Stochastic behavior of effective field theories across the threshold

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We explore how the existence of a field with a heavy mass influences the low energy dynamics of a quantum field with a light mass by expounding the stochastic characters of their interactions which take on the form of *fluctuations* in the number of (heavy field) particles created at the threshold, and *dissipation* in the dynamics of the light fields, arising from the back reaction of produced heavy particles. We claim that the stochastic nature of effective field theories is intrinsic, in that dissipation and fluctuations are present both above and below the threshold. Stochasticity builds up exponentially quickly as the heavy threshold is approached from below, becoming dominant once the threshold is crossed. But it also exists below the threshold and is, in principle, detectable, albeit strongly suppressed at low energies. The results derived here can be used to give a quantitative definition of the ''effectiveness'' of a theory in terms of the relative weight of the deterministic versus the stochastic behavior at different energy scales. [S0556-2821(97)06406-0]

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I. INTRODUCTION

The goal of this paper is to explore how the existence of a field with a heavy mass (a heavy sector, or heavy field, in short) influences the low energy dynamics of a quantum field with light mass (light sector, or light field). It is a well-known result from effective field theory [1–3] that at low energies the heavy fields effectively decouple. This means that it is possible to describe the low energy (infrared) physics through an effective action of the light fields, whereby no explicit reference to heavy fields is made.[1] This description breaks down as the energy gets close to the heavy particle mass threshold, where particle creation of the heavy field begins to get significant. Generally, the breakdown of the effective light theory is described in terms of the loss of predictive power of the theory, resulting from a proliferation of increasingly nonlocal interactions. We shall propose a new way to look at the threshold behavior of the light theory, based on the manifestation of stochastic features which we believe are intrinsic to effective theories [6,7].

This loss of predictability follows from the fact that the light field interacts in a complex way with quantum fluctuations of the heavy field. The stochastic characters of these interactions take on the form of *fluctuations* in the number of (heavy field) particles created at the threshold, and *dissipation* in the dynamics of the light fields, arising from the back reaction of produced heavy particles. The stochastic properties of effective field theory arise from particle creation, and the two processes it engenders, i.e., dissipation and noise, are related by the fluctuation-dissipation relation [8].

The appearance of stochastic behavior, such as dissipation and noise, is predicated upon the actual observational context which defines the system, and its interaction with the unobserved or unobservable variables which make up the environment. In fact, for observers in the limited range of validity of the system (say, at low energy), the existence of the environment (say, the heavy sector) can sometimes only be indirectly deduced by the modified behavior of the system, rendered by such restrictions.

Dissipation becomes obvious above the heavy mass threshold, where the light field self-energy becomes imaginary (in agreement with the optical theorem). As we shall show below, a proper analysis of the effective light theory shows that a dissipative theory must also be stochastic at some level. Our claim, based on the results of this paper, is that the stochastic nature of effective field theories is intrinsic, in that dissipation and fluctuations are present both above and below threshold. Stochasticity builds up exponentially quickly as the heavy threshold is approached, becoming dominant once the threshold is crossed. But it also exists below threshold, albeit strongly suppressed at low energy. This is in contradistinction to the conventional belief that such behavior changes discontinuously on threshold crossing. Using the expressions we derive here, one can quantitatively define the ''effectiveness'' of a theory in terms of the relative weight of the deterministic versus the stochastic behavior at different energy scales.

The presence of stochastic effects shows that the physics of the light fields is different in an effective theory, in small but important ways, from what would follow if the light action were fundamental, even at scales below the heavy threshold. The difference lies in the phenomena of dissipation and fluctuation generation, which are present in effective

[1]It should be clear from the discussion below that we regard the effective theory as a description of the actual physics accessible to an observer at a given energy scale including the effect of the higher mass sector, rather than as a formal construct obtained from the full theory by application of some approximation scheme such as the Schwinger-DeWitt proper-time quasilocal [4] or the Heisenberg-Euler [5] inverse-mass expansion of propagators. The nonperturbative effect we discuss here cannot be obtained by these approximations.

theories but absent in fundamental ones. On a more speculative level, we may argue that there is no real ''fundamental'' theory in nature [9]. A theory only appears to be fundamental at low energy in ignorance of the presence of other heavy constituents in nature because the stochastic components generated from such interactions are suppressed. At higher energies such features become more important and the presence of the heavy sector becomes more apparent. Thus, the magnitude of noise and dissipation can serve as a measure of the degree of resolution of the means of observation compared to the intrinsic mass or energy scales of the more complete theory. This viewpoint is a natural consequence from regarding an effective theory as an open system [6,7], which is what we used earlier in the analysis of the statistical-mechanical properties of particles and quantum fields [10–13] and semiclassical gravity [14–17].

Recent investigation of the statistical-mechanical aspect of gravitational systems and quantum fields began with the work of Bekenstein [18] and Hawking [19] on black hole entropy. Penrose's proposal of gravitational entropy and the Weyl curvature conjecture [20] were analyzed in the context of back reaction of particle creation by one of us [21]. Entropy of quantum fields associated with particle creation was discussed in [22,23] (see also [24]). The concept of field entropy was further explored in [25,26] and more recent works. Entropy of interacting fields defined by the truncation of a Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy and the factorization of higher order correlation functions was proposed in [27]. Noise, decoherence, fluctuations, and dissipation in this scheme were discussed in [12,13]. A common assumption in quantum theories of structure formation, i.e., quantum correlation functions directly go over to their classical counterparts, was shown to be incorrect [17] when the stochastic properties of quantum fields are carefully considered. As our present analysis further demonstrates, only those modes of the light field which are dynamically entangled with the quantum heavy fields can partake of the process of decoherence and quantum to classical transition to acquire a stochastic character. The relationship between dissipation and stochasticity has been further discussed in the context of decoherence and quantum to classical transition. Calzetta and Mazzitelli pointed out the connection between particle creation and decoherence [28]. Paz and Sinha showed that a decohered field must of necessity possess traits of randomness [29]. The stochastic aspects of classical theories emerging from quantum mechanics are discussed at length in an important paper by Hartle and Gell-Mann [30]. (See also [31].)

Our inquiry into the stochastic nature of effective field theory thus compels us to adopt a new and more complex viewpoint of quantum field theory, incorporating the statistical-mechanical properties of quantum fields. This means that we are more interested in the *causal development* of quantum fields than in the traditional scattering or transition amplitude aspects. For this we need the in-in [or closed time path (CTP) or Schwinger-Keldysh] [32] rather than the in-out (or Schwinger-DeWitt) [4] formulation. We will use the related influence functional (Feynman-Vernon) [33] formalism to extract the stochastic features of effective field theory. In addition, we will need to probe into the *nonperturbative effects*. By perturbative, we refer here specifically to expansions in the coupling constants of fields, rather than loop expansion, or adiabatic approximation. Nonperturbative calculations are exemplified by Schwinger's original derivation of particle creation in a constant electric field [34]. For noninteracting fields in curved spacetimes with regions where a vacuum for a field theory can be defined (asymptotically flat, or statically bounded evolution), Parker's [35] and Zel'dovich's [36] treatment of cosmological particle creation and Hawking's [19] derivation of black hole radiance by means of Bogoliubov transformations are nonperturbative, even though for more general situations where a well-defined global vacuum is lacking [37], one may need to appeal to approximate or perturbative concepts such as adiabatic vacuum [38,39]. For fields propagating in nontrivial spacetimes (such as the nonconformally flat spacetimes of the Bianchi universes [40,41]), or for interacting fields (e.g., [42,43]), one has to appeal to perturbative expansion of the interaction or coupling parameter (such as the λ in a ϕ^4 theory or the anisotropy). These nonperturbative effects may be quantitatively significant in the proper environment, such as during the reheating era in inflationary cosmology [44–48].

One main result of this paper is that a light field plane wave is always followed by a stochastic, slowly varying light ''echo.'' This ''echo'' is produced by the back reaction of heavy particles created from the seed light wave, and it is a nonperturbative effect. The amplitude and growth rate of the echo increases exponentially as the frequency of the seed wave approaches the heavy scales. We may understand this as a ''diamagnetic'' effect (in contrast with a paramagnetic effect), since it involves two steps: First, the polarization of the vacuum by the light seed wave, and then the coupling of the appropriate light modes to the polarized vacuum.

If the light field self-interacts, this effect shall be masked by the corresponding one originating from quantum fluctuations of the light field. In principle, these two effects could be disentangled by recourse to their different scale dependence. In any case, the presence of both effects underlies the fact that any field theory used in practice for the description of real physical systems is an effective theory. This heuristic observation may be put on a rigorous footing by casting a field theory as a theory of a background field interacting with a hierarchy of Wightman functions; the theory becomes effective when this hierarchy is truncated, either explicitly through some approximation scheme, or implicitly by the limited accuracy of specified observations [12,13].

