Baryogenesis from cosmic strings at the electroweak scale

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We explore the viability of baryogenesis from light scalar decays after the electroweak phase transition. A minimal model of this kind is constructed with new CP-violating interactions involving a heavy fourth family. The departure from thermal equilibrium must come from topological defects such as cosmic strings, and we show that almost any mechanism for producing the cosmic strings at the electroweak scale results in a viable theory. Baryogenesis occurs in the fourth generation but the baryon number is later transported to the visible generations. This mechanism of indirect baryogenesis allows us to satisfy experimental limits on the proton lifetime while still having perturbative baryon number violation at low energies. The fourth family has very small mixing angles which opens the possibility of distinct observable signatures in collider experiments. [S0556-2821(97)07106-3]

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I. INTRODUCTION

The experimental bounds on the ratio of baryon excess to the entropy of the universe are [1]

$$\eta=(2-8)\times 10^{-11}. \tag{1}$$

Baryogenesis is an attractive explanation of the observed fact that baryons are more abundant than antibaryons in the universe. The conditions necessary for baryogenesis were spelled out by Sakharov nearly three decades ago [2]. The three conditions are (i) existence of baryon number violating interactions, (ii) C- and CP-violating processes, and (iii) a departure from thermal equilibrium. The earliest models of baryogenesis were based on baryon number and CP-violating processes of grand unified theories (GUT's). The necessary departure from thermal equilibrium was achieved by having superheavy bosons decay by slow interactions that make them overabundant (in comparison to their thermal distribution) in a rapidly expanding universe [3,4].

It was realized subsequently [5], however, that anomalous baryon number violation in the electroweak theory itself [6] could wipe out any baryon asymmetry formed at the GUT scale unless the density of baryons minus leptons ($B-L$) is not zero. Another difficulty with GUT scale baryogenesis is inflation. Inflation is needed to get rid of heavy monopoles formed during GUT scale symmetry breaking, but it also inflates away any baryons produced at that scale.

Since then several other mechanisms for baryogenesis have been proposed that produce baryons at or after electroweak phase transition. The most notable is the mechanism of electroweak baryogenesis [7]. This mechanism, however, requires a sufficiently strong first order phase transition [7,8]. At present the question of the order of the electroweak phase transition remains unanswered. If the electroweak phase transition were weakly first order or second order, then alternative mechanisms of baryogenesis at or below the electroweak scale would become very attractive.

Such models can be classified broadly by asking the two fundamental questions: (i) What is the source of B violation? (ii) What is the reason for departure from thermal equilibrium?

The question of CP violation is not included among the above two criteria. Models of baryogenesis must go beyond the standard model to incorporate new CP-violating interactions. However, we do not see any obvious way of classifying the new CP-violating sectors.

The usual source of B violation is one of the following two: (A) the nonperturbative B violation in the standard model; (B) perturbative B violation in an extended standard model.

We would broadly classify the reason for departure from thermal equilibrium into[1] (a) out of equilibrium decay of massive excitations and (b) the expanding or collapsing wall. [The term ''wall'' refers to a sharp change in the value of the Higgs field's vacuum expectation value (VEV).]

The mechanism of electroweak baryogenesis falls into the classes (A) and (b), respectively. The ''wall'' is the expanding wall of a bubble of true vacuum at the electroweak phase transition. If we are looking for alternatives to electroweak baryogenesis but want baryogenesis to occur after the electroweak phase transition, then we either need to give up one or both of (A) and (b) or look for a model that falls in the classes (A) and (b) but does not require a first order phase transition.

There are in fact two mechanisms in the literature that are similar to electroweak baryogenesis in that they fall in the classes (A) and (b), but the ''wall'' is provided not by an expanding bubble of true vacuum but by a collapsing cosmic string. In one of them [9], anomalous baryon number violating processes take place inside collapsing cosmic strings. In another case [10], electroweak strings (or Z strings [11]) with magnetic Z flux are needed. Both these models need new CP-violating sectors. The first one also requires that sphaleron effects be appreciable inside the ''core'' of the

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[1]Other mechanisms have been suggested. For example, see Ref. [12].

cosmic strings. All models of this kind are therefore sensitive to the structure and properties of the strings. In some cases they may require a light Higgs boson which implies a first order phase transition so the question of finding an alternative to electroweak baryogenesis remains open. The second model needs metastable Z strings, for which no viable extension of the standard model exists so far.

