Schwinger mechanism, Unruh effect, and production of accelerated black holes

R. Parentani*
Laboratoire de Mathématique et Physique Théorique, C.N.R.S. UPRES A 6083, Faculté des Sciences, Université de Tours, 37200 Tours, France

S. Massar†
Raymond and Beverly Sackler Faculty of Exact Sciences, School of Physics and Astronomy, Tel-Aviv University, Tel-Aviv 69978, Israel

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We compute the corrections to the transition amplitudes of an accelerated Unruh ''box'' that arise when the accelerated box is replaced by a ''two level ion'' immersed in a constant electric field and treated in second quantization. There are two kinds of corrections: those due to recoil effects induced by the momentum transfers and those due to pair creation. Taken together, these corrections show that there is a direct relationship between pair creation amplitudes described by the Heisenberg-Euler-Schwinger mechanism and the Unruh effect, i.e., the thermalization of accelerated systems at temperature $a/2\pi$ where a is the acceleration. In particular, there is a thermodynamical consistency between both effects whose origin is that the Euclidean action governing pair creation rates acts *as* an entropy in delivering the Unruh temperature. Upon considering pair creation of charged black holes in an electric field, these relationships explain why black holes are created from vacuum in thermal equilibrium, i.e., with their Hawking temperature equal to their Unruh temperature. [S0556-2821(97)06006-2]

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I. INTRODUCTION

Quantum field theory predicts two remarkable phenomena when charged matter is accelerated in a uniform electric field. The first is the Heisenberg-Euler-Schwinger mechanism [1,2]. That is, vacuum instability due to spontaneous creation of charged pairs. When the electric field is turned on in a volume V during a period T, the norm of the overlap between the initial and the final vacua, which gives the probability not to produce pairs, decreases as

$$|\langle 0,\mathrm{out}|0,\mathrm{in}\rangle|^2 = e^{-\Gamma T},$$

where

$$\Gamma = V\left(\frac{E}{2\pi}\right)^{d/2} \ln(1+e^{-\pi M^2/E}), \tag{1}$$

where d is the number of space time dimensions, M is the mass of the scalar charged particle, $E=eE_0$ is the product of the charge e of the particle by the electric field E_0 (we put $\hbar=c=1$). To allow the comparison with the second effect we shall discuss below, it is appropriate to notice that the pairs produced by the Schwinger mechanism possess a well-defined spectrum. Indeed, the mean number of pairs $N(k_\perp)$ characterized by a transverse (with respect to the direction of the acceleration) momentum $k_\perp$ is given by

$$N(k_\perp) = Ce^{-\pi(M^2+k_\perp^2)/E}, \tag{2}$$

where the overall factor C takes into account the phase space factor arising from quantization in a volume V. For later convenience we reexpress Eq. (2) in such a way that the constant C cancels,

$$\frac{N(k_\perp)}{N(k_\perp=0)} = e^{-\pi k_\perp^2/E} = \frac{P(k_\perp)}{P(k_\perp=0)}, \tag{3}$$

where we have introduced the probability $P(k_\perp)$ that a detected particle produced from vacuum possesses that momentum. This latter concept is more intrinsic since it does not depend on global characteristics such as V and T nor on the rate of pair production.

The second effect concerns the physics which arises when (these) accelerated particles are coupled to quantum radiation, i.e., photons, or more generally to some massless field ϕ. Through this coupling, massive particles behave like detectors. By detector we mean a quantum system which has two (or more) energy levels separated by an energy gap Δm and which can make transitions by emitting or absorbing a photon ϕ. Therefore, these detectors probe the state of the ϕ field.

Using this concept of particle detector, Unruh [3] proved that when such a detector is uniformly accelerated, it perceives the vacuum state of the ϕ field to be thermally populated with the temperature

$$T_U = a/2\pi, \tag{4}$$

where a is the acceleration of the detector. Such an accelerated detector will eventually reach equilibrium with the heat bath, whereupon its two energy levels are populated with the Boltzmanian ratio

$$\frac{P_+}{P_-} = e^{-2\pi\Delta m/a} = \frac{R_{-\to+}}{R_{+\to-}}, \tag{5}$$

*Electronic address: parenta@parenta.phys.unit-tours.fr
†Electronic address: massar@ccsg.tau.ac.il

where $P_+(P_-)$ is the probability to be found in the excited (ground) state. We have also introduced the rates of transitions $R_{\pm\to\mp}$ which determine dynamically this equilibrium.

At first sight, besides the fact that charged particles propagate with uniform acceleration

$$a = eE_0/M = E/M \tag{6}$$

once they are produced in the constant electric field, the Schwinger and Unruh effects seem hardly related phenomena. Indeed the Schwinger effect requires a second quantized framework for the massive charged field ψ_M only and makes no references to quantized massless fields. On the contrary, in the Unruh effect, the propagation of the detector is described by a given classical trajectory (this is of course an approximation, see below) and only its internal transitions accompanied by the emission or absorption of quanta of the radiation field are treated quantum mechanically. Moreover, the Unruh effect can even be understood without introducing the detector at all. Indeed, it suffices to reexpress the vacuum state of the field ϕ, i.e., Minkowski vacuum, in terms of its ''Rindler'' particle content. By Rindler quanta, one designates the quanta of the radiation field associated to the eigenmodes of the boost operator which vanish beyond the horizon defined by the accelerated trajectory of the system [4,3]. Fulling found that Minkowski vacuum is a thermal distribution of Rindler quanta. Then, Unruh proved that accelerate detectors react to Rindler quanta as inertial detectors react to inertial (Minkowski) quanta. Therefore, accelerated systems find themselves in a thermal bath.

However, there are a number of questions which cannot be answered by this kinematical analysis based on Rindler modes only. To reveal the aspects inevitably missed by this analysis and to prove the necessity of considering more dynamical frameworks, we shall proceed in three steps by posing and answering questions.

(a) What is the energy balance of the accelerated-thermal equilibrium as seen by an inertial observer? In order to answer this question, it is mandatory to abandon the description in terms of the Rindler modes and to use instead the Minkowski modes of the radiation field. The main result is that, in spite of the equilibrium, Minkowski quanta are produced [5,6] and their total number equals [7] the number of internal transitions of the detector.

(b) Where does this energy come from? or more locally, What is the incidence of the energy-momentum transfer occuring when one emission process takes place? To answer these questions requires an enlargement of the dynamics. One must indeed quantize the center-of-mass position of the detector—and thus attribute it a finite mass M—and introduce an external force such that the detector accelerates uniformly in the absence of transitions. This is precisely the role of a constant electric field. The main results of this enlarged dynamical framework are the following [8]: (1) The transition probabilities between excited and ground state still satisfy Eq. (5) when $\Delta m \ll M$. (2) Due to recoil effects, the energy flux emitted by the detector becomes rapidly incoherent and positive. Moreover it is accompanied by a constant drift from uniformly accelerated trajectories which expresses the dissipation of potential electric energy into radiation.

(c) What are the consequences of ''second quantizing the detector,'' i.e., of taking into account amplitudes of producing pairs of charged detectors in the electric field? Indeed a *complete* description of a quantum relativistic system in an external field can only be obtained by working in a second quantized framework [the answers delivered in point (b) were based on an approximate first quantized treatment (WKB) in which corrections in $e^{-\pi M^2/E}$ were neglected [8]]. This further enlargement of the dynamics *implies* that both the Schwinger and the Unruh effect are encompassed in the same model. To analyze the consequences of this enlargement is the central problem addressed in the present article.