To connect with earlier theoretical work (developed mainly in the 1970s and 1980s) we shall begin in Sec. II with a discussion of conventional effective field theories [1,3]. We shall show how the features of light physics we want to develop appear already in the conventional (in-out) formulation, albeit in a somewhat obscure form. We shall then in Sec. II B go over to the more suitable causal (in-in) formulation of quantum field theory. In Sec. II C we show how the stochastic characters of an effective field theory can be identified from the in-in effective action with the aid of the Feynman-Vernon formalism, and how both dissipation and fluctuations can be related to particle creation above threshold. In Sec. III we show how such features remain in the below-threshold regime and derive the new effects which can, in principle, be used to discern an effective theory from a ''fundamental'' theory. Appendices contain the details of the derivation of the heavy field quantization in the background of a light plane wave.

II. STOCHASTIC BEHAVIOR NEAR THE THRESHOLD

As could be expected, the problems with conventional light effective theories are most acute if we attempt to implement them in the above-threshold regime; in this range, dissipation and noise appear even at the perturbative level. We shall take advantage of this fact to introduce the main concepts of dressed field, dissipation, noise kernels, etc., in the familiar setting of one-loop Feynman graphs. We shall then proceed in the next section to discuss these effects in the physically more relevant, below-threshold regime.

A. Effective theory of the light particles

To simplify the technical burden, we shall work on a toy model of quantum field theory consisting of two real scalar fields ϕ and Φ. We have used this model to treat the dissipation of quantum fields via particle creation by the CTP method before [41,43]. For more general models, we refer the reader to [10]. Similar consideration of the dissipative and noise properties of two field interactions can be found in [44–46,48–51].

The classical action is given by

$$S=S_l+S_H+S_{lH}, \tag{2.1}$$

$$S_l=\int d^4x\left(-\tfrac{1}{2}\right)[\partial\phi\partial\phi+m^2\phi^2-g\langle\Phi^2\rangle_0\phi], \tag{2.2}$$

$$S_H=\int d^4x\left(-\tfrac{1}{2}\right)[\partial\Phi\partial\Phi+M^2\Phi^2], \tag{2.3}$$

$$S_{lH}=\int d^4x\left(-\tfrac{1}{2}\right)g\phi\Phi^2, \tag{2.4}$$

where the subscripts l,H are used to denote the light and heavy fields. We use signature $-+++$, and ignore terms necessary for renormalization purposes (other than the term linear in the ϕ field). Here, $\langle\Phi^2\rangle_0$ stands for the vacuum expectation value in the absence of background field, i.e., $\langle\Phi^2\rangle_{\phi=0}$. We also assume $m\ll M$.

The effective theory of light particles (we may, henceforth, call it light effective theory) is defined by the action functional

$$S_{\mathrm{eff}}=S_l+\delta S, \tag{2.5}$$

where

$$\delta S[\phi]=-i\ln\int D\Phi\quad e^{i\{S_H(\Phi)+S_{lH}(\phi,\Phi)\}}. \tag{2.6}$$

Formally, δS is the effective action for the heavy fields propagating on the light background field (considered as an external field), evaluated at the vacuum expectation value (VEV) [52]. In our case, this VEV vanishes by symmetry, so we shall not mention it explicitly. Perturbatively, δS is the sum of all one particle irreducible (1PI) vacuum bubbles, with light field insertions but only heavy internal lines.

To second order in the coupling constant g, we find

$$\delta S[\phi]=\left(\frac{-g}{2}\right)\int d^4x\,\phi(x)\Delta_F(x,x)+\left(\frac{ig^2}{4}\right)\int d^4x d^4x'\,\phi(x)\phi(x')\Delta_F^2(x,x'), \tag{2.7}$$

where

$$\Delta_F(x,x')=\frac{\langle\mathrm{out}|T[\Phi(x)\Phi(x')]|\mathrm{in}\rangle}{\langle\mathrm{out}|\mathrm{in}\rangle}=(-i)\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\frac{1}{k^2+M^2-i\epsilon} \tag{2.8}$$

is the Feynman propagator for the heavy particles. The linear term is canceled by an appropriate counterterm. For the quadratic term, we find

$$\Delta_F^2(x,x')=\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\left\{(i)(k^2+m^2)\times\int_{4M^2}^{\infty}\frac{ds}{(s-m^2)}\frac{h(s)}{(k^2+s-i\varepsilon)}\right\}, \tag{2.9}$$

where we have performed the necessary subtractions to ensure that m^2 remains the physical mass of the light field, and

$$h(s)=\frac{1}{(4\pi)^2}\sqrt{1-\frac{4M^2}{s}}. \tag{2.10}$$

An effective light theory deals with the $k^2\to 0$ limit. We can obtain a formal expression for the effective action by expanding Eq. (2.9) in inverse powers of the heavy mass; this expansion is analogous to the Heisenberg-Euler Lagrangian for the electromagnetic field [5]. At any finite order, we obtain a higher derivative theory [53]. Such approximation will not show dissipation nor fluctuations in the light field.

In this limit, δS is analytic in k and real. However, this ceases to be the case as soon as we cross the heavy particle threshold $4M^2$. Above the threshold, δS is neither analytical nor real. In this regime a light effective theory is not only cumbersome because of the proliferation of nonlocal terms, rather, the whole concept of an effective action breaks down.

A striking feature of the light action, if we insist on taking it seriously above threshold, is that it leads to complex and noncausal equations of motion. This follows from the in-out boundary conditions built in the path integral equation (2.6). The imaginary part of the effective action is related to the imaginary part of the Feynman graph. Because of the optical theorem, we know this, in turn, is related to pair creation from the light particles. Therefore, a complex action would give rise to dissipative terms in the equation of motion of the light field, and the fluctuations in the particle creation would measure the breakdown of the (low energy) effective theory. The unitarity of the full quantum theory is broken in the effective theory. However, the chosen in-out boundary conditions obscure this fact, since they make it hard to discern the arrow of time arising from dissipation [40]. For this and other reasons, one should use the causal in-in boundary conditions, as introduced by Schwinger [32,41,43].

B. Causal effective field theory

Let us derive the causal and real equations of motion which describe the evolution of physical perturbations of the light field. This is achieved by doubling the degrees of freedom to two fields $\phi^{+,-}$, or rather, by assuming the field is actually defined on a closed time path. The equations of motion are found by taking the variation with respect to ϕ^+ of the CTP action functional

$$S_{\rm eff}^{\rm CTP}=S_l[\phi^+]-S_l[\phi^-]+\delta S^{\rm CTP}[\phi^+,\phi^-], \qquad (2.11)$$

where [32,41]

$$\delta S^{\rm CTP}[\phi^+,\phi^-]=-i\ln\int D\Phi^+D\Phi^- \quad e^{i\{S_H(\Phi^+)-S_H(\Phi^+)+S_{lH}(\phi^+,\Phi^+)-S_{lH}(\phi^-,\Phi^-)\}}. \qquad (2.12)$$

The quadratic terms in the effective action, to second order in the coupling constant, are

$$\frac{ig^2}{4}\int d^4xd^4x'\{\phi^+(x)\phi^+(x')\Delta_F^2(x,x')-2\phi^+(x)\phi^-(x')\Delta_-^2(x,x')+\phi^-(x)\phi^-(x')\Delta_D^2(x,x')\}, \qquad (2.13)$$

where the propagators are the expectation values taken with respect to the ''in'' vacuum defined by

$$\Delta_F(x,x')=\langle{\rm in}|T[\Phi(x)\Phi(x')]|{\rm in}\rangle$$
$$=(-i)\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\frac{1}{k^2+M^2-i\varepsilon}, \qquad (2.14)$$

$$\Delta_D(x,x')=\langle{\rm in}|\widetilde{T}[\Phi(x)\Phi(x')]|{\rm in}\rangle$$
$$=(i)\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\frac{1}{k^2+M^2+i\varepsilon}, \qquad (2.15)$$

$$\Delta_-(x,x')=\langle{\rm in}|\Phi(x')\Phi(x)|{\rm in}\rangle$$
$$=(2\pi)\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\delta(k^2+M^2)\,\theta(-k^0). \qquad (2.16)$$

The equations of motion become (but see below)

$$(-\Box+m^2)\phi(x)+g^2\int d^4x' \quad D(x,x')\phi(x')=0, \qquad (2.17)$$

where

$$D(x,x')=\frac{i}{2}[\Delta_F^2(x,x')-\Delta_-^2(x,x')]. \qquad (2.18)$$

Since from the definitions

$$\Delta_-(x,x')=\Delta_F(x,x') \quad \text{if} \quad t'>t, \qquad (2.19)$$

while

$$\Delta_-(x,x')=\Delta_F^*(x,x') \quad \text{if} \quad t>t', \qquad (2.20)$$

it is obvious that Eq. (2.17) is real and causal. Explicitly,

$$D(x,x')=\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\left\{\left(\frac{-1}{2}\right)(k^2+m^2)\times\int_{4M^2}^\infty\frac{ds}{(s-m^2)}\frac{h(s)}{[(k+i\varepsilon)^2+s]}\right\}, \qquad (2.21)$$

with the same h as in Eq. (2.10), and $(k+i\varepsilon)^2=-(k^0+i\varepsilon)^2+\vec{k}^2$, carrying the causal boundary conditions.