The limitations of the above models lead us to explore other alternatives to electroweak baryogenesis where we give up one or both of (A) and (b). The GUT scale baryogenesis from heavy boson decays in fact falls under the classes (B) and (a). These models automatically have the virtue of being insensitive to the order of the electroweak phase transition. The question arises, whether there are viable models falling under these classes that can produce baryons below the scale of the electroweak phase transition.

Seemingly, there are two hurdles to having this mechanism work at such low energy scales. The first is the constraint from proton decay. The stability of the proton implies that baryon number violating interactions must couple to the first generation quarks with a very small coupling. It seems that to make this ratio small, the baryon number violating interactions must involve superheavy particles (mass$>10^{16}$ GeV). The second obstacle is that for a massive excitation to decay out of equilibrium, its decay (and annihilation) rates should be smaller than the expansion rate of the universe. With $SU(3)_C \times SU(2)_W \times U(1)_Y$ couplings or with Yukawa couplings of the order of $10^{-3}-10^{-5}$, the decay (or annihilation) rates are usually greater than the Hubble expansion rate unless the universe is at temperatures as high as 10^{10} GeV. In Ref. [13] these two hurdles were overcome by a model where proton decay was forbidden by lepton number conservation and some heavy excitations were required to be $SU(3)_C \times SU(2)_W \times U(1)_Y$ singlets. The model needs colored scalars as well as a pair of massive Majorana fermions which have no gauge interactions. In this paper we seek an alternative particle decay mechanism with, what we believe, a simpler spectrum that may still have distinct experimental signatures.

Our motivation is in fact twofold. Firstly we would like to incorporate the merits of boson decay models in a model that can be completely described by an effective theory at the electroweak scale. By bringing down the scale of B and CP violation to the electroweak scale, one improves the testability of the theory compared to GUT scale models. We would also like to restrict the fermion content to simple sequential families and would not require any of them to be gauge singlets. Naturally the model should be viable even if the electroweak phase transition is second order.

Secondly, extensions of the standard model, such as models of top-color assisted technicolor [14], suggest the possibility of having symmetries under which quarks of different families transform differently. These symmetries must be broken above the electroweak scale to permit quark mixing at low energies. However, the existence of these symmetries opens up the possibility of having very small quark mixing angles. Thus the problem of proton decay that must be addressed in models of baryogenesis through perturbative B violation may find a new solution through small mixing angles (10^{-4}–10^{-6}) between new heavy quarks that have B-violating interactions and the light quarks that constitute the proton. If a viable model of baryogenesis makes use of this mechanism, the extra fermions needed in the CP-violating sector would simply consist of copies of the observed fermions, yet can have distinct signatures in collider experiments which would point strongly to an underlying theory with perturbative baryon number violation at low energies.

In this paper we show that the idea of small mixing angles described above does indeed provide a natural answer to the needs of any model of baryogenesis based on light boson decays. In our model, although baryogenesis happens through the decay of scalar bosons, the departure from thermal equilibrium can be obtained only by having topological defects like cosmic strings. This is a consequence of making the theory insensitive to the order of the electroweak phase transition [15]. However, we show that once the simplest B- and CP-violating sector incorporating the above ideas is constructed, the cosmic strings can be obtained as an inevitable bonus. In addition, the mechanism is not sensitive to the structure and properties of the cosmic strings.

The paper is organized as follows. In Sec. II we lay out the basic picture of how the model works. In particular we point out the various parts of the mechanism that one must check carefully to compute the baryon asymmetry generated in the end. The phenomenological viability of the model is then shown part by part in the following sections.

