Interpolating the stage of exponential expansion in the early universe: Possible alternative with no reheating

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In the standard picture, the inflationary universe is in a supercooled state which ends with a short time, large scale reheating period, after which the universe goes into a radiation-dominated stage. An alternative is proposed here in which the radiation energy density smoothly decreases all during an inflationlike stage and with no discontinuity enters the subsequent radiation-dominated stage. The scale factor is calculated from standard Friedmann cosmology in the presence of both radiation and vacuum energy density. A large class of solutions confirm the above identified regime of nonreheating inflationlike behavior for observationally consistent expansion factors and not too large a drop in the radiation energy density. One dynamical realization of such inflation without reheating is from warm inflation-type scenarios. However the solutions found here are properties of the Einstein equations with generality beyond slow-roll inflation scenarios. The solutions also can be continuously interpolated from the nonreheating-type behavior to the standard supercooled limit of exponential expansion, thus giving all intermediate inflationlike behavior between these two extremes. The temperature of the universe and the expansion factor are calculated for various cases. Implications for baryongenesis are discussed. This nonreheating, inflationlike regime also appears to have some natural features for a universe that is between nearly flat and open. [S0556-2821(97)04606-7]

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I. INTRODUCTION

In the original conception of inflation [1], it was assumed that the universe underwent isentropic expansion during the stage of rapid growth of the scale factor. The entropy required to make the post-inflationary universe consistent with observation was assumed to be generated in a short-time reheating period. However, it is clear that for a range of moderate thermodynamic conditions, the cosmological horizon and flatness problems, which are explained by inflation, require only the kinematic property that the scale factor grows rapidly. More recently [2] it was realized that these kinematic conditions could still arise in the presence of a sustained radiation component during inflation. Specifically, in [2] it was shown that under certain isothermal conditions inflation could still occur. More so, it was shown there that within these limits the initial seeds of density perturbations could be dominantly of thermal instead of quantum origin. A realization of an isothermal or warm inflation scenario, in the context of slow-roll scalar field dynamics for parametrically large dissipation, was shown in [3] to be consistent with observational constraints for the amplitude and expansion factor, without requiring an ultraflat Coleman-Weinberg potential, which in order to form requires the coupling constant to be fine-tuned. Questions about the fundamental origin of large dissipation are still left open.

The warm inflation scenario served as a explicit demonstration of an otherwise true but ambiguous statement, that inflation can occur in the presence of a thermal component. That this is true is self-evident, as, for example, within the context of scalar field theory. Here the requirements for exponential expansion are

$$\rho_v \gg \delta\rho_\phi \tag{1}$$

and

$$\rho_v \gg \rho_{\rm kinetic}, \tag{2}$$

with

$$\rho_{\rm kinetic} \equiv \tfrac{1}{2}\dot{\phi}^2 + \rho_r . \tag{3}$$

Here $\delta\rho_\phi$ is the energy density perturbation, and ρ_v and ρ_r are the background vacuum and radiation energy densities, respectively. Thus energetics alone does not prohibit the relation

$$\rho_v \gg \rho_r \gg \tfrac{1}{2}\dot{\phi}^2, \quad \delta\rho_\phi . \tag{4}$$

By itself this inequality gives no indication of the extent that radiation can modify the supercooled scenario. However, the warm inflation scenario [3] demonstrated that at least in the limit of near thermal equilibrium, the effect is nontrivial. By reexamining this scenario solely in terms of energetics in [4], it became evident that both supercooled and thermal slow-roll scenarios could be viewed as limiting cases of a class of nonequilibrium kinetic possibilities. A preliminary step to a nonequilibrium study is determining the possible kinematic behaviors of the scale factor for a universe in a mixed state of radiation and vacuum energy. This is the first motivation that leads us to examine the scale factor in this paper.

In light of this, we find it useful to distinguish between the behavior of the scale factor, which we consider kinematics, from the underlying dynamics that induces this behavior. The classification of scale factor behavior is considered kinematic, because it involves characterizing different solutions and different regimes of a given solution, all arising from a particular equation. Besides inflation, common among these are radiation-dominated and matter-dominated behavior. Originally inflation was associated with an exponentially growing scale factor [1]. Subsequently any form of accelerated expansion $[\ddot{R}(t)>0]$ has become associated with inflation.

Dynamics enters in determining the time evolution of the background stress-energy tensor, which is the driving source in the scale factor equation. In the context of dynamics, inflation may or may not arise due to a phase transition. In general, dynamics is stochastic, although the degree of stochasticity may well be approximated by pure dynamics or near-equilibrium statistical dynamics.[1] Inflation scenarios realized in a supercooled regime are examples of the former, whereas warm inflation scenarios [3] are examples of the latter.

The present most successful formulation of supercooled scenarios is new inflation [5,6]. Although several variants of the original scenario have been formulated (for a review please see [7,8]), up to observation the basic assumptions and mechanism are the same. The new inflation assumptions are that dynamics can be described by a suitable potential with a suitable order parameter, known as the inflaton, and that evolution is governed by the Lagrangian equations of motion. The basic mechanism of new inflation is slow-roll dynamics at supercooled temperatures.

In the simplest form of new inflation, the inflaton is a scalar field. The conventional treatment of scalar field dynamics assumes that it is pure vacuum energy dominated. The various kinematic outcomes are a result of specially chosen Lagrangians. In most cases the Lagrangian is unmotivated from particle phenomenology. Clear exceptions are the Coleman-Weinberg potential with an untuned coupling constant, which is motivated by grand unified theories [9], and supersymmetric potentials, although in the latter case, the choice of the supersymmetric potential is again arbitrary, and in the former case new inflation is inconsistent with observation. Making one extension to the new inflation picture, the behavior of the scale factor can also be altered for any given potential when radiation energy is present. Out of pure kinematic interest, this effect has reason to be examined.

More so than just this reason, one may also project to circumstances sometime in the future when observational data will allow determination of the optimal potential among the candidate choices (for examples of recent attempts please see [10,11]). If one accepts the new inflation approximation that the relaxational dynamics of the inflaton can be described by a potential, the next question is what is the microscopic origin of this so preferred potential. If one were restricting oneself to supercooled scenarios, one argument is that the so preferred potential happens to be the one that formed during the rapid quench at the onset of inflation. Another argument is that this is a fundamental zero temperature potential of an elementary field in the Lagrangian. Since for supercooled new inflation scenarios, one of the unanswered questions is that no potential that is suitable for inflation has an already known phenomenological origin, the second argument is highly predictive. Yet to substantiate either claim, one would need to study the evolution of the potential from its high temperature state during the quench. This would lead to examining the interplay between radiation and vacuum energy density at the onset of inflation.

Having appreciated this point, the time interval in which this transition occurs becomes important. The short-time regime is relevant to supercooled new inflation scenarios and the extent to which this interval can be extended is relevant to warm inflation scenarios. Thus, whether stated in the conventional sense of new inflation or the extended sense of warm inflation, the out-of-equilibrium evolution of the inflationary potential will require study, and as an initial step, the scale factor dynamics needs to be examined in a mixed state of background vacuum and radiation energy density. This additional connection to supercooled scenarios provides a second motivation for this study.