The light field ϕ described by the wave equation (2.17) is clearly no longer the classical light field, but is now dressed through the interaction with the quantum fluctuations of the heavy field. A different approach to the dynamics clarifies this point. The Heisenberg equations of motion for the light field are

$$(-\Box+m^2)\phi(x)+\left(\frac{g}{2}\right)[\Phi^2-\langle\Phi^2\rangle_0]=0, \qquad (2.22)$$

where we have subtracted the expectation value of Φ^2, computed at vanishing light fields, to make $\phi=0$ the true light vacuum. Comparing Eqs. (2.17) and (2.22), we see that the former amounts to the approximation

$$[\Phi^2-\langle\Phi^2\rangle_0]\approx 2g\int d^4x'D(x,x')\phi(x'). \qquad (2.23)$$

On the other hand, a direct calculation shows that

$$2g \quad D(x,x')\equiv\left.\frac{\delta\langle\Phi^2\rangle(x)}{\delta\phi(x')}\right|_{\phi=0} \qquad (2.24)$$

so that, within the present accuracy,

$$2g\int d^4x'D(x,x')\phi(x')\simeq\langle\Phi^2\rangle_\phi-\langle\Phi^2\rangle_0. \qquad (2.25)$$

Here, $\langle\Phi^2\rangle_\phi$ stands for the vacuum expectation value evaluated with respect to the background of a nonzero light field ϕ. So, in this approximation, the q number Φ^2 in the Heisenberg equation of motion is substituted by its expectation value, computed as a causal functional of the light background. (In quantum open systems language, the heavy field is said to be ''slaved'' to the light one [13].)

We wish to point out that the equation of motion, Eq. (2.17), for the expectation value of the light field is not the classical equation of motion, but includes the dissipation kernel D, which accounts for the averaged effect of back reaction of the heavy field on the light one. In the linearized theory, of course, we use the approximation equation (2.21) for D to first order in g, but this is still the full (i.e., mean) back reaction. Actually, Eq. (2.17) is simply the expectation value, taken with respect to the in vacuum, of the Heisenberg equation of motion for the light field deduced from the action equations (2.1)–(2.4), and thus it provides a consistent and full description of the dynamics of the expectation value of the light field to this order.

Now, at issue is whether physically this degree of accuracy offers a sufficient depiction of one's problem at hand. There are many examples where a mean field description fails. An important class of problems is critical fluctuations. As we shall show presently, the theory based on Eq. (2.17), no matter how accurately the back reaction (that is, D) is computed, cannot provide sensible answers to the more complex problem of the physics of fluctuations around the mean field.

There are two possible answers to the inaccuracy of the theory based on Eq. (2.17) when dealing with fluctuations. [Again, we stress that this is not a matter of accounting for back reaction, since Eq. (2.17) would be unsuitable to describe the full dynamics of the light field even if D could be computed exactly.] The always correct one is to compute the joint evolution of the coupled light-heavy field exactly. This is usually difficult if not impossible, and is indeed the *raison d'être* for devising an effective theory; i.e., one wishes to achieve a description of the light field dynamics based largely on the light field alone (e.g., effective renormalizability [1]). The second response, the one we shall explore in the following, is to enlarge the usual effective theory framework to explicitly include stochastic terms. We will show how this extension can be done in a way which is consistent both with field theory and with statistical mechanics.

In a free theory, it would be enough to introduce random initial conditions for the light field to properly account for these fluctuations. Interacting theories are intrinsically more complex, since in addition to the first order effect associated with uncertainty in the initial conditions, we have a ''second order'' effect produced by the driving of the light field by heavy particles created from the initial light field. The second order effect has not only a deterministic part (accounted for by the dissipation kernel D), but also a stochastic part, because the actual particle production process is not deterministic, and the number of particles displays irreducible fluctuations. This effect, moreover, changes in character as the actual state of the heavy field deviates from the vacuum. It is in this way that an arrow of time appears in the theory. The fact that pair creation brings forth dissipation, and the fluctuations in back reaction associated to fluctuations in particle number manifest as noise, is the physical underpinning of the fluctuation-dissipation theorem, as we shall show below.

Before we continue, however, let us elaborate on our earlier claim that Eq. (2.17) cannot give a satisfactory account of the physics of the light field, where satisfactory means that not only the mean field evolution, but also the fluctuations around the mean field are accounted for. We can see this, for example, by consideration of the light field fluctuations. These are described by the Hadamard kernel

$$G_1(x,x')=\langle \mathrm{in}|\{\phi(x),\phi(x')\}|\mathrm{in}\rangle. \tag{2.26}$$

The Fourier transform $G_1(k)$ is related to that of the expectation value of the commutator of two fields G by the zero temperature Kubo-Martin-Schwinger (KMS) formula [57]

$$G_1(k)=\mathrm{sgn}(k^0)G(k). \tag{2.27}$$

If we assume canonical equal-time commutation relations, there is a simple relationship between G and G_{ret},

$$G_{\mathrm{ret}}(x,x')=(-i)G(x,x')\,\theta(x^0-x^{0'}), \tag{2.28}$$

which translates into

$$G(k)=2\,\mathrm{Im}G_{\mathrm{ret}}(k). \tag{2.29}$$

The retarded propagator is simply the inverse of Eq. (2.17) with causal boundary conditions. It has a pole at $-k^2=m^2$ and a branch cut from $-k^2=4M^2$ on. Therefore,

$$G_{\mathrm{ret}}(k)=\frac{B}{[(k+i\varepsilon)^2+m^2]}+\left(\frac{g^2}{2}\right)\int_{4M^2}^{\infty}ds\,\frac{h(s)|G_{\mathrm{ret}}(s)|^2}{[(k+i\varepsilon)^2+s]}, \tag{2.30}$$

where B is the residue at the pole, and $G_{\mathrm{ret}}(s)$ stands for the propagator evaluated on the $-k^2=s$ shell. We conclude

$$G(k)=2\pi\left\{B\,\delta(k^2+m^2)+\left(\frac{g^2}{2}\right)h(-k^2)|G_{\mathrm{ret}}(k)|^2\times\theta(-k^2-4M^2)\right\}\mathrm{sgn}(k^0), \tag{2.31}$$

$$G_1(k)=2\pi\left\{B\,\delta(k^2+m^2)+\left(\frac{g^2}{2}\right)h(-k^2)|G_{\mathrm{ret}}(k)|^2\times\theta(-k^2-4M^2)\right\}. \tag{2.32}$$

But this result is inconsistent with Eq. (2.17): since modes above threshold are damped, they could not possibly sustain a time translation invariant autocorrelation such as Eq. (2.32). However, addition of a stochastic source $g\xi$ to the right-hand side of Eq. (2.17) removes the contradiction: we can identify the first term in Eq. (2.32) as the ''natural'' light quantum fluctuations, and the second as the fluctuations induced by the action of the external source. If the statistics of this new term is chosen correctly, the stochastic source will feed onto the light field precisely the amount of fluctuation necessary to keep the noise level, as required by the fluctuation-dissipation relation. We shall show now how to set up a theory whereby this desirable result is built in.

C. Above threshold: Fluctuations and dissipation

As we have just remarked, the expectation value alone does not capture the full effect of the heavy fields on the light ones; to do so consistently demands that an extra stochastic source term $\xi(x)$ should be present in Eq. (2.17), as

$$(-\Box+m^2)\phi(x)+g^2\int d^4x'D(x,x')\phi(x')=g\xi(x), \tag{2.33}$$

which takes into account the fluctuations of the heavy fields: namely,

$$\xi(x)\sim(\tfrac{1}{2})[\Phi^2-\langle\Phi^2\rangle_\phi](x). \tag{2.34}$$

The external source vanishes on average, but has a rms value

$$\langle\xi(x)\xi(x')\rangle\equiv N(x,x')=(\tfrac{1}{8})[\langle\{\Phi^2(x),\Phi^2(x')\}\rangle_0 -2\langle\Phi^2\rangle_0^2]. \tag{2.35}$$

Or, explicitly,

$$N(x,x')=\int\frac{d^4k}{(2\pi)^4}e^{ik(x-x')}\left(\frac{\pi}{2}\right)h(-k^2). \tag{2.36}$$

Following Feynman and Vernon [33], as we have done in related problems [17], we can show that a Langevin-type equation (2.33) is properly derived from the CTP effective action, rather than the more familiar deterministic equation (2.17).

