11}(1535)$ on the proton target in units of $\mathrm{GeV}^{-1/2}$.

at CEBAF, MAMI, and SPring-8.

From the above observation, we can see that the relativistic effect works to shift the crossing point, i.e., zero in the polarized structure function $\Gamma_1^p(Q^2)$, from 0.45 to around '1.0 GeV^2 compared to the calculation with the nonrelativistic model [14]. Moreover, the calculated polarized structure function $\Gamma_2^p(Q^2)$ has no crossing point: The polarized structure function $\Gamma_2^p(Q^2)$ decreases from a positive value (around 0.16) at $Q^2=0$ to zero at the DIS region, and is always positive in the finite Q^2 region. This result are in agreement with the Q^2 dependence of the sum rule of $\Gamma_2(Q^2)$ derived in Ref. [13], while it is not the case for the nonrelativistic model [14]. It is very interesting to see if future experiments confirm or rule out the existence of the crossing point in $\Gamma_2^p(Q^2)$. In addition, the sum rule values for the proton and neutron calculated with the RHOM is a little smaller than the values in the nonrelativistic approximation, and the Q^2 evolution in the RHOM is slower than that in nonrelativistic calculations. We hope that all the differences between the two models could be tested in forthcoming experiments at CEBAF, MAMI, and SPring-8.

It should be pointed out that our calculation in this paper is preliminary. Here, only contributions from the resonances to the polarized structure functions are considered. In the works of Li [32], it has been stressed that nonresonance contributions will increase with the increasing of the momentum transfer Q^2 and that at moderate Q^2 the sum rule of the polarized structure functions is not sufficiently satisfied by the resonance contributions alone. Therefore, the quantitative prediction for the Q^2 evolution of the polarized structure functions in the finite Q^2 range should include both contributions from resonances and nonresonances. It has been proved [32] that consideration of the resonances would give us better agreement with the Ellis-Jaffe sum rule and the Bjorken sum rule simultaneously at large Q^2. It means that the effect of the DHG sum rule should be taken into account in extracting the sum rules of the polarized structure functions from the data in the DIS range and the study of the polarized structure functions in the resonance region is important.

Finally, it should be mentioned that some other effects, such as the configuration mixing effect, nonperturbative QCD effect including instanton effects, etc., are not included. Full relativistic calculations using a relativistic Hamiltonian are also advisable. The study of the polarized spin-structure functions with the baryon current matrix elements in a light-front framework [31] is another interesting topic. A more sophisticated study on these subjects will be given in our future works.

ACKNOWLEDGMENTS

One of the authors (Y.B.D.) wishes to acknowledge support from the JSPS of Japan. He also thanks the physics members of the Faculty of Human Development, Kobe University, for their kind hospitality during this work.

[1] J. Ashman *et al.*, Phys. Lett. B **206**, 1 (1989); Nucl. Phys. **B206**, 364 (1988); G. Baum *et al.*, Phys. Rev. Lett. **45**, 2000 (1980); Spin Muon Collaboration, B. Adeva *et al.*, Phys. Lett. B **302**, 533 (1993); SLAC-E142 Collaboration, P. L. Anthony *et al.*, Phys. Rev. Lett. **71**, 959 (1993).

[2] D. Adams *et al.*, Phys. Lett. B **329**, 399 (1994).

[3] S. D. Drell, in *Lepton-Hadron Scattering*, Proceedings of the SLAC Summer Institute, Stanford, California, 1991, edited by J. Hawthorne (SLAC Report No. 398, Stanford, 1992); F. E. Close, in *Hadronic Physics in the 1990's with Multi-GeV Electrons*, Proceedings of the first European workshop, Seillac, France, 1988, edited by B. Frois *et al.* [Nucl. Phys. **A497**, 109c (1989)]; B. L. Ioffe, Surv. High Energy Phys. **8**, 107 (1995); M. Anselmino, B. L. Ioffe, and E. Leader, Sov. J. Nucl. Phys. **49**, 136 (1989); V. Burkert and Z. J. Li, Phys. Rev. D **47**, 46 (1993); V. Burkert and B. L. Ioffe, Phys. Lett. B **296**, 223 (1992); D. Drechsel *et al.*, Few-Body Syst. **15**, 99 (1993); Few-Body Syst. **16**, 99 (1994).

[4] A. L. Ioffe *et al.*, *Hard Processes* (North-Holland, New York, 1984); F. E. Close, *Introduction to Quarks and Partons* (Academic, New York, 1978).