To completely study the nonequilibrium dynamics, the problem divides into two steps. The first step is determining the regimes in which accelerated expansion and pure inflation can occur and characterizing the behavior of the scale factor in these regimes. The second step is understanding within the allowed regimes, the class of spectra of primeval energy density perturbations. The first step is moderately model dependent and mainly involves energetics and Friedman cosmology. The second step is a more acute problem of dynamics. Although we will only address the first step in this paper, let us make a few comments about the second step.

In general there is no unique formulation of nonequilibrium dynamics for almost any system. The first step in formulating any approach requires understanding the scales in one's problem. For inflation the simplest assumption is that there are two scales: a long-time, long-distance scale associated with vacuum energy dynamics and a single short-time, short-distance scale associated with a random force component. The Hubble time during inflation, $1/H$, appropriately separates the two regimes. For grand unified theory [9], this time interval is $1/H \approx 10^{-34}$ sec.

The assumption of a long-time scale for the evolution of the vacuum is based on observation. Otherwise inflation would not sustain itself sufficiently long and nor would the energy release maintain smoothness. Accepting this as an empirical constraint, the relaxational dynamics of the inflaton's order parameter justifiably could be described by a free-energy functional. What the specific functional is requires dynamics. In the presence of a radiation component, the functional need not have any similarity to a fundamental potential from the underlying quantum field theory. Furthermore, in grand unified theories as an example, the characteristic time scale of inflation, $1/H$, is about 10^{10} times faster than the characteristic hadronic interaction scale $(1/\Lambda_{\mathrm{QCD}})$, which is a comparison scale where there is good empirical understanding about matter. Thus, at the inflation scale, familiar concepts about matter and from field theory about near-thermal-equilibrium-motivated effective potentials also need not be appropriate.

The problem here has similarities to certain phase separation problems commonly known in association with binary alloys, and the name synonymous with them, spinodal decomposition, has been used before in new inflation cosmol-

[1]The role of stochasticity in cosmology has been emphasized by the Maryland school. For a review please see [31,32].

ogy.[2] The similarity in both cases is that the system is being cooled faster than its characteristic response time to equilibrate. Of course, if this analogy is meant to be complete, the cooled system should still be at a non-negligible temperature, since, at least in the binary alloy problem, the relaxational dynamics is driven by short-ranged thermally excited fluctuations. The analogy to spinodal decomposition not only gives a nice guiding picture, but it also has a type of consoling appeal, which covers for our ignorance about matter, much less quantum field theory, under such extreme conditions, since at least in the context of alloys the problem is considered sufficiently complex to make phenomenological modeling of the nonequilibrium potential an accepted practice. If viewed in the same way, the several scalar inflationary potentials that have been suggested could be interpreted as the cosmologist's attempt at nonequilibrium phenomenology.

A. Hypothesis

Although the reasons given above well motivate examination of the scale factor, I will now describe an alternative to the standard inflationary universe scenario. Consider the following possibility which will be demonstrated in the sequel. It should be easy to convince oneself that a radiation energy density $\rho_r(t)$ of, say, 1 part in 10 000 to the vacuum energy density $\rho_v(t)$ probably should not alter too much the inflationlike behavior of the scale factor. However, when looked upon in terms of the temperature T_r of the radiation energy density, this implies that T_r is only an order of magnitude below the scale of the vacuum energy density. If such a state for the radiation energy density could be maintained by the mutual effects of constant vacuum energy decay and a steadily decreasing acceleration of the scale factor, it could be possible for an inflationlike stage to smoothly enter into a radiation-dominated stage without any discontinuities in $\rho_r(t)$. This possibility was suggestive from formulating the warm inflation scenario [3].

In this paper evidence is presented for inflationlike trajectories of the scale factor which solve the horizon and flatness problems, but for which the radiation energy density monotonically enters the post-inflation radiation-dominated stage with in particular no intermediate reheating stage. This is a regime in between the radiation-dominated and inflation regimes, which has features similar to both a big-bang-like explosion and an inflationlike expansion.

The paper is organized as follows. In the next section the problem is formulated, general solutions are given in Sec. III, special examples are given in Sec. IV, and finally the conclusions are in Sec. V. From the class of solutions that we find for the scale factor, supercooled expansion, which we call supercooled inflation, is a limiting case. This is a kinematic identification. A particular and most noteworthy dynamic realization of supercooled inflation scenarios is the class of new inflation scenarios. Supercooled inflation has associated with it also a range of power law [12], quasiexponential [13], and exponential [5,6] behavior for the scale factor. However, these varied behaviors arise from the specifics of the particular Lagrangian that is being considered. In what we will examine, for any given Lagrangian, a large range of behavior may still arise, depending on the radiation energy density content.

There is one other problem that the present work may help clarify. We will discuss it briefly here. However, it gets into the realm of field theory dynamics, which this paper will mainly avoid. The most notable shortcoming of new inflation is in explaining small scale energy density inhomogeneities [14,6,15–17]. The problem is sometimes referred to as the amplitude fluctuation problem [15]. The warm inflation scenario in [3] is a solution to this problem. However, our formulation there did not detail a time history for an inflation-like state with radiation. The present work does and in fact was its starting motivation. However, as a result of the generality of the solutions given here, it appears better to consider warm inflation as a particular dynamic realization within the big-bang-like inflation regime.

Our equations can also be examined for the initial stage of entering into the rapid expansion state, but we will not study that here.

II. FORMULATION

We are interested in the scale factor from some short time after the initial singularity, when quantum gravitational effects become negligible. We assume that space is homogeneous and isotropic, and restrict ourselves to Friedman cosmology with the Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \quad (5)$$

For notational convenience, the origin of cosmic time is defined as the beginning of our treatment.

Let us start with the standard equations of Friedman cosmology [16,18] for the scale factor $R(t)$ in the presence of vacuum energy density $\rho_v(t)$ and radiation energy density $\rho_r(t)$. The equations of state which relate the energy density ρ to the pressure p are

$$p_v(t) = -\rho_v(t), \quad (6)$$

$$p_r(t) = \tfrac{1}{3}\rho_r(t). \quad (7)$$

In Friedman cosmology the ten Einstein equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ reduce to two independent ones, which are from the time-time component, also known as Friedmann's equation

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8\pi G}{3}\rho, \quad (8)$$

and from any of the three diagonal space-space components, all of which give

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi G p. \quad (9)$$

[2] For scalar field inflaton dynamics, the analogy actually is to spinodal decomposition for a nonconserved order parameter, such as found in certain domain growth problems [33,34], whereas the binary alloy problem involves a conserved order parameter.