To this end, let us first replace the field variables $\phi^{+,-}$ by the average and difference variables

$$[\phi]=\phi^+-\phi^-,\quad \{\phi\}=\phi^++\phi^-. \tag{2.37}$$

With the identity

$$S_{\rm eff}^{\rm CTP}(\{\phi\},[\phi]=0)\equiv0, \tag{2.38}$$

it follows that the equation of motion is

$$\frac{\delta S_{\rm eff}^{\rm CTP}}{\delta[\phi]}(\{\phi\}=\phi,[\phi]=0)=0. \tag{2.39}$$

The quadratic terms in the effective action, Eq. (2.13), may be written as

$$\frac{g^2}{2}\int d^4xd^4x'\{-[\phi(x)]D(x,x')\{\phi(x')\} +i[\phi(x)]N(x,x')[\phi(x')]\}. \tag{2.40}$$

It may seem that the noise kernel N does not contribute to the equations of motion. However, by virtue of the identity

$$\exp\left\{-\frac{g^2}{2}\int d^4xd^4x'[\phi(x)]N(x,x')[\phi(x')]\right\} \equiv\int D\xi P[\xi]\exp\left\{-ig\int d^4x\xi(x)[\phi(x)]\right\} \tag{2.41}$$

for some probability density P, with

$$\langle\xi(x)\xi(x')\rangle\equiv N(x,x') \tag{2.42}$$

we may substitute the quadratic term in the light effective action by coupling the field to a stochastic source whose autocorrelation is given by the noise kernel N. The fact that both dissipation and noise kernels can be expressed in terms of the same function h in this example is the origin of the fluctuation-dissipation theorem.

We should stress that the field ϕ in Eq. (2.33) does not allow the same physical interpretation as ϕ in Eq. (2.17). The latter is the expectation value of the Heisenberg light field operator, while the former includes also the fluctuations around the mean field, we may call it the ‘‘dressed’’ light field. The dressed light field is *prima facie* a quantum field, and the stochastic driving force from the environment is also quantum in nature. Since light and heavy modes are dynamically entangled, interference effects abound, but are extremely hard to display for observations carried out at low energy. A heavy sector serving as an environment to the light sector can decohere it, and induce a quantum to classical transition. After decoherence the open system variables obey an effectively classical equation of motion, but driven by stochastic source terms, such as in a Langevin equation. Indeed, the amount of noise in this open system is a direct measure of the degree of entanglement with the unobserved sector. (This delicate borderline between classical and quantum physics is a general feature of quantum noisy systems [54].) The issue of decoherence is an important one lying at the foundation of quantum mechanics and has been studied by many people in recent years [55,56]. We have also discussed this issue for model field theories [14,17]. By the same reasoning, we can assume safely for our considerations here that the light field has been decohered and behaves like a classical stochastic field.

To drive this point deeper, observe that the calculation above relied on ordinary quantum-mechanical rules, such as the KMS theorem, so it would be correct to consider the fluctuations in the dressed light field as quantum in nature. But this is the point of view of an observer who is aware of the existence of the heavy field. Since the high momentum modes are entangled with the heavy field and become correlated through particle creation, only such an observer could effect interference between modes above the threshold. For observers who cannot operate on the heavy field, the interference of these modes is not observable, and their answer could be that these are classical fluctuations. As in many other situations in quantum physics, questions such as this can have different answers depending on the specific observational context.

Our perturbative treatment so far suggests that noise and dissipation only turn on above threshold. We now wish to show that they are indeed present below the threshold, albeit exponentially suppressed.

III. STOCHASTIC BEHAVIOR BELOW THE THRESHOLD

The analysis of the previous section highlighted the main elements of the light field theory, namely, the dressing of the light field by the heavy quantum fluctuations, the onset of dissipative processes, and the decoherence and noise generation thereof. However, this analysis is based on a hypothetical light field with momenta above the heavy threshold, a regime where light effective theory would be of theoretical rather than practical (observational) interest. What makes the ongoing discussion relevant is that the same phenomena are actually occurring at the light scales, albeit strongly suppressed. They are not revealed in a perturbative theory. In this section, we shall describe some of the most conspicuous manifestations of noise and dissipation in the infrared regime.

To simplify the calculation, we shall assume the existence of a seed classical light background field, in the form of a monochromatic plane wave. It may arise through the action of some external agent, or as an outcome of the previous history of the system. We also assume that the interaction between the light background and the heavy quantum field is adiabatically switched off in the past, so that there is a well defined in vacuum for the heavy fields, and that no substantial particle creation occurs prior to a given time (conventionally, taken as $t=0$), so that at this time the heavy field is still in the in vacuum state. Our aim is to compute the amplification of the heavy quantum fluctuations due to parametric resonance, and the light fluctuations arising from the back reaction thereof.

This situation actually arises in many cases of interest, such as the background gravitational field in the early Universe interacting with quantum matter fields. In this case, detailed studies show that indeed the gravitational field is prone to decohere earlier than the matter fields, so the classical-quantum distinction is unambiguous [29]. In the case of multiparticle production in heavy ion collisions, for example, particle currents are applied externally, while the gauge fields take the role of the ''irrelevant'' heavy fields [58] which are coarse grained. If we study the generation of a cosmic background magnetic field, on the other hand, a seed magnetic field comes from the past (for example, through amplification of vacuum fluctuations during inflation) and is further amplified through interaction with charged particles in the radiation era [59]. To give yet another example, we could model a laser as a light field (the electromagnetic field in a resonant cavity) interacting with heavy fields (the creation operators for the gas in the cavity, in different possible internal states). Then, the seed is the externally provided pumping [60].

While the situation we shall discuss is at best a toy model for these relevant systems, it will allow us to show in detail how the back reaction of the heavy field on the light one leads to the onset of a distinct, inhomogeneous, stochastic structure, whose amplitude, growth, and coarsening rates depend exponentially on the ratio of the light to the heavy scales. Thus, the light theory will have a stochastic character, even for observers confined to infrared phenomenology.

A. Nonperturbative equations of motion

Let us return to the fundamental definitions

$$S_{\rm eff}^{\rm CTP}=S_l[\phi^+]-S_l[\phi^-]+\delta S^{\rm CTP}[\phi^+,\phi^-], \qquad (3.1)$$

$$\delta S^{\rm CTP}[\phi^+,\phi^-]=-i\ln\int D\Phi^+D\Phi^- \quad e^{i\{S_H[\Phi^+]-S_H[\Phi^+]+S_{lH}[\phi^+,\Phi^+]-S_{lH}[\phi^-,\Phi^-]\}}. \qquad (3.2)$$

We shall now attempt a nonperturbative evaluation of this path integral. Using the sum and difference field variables

$$[\phi]=\phi^+-\phi^-, \quad \{\phi\}=\phi^++\phi^-, \qquad (3.3)$$

we can extract the ''deterministic'' part as

$$\delta S^{\rm CTP}[\phi^+,\phi^-]=\left(\frac{-g}{2}\right)\int d^4x\langle\Phi^2\rangle_{\{\phi\}}(x)[\phi] + \Delta S(\{\phi\},[\phi]), \qquad (3.4)$$

where, as defined in the previous section, the subscript $\{\phi\}$ denotes averaging with respect to the $\{\phi\}$ field. We perform a functional Fourier transform

$$\exp\{i\Delta S[\{\phi\},[\phi]]\}=\int D\xi e^{ig\int\xi[\phi]}P[\xi,\{\phi\}]. \qquad (3.5)$$

Observe that

$$\langle\xi(x)\rangle=0, \qquad (3.6)$$

$$\langle\xi(x)\xi(x')\rangle\equiv N(x,x')=(\tfrac{1}{8})[\langle\{\Phi^2(x),\Phi^2(x')\}\rangle_{\{\phi\}} - 2\langle\Phi^2\rangle_{\{\phi\}}(x)\langle\Phi^2\rangle_{\{\phi\}}(x')], \qquad (3.7)$$

where

$$\langle f\rangle\equiv\int D\xi fP[\xi,\{\phi\}]. \qquad (3.8)$$

This is to be contrasted with the result in the perturbative treatment equation (2.36).

The functional $P[\xi,\{\phi\}]$ must be real (as follows from $\Delta S[\{\phi\},-[\phi]]=-\Delta S[\{\phi\},[\phi]]^*$) and it is non-negative to one-loop approximation. We may think of it as a functional Wigner transform of the effective action [61], and thereby as a probability density ''for all practical purposes.'' Observe that P will not be Gaussian in general. In our concrete application, nevertheless, the effective action is one-loop exact, so the identification of P as a Gaussian probability density poses no difficulty.