In Sec. III we describe the CP- and B-violating interactions and review the boson decay mechanism of [3] that forms the core of the present mechanism. We then consider phenomenological constraints on the mixing angles of the model from proton decay experiments. Finally we comment on the potentially observable experimental signatures predicted by the model.

In Sec. IV we present an estimate of the number density of the scalar bosons generated by the decay of cosmic strings in this model.

In Sec. V we consider the Boltzman's equations for the evolution of baryon number after the electroweak phase transition and show that for a particular range of parameters the baryon asymmetry generated immediately after the phase transition can survive until the present time.

II. BASIC MECHANISM

Our starting point is the boson decay mechanism. Simplest models of baryogenesis need at least two bosons with B- and CP-violating interactions. In the next section we will give a brief review of this mechanism. As mentioned earlier, for successful baryogenesis there must be departure from thermal equilibrium. However, our primary motivation is to avoid having superheavy scalar bosons. In the usual picture of cosmological evolution, TeV scale excitations do not go out of thermal equilibrium unless they are practically noninteracting. Therefore we are led to consider the following scenario. Suppose cosmic strings are formed at or slightly above the electroweak phase transition. Some of these strings will be in the form of loops which will subsequently decay into particles. If a large number of these loops decay into scalar bosons that are sufficiently heavy, then there may be an overabundance of the scalar bosons. The decay of the overabundant scalar bosons can then be the reason for the necessary

departure from thermal equilibrium.

We would like to construct the minimal B-violating sector of scalar bosons of the above kind. As shown in Ref. [3] there will be at least two of these bosons. If they have direct B-violating couplings to light quarks then we have a large width for proton decay unless the couplings are very small. But if the couplings are very small then we do not seem get sufficient B violation and there is no baryogenesis.

A possible solution of the above problem is to have a fourth generation of quarks. The fourth generation can also be used to include new CP violation which is a necessary ingredient of baryogenesis. The scalar bosons can decay into the new quarks through B-violating processes leading to baryogenesis. However, the fourth generation quarks must mix with the lighter generations for baryogenesis to occur in the visible generations. In the next section we show that it is possible to have small mixing angles ($\leqslant 10^{-4}$) between the B-violating fourth generation and the other three generations such that the width for proton decay is within experimental bounds.

Thus the basic picture is the following. Immediately after the electroweak phase transition there must be a network of decaying string loops that produce a thermally overabundant quantity of scalar bosons. These scalar bosons decay into a fourth family of quarks and leptons while generating a baryon asymmetry. Finally the fermions of the fourth family must decay into fermions of the other families. This is the minimal structure for baryogenesis from light scalar decays after the electroweak phase transition.

There are several cosmological and phenomenological constraints that the model must satisfy. Here we list the main constraints that will be shown to be satisfied by the model in the next sections.

(i) All couplings must be natural. We do not address the hierarchy problem due to scalars that also exists in the standard model.

(ii) Fermions of the fourth family must have experimentally viable masses.

(iii) There should not be a large width for proton decay. This implies small mixing angles between the fourth and the other families.

(iv) Cosmic strings should be naturally incorporated in the theory.

(v) All heavy particles (the scalar bosons and the members of the fourth family) must have large decay widths so that they decay within a cosmologically acceptable period. A long lived heavy particle may cause the problem of the overclosing of the universe. In particular, the fourth generation leptons must mix with the other three generations in order to decay. The lepton mixing must be large enough from this point of view.

(vi) The baryon asymmetry, once created, should not suffer a wash out from inverse decays.

Each of the above constraints is fairly restrictive. Indeed it is not obvious that (iii) and (v) can be satisfied simultaneously. However, as we show now, the minimal model built with the motivations that we have mentioned earlier satisfies all the above conditions for viability. The rest of the paper is devoted to the viability proof of this mechanism.

III. B VIOLATION AND A FOURTH GENERATION

A. B and CP violation

The minimal light scalar boson decay model must have (i) two scalar bosons X_1 and X_2 and (ii) a fourth family of quarks and leptons $(\nu_4,e_4)_L,\nu_{4R},e_{4R},(t',b')_L,t'_R,b'_R$.