For our purposes it is preferable to use two other equations obtained from these, the scale factor equation

$$\frac{\ddot{R}}{R}=\frac{8\pi G}{3}[\rho_v(t)-\rho_r(t)] \tag{10}$$

and the stress energy conservation equation

$$\dot{\rho}_r(t)=-4\rho_r(t)\frac{\dot{R}(t)}{R(t)}-\dot{\rho}_v(t), \tag{11}$$

where we have used the equations of state (6) and (7). We aim to solve for $R(t)$ and $\rho_r(t)$ in Eqs. (10) and (11), for a prescribed $\rho_v(t)$, for $t>0$, and with arbitrary initial conditions for $R(t)$, $\dot{R}(t)$, and $\rho_r(t)$ up to the constraints

$$R(0)>0 \tag{12}$$

$$\dot{R}(0)>0 \tag{13}$$

and

$$\rho_r(0)>0. \tag{14}$$

By taking the sum and difference of Friedman's equation (8) and the scale factor equation (10), the vacuum and radiation energy densities can be separately expressed in terms of the scale factor as [19]

$$\rho_v(t)=\frac{3}{16\pi G}\left[\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}\right], \tag{15}$$

$$\rho_r(t)=\frac{3}{16\pi G}\left[-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}\right]. \tag{16}$$

For an arbitrary test vacuum function $\rho_v(t)$, one can use Eq. (15) to solve for $R(t)$.

We make the substitution

$$s(t)=R^2(t). \tag{17}$$

Equation (15) then becomes the inhomogeneous wave equation with time-dependent frequency

$$\ddot{s}-\frac{32\pi G}{3}\rho_v(t)s=-2k. \tag{18}$$

This equation has been widely studied [20,21]. Again we are interested in the solutions to Eq. (18) for $t>0$ with arbitrary initial conditions for $s(t)$ up to the constraints from Eqs. (12)–(14) and (17) which imply

$$s(0)>0 \tag{19}$$

and

$$\dot{s}(0)>0. \tag{20}$$

III. SOLUTION

In this section solutions are obtained for the scale factor from Eqs. (10) and (11) for a large class of vacuum energy decay functions. Even before getting this specific, there are two general features, one at short and one at long time, which are recurring themes to the existence of the big-bang-like inflation regime. At long time, if $\rho_v(t)$ goes to zero sufficiently fast, from Eqs. (18) and (17) one can see that $R(t\to\infty)\sim t^{1/2}$, thus tending to a radiation-dominated behavior. At short time, for an initially radiation-dominated universe,

$$\rho_r(t\sim 0)\gg\rho_v(t\sim 0), \tag{21}$$

Eqs. (10) and (11) imply $R(t\sim 0)\sim(a+bt)^{1/2}$. Alternatively, this can be seen from Meissner's separation, Eqs. (15) and (16), since Eq. (21) implies from Eqs. (15) and (16) that

$$\dot{s}^2(t\sim 0)\gg\ddot{s}(t\sim 0). \tag{22}$$

Taylor expanding $s(t)$ about the origin as $s=s_0+s_1t+s_2t^2/2+\cdots s_nt^n/n!+\cdots+$, Eq. (22) implies $s_1^2\gg s_2$. Using this and Eq. (18), one can study the initial condition dependence of entering the inflationlike stage, but we will not pursue that here.

Let us now turn to specific solutions. Although this paper is focused on the kinematic possibilities for the scale factor, independent of justification from any specific field theory, we will motivate a class of vacuum decay functions from a general class of scalar field dynamics. In fact as we will show below, in the limit of strong dissipation, this motivation can be partly justified.

We consider stochastic evolution for the inflaton governed by the Langevin-like equation

$$\ddot{\phi}(t)+\left[\Gamma+3\frac{\dot{R}(t)}{R(t)}\right]\dot{\phi}(t)+V'(\phi(t))=\eta(t), \tag{23}$$

where $\eta(t)$ is a random force function with vanishing ensemble averaged expectation value

$$\langle\eta(t)\rangle=0. \tag{24}$$

The effect of the inflaton's interaction with radiation is represented by the dissipative constant Γ and the random force function $\eta(t)$. In a simple model for the radiation system and in the limit of pure inflation ($\dot{R}/R=$const) this equation was obtained from quantum field theory in [4].

We are interested in the limit of strong dissipation,

$$\Gamma\gg\frac{\dot{R}}{R}, \tag{25}$$

and the slow-roll regime

$$\Gamma|\dot{\phi}|\gg|\ddot{\phi}|. \tag{26}$$

For our present purposes, the ensemble-averaged equation of motion is all that we need. Thus in the above specified limits, Eq. (23) becomes

$$\frac{d\phi}{dt}=-\frac{1}{\Gamma}\frac{dV(\phi)}{d\phi}. \tag{27}$$

Let us consider potentials of the form

$$V(\phi)=\lambda M^{4-n}(M-\phi)^n \tag{28}$$

in the region

$$0<\phi<M, \tag{29}$$

where λ is dimensionless. For inflation driven dynamically at the grand unified scale $M\sim M_{\mathrm{GUT}}\approx 10^{14}$ GeV.

Globally all of the potentials in Eq. (28) are improper for slow-roll inflation scenarios, since they fail to represent symmetry breaking. However, our present interest is the behavior of the scale factor for a large class of slow-roll conditions. In this sense Eq. (28) represents a class of local approximating potentials from which an arbitrary potential can be piecewise constructed. Thus it is also not a concern that such potentials have no minima for odd n and are nonanalytic for noninteger n when $\phi=M$.

In fact near the global minima, where the vacuum energy goes to zero, quadratic ($n=2$) dependence would be the normal expectation for any generic free-energy functional. This case is not only of special physical interest but is also mathematically a little different. We will differentiate this case of $n=2$ from all others and refer to it as the quadratic limit.

To keep our discussion explicit, we will express the results that follow in the context of the slow-roll inflation scenario. However, it should be noted that the solutions for the scale factor given below carry a relevance beyond the slow-roll scenario. Let us briefly recall the slow-roll scenario. In the standard setting of the slow-roll transition, the inflaton starts near the origin and is making its decent to the symmetry-broken minima at $\phi=M$.

At the origin of cosmic time we will assume that the slow-roll transition begins with

$$\phi(0)=\epsilon M \tag{30}$$

and $\epsilon\ll 1$. With these initial conditions, the solutions of Eq. (27) for potentials in Eq. (28) are, for $n=2$,

$$\phi(t)=M\left[1-\exp\left(-\frac{B_2}{2}(t-t_{0_2})\right)\right] \tag{31}$$

and, for $n\neq 2$,

$$\phi(t)=M\{1-[B_n(t+t_{0_n})]^{1/(2-n)}\}, \tag{32}$$

with

$$B_2\equiv\frac{4\lambda M^2}{\Gamma} \tag{33}$$

and

$$B_n\equiv\frac{n(n-2)\lambda M^2}{\Gamma}. \tag{34}$$

Here t_{0_2} and t_{0_n} are suitably adjusted to satisfy Eq. (30). Equating the potential to the vacuum energy density

$$\rho_v(t)=V(\phi(t)) \tag{35}$$

implies, for $n=2$,

$$\rho_v(t)=\lambda M^4\exp[-B_2(t-t_{0_2})] \tag{36}$$

and, for $n\neq 2$,

$$\rho_v(t)=\lambda M^4[B_n(t+t_{0_n})]^{n/(2-n)}. \tag{37}$$

Substituting the above in Eq. (18) and solving the homogeneous (flat space) equation, we find for the scale factor from Eq. (17) for $n=2$,

$$R=\sqrt{C_1 I_0(z_2(t))+C_2 K_0(z_2(t))}, \tag{38}$$

and in the $n\neq 2$ case for all but $n=4$,

$$R=[B_n(t+t_{0_n})]^{1/4}\times\sqrt{C_1 I_{(2-n)/(4-n)}(z_n(t))+C_2 K_{(2-n)/(4-n)}(z_n(t))}, \tag{39}$$

where

$$z_2(t)\equiv\frac{4}{B_2}H_2\exp\left(\frac{-B_2}{2}t\right) \tag{40}$$

$$z_n(t)\equiv\frac{4(2-n)H_n t_{0_n}}{(4-n)}\left(\frac{t}{t_{0_n}}+1\right)^{(4-n)/(4-2n)}, \tag{41}$$

with

$$H_2\equiv\sqrt{\frac{8\pi G\lambda M^4}{3}}\exp\left(\frac{B_2 t_{0_2}}{2}\right), \tag{42}$$

$$H_n\equiv\sqrt{\frac{8\pi G\lambda M^4}{3}}(B_n t_{0_n})^{n/2(2-n)}. \tag{43}$$