At this point, it is appropriate to consider the emission rates of photons by bremsstrahlung from an electron accelerated in a constant electric field. These emission rates were analyzed by Nikishov [9,10] a few years before Unruh's seminal work. It is very interesting to notice that the point of view adopted by Nikishov was to consider these emission processes as describing corrections to the Schwinger effect rather than corrections to the Unruh effect induced by ''second quantizing the detector.'' This dual point of view clearly illustrates the entangled nature of both processes when studied in the enlarged framework. Nikishov showed that the ratio of the transition rates for an electron to jump from a state with transverse momentum $k_\perp$ to a state with zero momentum accompanied by the emission of a photon to the inverse transition satisfies

$$\frac{R_{k_\perp\to 0}}{R_{0\to k_\perp}} = e^{-\pi k_\perp^2/E} = \exp(-2\pi(k_\perp^2/2m_e)/a_e) \tag{7}$$

where $a_e = E/m_e$. In the second equality we have written $k_\perp^2/E$ as $k_\perp^2/2m_e \times 2/a_e$ in order to explicitize the relationship with Eq. (5). In the nonrelativistic limit $k_\perp^2 \ll m_e^2$, it is indeed legitimate to consider $k_\perp^2/2m_e$ as providing the energy levels of the ''detector states,'' see [11,12] and point (b) above. The manifest similarities between Eq. (7) and both Eq. (3) and Eq. (5) strongly invite to inquire into their dynamical origin, if any.

A first indication that there is a deep relation is furnished by an analysis of the Euclidean instanton[1] associated with the Schwinger process. This instanton is obtained by considering the classical dynamics of a relativistic particle of mass M and charge e in an electric field E_0. The classical (Lorentzian) trajectories have uniform acceleration either to the right (corresponding to particles) or to the left (for antiparticles) and the Euclidean orbits are closed trajectories, as in a magnetic field. The Euclidean action for completing an orbit is

$$S_{\rm Euclid} = \pi M^2/E. \tag{8}$$

Contact with the second quantized framework is made by the fact that the probability of creating pairs scales like $e^{-S_{\rm Euclid}}$, see Eq. (1).

[1]Remarkably, this analysis can be straightforwardly extended to black hole pair production and their subsequent thermal effects. Furthermore these relations between Euclidean gravity and thermal phenomena shed a new light on the thermodynamical approach to gravity [13] presented by Jacobson, see Sec. VII.

What really concerns us is the amount of Euclidean proper time necessary to complete this orbit. It is given by the Hamilton Jacobi relation

$$\tau_{\mathrm{Euclid}} = \partial_M S_{\mathrm{Euclid}} = \partial_M\left(\frac{\pi M^2}{E}\right) = \frac{2\pi}{a} = T_U^{-1}. \tag{9}$$

It equals the inverse Unruh temperature. At this point, it should be recalled that the quantum processes induced by the uniform acceleration and leading to Eq. (5) are *all* governed by lapses of proper time τ. By using Eq. (9), Eq. (7) can be written as

$$\frac{R_{k_\perp \to 0}}{R_{0\to k_\perp}} = e^{-(k_\perp^2/2m_e)\,\partial_m S_{\mathrm{Euclid}}}. \tag{10}$$

This strongly suggests that the Unruh process can be obtained from a first order comparison of two neighboring Schwinger processes, in a manner similar that canonical distributions characterized by a temperature are obtained from a first order change applied to microcanonical distributions characterized by energy only. The validity of this comparison with thermodynamics will be confirmed upon considering black hole pair production. It will then be clear that the Euclidean action behaves *as* an entropy in delivering the Unruh temperature, see Eq. (9), therefore it behaves like the Bekenstein entropy in the latter's determination of Hawking temperature.

In this paper we shall prove these interpretations are correct and we shall provide the physical rationale behind them. To this end, we shall use a simple model and proceed in several steps. These are presented in the next section.

II. THE MODEL AND THE STRATEGY

Instead of working with the transverse momentum, see Eqs. (3) and (7), to establish the relations between the Schwinger and Unruh effects, we shall use a two-dimensional model [8] composed of two charged fields ψ_M and ψ_m and the scalar massless field ϕ. The Hamiltonian which governs the transition amplitudes is

$$H_{\phi\psi} = \tilde{g}\int dx[\psi_M \psi_m^\dagger \phi + \mathrm{H.c.}], \tag{11}$$

where $\tilde{g}$ is a coupling constant. We have chosen that model because the expressions for the transition amplitudes are considerably simpler than in the four dimensional case, see [9,10]. Thus, they display more clearly the seeked relationships.

In Sec. V, we shall compute ''exactly'' the first order (in $\tilde{g}$) transition amplitudes. By exactly we mean to all orders in $\Delta m/M$ and a/M thereby taking into account recoil effects (first quantized effects) and vacuum instability (a second quantized effect). In [14] both recoils and pair creation effects were neglected, and in [8,15] only recoils effects were taken into account.

From the properties of these amplitudes under *crossing symmetry*, we prove that the ratio of the transition rate from the ground to the excited state $(m \to M)$ to the inverse transition rate $(M \to m)$ is *exactly* given by

$$\frac{R_{m\to M}}{R_{M\to m}} = e^{-\pi(M^2-m^2)/E} = e^{-2\pi(M-m)/\bar{a}}. \tag{12}$$

In the second equality, as in Eq. (7), we have rewritten the exponent in order to make contact with the Unruh expression controlled by an acceleration and an energy gap, see Eq. (5). The only difference is that the unique acceleration is replaced by the mean acceleration $\bar{a} = (M+m)/2E$. This should cause no surprise since the two levels M and m experience different accelerations. In fact the Unruh formula, Eq. (5), should always be considered as an approximate expression valid when it is legitimate to deal with a single acceleration. Strictly speaking, this requires that the limit $\Delta m/M \to 0$, $a/M \to 0$ be taken. More physically, it requires that the mass scales be well separated, i.e., $M - m \ll M$. In that case, the concept of a single[2] acceleration is meaningful, and therefore the concept of temperature as well.

However, even when the condition $M - m \ll m$ is not satisfied, *the two level ion reaches equilibrium with the population ratio of its excited and ground state given by the Schwinger mechanism*. Indeed, the ratio of the probabilities P_M and P_m to find a given particle produced from vacuum in the M or the m state is equal to the ratio of the mean numbers N_M and N_m of produced quanta of masses M and m, see Eqs. (2) and (3), and therefore given by

$$\frac{P_M}{P_m} = e^{-\pi(M^2-m^2)/E} = \frac{N_M}{N_m}. \tag{13}$$

The equality of the ratios of the transition rates, Eq. (12), and of the probabilities, Eq. (13), proves that there is a consistency between the Schwinger mechanism and the extended-Unruh-effect defined by *keeping* the finite corrections in a/M and in $\Delta m/M$ into account. When the mass ratio $\Delta m/M$ is negligible, one fully recovers the thermal equilibrium governed by a temperature, as in the original Unruh description. When $\Delta m/M$ is not negligible, even though it is illegitimate to deal with a single accelerated trajectory, the equilibrium distribution still exists and still coincides with the Schwinger distribution. In this sense, pairs are born in equilibrium [14].

In Sec. VI, we determine the origin of the equality between Eqs. (12) and (13). We show that this equality is *dictated* by the analytical properties of the amplitudes governing pair creation under crossing symmetry and *CPT*.

To understand this result, it is necessary to first consider the amplitude for a particle to propagate from $t = -\infty$ to $t = +\infty$ [Fig. 1(a)]. By *crossing symmetry*, one replaces the incoming particle by an outgoing antiparticle, hence one ob-

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[2]These features arise whenever one enlarges the dynamics so as to abandon the background field approximation wherein it is postulated that all processes can be described by a quantum system coupled to a single trajectory. Indeed, upon studying *afterwards* the semiclassical regime to determine how the background field approximation re-emerges, one explicitizes the conditions which must prevail to validate that approximation. The interested reader will consult [16] where this approach is applied to quantum cosmology to determine the validity of the semiclassical Einstein equations.