Note that we have a right-handed neutrino in the fourth family which is required by experimental bounds on the number of light neutrino flavors (2.99 ± 0.04) in the standard model. We can have all the necessary B and CP violation with the above particle content if X_1 and X_2 are $\mathrm{SU}(2)_W$ singlets and $\mathrm{SU}(3)_C$ triplets and have hypercharge $Y=-2/3$. In addition to the coupling of all the fourth generation quarks and leptons to the standard Higgs boson, we can then have the following new Yukawa couplings:

$$L_Y=\cdots g_{l1}t'_Rb'_RX_l+g_{l2}\overline{b'_R}\ \overline{\nu_{4R}}X_l+g_{l3}\overline{t'_R}\ \overline{e_{4R}}X_l$$
$$+g_{l4}\overline{l_{4L}}\ \overline{q_{4L}}X_l+g_{l5}q_{4L}q_{4L}X_l+\mathrm{H.c.}+\cdots \qquad (2)$$

with $l=1$, 2 and q_4 representing the new quark doublet.

With these interactions the action has a nonanomalous $\Sigma_{i=1}^{3}(B-L)_i\times(B-L)_4$ symmetry, where $\Sigma_{i=1}^{3}(B-L)_i$ is the difference of the baryon and lepton numbers in the first three generations and $(B-L)_4$ is the difference of the baryon and lepton numbers in the fourth generation.

The CP violation in L_Y can be communicated to the ordinary quarks and leptons if the quarks and leptons of the fourth generation mix with the other generations. Note that lepton mixing is allowed since the neutrino in the fourth generation is massive. When these mixings are present, the quarks and leptons of the lighter generations must couple directly to the colored scalars X_1 and X_2 and the global nonanomalous symmetry is reduced to $(B-L)$. We require that terms in the action that violate $(B-L)_4$ by n units are suppressed by small couplings and mixings of the order of s^n where s is a small number $\sim(10^{-4}$–$10^{-6})$. One way to motivate this suppression is to think of this action to be a low energy limit of a theory that has a $(B-L)_4$ violating sector involving only massive fields. When these massive fields are integrated out, the resulting $(B-L)_4$ violation is suppressed by powers of their mass. In Appendix A we describe an explicit way of realizing this suppression by considering a simple theory with gauged $(B-L)_4-\Sigma_{i=1}^{3}(B-L)_i$. When this symmetry is broken, a low energy theory similar to the one described above is obtained. Other allowed couplings are gauge invariant quartic couplings between the new scalars and the standard Higgs boson. The suppression rule described above should apply to all these couplings.

The interactions (2) violate baryon and lepton numbers while preserving $B-L$. The interactions are in fact identical to the scalar boson interactions of Ref. [3], where it was shown that the out of equilibrium decays of X_1 and X_2 produce a baryon asymmetry. The amount of CP violation coming from the above terms was calculated in [3] and we briefly recapitulate the main results. At tree level, decays of X_1 and X_2 produce no CP violation since the cross section for $X_1\rightarrow$baryons is exactly equal to the cross section for $\overline{X_1}\rightarrow$antibaryons. The same statement applies to decays of X_2 and $\overline{X_2}$. However, at the one loop level there are several other processes contributing to the same decay modes. Con-

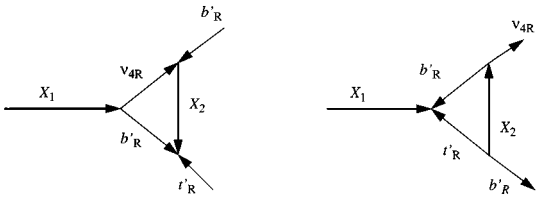

FIG. 1. One loop contributions to X_1 decays.