[5] S. B. Gerasimov, Sov. J. Nucl. Phys. **2**, 430 (1966); S. D. Drell and A. C. Hearn, Phys. Rev. Lett. **16**, 908 (1966).

[6] J. Schwinger, Proc. Natl. Acad. Sci. USA **72**, 1 (1975); **72**, 1559 (1975).

[7] H. Burkhardt and W. N. Cottingham, Ann. Phys. (N.Y.) **56**, 543 (1970).

[8] D. Drechsel, Prog. Part. Nucl. Phys. **34**, 181 (1995).

[9] R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, MA, 1972).

[10] R. L. Jaffe, Comments Nucl. Part. Phys. **19**, 239 (1990); R. L. Jaffe and X. D. Ji, Phys. Rev. D **43**, 724 (1991).

[11] Z. P. Li, Phys. Rev. D **47**, 1854 (1993); V. Burkert *et al.*, *ibid.* **47**, 46 (1993).

[12] V. D. Burkert and B. L. Ioffe, Sov. Phys. JETP **105**, 1153 (1994).

[13] J. Soffer and O. Teryaev, Phys. Rev. Lett. **70**, 3373 (1993); Phys. Rev. D **51**, 25 (1995).

[14] Z. P. Li and Y. B. Dong, Phys. Rev. D **54**, 4301 (1996).

[15] M. Bourdeau and N. Mukhopadhyay, Phys. Rev. Lett. **58**, 976 (1987).

[16] T. Abdullah and F. E. Close, Phys. Rev. D **5**, 2332 (1972).

[17] M. Warns *et al.*, Z. Phys. C **45**, 627 (1990).

[18] R. Koniuk and N. Isgur, Phys. Rev. D **21**, 1868 (1980); N. Isgur and G. Karl, Phys. Lett. **72B**, 709 (1977); **74B**, 353 (1978); Phys. Rev. D **18**, 4187 (1978).

[19] V. D. Burkert, in *Excited Baryons 1988*, Proceedings of the workshop, Troy, New York, 1988, edited by G. Adams, N. C. Mukhopadhyay, and Paul Stoler (World Scientific, Singapore, 1989), p. 122; V. D. Burkert, Int. J. Mod. Phys. E **1**, 421 (1992).

[20] S. J. Brodsky and J. Primack, Ann. Phys. (N.Y.) **52**, 315 (1969); F. E. Close and L. A. Copley, Nucl. Phys. **B19**, 477 (1970); F. E. Close and H. Osborn, Phys. Rev. D **2**, 2127 (1970); H. Osborn, Phys. Rev. **176**, 1523 (1968); R. Faustov, Nuovo Cimento A **69**, 37 (1970); G. Feinberg and J. Sucher,

Phys. Rev. Lett. **35**, 35 (1975); J. Sucher, Rep. Prog. Phys. **41**, 1781 (1978); M. Warns *et al.*, Z. Phys. C **45**, 613 (1990); Phys. Rev. D **42**, 2215 (1990); S. Capstick, *ibid.* **46**, 1965 (1992); **46**, 2864 (1992).

[21] F. Foster and G. Hughes, Z. Phys. C **14**, 123 (1982).

[22] H. Yukawa, Phys. Rev. **91**, 416 (1953).

[23] R. P. Feynman *et al.*, Phys. Rev. D **3**, 2107 (1971).

[24] K. Fujimura *et al.*, Prog. Theor. Phys. **43**, 73 (1970); **44**, 93 (1970).

[25] R. G. Lipes, Phys. Rev. D **5**, 2849 (1972).

[26] Y. S. Kim *et al.*, Phys. Rev. D **12**, 129 (1975).

[27] Y. Kizukuri *et al.*, Prog. Theor. Phys. **61**, 559 (1979).

[28] V. V. Burov *et al.*, Z. Phys. A **306**, 149 (1982).

[29] A. I. Titov *et al.*, Chin. J. Phys. **32**, 1351 (1994).

[30] F. E. Close and Z. P. Li, Phys. Rev. D **42**, 2194 (1990); Z. P. Li and F. E. Close, *ibid.* **42**, 2207 (1990).

[31] S. Capstick and B. D. Keister, Phys. Rev. D **51**, 3598 (1995).

[32] Z. P. Li, Phys. Rev. D **48**, R3945 (1993); Z. P. Li *et al.*, *ibid.* **50**, 3119 (1994).