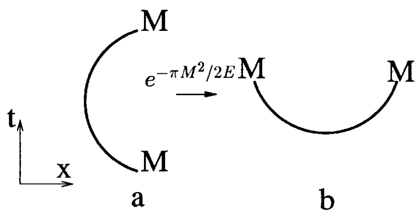

FIG. 1. Feynman diagrams representing (a) a particle of mass M and charge e propagating from $t=-\infty$ to $t=+\infty$ in the electric field E_0, and (b) a particle-antiparticle pair creation in the electric field. The particles are represented by curved lines because they are accelerated by the electric field. These diagrams are oriented both in space and time: the left-right symmetric of (a) would represent an antiparticle accelerated in the opposite direction, the up-down symmetric of (b) would represent particle-antiparticle annihilation. Amplitudes (a) and (b) are related by level crossing. The ratio of their norms is given by the Schwinger factor $e^{-\pi M^2/2E}$, see Eq. (19) in the text for the precise mathematical definition.

tains the amplitude for pair creation [Fig. 1(b)]. The currents associated with these two propagations are related by the factor $e^{-\pi M^2/E}$.

Now consider radiative processes involving a ϕ quantum, to first order in $\tilde{g}$. The amplitude (a) of Fig. 2 represents the deexcitation of a detector mass M accompanied by the emission of a massless quantum of energy ω. The norm squared of this amplitude determines the rate $R_{M\to m}$ of Eq. (12). On the other hand the rate $R_{m\to M}$ of spontaneous excitation of the detector is determined by the norm squared of the amplitude (d) of Fig. 2. Our aim is to understand how these two amplitudes are related.

First note that by using T symmetry, amplitude (d) is equal up to complex conjugation to the amplitude (e) for a detector in the excited state to absorb a light quanta and deexcite. Then note that amplitudes (a) and (e) are related by crossing symmetry for the light quantum, i.e., by taking $\omega\to e^{i\pi}\omega$. This latter relation will be exploited in Sec. V. However it does not help to understand how Eq. (12) relating the rate $R_{m\to M}$ to $R_{M\to m}$ is connected to the Schwinger process, Eq. (13), since it involves crossing symmetry applied to the photon only.

To obtain this understanding, we shall use instead the succession $a\to b\to c\to d$ which proceeds through crossing symmetry applied *twice* to the Schwinger process. First we use crossing symmetry for the ''excited'' detector of mass M so as to relate (a) to the amplitude to create from vacuum a deexcited detector, an excited anti–detector, and a massless quantum of energy ω [Fig. 2(b)]. The ratio of the norms of amplitudes (b) and (a) is given by the Schwinger factor $e^{-\pi M^2/2E}$ exactly as in the case considered in Fig. 1 above. Secondly, by using again crossing symmetry applied to the deexcited detector of mass m, one obtains the amplitude (c) for a deexcited antidetector to get excited and to emit a light quantum. The ratio of the norms of amplitudes (b) and (c) is given by the Schwinger factor $e^{\pi m^2/2E}$. In this case it is the mass m rather than the mass M which comes up in the exponential weight. Thirdly, we use the CP symmetry to map particles into antiparticles while leaving the electric field E_0 unchanged. Thus CP maps the amplitude (c) onto the sought for amplitude (d) for a deexcited detector to get excited and emit a quantum ω. The equality of Eqs. (12) and (13) and the connection between the (extended) Unruh effect and the Schwinger process is thus explained by this succession.

The motivation for our emphasis on CPT and crossing symmetry is that these analytical properties should hold irrespectively of the specific model under examination. As an illustration of this universality, in Sec. VII, we consider pair creation of charged black holes in a constant electric (or magnetic) field [17–19] and the subsequent emission of quanta through Unruh effect as well as through Hawking process [20]. We show that there is once more a complete thermodynamic consistency between the production of the black hole pairs and both of these radiative effects. This thermodynamic consistency illustrates that the Euclidean action Eq. (8) acts indeed as an entropy in delivering the Unruh temperature, see Eq. (9), since it is given in terms of a quarter of a change in area and occurs in amplitudes added to the Bekenstein entropy.

III. THE SCHWINGER EFFECT

We recall in this section the essential steps necessary to obtain the Schwinger effect. The reader unfamiliar with pair creation in an electric field may consult Refs. [9,10,14,21]. What differs in our presentation is the emphasis put on the use of *crossing symmetry* in defining and obtaining the Bogoliubov coefficients. The reason for this emphasis is that crossing symmetry will play a crucial role in Secs. V and VI.

We consider a massive charged scalar field ψ_M in a constant electric field E_0. In the homogeneous gauge ($A_t=0$, $A_z=E_0t$) ψ_M obeys the Klein Gordon equation

$$[\partial_t^2-(\partial_z-iEt)^2-\partial_y^2-\partial_x^2+M^2]\psi_M=0, \tag{14}$$

where $E=eE_0$. In this gauge three-momentum is conserved. The transverse momentum squared acts like a shift of the mass squared, see Eq. (2). From now on, however, for reasons of simplicity, we shall take it to vanish and work in 1+1 dimensions. Since the longitudinal momentum p is also conserved, ψ_M can be decomposed as a sum of $e^{ipz}\chi_{p,M}(t)$, where $\chi_{p,M}(t)$ obeys

$$[\partial_t^2+(p-Et)^2+M^2]\chi_{p,M}(t)=0. \tag{15}$$

There are two independent solutions of this equation and their asymptotic behavior must be used to identify which linear superpositions describe, *in* and *out*, particle and antiparticle states. Indeed, because of the time dependence of the frequency, *in* particle modes, i.e., solutions of Eq. (15) carrying unit positive current for $t\to-\infty$, will be a superposition of *out* particle and antiparticle modes for $t\to\infty$:

$$\chi^{\rm in}_{M,p}=\alpha_M\chi^{\rm out}_{M,p}+\beta_M\overline{\chi}^{\rm out*}_{M,-p}. \tag{16}$$

Current conservation requires

$$|\alpha_M|^2-|\beta_M|^2=1. \tag{17}$$

To obtain the asymptotic behaviors of the various modes, it is useful to use the following integral representation, see [22]. For instance, in modes are given by

$$\chi^{\text{in}}_{M,p}(t)=\alpha_M\int_0^\infty \frac{du}{\sqrt{8\pi^2}}(u)^{(-iM^2/2E)-(1/2)} \times e^{iE[u^2/4-(t-p/E)u+(t-p/E)^2/2]}, \qquad (18)$$

where the normalization factor is shown to be the coefficient α_M. One easily obtains the Bogoliubov coefficient β_M from this integral representation because, for large $|t|$, i.e., $|t|\gg M/E$, the integral receives all its contribution from saddle points at $u\to\infty$ and from the region $u\to 0$. One finds that the saddle point at $u\to\infty$ describes the outgoing particle wave carrying for $t\to\infty$ a current $|\alpha_M|^2$. Instead the contribution from $u\to 0$ describes, for $t\to-\infty$ the incoming branch carrying unit positive current, and for $t\to+\infty$ the antiparticle branch carrying negative current, see [22] for more details. β_M is the ratio of these latter contributions at small u. To evaluate this ratio, it is legitimate to neglect the term in u^2 in the exponential and one is left with the integral representation of Γ functions:

$$\beta_M=\frac{\int_0^\infty du \;\; u^{(-iM^2/2E)-(1/2)}e^{iE(-|t|u+t^2/2)}}{\int_0^\infty du \;\; u^{(-iM^2/2E)-(1/2)}e^{iE(|t|u+t^2/2)}}=-ie^{-\pi M^2/2E}. \qquad (19)$$

Note how it is the sign of the exponent of $e^{\pm iE|t|u}$ which governs the ratio of these integrals. By sending t to $e^{i\pi}t$ in the lower integral, the contribution of the incoming particle is replaced by the one of the outgoing anti-particle. This is what we designate by crossing symmetry, see Fig. 1. In Sec. VI, we shall see that it is this continuation used twice which implies the equality of Eqs. (12) and (13).