sider for instance the decay $X_1 \to \overline{t'_R} + \overline{b'_R}$ and $X_1 \to t'_R + \nu_{4R}$. The one loop diagrams contributing to the process have an internal X_2 propagator (Fig. 1). The interference of the tree order and one-loop diagrams produces a net CP violation for the decay of X_1 through these channels. The baryon number produced per decay through the channel "$X_1 \to t'_R + \nu_{4R}$" is [3]

$$\Delta B_R = \frac{4\,\mathrm{Im}(g_{11} g^*_{21} g^*_{12} g_{22})\,\mathrm{Im}(I)}{g_{11} g^*_{11}}, \tag{3}$$

where I is the relevant loop integral. If all the fermions have similar masses, the loop integrals in all the channels are the same. Taking all channels into account, one then has the expression

$$\Delta B = |g|^2 \mathrm{Im}(I)\,\epsilon, \tag{4}$$

where g is a typical Yukawa coupling and ϵ is a phase angle characterizing the average strength of the CP violation. The imaginary part of I is easily evaluated when all the fermions are massless:

$$\mathrm{Im}(I) = \left\{ 16\pi \left[1 - \rho^2 \ln\left(1 + \frac{1}{\rho^2} \right) \right] \right\}^{-1}, \tag{5}$$

where $\rho = m_{X_1}/m_{X_2}$ is the ratio of the masses of the two colored scalars. Clearly the CP violation is zero if the two scalars have the same mass. For comparable but unequal masses $\mathrm{Im}(I) \sim (10^{-2}$–$10^{-3})$. The value of $\mathrm{Im}(I)$ decreases if the fermions are not massless, but the order of magnitude estimate is unchanged.

The value of ϵ can be as large as 1 radian. Once a range of values for the masses and mixing angles are found, the allowed range of values for ϵ is fixed by the baryon to entropy ratio generated in the theory.

B. Masses, mixing angles, and proton decay

We shall see later that for baryogenesis to be successful it must be possible for the fourth generation quarks and leptons to decay into lighter quarks and leptons, while the reverse process must be prohibited. Therefore all fourth generation fermions need to be heavier than the Z, so that they can decay to a W or a Z and a light fermion. The mass difference between two members of an electroweak doublet can not be large by considerations of the ρ parameter. In this model, since all masses come from standard couplings to the Higgs bosons, there is no obstruction to satisfying this criterion. Masses of the scalars X_1 and X_2 need to be slightly higher than the electroweak scale. A mass of a few TeV seems to be necessary for sufficient baryogenesis.

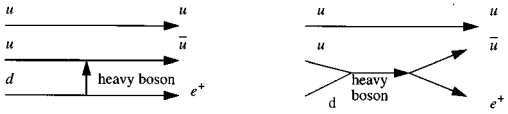

FIG. 2. Proton decay to pion and positron.

The mixing of the fourth generation quarks and leptons to members of the other three generations provides a way for the heavy quarks and leptons to decay. As we shall see later, with too small mixings baryogenesis never occurs in the visible sector. On the other hand if the mixings are too large, the decay width of the proton increases beyond experimental bounds. We show that there is an allowed region in the space of the mixing angles where baryogenesis is achieved while having acceptably small decay width for the proton.

In GUT theories where quarks of the first generation have baryon number violating couplings, the proton decays by processes shown in Fig. 2. To meet the experimental limit on the lifetime of proton ($>10^{33}$ years), the mass of the internally propagating GUT boson should be $>10^{16}$ GeV.

In the present model B-violating couplings involving the first three generations are down by a suppression factor. For instance, if the mixing angle between a fourth generation quark and a first generation quark is $\Theta_{q_4 q_1}$, a typical baryon number violating coupling involving first generation quarks will be down by two powers of this mixing:

$$L_Y = \cdots + g_{11} \Theta^2_{q_4 q_1} u_R d_R X_1 + \cdots . \tag{6}$$

We retain the coupling g_{11} to exhibit that the couplings now are smaller in comparison to the similar couplings involving only fourth generation quarks. The processes leading to proton decay still look like the one sketched in Fig. 2, except the baryon number violating vertices now have very small coupling constants. The amplitude for the process is

$$A \sim g^2 \Theta^3_{q_4 q_1} \Theta_{l_4 l_3} \frac{1}{m_X^2}, \tag{7}$$

where $\Theta_{q_4 q_1}$ is a typical mixing angle in the quark sector (between the first and fourth generations), $\Theta_{l_4 l_3}$ is the largest mixing angle in the lepton sector, and g^2 is the product of the typical coupling constants at the two baryon number violating vertices without the suppression factors.