In Eqs. (42) and (43) we have identified the Hubble parameter at $t=0$ based on the definition

$$H\equiv\sqrt{\frac{8\pi G\rho_v(0)}{3}} \tag{44}$$

for the respective vacuum energy densities in Eqs. (36) and (37). In the Appendix we have listed properties of modified Bessel functions that will be useful to us. Finally from the $n\neq 2$ cases for $n=4$ the solution is

$$R=|B_4(t+t_{0_4})|^{1/4}\sqrt{C_1|B_4(t+t_{0_4})|^{\mu}+C_2|B_4(t+t_{0_4})|^{-\mu}}, \tag{45}$$

where

$$\mu=\frac{1}{2}\left(1+\frac{128\pi G\lambda M^4}{3B_4^2}\right)^{1/2}. \tag{46}$$

The inhomogeneous wave equation in Eq. (18), which is for curved space $k\neq 0$, can be solved from the above solutions for the homogeneous equation by familiar methods [20,21]. Irrespective of the slow-roll scenario, the results, Eqs. (38) and (39), are valid for any scenario that motivates vacuum decay behavior as in Eqs. (36) and (37). Likewise for other types of vacuum decay functions, Eq. (18) can be solved.[3]

In the next section, the solutions Eqs. (38) and (39), will be studied through specific examples. Here some of their general features will be noted. The quadratic limit is examined first. The growing mode as $t \to \infty$ in Eq. (38) from Eqs. (A1) and (A2) is $K_0(z_2(t))$ with

$$R(t \to \infty) \sim t^{1/2}, \tag{47}$$

thus asymptotically exhibiting radiation-dominated behavior.

Inflationlike expansion at intermediate time is also governed by $K_0(z_2(t))$. From Eq. (A2) a large expansion factor of e^N with $N \geqslant 50$ will require

$$\frac{2H_2}{B_2} \sim N. \tag{48}$$

This is like what one would expect, since the vacuum energy density must decay sufficiently slowly relative to the expansion time for the scale factor, in order to be the driving source for inflationlike behavior in the Einstein equations.

In order to establish the dominance of the $K_0(z_2(t))$ term to the $I_0(z_2(t))$ term in Eq. (38), what remains is to show that there is no way for the initial conditions to force C_1 to be exponentially large relative to C_2. Treating $4H_2/B_2 \gg 1$ and using Eqs. (A5) and (A6), this follows from the constraints, Eqs. (19) and (20). Equation (20) could be satisfied for C_1 exponentially large but negative relative to C_2, but then Eq. (19) would not be satisfied. Having established the dominance of the $K_0(z_2(t))$ mode, let us estimate the expansion factor for a single $n=2$ section of the potential, Eq. (28), with $\epsilon=0$ in Eq. (30) so that $t_{0_2}=0$ in Eq. (31). We find for the asymptotic behavior

$$\frac{R(t \to \infty)}{R(0)} \sim \frac{(B_2 t/2)^{1/2} e^{2H_2/B_2}}{(\pi B_2/8H_2)^{1/4}}, \tag{49}$$

and so an expansion factor of e^{2H_2/B_2}.

Away from the quadratic limit ($n \neq 2$) from Eqs. (39), and (A5), (A6) we see that for

$$\frac{4-n}{2-n} > 0 \tag{50}$$

the solution will grow exponentially at large time, and thus never asymptotes into a radiation-dominated behavior. This corresponds to a vacuum decay function that decays slower than $1/t^2$ at large times in Eq. (18). Radiation-dominated behavior at large time is attained for

$$\frac{4-n}{2-n} < 0, \tag{51}$$

which implies potentials in Eq. (28) with

$$2 < n < 4 \tag{52}$$

or vacuum decay functions in Eq. (18) that decay faster than $1/t^2$. Finally for $n=4$, which corresponds to a vacuum decay falling off exactly as $1/t^2$, $R(t)$ has the same power law behavior throughout, with a growth bounded from below by $t^{1/2}$. As such, this case is not useful for our present purpose. This implies that the only symmetric potential about the symmetry broken point, $\phi = M$, that leads to radiation-dominated and not inflationlike asymptotic behavior is the quadratic case $n=2$. As an aside, note that the $n=4$ case is interesting since on either side are solutions with two very different types of asymptotic behavior.[4]

Returning to the cases in Eq. (52), the growing mode in Eq. (39) is $K_{(2-n)/(4-n)}(z_n(t))$. Let us estimate the expansion factor for a single $n \neq 2$ sector of the potential, Eq. (28), in the range Eq. (52) for $\epsilon=0$ in Eq. (30) so that

$$B_n t_{0_n} = 1 \tag{53}$$

in Eq. (32). The arguments are the same as above for the $n=2$ case with the final result

$$\frac{R(t \to \infty)}{R(0)} \sim (B_n t)^{1/2} \exp\left[\frac{2(n-2)H_n}{(4-n)B_n}\right]. \tag{54}$$

Before closing this section, one additional qualifying statement is needed about the solutions, Eqs. (38) and (39), if the vacuum decay functions, Eqs (36) and (37), are obtained from slow-roll scalar field dynamics. Recall that the energy density and pressure of the zero mode of the scalar field are

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi), \tag{55}$$

$$p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V(\phi). \tag{56}$$

Therefore, the equation of state (6) is valid in the limit that the potential energy dominates the kinetic energy

$$\tfrac{1}{2}\dot{\phi}^2 \ll V(\phi). \tag{57}$$

One must check that this kinetic energy suppression condition is always valid.

[3]One case is during reheating in supercooled scenarios. For this, the vacuum decay function in Eq. (18) should have the approximate time dependence $e^{-|\Gamma|t}(1+\cos Bt)$ with $B \sim M \gg H$. These types of equations are treated in [20]. The $|\Gamma|=0$ case is the Mathieu equation. The solutions of these equations describe the scale factor behavior during those stages of reheating when the equation of state (6) is valid for the inflaton. Another nontrivial aspect of scale factor behavior in supercooled scenarios is at the beginning where initial condition dependence on preinflationary radiation energy density can be studied. For this, the solutions, Eq. (39), for $n \sim 0$ are useful.