We conclude that the correct (in the sense of accounting for both mean field and the fluctuations around it), nonperturbative effective equation of motion for the light fields reads

$$(-\Box+m^2)\phi(x)+\left(\frac{g}{2}\right)[\langle\Phi^2\rangle_\phi-\langle\Phi^2\rangle_0](x)=g\xi(x). \tag{3.9}$$

Our goal is to show that any plane wave light field background will be followed by a slowly varying echo. Since the light mass is nonvanishing there is no loss of generality for our purpose if we assume the light field is homogeneous in space and harmonic in time: i.e.,

$$\phi(t)=\phi_0\sin 2\omega t. \tag{3.10}$$

The condition that the light four-momentum lies below the branch point at $-k^2=4M^2$ translates into $\omega\leqslant M$.

Of course, the field configuration equation (3.10) is a solution of the light equations of motion only insofar as dissipative effects can be neglected, which, as we shall show, is not the case close to threshold. Therefore, to extend our argument near threshold, we must assume that the background light field is sustained by some external agent. (This way of reasoning is not unusual, e.g., it appears in Kramer's calculation of decay rates, where it is assumed that ensemble distribution is kept stationary by such an external agent.) The point is that, even below threshold, external agent will have to do work to sustain the light field (dissipation) and that part of the dissipated energy will be returned to the field as a stochastic echo (fluctuations). Equally important, these phenomena do not appear all of a sudden as threshold is crossed, but build up gradually as we approach the critical scale from below.

To compute the nonperturbative noise kernel, we decompose the quantum heavy fields propagating on the light background field into normal modes. The amplitudes of each normal mode are complex, with

$$\Phi_{-\vec{k}}=\Phi^\dagger_{\vec{k}}. \tag{3.11}$$

They obey the wave equation

$$\partial_t^2\Phi_{\vec{k}}+\Omega_k^2\Phi_{\vec{k}}=0, \tag{3.12}$$

where

$$\Omega_k^2=\vec{k}^2+M^2+g\phi(t) \tag{3.13}$$

is the natural frequency of the $\vec{k}$th mode. Here, we shall disregard the possibility of Ω becoming imaginary through a large negative light field, i.e., we assume $g\phi_0\leqslant M^2$. Beyond this, we shall not make other assumptions on the strength of the interaction. The Eq. (3.12) is the exact Heisenberg equation of motion for the heavy modes, and the resulting light effective theory will be nonperturbative.

The strength of interaction between the light and heavy fields is measured by

$$\kappa_k=\frac{1}{2\Omega_k}\frac{d\Omega_k}{dt}=\frac{1}{4\Omega_k^2}\frac{d\Omega_k^2}{dt}. \tag{3.14}$$

We assume the heavy field is in the vacuum state at some initial time $t=0$. Since it is a free field, Wick's theorem holds, and our problem is to relate the field at abitrary times to the initial creation and destruction operators. Of course, without knowing the explicit evolution law for the light field, we cannot get the exact form, but have to find a suitable approximation scheme. The general relationship we seek is

$$\Phi_k(t)=f_k(t)a_k(0)+f_k^*(t)a^\dagger_{-k}(0), \tag{3.15}$$

where f_k is the positive frequency mode associated to the in particle model [14]. It can be decomposed into instantaneous positive and negative frequency parts as

$$f_k(t)=\frac{1}{\sqrt{2\Omega_k}}[\alpha_k(t)+\beta_k(t)]. \tag{3.16}$$

B. Stochastic features near threshold

Let us first consider the near threshold $(\omega\sim M)$, weak field regime, where Ω_k is essentially constant, and

$$\kappa_k\sim 2c_k\cos 2\omega t, \tag{3.17}$$

where

$$c_k\sim\frac{\omega g\phi_0}{4\Omega_k^2}. \tag{3.18}$$

The Bogoliubov coefficients α_k,β_k are calculated in Appendix A to be

$$\alpha_k(t)=\left(\frac{c_k}{2\gamma_k}\right)e^{-i\omega t}e^{\gamma_k t}(1+e^{i\delta_k}e^{-2\gamma_k t})e^{-i\delta_k/2}, \tag{3.19}$$

$$\beta_k(t)=\left(\frac{c_k}{2\gamma_k}\right)e^{i\omega t}e^{\gamma_k t}(1-e^{-2\gamma_k t}), \tag{3.20}$$

where

$$e^{i\delta_k/2}=\left(\frac{\gamma_k}{c_k}\right)+i\left(\frac{\Omega_k-\omega}{c_k}\right), \tag{3.21}$$

$$\gamma_k=\sqrt{c_k^2-(\Omega_k-\omega)^2}. \tag{3.22}$$

The nonperturbative character of these expressions should be clear; the approximations made involved keeping only the dynamically most relevant interactions, but we have consistently retained all powers of the coupling constant. In other words, in this theory the path integral defining the light effective action is one-loop exact, and our results, which amount to a calculation of this path integral, are correspondingly nonperturbative.

We observe that Eqs. (3.19) and (3.20) are formally valid in the whole range of frequencies. However, outside the parametric resonance regime, we have

$$\gamma_k\sim\pm i(\Omega_k-\omega) \tag{3.23}$$

and both α_k and β_k describe positive frequency oscillations above the heavy threshold. We are interested here in the opposite case, where three features stand out, namely, (1) the generation of the negative frequency components described by β_k, which is the physical basis for vacuum particle creation; (2) the exponential amplification due to ongoing particle creation, and (3) the phase locking of a whole range of wavelengths at the resonance frequency ω. As we shall now see, phase locking allows the generation of a low frequency, inhomogeneous stochastic field, which can be detected at the scale of the light sector. This is the main physical indication of the new features of dissipation and fluctuation below threshold we want to highlight in the context of effective field theory.

In order to find the noise kernel, let us decompose the Heisenberg operator Φ^2 into a c number, a diagonal (D) and a nondiagonal (ND) (in the particle number basis) part

$$\Phi^2=\langle\Phi^2\rangle_\phi+\Phi^2_{\rm D}+\Phi^2_{\rm ND}, \tag{3.24}$$

where the (D) and (ND) components are

$$\Phi^2_{\rm D}=\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\;e^{i(k+k')x}\{f_k(t)f^*_{k'}(t)a^\dagger_{-k'}a_k+f^*_k(t)f_{k'}(t)a^\dagger_{-k}a_{k'}\}, \tag{3.25}$$

$$\Phi^2_{\rm ND}=\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\;e^{i(k+k')x}\{f_k(t)f_{k'}(t)a_ka_{k'}+f^*_k(t)f^*_{k'}(t)a^\dagger_{-k}a^\dagger_{-k'}\}. \tag{3.26}$$

Observe that

$$\langle\Phi^2_{\rm D}\rangle_\phi=\langle\Phi^2_{\rm ND}\rangle_\phi=\langle\Phi^2_{\rm D}\Phi^2_{\rm ND}\rangle_\phi=\langle\Phi^2_{\rm D}\Phi^2_{\rm D}\rangle_\phi\equiv 0. \tag{3.27}$$

Therefore,

$$\begin{aligned}N(x,x')&=(\tfrac{1}{8})\langle\{\Phi^2_{\rm ND}(x),\Phi^2_{\rm ND}(x')\}\rangle_{\{\phi\}}\\&=(\tfrac{1}{2})\int\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}e^{i(k+k')(x-x')}\\&\quad\times{\rm Re}\{f_k(t)f_{k'}(t)f^*_k(t')f^*_{k'}(t')\}.\end{aligned}$$

If no particle creation occurred, the noise kernel would contain frequencies above threshold only. However, in the presence of frequency locking and a negative frequency part of the mode functions f, the noise kernel also contains a steady component

$$N_S(x,x')=\left(\frac{1}{2}\right)\int'\frac{d^3k}{(2\pi)^3\Omega_k}\frac{d^3k'}{(2\pi)^3\Omega_{k'}}\times e^{i(k+k')(x-x')}\Theta_{kk'}(t,t'), \tag{3.28}$$

where the integral is restricted to those modes where γ_k is real, and

$$\Theta_{kk'}(t,t')={\rm Re}\{[\alpha_k(t)\beta_{k'}(t)+\alpha_{k'}(t)\beta_k(t)][\alpha_k(t')\beta_{k'}(t')+\alpha_{k'}(t')\beta_k(t')]^*\}. \tag{3.29}$$

It is important to notice that Θ is slowly varying not only with respect to the heavy frequencies Ω, but also with respect to the locking frequency ω. Of course, we do not observe the noise kernel directly, but only through its effect on the light field. However, since the steady part of the stochastic source is slowly varying in space and time, to first approximation it induces a stochastic light field ϕ_S which is simply proportional to it

$$\phi_S\sim\left(\frac{g}{m^2}\right)\xi_S;\quad\langle\phi_S\phi_S\rangle\sim\left(\frac{g}{m^2}\right)^2N_S. \tag{3.30}$$

This is the echo we sought for. One can deduce the noise and its autocorrelation in this way.