The corresponding out mode with asymptotic unit final current directed towards $z=+\infty$ is obtained by replacing t by $-t$, p by $-p$ and by complex conjugation. Thus, its integral representation is

$$\chi^{\text{out}}_{M,p}(t)=\chi^{\text{in}*}_{M,-p}(-t)=\alpha_M\int_0^\infty \frac{du}{\sqrt{8\pi^2}}(u)^{(iM^2/2E)-(1/2)} \times e^{-iE[u^2/4+(t-p/E)u+(t-p/E)^2/2]}. \qquad (20)$$

One also shows that the anti-particle *in*- and *out*-modes are given by

$$\overline{\chi^{\text{in}}_{M,-p}}(t)=\chi^{\text{in}}_{M,p}(t), \quad \overline{\chi^{\text{out}}_{M,-p}}(t)=\chi^{\text{out}}_{M,p}(t). \qquad (21)$$

In the second quantized framework, the field operator ψ_M should be decomposed both in terms of the in modes and out modes:

$$\psi_M=\int_{-\infty}^{+\infty}dp \;\; e^{ipz}[b^{\text{in}}_{M,p} \;\; \chi^{\text{in}}_{M,p}+c^{\dagger\text{in}}_{M,-p} \;\; \overline{\chi^{\text{in}*}_{M,-p}}]$$
$$=\int_{-\infty}^{+\infty}dp \;\; e^{ipz}[b^{\text{out}}_{M,p} \;\; \chi^{\text{out}}_{M,p}+c^{\dagger\text{out}}_{M,-p} \;\; \overline{\chi^{\text{out}*}_{M,-p}}]. \qquad (22)$$

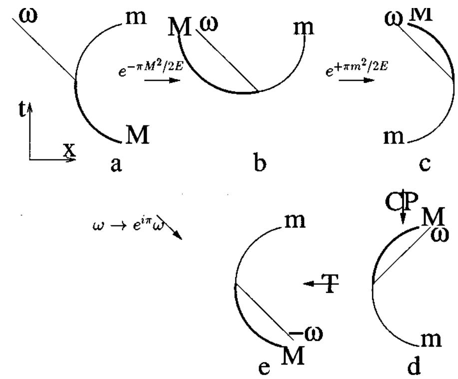

FIG. 2. Feynman diagrams representing (a) an accelerated detector which deexcites and emits a ϕ quantum of energy ω, (b) pair creation of a pair of detectors and the emission of a ϕ quantum, (c) spontaneous excitation of an anti−detector, (d) spontaneous excitation of a detector, and (e) deexcitation of a detector accompanied by the absorption of a ϕ quantum. The conventions are the same as in Fig. 1. A thick curved line designates an excited detector of mass M, a thin curved line a deexcited detector of mass m, and a straight line a ϕ quantum. The orientation of the straight lines (45 degrees to the right or left) corresponds to the momentum of the light quantum being $k_x=-\omega$ or $k_x=+\omega$. Amplitudes (a) and (b), and amplitudes (b) and (c) are related by level crossing. Upon passing from one to the other they acquire the Schwinger factor $e^{-\pi M^2/2E}$ and $e^{\pi m^2/2E}$ respectively. Diagrams (c) and (d) are related by CP and diagram (d) and (e) by T symmetry. Alternatively one can pass directly from (a) to (e) by taking ω to $e^{i\pi\omega}$. These relations shall be proven in Secs. V and VI.

Whereupon one obtains the in vacuum and the out vacuum, which are annihilated by the corresponding destruction operators:

$$b^{\text{in}}_{M,p}|0,\text{in}\rangle_M=c^{\text{in}}_{M,p}|0,\text{in}\rangle_M=0,$$
$$b^{\text{out}}_{M,p}|0,\text{out}\rangle_M=c^{\text{out}}_{M,p}|0,\text{out}\rangle_M=0. \qquad (23)$$

From the Bogoliubov transformation, Eq. (16), one obtains the mean number of produced pairs of momentum p:

$$N_M={}_M\langle 0,\text{in}|b^{\dagger\text{out}}_{M,p}b^{\text{out}}_{M,p}|0,\text{in}\rangle_M=|\beta_M|^2=e^{-\pi M^2/E}. \quad (24)$$

One can also express the in-vacuum in term of its out-particle content:

$$|0,\text{in}\rangle_M=Z_M\prod_p \; e^{-(\beta_M/\alpha_M)b^{\dagger\text{out}}_{M,p}c^{\dagger\text{out}}_{M,p}}|0,\text{out}\rangle_M, \qquad (25)$$

where Z_M is the amplitude not to produce pairs. Its norm square is

$$|Z_M|^2=|_M\langle 0,\text{out}|0,\text{in}\rangle_M|^2=\prod_p \left|\frac{1}{\alpha_M}\right|^2$$
$$=\exp\left(-\sum_p \; \ln(1+e^{-\pi M^2/E})\right). \qquad (26)$$

One recovers the Schwinger result, Eq. (1) by noting that $\Sigma_p=ELT/2\pi$ when the electric field is turned on in a box of size L during a time T (if $T,L\gg E^{-1/2}$), see [21].

Finally we note that the amplitudes represented in Fig. 1 are

$$\text{Fig. 1(a)} \quad \leftrightarrow \quad {}_M\langle 0,\text{out}|b^{\text{out}}_{M,p}b^{\dagger\text{in}}_{M,p}|0,\text{in}\rangle_M = Z_M/\alpha_M,$$

$$\text{Fig. 1(b)} \leftrightarrow {}_M\langle 0,\text{out}|b^{\text{out}}_{M,p}c^{\text{out}}_{M,-p}|0,\text{in}\rangle_M = -Z_M\beta_M/\alpha_M. \tag{27}$$

IV. THE UNRUH EFFECT

We recall the essentials of the Unruh effect for a two level atom, in the (1+1)-dimensional case. The reader may also wish to consult [3,21]. We shall again put emphasis on the use of *crossing symmetry* which allows, in this case, to determine the transition amplitude of the opposite channel in terms of an analytical continuation in the energy of the photon applied to the amplitude of the direct channel. The orientation of the continuation is such that the stability of the vacuum state is guaranteed.

The trajectory followed by the uniformly accelerated detector is

$$t = a^{-1}\sinh a\tau, \quad z = a^{-1}\cosh a\tau, \tag{28}$$

the detector is coupled to a massless field ϕ through the interaction Hamiltonian

$$\int dzdt \;\; H_{\text{int}} = g\int d\tau[e^{-i\Delta m\tau}\phi(t(\tau),z(\tau))|+\rangle \times\langle -| + \text{H.c.},] \tag{29}$$

where $|+\rangle$ and $|-\rangle$ are the excited and ground states of the detector, Δm is the energy gap between the two states and g is a coupling constant.