[4]It is also an interesting coincidence that $n=4$ separates renormalizable and nonrenormalizable scalar quantum field theory, with the nonrenormalizable side, $n>4$, corresponding to the observationally inconsistent non-radiation-dominated asymptotic scale factor behavior. Furthermore, the $n=4$ case neither asymptotes to radiation-dominated behavior nor is believed to be nonperturbatively a nontrivial quantum field theory [35]. Of course, for the inflaton, since it is coupled to gravity, the whole theory is always nonrenormalizable in any case.

For this, first recall that the exact equation of motion for the inflaton in the limit, Eq. (25), is the second order equation

$$\ddot{\phi}+\Gamma\dot{\phi}+V'(\phi)=0. \tag{58}$$

For the quadratic case $n=2$, this equation can be exactly solved and it can be verified that the slow-roll condition, Eq. (26), and the kinetic energy suppression condition, Eq. (57), are both valid for all $t>0$ provided

$$\frac{\lambda M^2}{\Gamma^2}\ll 0. \tag{59}$$

In addition, if $\rho_v(0)$ is required to be large in Eqs. (36) and (37), so that λ cannot be made tiny, Eq. (59) implies

$$\Gamma\gg M. \tag{60}$$

This is the large dissipative regime required for warm inflation [3].

For the cases $2<n<4$, to verify Eq. (57), first it will be shown that the solutions, Eq. (32), are consistent with the slow-roll condition, Eq. (26), for all $t>0$. Next a direct verification will be made that the solutions, Eq. (32), respect the condition, Eq. (57). Addressing step one, it is observed from Eq. (32) that for the entire range $2<n<4$, $\ddot{\phi}(t)$ vanishes faster than $\dot{\phi}(t)$ as $t\rightarrow\infty$. Thus Eq. (26) is satisfied under the same parametric restrictions as in the $n=2$ case, Eqs. (59) and (60). Proceeding to the second step, it can be verified from the slow-roll approximate solutions, Eq. (32), that $\dot{\phi}^2(t)$ vanishes faster than $V(\phi(t))$ as $t\rightarrow\infty$. Thus in the regime, Eq. (60), Eq. (57) is satisfied for all $t>0$ so that Eq. (6) is always valid.

To summarize, it has been verified that the equation of state (6) is valid for the scalar field for all $t>0$ and in the entire range $2\leq n<4$, when in the strong dissipative regime, Eq. (60). This type of slow-roll motion is analogous to an overdamped oscillator [4]. Note that confirming the validity of Eq. (6) for all $t>0$ is more than needed, since in any case $\rho_r(t)$ overtakes $\rho_v(t)$ at some much earlier stage.

The results presented in this section now demonstrate the existence of inflationlike scale factor trajectories which smoothly go into a radiation-dominated behavior without a discontinuous reheating stage.

IV. EXAMPLES

In this section we will examine some specific examples from the solutions for the scale factor in Eqs. (38) and (39). In these examples we will see how the radiation energy density eventually overtakes the vacuum energy density with no discontinuities, and in the processes the universe smoothly goes from an inflationlike to a radiation-dominated stage. We will also study the magnitude of decrease in the radiation energy density, thus the temperature of the universe, from before to after the inflationlike stage. In supercooled scenarios, the post-inflation temperature is referred to as the reheating temperature, but here it is better to call it the initial temperature after inflation, $T_{\rm AI}$. In particular the inflationlike stage is defined as the time period when the scale factor has positive acceleration

$$\ddot{R}(t)>0, \tag{61}$$

with the time ''just before,'' $t_{\rm BI}$, and ''just after,'' $t_{\rm AI}$, inflation being defined as the end points of the accelerated expansion interval

In Sec. IV(b) we will examine a particular $n\neq 2$ case from Eq. (39) which can be fully expressed with simple analytic functions. Then in Sec. IV(c) we will examine the quadratic limit. For this study, we will first convert to a set of dimensionless quantities.

A. Dimensionless theory

We will work with the dimensionless quantities defined as

$$a(\tau)\equiv\frac{R(\tau)}{R(\tau_{\rm BI})}, \tag{62}$$

$$b(\tau)\equiv\frac{\rho_v(\tau)}{\rho_v(\tau_{\rm BI})}, \tag{63}$$

$$c(\tau)\equiv\frac{\rho_r(\tau)}{\rho_v(\tau_{\rm BI})}, \tag{64}$$

where dimensionless time

$$\tau\equiv Ht, \tag{65}$$

$\tau_{\rm BI}$ is the time when accelerated expansion begins, and H is defined in Eq. (44) except with the vacuum energy density evaluated at $\tau_{\rm BI}$, $\rho_v(\tau_{\rm BI})$. Defining $s_a(\tau)\equiv a^2(\tau)$, the Meissner separation, Eqs. (15) and (16), in terms of $s_a(t)$ and the rescaled quantities is

$$b(\tau)=\frac{1}{4s_a(\tau)}\frac{d^2s_a(\tau)}{d\tau^2}+\frac{k}{2H^2s_a(\tau)} \tag{66}$$

and

$$c(\tau)=-\frac{1}{4s_a(\tau)}\frac{d^2s_a(\tau)}{d\tau^2}+\frac{1}{4s_a^2(\tau)}\left(\frac{ds_a(\tau)}{d\tau}\right)^2+\frac{k}{2H^2s_a(\tau)}. \tag{67}$$

The radiation energy density will be related to a temperature measure by the Stefan-Boltzmann radiation law

$$\rho_r(\tau)\sim T^4(\tau). \tag{68}$$

This law need not hold under far from equilibrium conditions, but we will nevertheless refer to $T(\tau)$ as the temperature of the universe at time τ. We will study the temperature of the universe in terms of the ratio

$$\alpha(\tau)\equiv\frac{T(\tau)}{T(\tau_{\rm BI})}. \tag{69}$$

The field theory quantities will also be rescaled. The natural scale for them is M not H and in general these two scales are different. This scale disparity is an inherent feature of scalar field slow-roll dynamics. A primitive source of the dilemmas encountered in slow-roll scenarios is its two-scale nature. The natural time scale in the field theory to release the vacuum energy, $1/M$, in general differs from the characteristic cosmological expansion time $1/H$. In grand unified theories this disparity works against theoretical preference, since

$$1/M \ll 1/H. \tag{70}$$

Had this inequality been reversed, it would have been parametrically satisfying and perhaps a strong argument for theoretical consistency between cosmology and particle physics. However, since this is not the case, it either means slow-roll dynamics is wrong, field theory dynamics for inflation at the grand unified scale is wrong, grand unified theory is incomplete or wrong, or that the physics needs further elaboration, perhaps from nonequilibrium methods. We will not address the dynamical problem here, but it is worthwhile to keep track of the scale disparity. Thus we will rescale everything with respect to H, but for quantities where M is the natural scale, the rescaling will include the additional factor

$$\beta \equiv \frac{M}{H}. \tag{71}$$

The field theory quantities are rescaled as

$$\Gamma \equiv \gamma \beta H \tag{72}$$

and

$$\phi \equiv \sigma \beta H. \tag{73}$$

B. $n=8/3$

Let us consider the case $n=8/3$ from the $n\neq 2$ class of potentials in Eq. (28) which in rescaled parameters is