It is interesting to show the actual form of the noise kernel in the opposite limits of very long and very short times, as we now do.

1. Long time limit

At long times, the correlation function is dominated by the very long wavelength modes. We may thus approximate

$$c_k\sim c_0\sim\frac{\omega g\phi_0}{4M^2}, \tag{3.31}$$

$$\gamma_k\sim\gamma_0-\frac{\sigma^2k^2}{2} \tag{3.32}$$

in the exponents, where

$$\gamma_0=\sqrt{c_0^2-(M-\omega)^2}, \tag{3.33}$$

$$\sigma^2=\frac{1}{\gamma_0}\left[1-\frac{\omega}{M}+\frac{2\omega^2g^2\phi_0^2}{M^6}\right], \tag{3.34}$$

neglecting k^2 elsewhere. Moreover, for $\gamma_0t\gg1$, we neglect the decaying modes, and extend the integral to all k space. The result is

$$\alpha_k(t)\sim\left(\frac{c_0}{2\gamma_0}\right)e^{-i\omega t}e^{\gamma_kt}e^{-i\delta_0/2}, \tag{3.35}$$

$$\beta_k(t)=\left(\frac{c_0}{2\gamma_0}\right)e^{i\omega t}e^{\gamma_kt}, \tag{3.36}$$

$$\alpha_k(t)\beta_{k'}(t)+\alpha_{k'}(t)\beta_k(t)\sim2\left(\frac{c_0}{2\gamma_0}\right)^2e^{(\gamma_k+\gamma_{k'})t}e^{-i\delta_0/2}, \tag{3.37}$$

$$\Theta_{kk'}(t,t')=4\left(\frac{c_0}{2\gamma_0}\right)^4e^{(\gamma_k+\gamma_{k'})(t+t')}, \tag{3.38}$$

$$N_S(x,x')=\left(\frac{1}{8(2\pi)^6M^2}\right)\left(\frac{c_0}{\gamma_0}\right)^4e^{2\gamma_0(t+t')}\times\left[\int d^3ke^{ik(x-x')}e^{-\sigma^2k^2(t+t')/2}\right]^2. \tag{3.39}$$

Performing the Gaussian integrals,

$$N_S(x,x')\sim\left(\frac{\omega g\phi_0}{4M^2}\right)^4\frac{e^{2\gamma_0(t+t')}}{\gamma_0^4M^2}\,\frac{e^{-(x-x')^2/\sigma^2(t+t')}}{(2\pi\sigma^2(t+t'))^3}. \tag{3.40}$$

We observe that a large scale, inhomogeneous stochastic structure in the light sector emerges from the back reaction of the created pairs of the heavy field. This structure takes the form of domains where the field is aligned, and the characteristic size of these domains grows as the square root of time.

2. Short time limit

In the opposite, very short time limit, we find

$$\alpha_k\approx e^{-i\omega t}, \tag{3.41}$$

$$\beta_k(t)=c_k t e^{i\omega t}, \tag{3.42}$$

$$\alpha_k(t)\beta_{k'}(t)+\alpha_{k'}(t)\beta_k(t)\sim(c_k+c_{k'})t, \tag{3.43}$$

$$\Theta_{kk'}(t,t')=(c_k+c_{k'})^2tt', \tag{3.44}$$

$$N_S(x,x')=\left(\frac{tt'}{2}\right)\int{}'\frac{d^3k}{(2\pi)^3\Omega_k}\,\frac{d^3k'}{(2\pi)^3\Omega_{k'}}\;e^{i(k+k')(x-x')}\times(c_k+c_{k'})^2. \tag{3.45}$$

Approximately,

$$N_S(x,x')\sim\left(\frac{c_0^2k_0^6}{2\pi^4\Omega_0^2}\right)tt'f^2(k_0r), \tag{3.46}$$

where $r=|\vec{x}-\vec{x}'|$,

$$f(u)=\left(\frac{1}{u^3}\right)[u\cos u-\sin u], \tag{3.47}$$

and k_0 marks the boundary of the resonant zone,

$$k_0\sim\sqrt{(\omega+c_0)^2-M^2}. \tag{3.48}$$

As before, we should stress that the scale of the stochastic echo is much lower than threshold. Even in the $\omega\to M$ limit, we find

$$k_0\sim\omega\sqrt{\frac{g\phi_0}{2M^2}}\ll\omega. \tag{3.49}$$

C. Stochastic behavior far below threshold

Let us now consider the physically most relevant case, when the frequency of the light background wave is far below the heavy threshold. As before, we assume $\phi(t)=\phi_0\sin 2\omega t$, so that

$$\Omega_k\sim\Omega_{k0}+\delta\Omega_k, \tag{3.50}$$

$$\delta\Omega_k\sim\frac{g\phi_0}{2\Omega_{k0}}\sin 2\omega t, \tag{3.51}$$

$$\kappa_k=\frac{1}{2\Omega_k}\,\frac{d\Omega_k}{dt}\sim 2c_k\cos 2\omega t, \tag{3.52}$$

where

$$c_k=\frac{g\phi_0}{4\Omega_{k0}^2}\omega. \tag{3.53}$$

We are interested in the case where $c_k\ll\omega$, and we assume

$$\frac{\Omega_{k0}}{\omega}=(2N+1)(1+\delta), \tag{3.54}$$

with $N\gg 1\gg\delta$.

As we show in Appendix B, the Bogoliubov coefficients are given by

$$\alpha_k(t)=\left(\frac{C_k}{2\Gamma_k}\right)e^{\Gamma_k t}(1+e^{i\Delta_k}e^{-2\Gamma_k t})e^{-i\Delta_k/2}e^{-i\Theta_k}, \tag{3.55}$$

$$\beta_k(t)=\left(\frac{C_k}{2\Gamma_k}\right)e^{\Gamma_k t}(1-e^{-2\Gamma_k t})e^{i\Theta_k}, \tag{3.56}$$

where

$$e^{i\Delta_k/2}=\left(\frac{1}{C_k}\right)\{\Gamma_k+i[\Omega_{k0}-(2N+1)\omega]\}, \tag{3.57}$$

$$\Gamma_k=\sqrt{C_k^2-[\Omega_{k0}-(2N+1)\omega]^2}, \tag{3.58}$$

$$\Theta_k=(2N+1)\omega t-\left(\frac{\Omega_k c_k}{\omega^2}\right)\cos 2\omega t, \tag{3.59}$$

and

$$C_k=c_k\mathrm{J}_{2N}\left(\frac{2\Omega_{k0}c_k}{\omega^2}\right), \tag{3.60}$$

where J represents the usual Bessel function. When N is large, the asymptotics of Bessel functions yield [62]

$$C_k\sim\left(\frac{c_k}{\sqrt{\pi N\tanh a}}\right)e^{-2N(a-\tanh a)}, \tag{3.61}$$

where $\cosh a=\omega/2c_k(1+\delta)$, or, in short,

$$a\sim\ln\left(\frac{4\Omega_{k0}^2}{g\phi_0}\right). \tag{3.62}$$

As expected, both the amplitude and the growth rate of the stochastic ‘‘echo’’ are exponentially suppressed. In terms of the analysis of the previous subsection, this case always falls in the ‘‘short time’’ limit. The amplitude and growth rate, as well as the inverse size, of a stochastic domain shall be given by C_0. At truly low scales, the effect is extremely feeble, but it builds up exponentially as we reach for the heavy threshold. Since in a realistic situation this effect may be masked by self-interactions, this exponential scale dependence may be essential to its detectability, as one carries out measurements at successively higher energies in this below-threshold region.

The exponential suppression of particle creation and its back reaction below the threshold brings to mind the analogy with quantum tunneling phenomena, which also depend exponentially on the height of the potential barrier.[2] In both cases, though, these quantitatively small effects become important because of their qualitative impact on the physics of the system, and because no other perturbative effects are there to mask them. This analogy also shows that an exponentially small efect is not necessarily nonobservable. The analogy to tunneling is also striking because tunneling dynamics is extremely sensitive to dissipation [63] and, therefore, to the kind of phenomena we are discussing.

Even if the phenomena we have described would not appear in this same form in nature, for example, because of the absence of an external agent to sustain the light background field against dissipation, a second conclusion from our work is equally important, namely, that the breakdown of the light effective theory is not a discrete event occurring at threshold energies, but rather a progressive event unfolding as we reach the threshold from below, with the relative strength of the random to the deterministic parts providing a quantitative measure of the range of applicability of effective theory in any given context.