The second quantized field ϕ obeys the massless Klein Gordon equation in 1+1 dimensions

$$[\partial_t^2 - \partial_z^2]\phi = 0. \tag{30}$$

The complete set of solutions with positive Minkowski frequency ω are

$$\varphi_{k_\omega} = \frac{e^{-i\omega t}e^{ik_\omega z}}{\sqrt{4\pi\omega}} \quad \omega = |k_\omega|, -\infty < k_\omega < +\infty. \tag{31}$$

ϕ can therefore be decomposed as

$$\phi = \int_{-\infty}^{+\infty} dk_\omega[a_{k_\omega}\varphi_{k_\omega} + \text{H.c.}]. \tag{32}$$

The Minkowski vacuum is annihilated by all a_{k_ω} operators

$$a_{k_\omega}|0\rangle = 0. \tag{33}$$

To first order in g the amplitude for the detector to deexcite and emit a right moving Minkowski quantum (i.e., $k_\omega = \omega$) is

$$\begin{aligned} A(\Delta m,\omega,a) &= -i\left\langle -\left|\left\langle 0\left|a_{k_\omega}\left[\int dzdtH_{\text{int}}\right]\right|0\right\rangle\right|+\right\rangle \\ &= -iga\int_{-\infty}^{\infty} d\tau e^{-i\Delta m\tau}\frac{e^{i\omega e^{-a\tau}/a}}{\sqrt{4\pi\omega}} \\ &= -ig\int_0^{\infty}\frac{du}{u}u^{i\Delta m}\frac{e^{-iu\omega}}{\sqrt{4\pi\omega}} \\ &= -ig\Gamma(i\Delta m/a)e^{\pi\Delta m/2a}\frac{(\omega)^{i\Delta m/a}}{\sqrt{4\pi\omega}}, \end{aligned} \tag{34}$$

where the light like variable $u = t - z$ is related to the proper time τ by $au = -e^{-a\tau}$.

Similarly the amplitude for an excited detector to *absorb* this right moving quantum and to get deexcited is

$$B(\Delta m,\omega,a) = -i\left\langle -\left|\left\langle 0\left|\left[\int dzdtH_{\text{int}}\right]a^\dagger_{k_\omega}\right|0\right\rangle\right|+\right\rangle. \tag{35}$$

This amplitude is related to that usually considered in the Unruh effect, namely the amplitude for a deexcited detector to get spontaneously excited, by T symmetry, that is complex conjugation.

We shall not computed $B(\Delta m,\omega,a)$ directly since it is more instructive to determine it from the amplitude $A(\Delta m,\omega,a)$ by exploiting their analytical properties under *level crossing*, i.e., by taking $\omega \to e^{i\pi}\omega$. Indeed, $B(\Delta m,\omega,a)$ is given by

$$B(\Delta m,\omega,a) = A(\Delta m,e^{-i\pi}\omega,a)i. \tag{36}$$

Using the fourth line of Eq. (34), one obtains

$$B(\Delta m,\omega,a) = A(\Delta m,\omega,a)e^{-\pi\Delta m/a}. \tag{37}$$

Therefore the ratio of the transition rates is

$$\frac{R_{-\to+}}{R_{+\to-}} = \left|\frac{B(\Delta m,\omega,a)}{A(\Delta m,\omega,a)}\right|^2 = e^{-2\pi\Delta m/a} \tag{38}$$

since $|B/A|$ is independent of the energy ω of the photon. This is exactly what one would have obtained in a thermal bath at temperature $T_U = a/2\pi$, see Eq. (5).

V. THE SCHWINGER MECHANISM AND THE UNRUH EFFECT

By using the model of the accelerated two level ion presented in Sec. II, we shall show to order $\tilde{g}^2$ that the ratio of the transition rates $R_{M\leftrightarrow m}$ to emit a photon starting from the ground state (m) or the excited state (M) satisfies Eq. (12) even when the vacuum instability with respect to pair creation is fully taken into account. In the next Section, we shall rederive the same ratio from the sole analytical properties of the *pair creation amplitudes* under CPT and crossing symmetry. It is essential that these latter amplitudes do not vanish (i.e., $\beta_M \neq 0$) in order to determine the transition rates through this second indirect procedure.

We first compute the amplitude $\mathcal{A}(\Delta m,p,\omega)$ [depicted in

Fig. 2(a)] to emit a massless quantum starting from the heavier state M. This amplitude corresponds to the amplitude $A(\Delta m,\omega,a)$ of Eq. (34). To first order in $\tilde{g}$, using the momentum conservation, it is given by

$$\mathcal{A}(\Delta m,p,\omega)\,\delta(p-p'-k_\omega)$$
$$=-i_M\,\langle 0,\mathrm{out}|_m\,\langle 0,\mathrm{out}|\,\langle 0|a_{k_\omega}b^{\mathrm{out}}_{m,p'}H_{\psi\phi}b^{\dagger,\mathrm{in}}_{M,p}$$
$$\times|0\rangle|0,\mathrm{in}\rangle_m\,|0,\mathrm{in}\rangle_M$$
$$=-i\tilde{g}Z_M Z_m\delta(p-p'-k_\omega)\,\alpha_M^{-1}\alpha_m^{-1}$$
$$\times\int_{-\infty}^{\infty}dt\;\;\chi^{\mathrm{in}*}_{m,p-k_\omega}(t)\chi^{\mathrm{out}}_{M,p}(t)\;\;\frac{e^{i\omega t}}{\sqrt{4\pi\omega}}. \tag{39}$$

The overall factor $Z_M Z_m$ is the product of the in-out overlaps, see Eq. (26). It appears because the scattering process happens in the presence of pair production of charged quanta.

Notice that in the limit $M^2/E\to\infty$ at fixed $M-m=\Delta m$ and $M/E=1/a$, the integrand of Eq. (39) tends uniformly to the WKB expression studied in [8]. Therefore, by virtue of the analysis of that paper, $\mathcal{A}(\Delta m,p,\omega)$ tends to the ''Unruh'' amplitude $A(\Delta m,\omega,a)$, Eq. (34).

In terms of the integral representations of the χ modes, see Sec. III, and for $k_\omega=\omega$, we obtain

$$\frac{\mathcal{A}(\Delta m,p,\omega)}{Z_M Z_m}$$
$$=-i\tilde{g}\int_0^\infty\frac{du_1}{\sqrt{2\pi}}\int_0^\infty\frac{du_2}{\sqrt{2\pi}}\;\;u_1^{(iM^2/2E)-(1/2)}$$
$$\times u_2^{(im^2/2E)-(1/2)}\frac{e^{i\omega p/E}}{\sqrt{4\pi\omega}}\frac{1}{2}\int_{-\infty}^{\infty}d\tilde{t}e^{i\omega\tilde{t}}$$
$$\times e^{-iE[u_1^2/4+\tilde{t}u_1+\tilde{t}^2/2+u_2^2/4-(\tilde{t}+\omega/E)u_2+(\tilde{t}+\omega/E)^2/2]}, \tag{40}$$

where we have defined $\tilde{t}=t-p/E$. Performing the Gaussian integration over $\tilde{t}$ and introducing the variable $\delta=Eu_1u_2/2$ one has

$$\frac{\mathcal{A}(\Delta m,p,\omega)}{Z_M Z_m}=-i\tilde{g}\int_0^\infty\frac{du_1}{\sqrt{2\pi}}\int_0^\infty\frac{du_2}{\sqrt{2\pi}}$$
$$\times u_1^{(iM^2/2E)-(1/2)}u_2^{(im^2/2E)-(1/2)}$$
$$\times\frac{e^{i\omega p/E}}{4\sqrt{E\omega}}e^{-iE[u_1u_2/2-u_2\omega/E+\omega^2/2E^2]}$$
$$=-i\frac{\tilde{g}}{\sqrt{2E}}e^{i(\omega p-\omega^2/2)/E}$$
$$\times\int_0^\infty\frac{du_2}{u_2}u_2^{-i(M^2-m^2)/2E}\frac{e^{iu_2\omega}}{\sqrt{4\pi\omega}}$$
$$\times\left[\int_0^\infty\frac{d\delta}{\sqrt{2\pi E}}(2\delta/E)^{(iM^2/2E)-(1/2)}e^{-i\delta}\right]. \tag{41}$$

We postpone the evaluation of this expression since the determination of the equilibrium requires to know the ratio of the transition rates only, see Eq. (5). Therefore we shall *relate* $\mathcal{A}$ to the amplitude of the inverse process. One can either consider the amplitude to emit the same quantum starting from the ground state (m) [see Fig. 2(d)], or the amplitude to *absorb* this photon starting with the excited detector state (M) [see Fig. 2(e)], since one is the time reversal (T symmetry) of the other. As in Sec. IV, we consider the second amplitude, denoted by $\mathcal{B}(\Delta m,p,\omega)$, since it is *given* by

$$\mathcal{B}(\Delta m,p,\omega)=\mathcal{A}(\Delta m,p,e^{i\pi}\omega)i \tag{42}$$

by virtue of the stability of the vacuum of the photon field, see Eq. (36).