$$V(\sigma)=\lambda M^4(1-\sigma)^{8/3}. \tag{74}$$

Solving the slow-roll equation of motion

$$\frac{d\sigma}{d\tau}=\frac{3}{2}\kappa_{8/3}(1-\sigma)^{5/3}, \tag{75}$$

where using Eqs. (34)

$$\kappa_{8/3} \equiv \frac{B_{8/3}}{H_{8/3}}=\frac{16\lambda\beta}{9\gamma} \tag{76}$$

and with the initial condition

$$\sigma(0)=0, \tag{77}$$

we find

$$\sigma(\tau)=1-\frac{1}{(\kappa_{8/3}\tau+1)^{3/2}}. \tag{78}$$

This implies that the rescaled vacuum energy density is

$$b(\tau)=\frac{1}{(\kappa_{8/3}\tau+1)^4}. \tag{79}$$

Solving the homogeneous wave equation (18) using Eq. (79) we find

$$a(\tau)=\sqrt{(\kappa_{8/3}\tau+1)\left[C_1\exp\left(\frac{2\tau}{(\kappa_{8/3}\tau+1)}\right)+C_2\exp\left(\frac{-2\tau}{(\kappa_{8/3}\tau+1)}\right)\right]}. \tag{80}$$

Here $\tau_{\rm BI}$, C_1, and C_2 are determined by the initial radiation energy density r,

$$\frac{c(0)}{b(0)}=r, \tag{81}$$

the defining relation for $\tau_{\rm BI}$,

$$b(\tau_{\rm BI})=c(\tau_{\rm BI}), \tag{82}$$

and from Eqs. (62), which implies

$$a(\tau_{\rm BI})=1. \tag{83}$$

From Eqs. (66) and (67) and restricting to flat space, explicitly the first two conditions above imply, from Eq. (81),

$$\frac{1}{4s_a^2(0)}\left(\frac{ds_a(0)}{d\tau}\right)^2=r+1 \tag{84}$$

and, from Eq. (82),

$$\frac{1}{8s_a^2(\tau_{\rm BI})}\left(\frac{ds_a(\tau_{\rm BI})}{d\tau}\right)^2=b(\tau_{\rm BI}). \tag{85}$$

In Eq. (85), $s_a^2(\tau_{\rm BI})=1$, but we retain it explicitly since the same equality holds at $\tau_{\rm AI}$, and we will use it below to determine $\tau_{\rm AI}$.

Let us verify the various general features discussed in earlier sections for this specific example. At large time, by inspection of Eq. (80) one finds

$$a(\tau\rightarrow\infty)\sim\tau^{1/2}, \tag{86}$$

which verifies an asymptotic radiation-dominated behavior. Turning to the growth of the scale factor, to obtain an exponentially large one from Eq. (80) at large time, the only way is from the first term (the growing mode) and only if $1/\kappa_{8/3}\gg 1$. The constraints, Eqs. (19) and (20), imply $C_1>|C_2|$, so that the growing mode cannot be suppressed due to an exponentially small coefficient C_1 relative to C_2. To leave no ambiguity, let us focus on one case among a large class which are all about the same for what we want to study. For simplicity, in the case we will consider, arrange the initial conditions so that inflation begins at the origin of cosmic time $\tau_{\rm BI}=0$. One can confirm from Eqs. (83) and (84) that C_1 and C_2 are about the same order of magnitude, so that at large time, only the growing mode need be retained. For completeness we find

$$C_1=\frac{1}{2}\left(1+\sqrt{2}-\frac{\kappa_{8/3}}{2}\right) \tag{87}$$

and

$$C_2=\frac{1}{2}\left(1-\sqrt{2}+\frac{\kappa_{8/3}}{2}\right). \tag{88}$$

From the growing mode in Eq. (80) we see that to obtain N_e e-folds of expansion requires

$$\frac{1}{\kappa_{8/3}}=N_e, \tag{89}$$

which agrees with our general $n\neq 2$ approximation formula, Eq. (54). In grand unified theories, for the Coleman-Weinberg potential with an untuned coupling constant, one has $\beta\sim 10^4$ and $\lambda\sim 1$ so that from Eqs. (76) and (89) this implies

$$\gamma\sim N_e\times 10^4, \tag{90}$$

which for $N_e\sim 50$ implies $\gamma\sim 10^6$. This gives an overviewed explanation for the large dissipative constant found in the warm inflation scenario of [3]. However, the estimate here for γ is a little higher, because in the actual scenario, the finite-temperature Coleman-Weinberg potential has a smaller curvature. In this simplified discussion, this means β is smaller.

We see once again that the largeness of β makes a seemingly undesired appearance in the dynamics. Whereas in supercooled scenarios it forces a fine-tuning of the coupling constant, here it forces the dissipative constant to be larger than one would naively want. It remains a theoretical question whether such a large dissipative constant has an explanation. However, up to the parametric level, the warm inflation approach still has a hope of salvaging the simplest grand unified theory motivated Coleman-Weinberg model for inflaton dynamics. In new inflation, this possibility is not even parametrically satisfying, since fine-tuning the coupling constant also implies that the vacuum energy, and so the Hubble constant during inflation, drops substantially.

Turning to the radiation energy density next, our interest is to compute from the ratio in Eq. (69), $\alpha(\tau_{\rm AI})$. To keep an explicit example in mind, we again take $\tau_{\rm BI}=0$ with C_1 and C_2 as given in Eqs. (87) and (88). Here $\tau_{\rm AI}$ is determined by a solution once again to $b(\tau)=c(\tau)$, and so for a second solution to Eq. (85) for $\tau_{\rm AI}>\tau_{\rm BI}=0$. One clearly expects a second intersection, since $b(\tau)$, which goes as $(\kappa_{8/3}\tau+1)^{-4}$, is falling off faster than $c(\tau)$, which at large time goes as $(\kappa_{8/3}\tau+1)^{-2}$, and after the first intersection at $\tau_{\rm BI}$, $b(\tau)>c(\tau)$. To determine $\tau_{\rm AI}$, only the growing mode of $s_a(\tau)$ is retained from $[ds(\tau)/d\tau]^2$, to give the relation

$$\frac{\kappa_{8/3}^2}{4}(\kappa_{8/3}\tau_{\rm AI}+1)^2+\kappa_{8/3}(\kappa_{8/3}\tau_{\rm AI}+1)-1=0. \tag{91}$$

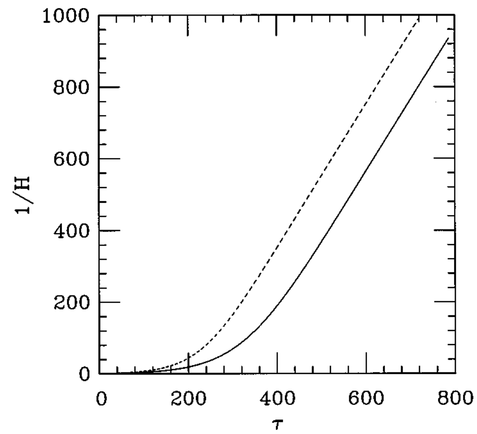

FIG. 1. The inverse Hubble parameter for a quadratic slow-roll potential with vacuum decay coefficients for the solid (dashed) cases $\kappa_2=0.03$ (0.04). The initial conditions are $c(0)=1.5$.