D. Dissipation below threshold

As discussed in the Introduction, a noisy theory should also be dissipative. It is interesting then to conclude our treatment of fluctuations with a brief account of dissipative phenomena at low energies.

Dissipation is associated with the nonperturbative deterministic part of the equation of motion, Eq. (3.9), namely,

$$\left(\frac{g}{2}\right)[\langle\Phi^2\rangle_\phi-\langle\Phi^2\rangle_0](x). \tag{3.63}$$

It is straightforward to show that

$$\langle\Phi^2\rangle_\phi=\int\frac{d^3k}{(2\pi)^3 2\Omega_k}\{1+2|\beta_k|^2+2\mathrm{Re}[\alpha_k\beta_k^*]\}. \tag{3.64}$$

Neglecting the dependence of Ω_k on the light field, the vacuum subtraction amounts to deleting the first term within brackets. The second term induces a deterministic, homogeneous shift in the low frequency light field. However, this effect is not associated with dissipation, being a reversible vacuum polarization effect much alike the Casimir energy between conducting plates [64].

It is the third term which depicts the truly dissipative effects. At short times it amounts to a viscous force

$$f=\left(\frac{g}{2}\right)\int'\frac{d^3k}{(2\pi)^3\Omega_k}\{c_k t\cos\omega t\} \tag{3.65}$$

[cf. Eqs. (3.41) and (3.42)]. The integral is restricted to those modes where particle creation is effective. This force dissipates energy from the oscillating light field, which must be provided by the external agency sustaining the plane wave background. The energy dissipated per unit volume is

$$\delta\varepsilon=\int dt f\dot\phi\sim\left(\frac{g}{2}\right)\int'\frac{d^3k}{(2\pi)^3\Omega_k}\left\{\frac{\omega c_k\phi_0 t^2}{2}\right\}$$
$$\equiv\int\frac{d^3k}{(2\pi)^3}\Omega_k|\beta_k|^2 \tag{3.66}$$

[cf. Eqs. (3.10), (3.18), and (3.42)]. This establishes the link between dissipation and particle creation, and is essentially the same result as obtained earlier via the perturbative approach (e.g., [43]).

A fraction of the dissipated energy is returned to the system, degraded into stochastic fluctuations. The stochastic source produces a total amount of work per unit volume

$$\delta W\sim g\int dt\langle\xi\dot\phi_S\rangle\sim\left(\frac{g}{m}\right)^2\int dt\frac{\partial}{\partial t'}N_S(t,t')|_{t'\to t}$$
$$\sim\left(\frac{g^2c_0^2k_0^6}{m^2\Omega_0^2}\right)t^2 \tag{3.67}$$

[cf. Eq. (3.46)].

Under equilibrium conditions, the sum total of the dissipated energy equals the total work done by the stochastic force integrated over time. This is a manifestation of a nonlinear fluctuation-dissipation relation. A precise statement of this involves the simultaneous consideration of several light modes, a task perhaps for future investigations.

IV. DISCUSSIONS

In this paper, we have presented a new way of looking at effective field theories, bringing forth their intrinsically dissipative and stochastic aspects. We have shown that dissipation and noise are generic features of such theories, both below and above the energy threshold of the heavy mass which defines their limit of applicability. As the threshold is crossed, the character of the light theory does not change discontinuously, as commonly believed, but is a continuous extension of what is already present below the heavy scales. The stochastic features of the light theory (including the build up of randomness and the breakdown of unitarity), though exceedingly small, will manifest themselves at an exponentially increasing rate as the energy is raised.

In the Introduction we have stressed the relevance of the observational context in the definition of an open system, and in interpreting the physical meaning of what is measured (e.g., appearance of dissipation and fluctuations in an open system, but absence in a closed system) in the restricted range of validity of the effective theory. In the same vein we understand a light field as a representation of the full quantum field observed at low energy. Standard texts tell us that this physical field is obtained from the bare fields of the theory through the renormalization process. However, it is instructional to reexamine the meaning of renormalization in an effective field theory from the open-system viewpoint. Technically, renormalization means that the effects of certain quantum degrees of freedom are added to the bare quantities, and one regards these renormalized quantities as the actual physically measurable ones (e.g., the mean energy in the Maxwell field is added to the bare electron mass to make up its physical mass). In the open-system viewpoint, which is

[2]We thank Diego Mazzitelli for this observation.

closer to observation than the formally complete yet unrealistic closed-system description (of all the constituents at all energies), renormalization is a coarse-graining operation: certain ''irrelevant'' modes (in the above example, the virtual photons surrounding an electron), considered as the environment, are ''slaved'' (for definition see [13]) to the ''relevant'' modes of the particle, which constitute the (open) system, thus enabling one to compute their mean effect on the relevant physics and come up with an effective theory for the (open) system. Because this is an essentially statistical operation, it carries with it the well-known statistical consequences: first, there is a gap between the mean value of a system mode and the actual value which includes the back reaction of the ''irrelevant'' modes, and for this difference the system will be subject to a random source from the environment. Second, the approximations on which the slaving procedure is based (for example, to compute the quantum averages of the environment variables it is often necessary to ignore or to downplay the back reaction of these modes on the system variables), lose accuracy as the influence of the ''irrelevant'' sector becomes large, as is the case when the fluctuations become significant, their coupling becomes strong, or, generally, in the long-time limit.

If one intends to have the light field represent the physical field, in a strict sense, the setup of the effective field theory should include a detailed account of the observational context, or at least of the renormalization procedure involved. Presumably, there would be transformation rules to translate the results from different renormalization prescriptions, and these could eventually take the form of renormalization group equations [65,66]. However, in the presence of a sizable gap between a light and a heavy scale, as in the case studied here, sensible prescriptions will label most of the heavy field modes as environment, and most of the light modes as system. Thus, we have adopted in the above the somewhat simplistic view of treating renormalization as the dressing of the light fields by the heavy quantum fluctuations. One shortcoming of this assumption is that, for example, if the light fields self-interact, this prescription will not eliminate all infinities from the theory.

In the region where the system and environment get progressively entangled, the system dynamics will acquire a stochastic component, and become dissipative. An arrow of time will also emerge in the effective theory. It is of interest to develop a renormalization group theory for dissipative systems. Some of the traditional concepts would need a newer and broader interpretation. The breakdown of an effective theory in the threshold region is theoretically related to the crossover behavior in critical phenomena studied in depth by O'Connor and Stephens [65]. Their observation on how the relevant degrees of freedom of a physical theory are dependent on the scales at which the theory is probed will be useful for the construction of open systems which are sensitive to the energy and observation scales. These are important questions at the foundation of statistical mechanics and field theory which we hope to probe into.

In the low energy regime where effective light theory works, the stochastic effects we have described are very weak and may be unobservable. However, for the effective theory concept to be fruitful in a broader range, it is highly desirable to find ways to extrapolate the low energy results to the threshold region, where contact with the fundamental theory can be made. The conventional approach is inadequate for this purpose because, as our analysis shows, dissipative and stochastic effects will assert themselves even below that scale.

The formalism we have presented is an improvement on the conventional one, not that it provides a better answer to the same question (the usual formalism essentially asks for the dynamics of the mean light field interacting with the quantized heavy field), but because it enables us to ask a different and deeper question, namely, the dynamics of the light mean field and the fluctuations around that mean. While keeping the light field as external, not only the mean effect of the heavy field, but also the fluctuations around the mean are computed. This allows one to keep track of the fluctuations of the light field induced by its interaction with the heavy field in full consistency, as required by and embodied in the fluctuation-dissipation relation.

The phenomena we have discussed, in particular the random features of the back reaction of the heavy field on the light ones, are, of course, a consequence of quantum theory and are, in principle, retrievable in other formulations. The stochastic method we used has the advantage that it highlights and systemizes these effects, which would otherwise have been much harder to decipher.

The ideas presented in this paper can lead to several directions of further development. At a basic level, there is the question about the fundamental nature of any realistic physical system described by quantum field theories. It is the view of the authors that in nature there is no irreducibly ''fundamental'' theories in the absolute sense, just as the existence of an absolute closed system is more in the hypothetical rather than in the physical realm. (Even for the Universe, it is a closed system only in the ontological rather than the physical sense.) All realistic theories describing open systems are to varying degrees noisy and dissipative. They are depicted by stochastic rather than strictly deterministic equations. Only when noise and dissipation are small can one describe in approximate terms the system by the usual tenets (e.g., effective action) of unitary field theory. The criterion of validity of an effective field theory is derived in this paper. In terms of structures, we also think that there are no irreducibly elemental theories or constituents in the absolute sense. The presence of noise, albeit in small amounts, points to the presence of a deeper layer of structure. (To give a historical example, Brownian motion marks the boundary between hydrodynamics and many-body theory, as it discloses the graininess of a seemingly continuous fluid.) Indeed, this point of view can be used to guide the probing into possible deeper and unknown layers of structures from a better-known, lower energy domain. The case of gravity, from the better-known, semiclassical regime to the unknown quantum regime, was what motivated us into examining the general properties of quantum open systems and effective theories in the first place [6]. Noise and fluctuations could then in this sense serve as a trace detector which allows us to obtain a glimpse of the deeper structures.