From the dependence in ω in Eq. (41), exactly like in the third line of Eq. (34), one deduces immediately that the square of the amplitudes which determines both the rates $R_{M\leftrightarrow m}$ and the equilibrium probabilities $P_{M(m)}$, satisfy

$$\left|\frac{\mathcal{B}(\Delta m,p,\omega)}{\mathcal{A}(\Delta m,p,\omega)}\right|^2=\frac{R_{m\to M}}{R_{M\to m}}=e^{-\pi(M^2-m^2)/E}=\frac{P_M}{P_m}. \tag{43}$$

Therefore we have proven Eq. (12) and the fact that the equilibrium probabilities P_M and P_m defined by these radiative processes are equal to those defined by the Schwinger process in Eq. (13).

Furthermore, when the mass gap Δm satisfies $\Delta m\ll M$, it is meaningful to write Eq. (43) as

$$\left|\frac{\mathcal{B}(\Delta m,p,\omega)}{\mathcal{A}(\Delta m,p,\omega)}\right|^2=e^{-2\pi\Delta m/\bar{a}}=\left|\frac{B(\Delta m,\omega,\bar{a})}{A(\Delta m,\omega,\bar{a})}\right|^2, \tag{44}$$

where $\bar{a}=2E/(M+m)=a_M(1-\Delta m/2M)^{-1}$. Thus, under the above inequality, one fully recovers the Unruh equilibrium, see Eq. (38), governed by a single acceleration since $a_M=E/M\simeq E/m\simeq\bar{a}$.

Notice that it is the first time that the concept of acceleration is brought to bear. It appears through a first order change in the exponential factor. This is exactly like the recovery of classical trajectories from wave packets. Indeed the stationarity condition is a first order change in the energy (or the momentum) applied to the phase of the wave packet. This emergence of the classical concepts of acceleration and temperature also bears many similarities with statistical mechanics since it is also through a first order change in the energy that the concept of equilibrium temperature arises from microcanonical ensembles. For further discussions see Sec. VII.

For completeness, we now compute the amplitude $\mathcal{A}$ itself, see Eq. (41). Performing the integrations one gets

$$\frac{\mathcal{A}(\Delta m,p,\omega)}{Z_M Z_m}=-i\frac{\tilde{g}}{2E}\frac{e^{i(\omega p-\omega^2/2)/E}}{\sqrt{4\pi\omega}}\Gamma(-i(M^2-m^2)/2E)$$
$$\times e^{\pi(M^2-m^2)/2E}(\omega)^{i(M^2-m^2)/2E}$$
$$\times\left[\Gamma\left(i\frac{M^2}{2E}+\frac{1}{2}\right)e^{\pi M^2/2E}\frac{(E/2)^{-iM^2/2E}}{\sqrt{2\pi}}\right]. \tag{45}$$

In terms of the mean acceleration $\bar{a}=2E/(M+m)$, one finds

$$\frac{\mathcal{A}(\Delta m,p,\omega)}{Z_M Z_m}=\left[\frac{\tilde{g}}{2gE}\right] A(\Delta m,\omega,\overline{a})$$
$$\times e^{i(\omega p-\omega^2/2)/E}[\alpha_M^{-1}(E/2)^{-iM^2/2E}], \tag{46}$$

where $A(\Delta m,\omega,\overline{a})$ is the ''Unruh'' amplitude Eq. (34) for a two-level system to emit a quantum from the heavier state when it follows a classical trajectory of uniform acceleration $\overline{a}$.

From this expression, one can determine what are the physical processes that cannot be described by the *approximate* amplitudes $A(\Delta m,\omega,\overline{a})$ and $B(\Delta m,\omega,\overline{a})$ based on the hypothesis that one can work with a single classical trajectory. An example of such a quantity is given in [8]. It is shown that the dynamical additional phase $e^{i(\omega p-\omega^2/2)/E}$ leads to decoherence effects which in turn lead to a positive local flux after a finite amount of proper time. This positive flux cannot be described in the treatment based a classical trajectory because there cannot be any loss of coherence in the over restricted dynamical framework wherein only the ϕ field carries momentum.

VI. *CPT* AND CROSSING SYMMETRY

The aim of this section is to rederive Eq. (43) from the amplitudes governing the vacuum instability under pair creation. We shall thereby understand why Eq. (12) and Eq. (13) coincide.

To this end we shall proceed as explained in Sec. II, see also Fig. 2. We introduce two other amplitudes related to the original amplitude $\mathcal{A}(\Delta m,p,\omega)$, Eq. (39), by crossing symmetry. The first one is obtained by replacing the incoming particle created by $b^{\dagger,\mathrm{in}}_{M,p}$ by an outgoing anti-particle destroyed by $c^{\mathrm{out}}_{M,-p}$. This matrix element, denoted by $\mathcal{A}^{(2)}(\Delta m,p,\omega)$ [see Fig. 2(b)], gives the amplitude to create a pair of charged quanta accompanied by the emission of the massless ω quantum. The second one is defined by replacing in $\mathcal{A}^{(2)}$ the outgoing particle destroyed by $b^{\mathrm{out}}_{m,p'}$ by an incoming antiparticle created by $c^{\dagger,\mathrm{in}}_{m,-p'}$. This is the amplitude, denoted by $\mathcal{A}^{(3)}(\Delta m,p,\omega)$ [see Fig. 2(c)], for an accelerated antiparticle of initial mass m to emit an ω quantum.

The second amplitude is given by the following matrix element:

$$\mathcal{A}^{(2)}(\Delta m,p,\omega)\,\delta(p-p'-k_\omega)$$
$$=-i_M\,\langle 0,\mathrm{out}|_m\,\langle 0,\mathrm{out}|\,\langle 0|a_{k_\omega}b^{\mathrm{out}}_{m,p'}c^{\mathrm{out}}_{M,-p}$$
$$\times\tilde{H}_{\psi\phi}\,|0\rangle\,|0,\mathrm{in}\rangle_m\,|0,\mathrm{in}\rangle_M$$
$$=-i\tilde{g}2\pi\delta(p-p'-k_\omega)$$
$$\times\frac{Z_M Z_m}{\alpha_M\alpha_m}\int_{-\infty}^{\infty}dt\;\;\chi^{\mathrm{in},*}_{m,p-k_\omega}(t)\;\;\chi^{\mathrm{in},*}_{M,p}(t)\;\;\frac{e^{i\omega t}}{\sqrt{4\pi\omega}}. \tag{47}$$

The second equality follows from the fact that in the homogeneous gauge, the temporal part of the wave function of an antiparticle of momentum $-p$ is equal to the wave function of the particle of momentum p, see Eq. (22). Then, as in Eq. (19), the only difference with the integrand of Eq. (40) is the sign flip in the factor $e^{-iE\tilde{t}u_1}$ which arises from the replacement[3] of $\chi^{\mathrm{out}}_{M,p}(t)$ by $\chi^{\mathrm{in},*}_{M,p}(t)$, where the χ modes are expressed in their integral representation, see Eqs. (18) and (20). Therefore, exactly as in Eq. (19), one has

$$\frac{\mathcal{A}^{(2)}(\Delta m,p,\omega)}{\mathcal{A}(\Delta m,p,\omega)}=\beta_M=-ie^{-\pi M^2/2E}. \tag{48}$$

This relation may be understood qualitatively from the fact that $\mathcal{A}$ is the decay amplitude $M\to m+\omega$ whereas $\mathcal{A}^{(2)}$ can be envisaged as describing the production of a pair of heavy particles followed by the decay of one of them into $m+\omega$. Thus one expects $\mathcal{A}^{(2)}\simeq\mathcal{A}\;\;e^{-\pi M^2/2E}$.