Solving for $\tau_{\rm AI}$ with the constraint $\tau_{\rm AI}>0$ we find

$$\tau_{\rm AI}=\frac{1}{\kappa_{8/3}}\left[\frac{2}{\kappa_{8/3}}(\sqrt{2}-1)-1\right]\approx\frac{2(\sqrt{2}-1)}{\kappa_{8/3}^2}, \tag{92}$$

so that, from Eq. (67),

$$c(\tau_{\rm AI})\approx\frac{\kappa_{8/3}^4}{16(\sqrt{2}-1)^4} \tag{93}$$

and

$$\alpha(\tau_{\rm AI})=\left(\frac{c(\tau_{\rm AI})}{c(\tau_{\rm BI})}\right)^{0.25}\approx 1.2\kappa_{8/3}. \tag{94}$$

Recalling from Eq. (89) that N_e e-folds of expansion require $\kappa_{8/3}=1/N_e$, if $N_e\sim 50$–70, we find that the temperature drops by a factor 1/50–1/70 from the beginning to the end of inflation with the duration of inflation being $\tau_{\rm AI}-\tau_{\rm BI}\sim N_e^2\sim$ 2500–4900.

To summarize, in this subsection we have presented an example that can be verified by inspection in which the scale factor expands sufficiently and then smoothly tends towards a radiation-dominated behavior at large time. In the course of this, the temperature of the universe during the inflationlike stage drops between one and two orders of magnitude.

C. Quadratic limit

Let us next examine the quadratic limit with the vacuum decay function

$$b(\tau)=\exp(-\kappa_2\tau). \tag{95}$$

The plots are in Fig. 1 for the inverse Hubble parameter

$$\frac{1}{H}\equiv\frac{a(\tau)}{da(\tau)/d\tau} \tag{96}$$

and in Fig. 2 for the temperature ratio $\alpha(\tau)$ defined in Eq. (69). In both figures the solid (dashed) curve is for the vacuum decay coefficient $\kappa_2=0.03$ (0.04). The calculations were done by a numerical integration of the coupled scale

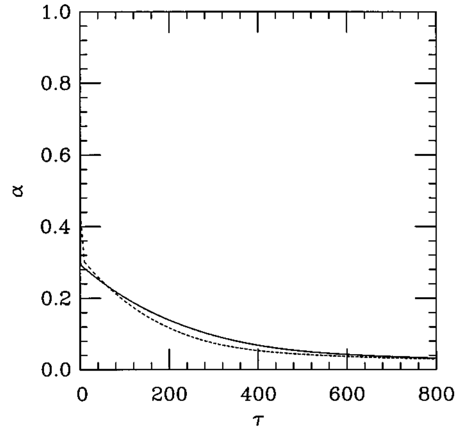

FIG. 2. The ratio $\alpha(\tau)$ of the universe's temperature at cosmic time τ to that at the beginning of inflation, τ_{BI}, for the same cases as in Fig. 1. For the solid (dashed) curve, the inflationlike stage begins at $\tau_{BI}=0.073$ (0.074) and ends at $\tau_{AI}=323$ (228). In both cases the temperature of the universe drops by about a factor of 10, with e-folds 67 (51).

factor equation (10) and stress-energy conservation equation (11), with the vacuum decay function in Eq. (95). As a cross-check, at each iteration the so computed scale factor was substituted into the left-hand side of Friedmann's equation (8), with the resulting energy density found from the right-hand side compared with that from the numerical integration. This also means the results cross-check separately for $b(\tau)$ and $c(\tau)$ from Eqs. (66) and (67), but since this requires $\ddot{a}(\tau)$, it is not a more stringent test.

For both cases in Fig. 1, the inverse Hubble parameter starts out flat, which is characteristic of inflationlike behavior. It then veers up at a time of order $\tau\sim 33$ (25) in the solid (dashed) case and finally tends to a slope of 2 at large times, thus becoming a radiation dominated universe on schedule. Inflation begins in the solid (dashed) case at $\tau_{BI}=0.073$ (0.074) and ends at $\tau_{AI}=323$ (228), so that the duration of inflation is $\tau_{AI}-\tau_{BI}\approx 323$ (228). During the inflationary period, the scale factor expands rapidly with total e-folds $N_e=67$ (51), which agrees with estimates from our approximation formula, Eq. (48), and the temperature drops by a factor $\alpha(\tau_{AI})=11$ (10).

The numerical results for τ_{AI} and $\alpha(\tau_{AI})$ also can be cross-checked to approximate analytic expressions, Solving Eq. (85) for the second solution at $\tau_{AI}>\tau_{BI}$, noting from the Appendix that $K_0(z_2(\tau))$ dominates in the solution, Eq. (38), at long time, and using $dK_0(z)/dz=-K_1(z)$, one finds for any κ_2 the general relation

$$\frac{K_1(z_2(\tau_{AI}))}{K_0(z_2(\tau_{AI}))}=\sqrt{2}. \tag{97}$$

From [22] one finds that this is satisfied for $z_2\approx 1.05$, so that from Eq. (40) we obtain the approximate formula

$$\tau_{AI}\approx -\frac{2}{\kappa_2}\ln\left(\frac{1.05\kappa_2}{4}\right). \tag{98}$$

Assuming $\tau_{BI}\ll 1$ so that $c(\tau_{BI})\approx 1$, as for the numerical cases presented above, and using Eq. (98) we obtain

$$\alpha(\tau_{AI})\approx \tfrac{1}{2}\sqrt{1.05\kappa_2}. \tag{99}$$

One can verify that the approximation formulas, Eqs. (98) and (99), reproduce the results for τ_{AI} and $\alpha(\tau_{AI})$, respectively, that were quoted above from the numerical calculations. Similar approximation formulas can also be obtained for the $n\neq 2$ cases.

In Fig. 2 observe the initial steep drop in $\alpha(\tau)$ for $\tau<1$. There is a very short initial transient period in which the initial radiation energy density stabilizes, followed by a steady state stage. For both cases in Fig. 2, we started with a radiation energy density $c(0)=1.5$. Thus initially the first term on the right-hand side of Eq. (11) (the ''sink term'') rapidly depletes $\rho_r(t)$ [equivalently $c(\tau)$ in the rescaled theory] until an approximate balance is reached by the second term (the ''source term''), after which steady state is reached. The initial conditions on the radiation energy density have a mild effect on the long-time behavior. For example, increasing $c(0)$ by a factor of 500 has less than a 1% effect on N_e. Without the source term, which arises from vacuum energy depletion, all the radiation energy would rapidly redshift away, as in supercooled scenarios.