There are also many physical situations where the mechanisms of fluctuation generation and structure growth described in this paper could be put to practical use. A particularly fertile ground is the physics of the early universe, modeled by a theory of massless fields (gravitons, neutrinos, and gauge bosons), interacting with heavy fields such as electrons, quarks, and cold dark matter candidates. The gravitational background will create particles of the heavy fields (while neutrinos and gauge bosons are shielded by conformal invariance), which in turn will react on the light fields, resulting in the generation of primordial gravitational fluctuations [17] and gauge fields. Closer to home, the theory of heavy ion collisions also presents a situation where a color field background interacts with the massive quark fields, resulting in the formation of a quark-gluon plasma, which could be investigated using the framework of this paper [58]. We hope to report on the result of this and related research in later publications.

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APPENDIX A: DERIVATION OF α AND β

Our method can be described as a translation into Hamilton-Jacobi language of the classical averaging method, as found in the textbooks by Landau and Lifshitz [67] and Bogoliubov and Mitropolsky [68] (it is close to the methods used by [47]). We shall borrow some tools of classical mechanics to approach our problem. The mode equation follows from the Hamiltonian

$$H=P_{-k}P_k+\Omega^2\Phi_{-k}\Phi_k, \tag{A1}$$

where Φ_k and Φ_{-k} are independent canonical variables and P_k, P_{-k} their conjugate momenta, respectively. We introduce creation and destruction operators through

$$\Phi_k=\frac{1}{\sqrt{2\Omega}}[a_k+a^*_{-k}], \tag{A2}$$

$$P_k=i\sqrt{\frac{\Omega}{2}}[a^*_k-a_{-k}]. \tag{A3}$$

We adopt the destruction operators as new canonical variables, with conjugated momenta

$$p_{\pm k}=ia^*_{\pm k}. \tag{A4}$$

The Hamiltonian expressed in terms of these new variables is

$$K=-i\Omega[a_kp_k+a_{-k}p_{-k}]-i\kappa[a_ka_{-k}+p_kp_{-k}], \tag{A5}$$

where, as before,

$$\kappa=\frac{1}{2\Omega}\frac{d\Omega}{dt}. \tag{A6}$$

The Bogoliubov transformation linking the destruction and creation operators at time t with those at time $t=0$ is given by

$$a_k(t)=\alpha(t)a_k(0)+\beta^*(t)a^\dagger_{-k}(0), \tag{A7}$$

$$a^\dagger_k(t)=\alpha^*(t)a^\dagger_k(0)+\beta(t)a_{-k}(0), \tag{A8}$$

where the coefficients satisfy the Wronskian condition

$$|\alpha|^2-|\beta|^2=1. \tag{A9}$$

In terms of the canonical variables, the Bogoliubov transformation takes the form

$$a_k(t)=\alpha(t)a_k(0)-i\beta^*(t)p_{-k}(0), \tag{A10}$$

$$p_k(t)=\alpha^*(t)p_k(0)+i\beta(t)a_{-k}(0). \tag{A11}$$

This is a canonical transformation with generating functional

$$S=G(t)[a_kp_k(0)+a_{-k}p_{-k}(0)]+F(t)a_ka_{-k}+E(t)p_k(0)p_{-k}(0), \tag{A12}$$

where (from now on, we shall occasionally omit the k subindices, to simplify the appearance of our formulas)

$$G=\frac{1}{\alpha},\quad E=\frac{i\beta^*}{\alpha},\quad F=\frac{i\beta}{\alpha}. \tag{A13}$$

S satisfies the Hamilton-Jacobi equation

$$-i\Omega\left[a_k\frac{\partial S}{\partial a_k}+a_{-k}\frac{\partial S}{\partial a_{-k}}\right]-i\kappa\left[a_ka_{-k}+\frac{\partial S}{\partial a_k}\frac{\partial S}{\partial a_{-k}}\right]+\frac{\partial S}{\partial t}=0. \tag{A14}$$

Therefore,

$$\frac{dE}{dt}-i\kappa G^2=0, \tag{A15}$$

$$\frac{dG}{dt}-i[\Omega+\kappa F]G=0, \tag{A16}$$

$$\frac{dF}{dt}-2i\Omega F-i\kappa[1+F^2]=0. \tag{A17}$$

Short of an exact solution, the conventional appproach to solving this equation would be to expand in powers of κ. This leads to the usual adiabatic approximation [38,39], since $\kappa\sim O(g)$, which is precisely what we should avoid for the present purpose.

As before, let us assume Ω is essentially constant, and

$$\kappa\sim 2c\cos 2\omega t, \tag{A18}$$

where

$$c \sim \frac{\omega g \phi_0}{4\Omega_k^2}, \tag{A19}$$

with $\omega \ll \Omega_k$. The idea is to retain only the most resonant terms in Eq. (A17); namely, we write

$$\frac{dF}{dt} - 2i\Omega_k F - ic[e^{2i\omega t} + e^{-2i\omega t}F^2] = 0. \tag{A20}$$

This equation allows a solution of the form

$$F = \frac{i}{c} e^{2i\omega t} \frac{\dot{u}}{u}, \tag{A21}$$

where u satisfies the ordinary equation

$$\frac{d^2u}{dt^2} - 2i(\Omega - \omega)\frac{du}{dt} - c^2 u = 0. \tag{A22}$$

The solutions are

$$u_{\pm} \sim e^{\pm \gamma t} e^{i(\Omega - \omega)t}, \tag{A23}$$

where

$$\gamma = \sqrt{c^2 - (\Omega - \omega)^2}. \tag{A24}$$

The case of interest to us is when γ is real.

To find α we integrate Eq. (A16) under the approximation

$$\kappa F \sim c e^{-2i\omega t} F \equiv i \frac{\dot{u}}{u}, \tag{A25}$$

that is, we keep only the slowly varying term. The integration is then trivial, and we get

$$G = \frac{e^{i\Omega t}}{u}, \quad \alpha = u e^{-i\Omega t}. \tag{A26}$$

Given F and α, finding β is a matter of algebra

$$\beta = -i\alpha F \equiv \frac{\dot{u}}{c} e^{i(2\omega - \Omega)t}. \tag{A27}$$

We thus find the boundary conditions $u(0) = 1$, $\dot{u}(0) = 0$. The solution is

$$u = \left(\frac{c}{2\gamma}\right)[e^{-i\delta/2} e^{\gamma t} + e^{i\delta/2} e^{-\gamma t}] e^{i(\Omega - \omega)t}, \tag{A28}$$

leading to Eqs. (3.19)–(3.21) as given in the main text.

APPENDIX B: PARTICLE CREATION FAR BELOW THRESHOLD

When the frequency ω of the normal modes of the light field is far below M, the above analysis is valid up to Eq. (A17), but care must be taken to identify the resonant terms. Let us decompose the frequency Ω into constant and fluctuating parts

$$\Omega = \Omega_0 + \delta\Omega. \tag{B1}$$

Then, from

$$\kappa = \frac{1}{2\Omega}\frac{d\Omega}{dt} \sim 2c\cos 2\omega t, \tag{B2}$$

we get

$$\delta\Omega \sim \left(\frac{2\Omega_0 c}{\omega}\right)\sin 2\omega t. \tag{B3}$$

Let us assume

$$\frac{\Omega_0}{\omega} \equiv (2N+1)(1+\delta), \tag{B4}$$

where N is an integer and $\delta \ll 1$. Then, resonance occurs at the frequency $(2N+1)\omega$.

Instead of Eq. (A21), we now try

$$F = \left(\frac{i(-1)^N e^{2i\Theta}}{C}\right)\left[\frac{\dot{U}}{U}\right], \tag{B5}$$

where

$$\Theta = (2N+1)\omega t - \left(\frac{\Omega c}{\omega^2}\right)\cos 2\omega t. \tag{B6}$$

Expanding the exponential as a Fourier series, and keeping only the resonant term, we find

$$\kappa e^{\pm 2i\Theta} \sim (-1)^N c J_{2N}\left(\frac{2\Omega c}{\omega^2}\right) \equiv (-1)^N C. \tag{B7}$$

The equation for U reads

$$\ddot{U} - 2i[\Omega - (2N+1)\omega]\dot{U} - C^2 U = 0. \tag{B8}$$

From here on, the argument exactly reproduces the previous case, leading to the results reported in the text.

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