Similarly, upon considering the amplitude $\mathcal{A}^{(3)}(\Delta m,p,\omega)$ defined by

$$\mathcal{A}^{(3)}(\Delta m,p,\omega)\,\delta(p-p'-k_\omega)$$
$$=-i_M\,\langle 0,\mathrm{out}|_m\,\langle 0,\mathrm{out}|\,\langle 0|a_{k_\omega}c^{\mathrm{out}}_{M,-p}\tilde{H}_{\psi\phi}c^{\dagger,\mathrm{in}}_{m,-p'}\,|0\rangle$$
$$\times|0,\mathrm{in}\rangle_m\,|0,\mathrm{in}\rangle_M$$
$$=-i\tilde{g}2\pi\delta(p-p'-k_\omega)\frac{Z_M Z_m}{\alpha_M\alpha_m}\int_{-\infty}^{\infty}dt\;\;\chi^{\mathrm{out}}_{m,p-k_\omega}(t)$$
$$\times\chi^{\mathrm{in},*}_{M,p}(t)\;\;\frac{e^{i\omega t}}{\sqrt{4\pi\omega}} \tag{49}$$

one finds that the sign of the linear term in u_2 appearing in the Gaussian factor has flipped. Therefore

$$\frac{\mathcal{A}^{(2)}(\Delta m,p,\omega)}{\mathcal{A}^{(3)}(\Delta m,p,\omega)}=\beta_m=-ie^{-\pi m^2/2E}. \tag{50}$$

As in Eq. (48), this may be understood from the fact that the amplitude $\mathcal{A}^{(2)}$ can also be envisaged as describing the creation of a pair of light particles followed by the spontaneous excitation of one of them, whereas $\mathcal{A}^{(3)}$ is the spontaneous excitation amplitude.

Now, by *CPT* invariance, one obtains

$$\mathcal{A}^{(3)}(\Delta m,p,\omega)=\mathcal{B}(\Delta m,p,\omega) \tag{51}$$

given in Eq. (42). Indeed one verifies that the integrand of $\mathcal{A}^{(3)}(\Delta m,p,\omega)$ coincides with the one of $\mathcal{B}(\Delta m,p,\omega)$ under the change of the dummy variable $\tilde{t}=-\tilde{t}$. Therefore, combining this latter relation with Eqs. (48) and (50), one obtains

$$\frac{\mathcal{B}(\Delta m,p,\omega)}{\mathcal{A}(\Delta m,p,\omega)}=\frac{\beta_M}{\beta_m}=e^{-\pi(M^2-m^2)/2E}. \tag{52}$$

[3]It should be noted that this product of in-modes appears systematically upon evaluating any amplitude under the double condition (pre- and post-selection in the Aharonov language) that the initial state of the system was the in-vacuum and that the final state contains one specific pair of charged quanta, see [23,21].

Thus the ratio of the scattering amplitudes is equal to the ratio of the Schwinger factors obtained by using crossing symmetry twice. This is what guaranteed that Eqs. (12) and (13) coincide. QED.

In the above calculation we considered the emission or absorption of a right moving quantum $e^{-i\omega(t-z)}$. Had one considered left moving quanta $e^{-i\omega(t+z)}$, different transition amplitudes would have been obtained since parity P is explicitly broken in our model by the external electric field. However the ratio of amplitudes for left and right moving is a constant

$$\frac{\mathcal{A}(\text{right})}{\mathcal{A}(\text{left})}=\frac{\mathcal{A}^{(2)}(\text{right})}{\mathcal{A}^{(2)}(\text{left})}=\frac{\mathcal{A}^{(3)}(\text{right})}{\mathcal{A}^{(3)}(\text{left})}=\frac{\alpha_M}{\alpha_m}\ e^{i\omega^2/2E}. \tag{53}$$

Therefore the ratios Eqs. (48), (50), and (52) and the equilibrium distribution Eq. (43) are independent of whether left or right moving particles are emitted.

VII. PAIR CREATION OF BLACK HOLES

We consider how the above analysis applies to pair creation of charged black holes in an external electric field which was considered in Refs. [17–19]. In the black hole case, the picture is more complicated because black holes have themselves an intrinsic temperature, the Hawking temperature, and because the semiclassical description of the production requires that their Unruh and Hawking temperature coincide. We recall that this condition arises from the requirement that the Euclidean instanton have no conical singularity. For a given electric field E_0, the charge Q of the hole is a function of its mass M. Thus only the probability to produce this one parameter family of black holes can be obtained by this semiclassical treatment.

Following [24,25], we express the probability to create a pair of black holes which belong to this family as

$$P_{M,Q,E_0}=Ce^{(\Delta\mathcal{A}+\mathcal{A}_{\rm BH})/4}, \tag{54}$$

where $\mathcal{A}_{\rm BH}(M,Q)$ is the area of the black hole horizon, $\Delta\mathcal{A}(M,Q,E_0)$ is the *change* of the area of the acceleration horizon induced by the creation of the black hole pair and C a constant which takes into account the appropriate phase factors, see Eq. (2). As emphasized in [24,25], $\mathcal{A}_{\rm BH}/4$ appears in this expression as furnishing the density of black hole states with mass M and charge Q thereby confirming the Bekenstein interpretation of $\mathcal{A}_{\rm BH}/4=S_{\rm BH}$ as the black hole entropy.

The domain of the one parameter family which can be compared with the Schwinger mechanism is the one in which the black holes are small compared to the inverse acceleration, i.e., in the point particle limit. Then, the change of the area reduces to

$$\frac{\Delta\mathcal{A}}{4}=-S_{\rm Euclid}=-\pi M^2/QE_0, \tag{55}$$

i.e., minus the Euclidean action to complete an orbit, Eq. (8).

In order to make contact with Eq. (9) and therefore to show that $S_{\rm Euclid}$ acts as an entropy in delivering the Unruh temperature, we consider the black hole pair creation probability from another point of view: We assume that Eq. (54) is valid for *all* values of M and Q and not only for the black holes which belong to the one parameter family. Then, one can make independent variations of M and Q and determine the most probable mass $\underline{M}$ at fixed Q by extremizing P_{M,Q,E_0} with respect to M. Using Eq. (55), one gets

$$\partial_M P_{M,Q,E_0}=P_{M,Q,E_0}\left[\partial_M\left(\frac{-\pi M^2}{QE_0}\right)+\partial_M\left(\frac{\mathcal{A}_{\rm BH}(M,Q)}{4}\right)\right]$$

$$0=P_{\underline{M},Q,E_0}\left[-\frac{1}{T_U(\underline{M},QE_0)}+\frac{1}{T_H(\underline{M},Q)}\right], \tag{56}$$

where we have defined the Hawking temperature as usual by $d\mathcal{A}_{\rm BH}/4=dM/T_H$. Therefore the equality of the Hawking and the Unruh temperature which defined the one-parameter family is recovered here as determining the most probable mass $\underline{M}$. Indeed one verifies that $\underline{M}$ constitutes a maximum of P_{M,Q,E_0} at fixed[4] Q and E_0. In this determination, the Euclidean action $S_{\rm Euclid}$ acts exactly like the Bekenstein entropy $\mathcal{A}_{\rm BH}/4$. This strongly suggests that the equality of Hawking and Unruh temperatures should be understood in the *mean* and not as a *necessary* condition that M and Q must satisfy in order to have black hole production. (A similar point of view has been put forward, but some how less explicitly, in [24].)