V. CONCLUSION

There are two possibly concerning or possibly predictive outcomes of scenarios occurring entirely in the big-bang-like inflation regime. We believe they are general features of such scenarios, although we do not have proof. First, to attain an observationally consistent expansion factor, it does not appear possible for the post-inflation temperature T_{AI} to be the same order of magnitude as that just before inflation, T_{BI}. In supercooled scenarios this is referred to as a perfect reheating and can be achieved by adjusting the decay width, which controls the reheating time period, to be sufficiently large [23,16]. In big-bang-like inflation scenarios, for observationally sufficient expansion, we generally find that T_{AI} is at least one order of magnitude below T_{BI}. Thus for inflationary dynamics at the grand unified scale one expects $T_{AI}\sim$ (0.1–0.01)M_{GUT}. In the context of grand unified theory, this implies that the X boson, with $M_X\sim M_{GUT}$, would not participate in post-inflation baryongenesis, although the lighter Higgs boson still could [24]. Moreover, the picture is further altered since in the big-bang-like class of inflation scenarios, there would be no violent discontinuities in $\rho_r(t)$ at the end of the inflationlike stage. This implies that baryongenesis could commence within the inflationlike stage and smoothly carry on afterwards. One can also consider long sustained big-bang-like inflation scenarios, in which the temperature drops by a few orders of magnitude during the inflationlike stage. For such scenarios, studies of baryongenesis from out-of-equilibrium decay processes at temperatures well below M_{GUT} may be useful [25]. As a final complementary note to this concern pertaining to baryongenesis, the lower temperature condition implies that magnetic monopole suppression works effectively.

The second point of concern for big-bang-like inflation scenarios with not too large a drop in the temperature during the inflationlike stage is that they generally appear to have not very large upper bounds on the expansion factor with e-folds $N_e^{max}\sim 1000$, but $N_e^{max}\sim 100$ being typical. Since observation indicates $N_e > 50$–70, this is still acceptable. For comparison, in the solution of the Coleman-Weinberg model in new inflation, it is found that $N_e \sim 10^7$ [6]. In general new inflation models are reported to predict very large e-folds N_e [16]. As one optimistic interpretation about the small e-fold constraint for scenarios within the big-bang-like inflation regime, this is preferred if the universe is between nearly flat and open [26].

In this paper we have shown that for a large range of vacuum energy density decay trajectories, an early universe initially in a radiation dominated stage can enter an inflationlike stage and finally enter back into a radiation-dominated stage with the radiation energy density suffering no sharp alterations during this motion and with a post-inflationary temperature within a range consistent with observation and theory. This regime differs from the standard inflation regime where the radiation energy density quickly vanishes at the onset of inflation and then is quickly regenerated at the end of inflation in a short-time reheating era. We reemphasize that the solutions we have found are properties of the Einstein equations, independent of quantum field theory. We also reiterate that in the presence of non-negligible radiation, one need not be restricted to familiar near equilibrium quantum field theory methods in searching for dynamical models. This is not to preclude conventional treatments. In fact, despite our emphasis on the kinematic properties of the final answer and its model-independent origin, one should note that we motivated all our results from the conventional dynamical picture.

One cannot say without further investigation what the relevance of the present results are. It has been established by this study that sufficiently rapid expansion behavior is more general than only that found in the inflation regime. As with any generalization, there is always a danger that it is nothing more than a mathematical novelty offering no new physical insight. In the present case, this does not appear to be a correct statement. First, in light of the new list of options, there seems no special reasons that favor the supercooled limit to any of the other possibilities demonstrated here. Also, if the naturality principle carries the interpretation that any possibility not otherwise ruled out by observation nor theoretical common sense is a candidate solution, then again the present generalization has substance. Finally, in conjunction with warm inflation [3], a suggestive solution to the amplitude fluctuation problem presents itself. However, a dynamical explanation for large dissipation, which is needed for that scenario, requires investigation. On the other hand, cosmic string formation [27–29,7], which is typically considered a post-inflation mechanism for large scale structure or beginning at the end stage of a supercooled scenario [30], could be a possible mechanism within a large period of a big-bang-like inflation scenario.

The most interesting result from this study is the finding of an inflationlike regime of scale factor behavior that asymptotes to the radiation-dominated regime without a reheating stage. However, returning to the introductory comments, the solutions presented here have applicability also to the class of supercooled inflation scenarios. In addition, by decreasing any of the coefficients B in Eqs. (33) and (34), one can smoothly interpolate from the big-bang-like stage of inflation to the supercooled stage of exponential expansion. There are many possibilities suggested by our results from mild but long sustained accelerated expansion to the standard exponential inflation. With any of these, due to the presence of radiation, the dynamic explanation may require the range from familiar methods of finite temperature quantum field theory to a full nonequilibrium statistical mechanical treatment.[5] Both further theoretical research and experimental information is needed to narrow the possibilities.

There are two extreme points of view that one might adopt. One is that the very early universe mostly wants to be radiation dominated but just sneaks into a inflationlike phase for a little while. The other is that the very early universe has trouble containing radiation energy just after the initial singularity, and so copiously inflates until some heating or reheating mechanism finally stabilizes the radiation energy. The former is the mildest modification of preinflation era thinking and the latter reflects present thinking. For now, experiment and theory do not indicate a strong preference for either viewpoint. However, it is a useful exercise to view the problem from both extremes, since from either end the other looks like a remote limiting case. This in our opinion is symptomatic of a misunderstanding about the radiation energy content in the very early universe. As such, we believe theories that make no presumptions about the radiation content all during the early universe better represent the present status of experimental information about this time period. Thus, allowing for any of the possible scale factor behaviors derived here appears a more realistic starting point to further study.

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APPENDIX

This appendix contains some properties of the modified Bessel functions $K_\nu(z)$ and $I_\nu(z)$, which are useful for the results in the text. At small z, the asymptotic behaviors for $\nu=0$ are

$$I_0(|z|\rightarrow 0)\sim 1, \tag{A1}$$

[5]Some considerations for reheating, such as in [36], may be useful also here. In addition further examination could be made of the possible nonequilibrium potentials (or free energy functionals) that can form. An example from spinodal decomposition is [37], which starts from a master equation and attempts to deduce the free energy functional. This treatment was for a conserved order parameter, whereas for inflaton dynamics a similar treatment is needed for a nonconserved order parameter. Other approaches which may be useful are in [38]. Finally the works in [39] offer guidance in formulating the nonequilibrium problem in an expanding universe.

$$K_0(|z|\to 0)\sim -\ln|z|, \tag{A2}$$

and, for $\nu\neq 0$,

$$I_\nu(|z|\to 0)\sim \frac{(\frac{1}{2}|z|)^\nu}{\Gamma(\nu+1)}, \tag{A3}$$

$$K_{|\nu|}(|z|\to 0)\sim \frac{1}{2}\Gamma(\nu)\left(\frac{1}{2}|z|\right)^{-|\nu|}, \tag{A4}$$

where Eq. (A3) is valid for all ν except $\nu\neq -1,-2,\ldots$.

At large z, for all ν,

$$I_\nu(|z|\to\infty)\sim \frac{\exp(|z|)}{\sqrt{2\pi|z|}}, \tag{A5}$$

$$K_\nu(|z|\to\infty)\sim \sqrt{\frac{\pi}{2|z|}}\exp(-|z|). \tag{A6}$$

Finally recall that

$$K_\nu(z)=K_{-\nu}(z), \tag{A7}$$

and for a negative argument

$$I_\nu(-|z|)=e^{i\nu\pi}I_\nu(|z|), \tag{A8}$$

$$K_\nu(-|z|)=e^{-i\nu\pi}K_\nu(|z|)-i\pi I_\nu(|z|). \tag{A9}$$

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