We now turn to the radiative processes which the black holes undergo as they are accelerated. Indeed the black holes will both emit radiation through the Hawking process and will interact with the Unruh heat bath of Rindler quanta. Because of the thermodynamic nature of the equilibrium condition Eq. (56), one expects that it should be preserved when radiative processes are taken into account. We now show that this is indeed the case, and more importantly that the rates of emission and absorption of photons can be deduced from the pair creation probability Eq. (54). To this end, we define the rate $R^-_{M,\nu}$ for an accelerated black hole of mass M to emit a massless quanta of boost energy ν thereby decreasing its mass by ν. Similarly we define the rate $R^+_{M-\nu,\nu}$ for the inverse transition, that is the absorption rate of quanta of boost energy ν by a black hole of mass $M-\nu$, see the amplitudes A and B in Sec. IV.

On the basis of our analysis of accelerated detectors presented in Sec. V, we conjecture that the amplitudes for these processes are related by level crossing and *CPT* to the amplitudes of producing pairs of black holes, Eq. (54) continued outside of the one parameter family. If this is correct, the ratio of the transition rates can be expressed as

$$\frac{R^+_{M-\nu,\nu}}{R^-_{M,\nu}}=\frac{P_{M,Q,E_0}}{P_{M-\nu,Q,E_0}}=e^{-\delta S_{\rm Euclid}+\delta\mathcal{A}_{\rm BH}/4} \tag{57}$$

[4]Together with Spindel and Gabriel, we are presently investigating more general variations in which Q also varies. Then the chemical potential induced by the electric field also participates to the determination of the equilibrium in the usual thermodynamical way.

for all values of ν and of M, i.e., for values no longer restricted to $\nu \ll M$ nor to $M = \underline{M}$, (both conditions being required for the semiclassical approximation [26] to be valid).

The factor $e^{-\delta S_{\rm Euclid}}$ expresses both the conjecture that transition rates of charged black holes coincide to those of the corresponding pointlike charged particles (same masses, same charge) and the fact that the latter's transition rates are governed by $\delta S_{\rm Euclid} = S_{\rm Euclid}(M, QE_0) - S_{\rm Euclid}(M-\nu, QE_0)$, see Eq. (43), and not by the (canonical) expression $2\pi(M-\nu)/a$ as in the semiclassical treatment, see Eq. (38). The factor $e^{\delta \mathcal{A}_{\rm BH}/4}$ expresses the conjecture that black holes behave like pointlike particles characterized by a degeneracy given by $e^{\mathcal{A}_{\rm BH}/4}$. This second conjecture has been recently proven for Schwarzshild black holes in [27]. Notice that Eq. (57) reduces to the semiclassical calculation when $\nu \to 0$ and when $M = \underline{M}$. Indeed, using Eq. (56), one obtains directly

$$\frac{R^+_{\underline{M}-\nu,\nu}}{R^-_{\underline{M},\nu}} = \frac{P_{\underline{M},Q,E}}{P_{\underline{M}-\nu,Q,E}} \to_{\nu\to 0} e^{-\nu/T_U + \nu/T_H} = 1 \tag{58}$$

as in [26].

Thus we see that not only the area of the black hole horizon acts as a *reservoir* entropy in delivering the properties of the radiation (for a recent expose which makes clear the passage from a microcanonical ensemble to canonical considerations in black hole thermodynamics, see Chap. 3.6 in [21]), but more surprisingly, by virtue of Eq. (55), the (change in) area of the acceleration horizon acts in the same way. Therefore the transition rates are directly determined by the sum of the change of horizon areas:

$$\frac{R^+_{M-\nu,\nu}}{R^-_{M,\nu}} = \exp(\delta \mathcal{A}_{\rm total}/4). \tag{59}$$

The present analysis sheds new light on the thermodynamical approach to gravity recently presented by Jacobson [13]. We recall that his approach is based on two main hypothesis, namely that changes in area are linearly related to changes in entropy and that the surface gravity is related to the temperature seen by accelerating observers. From these hypothesis, he deduced Einstein equations in the limit of small fluxes. The present analysis can be conceived as providing statistical (microcanonical) foundations to his thermodynamical approach, at least for the restricted set of phenomena considered in this paper. Indeed, both of his hypothesis are now derived from the fact that transition probabilities are given by the change in horizon area, Eq. (59). (Notice that this expression necessitates a choice of the action that governs the *transition amplitudes* of gravity, most likely that gravity is described by the Einstein-Hilbert action.) From Eq. (59) one obtains, first, that the area of the horizon indeed behaves like an entropy in its determination of transition rates and equilibrium configurations, and, secondly, that acceleration and temperature are correctly related.

Finally we note that the local interactions between the radiation field and the black holes lead to a *decoherence* of the black holes states. To understand this decoherence, note that before the first photon is emitted, one has a strict Einstein-Podolsky-Rosen (EPR) correlation between the momenta (and the other quantum numbers) of the two black holes since they are produced from vacuum. This correlation is, however, necessarily destroyed by photons since the interactions among the radiation field and the black holes are *local* in the sense that the inverse acceleration characterizing the mean wave length of the radiation emitted or absorbed is much larger than the horizon radius. Thus their masses will spread independently around the mean. However, in spite of this destruction of the initial correlations, the equilibrium distribution of the decohered momenta and masses is identical to the initial distribution when they were still exactly correlated, since the radiative processes maintain the ''pair creation'' equilibrium, see Eq. (58).

Moreover, this decoherence is just what it is necessary to invalidate the conclusions of the analysis of Yi [26,28]. He argued that accelerated black holes no longer emit radiation when their Hawking and Unruh temperatures coincide. His reasoning was based on coherently interfering amplitudes, a misleading feature arising when one works in a single classical background, i.e., by neglecting all quantum recoil effects. As stressed in [29] and the last remark of Sec. V, the amplitudes evaluated in the background field approximation are approximations which neglect the important phase appearing in Eq. (46). Taking into account this phase completely modifies the local properties of the emitted radiation [8].

In summary we have shown that there is a thermodynamic consistency between the Schwinger and Unruh effects. The classical concept of acceleration, and the thermodynamic concept of temperature, arise upon taking first order changes in the energy applied to the exponential factor appearing in transition amplitudes, see the remark made after Eq. (43). This is a universal feature. For instance the emergence of time in quantum cosmology also results from first order treatment of exponential factors [16] (the analogous treatment of exponential factors in statistical mechanics is also discussed in this paper). In the case of accelerated black holes the consistency with thermodynamics is enlarged once the additional fact that the black holes have an intrinsic entropy is properly taken into account. This enlarged consistency can probably be derived by appealing to the analytical properties of the amplitudes to produce black holes and to emit Hawking radiation under crossing symmetry and *CPT*, in close analogy to what we proved for accelerated particles. This might shed new light on the debate about whether black hole evolution can be described by a unitary S matrix [30].

Thus the outcome of our analysis is that upon enlarging the dynamical framework and going beyond the semi classical approximation, apparently unrelated phenomena such as the Unruh effect, the Schwinger effect, Hawking radiation, are described in one thermodynamicaly consistent whole. And the area of causal horizons seem to play an essential role in bringing about this unified description. We shall report further on this aspect in a forthcoming publication [31].

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