Using $m_X \sim 10^3$ GeV, we find the following inequality for the mixing angles if experimental bounds on proton decay are to be satisfied:

$$|g^2 \Theta^3_{q_2 q_1} \Theta_{l_2 l_1}| \leqslant 10^{-28}. \tag{8}$$

In principle it is possible to have greater quark mixings if the lepton mixing is smaller. Also, $\Theta_{q_4 q_3}$ and $\Theta_{q_4 q_2}$ are likely to be greater than $\Theta_{q_4 q_1}$ by one and two orders of magnitude, respectively.

The spectrum of masses and the mixing angles allowed for the fourth family makes for interesting and distinct experimental signatures. The presence of a sequential fourth generation is not ruled out by experiments. The D0 Collaboration puts a limit of $m_{q_4} > 131$ GeV from the charged cur-

rent (CC) decay modes of the fourth generation quarks t' and b' ($t' \to b+W, b' \to t+W$). However if the b' is lighter than the top quark but heavier than the Z, then its dominant decay mode is the flavor-changing neutral current (FCNC) mode $b' \to b+Z$ [16]. A search for b' through this mode is currently feasible and one expects some experimental results in the near future (see Ref. [17]).

The ν_4 is also potentially observable through the tri-lepton decay mode $\nu_4 \to l\nu\mu$. The CERN e^+e^- collider LEP I bound on the heavy neutrino mass remains $m_{\nu_4} > 46$ GeV. Some restrictions on the mixing angles of the neutrino have been placed by the D0 Collaboration [18] which are consistent with the mixing angles allowed in the present model. As we shall show later, the mixing angles between the fourth and the third generation quarks and leptons can be in the range $10^{-13} \leqslant \Theta_{q_4 q_3} \approx \Theta_{l_4 l_3} \leqslant 10^{-4}$. Very small mixing angles will make the lightest fourth generation quark effectively stable inside the detector and may lead to peculiar signatures. Even if the mixing angles were measured to be closer to the upper bound of the above range, the obvious natural explanation for their smallness would be the existence of extra symmetries at scales higher than 1 TeV which forbid the mixing between the fourth generation and the other generations. Then, with the standard gauge and Higgs couplings, there is an extra symmetry in the form of conserved $B_4 - L_4$ above the TeV scale. For the small quark mixings in the fourth generation to exist, this symmetry must be broken at the TeV scale and a likely result would be perturbative violation of baryon number in the fourth generation. Therefore experimental signatures that are consistent with the mixing angles predicted in this model point very strongly to a mechanism of baryogenesis through perturbative B violation.

IV. COSMIC STRINGS AND BARYOGENESIS

With the B violation and CP violation in place, all we need to produce baryons is an out-of-equilibrium decay of the scalar bosons. To achieve this there has to be a mechanism for making the scalar bosons overabundant immediately after the electroweak scale. This can happen through the formation and decay of cosmic strings. The cosmic strings must form close to the electroweak phase transition. Strings formed much earlier will have a distribution with a correlation length that is too large and their number density will be too small to generate the baryon excess we see today. Since the symmetry breaking in the standard model does not produce any cosmic strings, extra broken gauge symmetries must exist. However, as described in Appendix A, the broken symmetry can be the approximately conserved $(B-L)_4 - \Sigma_{i=1}^{3}(B-L)_i$. If this is a gauged symmetry that is spontaneously broken at the electroweak scale, the smallness of the mixing angles and the presence of the cosmic strings can be explained simultaneously.

Moreover, besides being candidates for seeding galaxies, cosmic strings occur in many, independently motivated extensions of the standard model. For instance, the Aspon model [19] has an extra U(1) that breaks close to the electroweak scale. The motivation there is to provide an explanation for the small value of the vacuum angle θ in QCD. Supersymmetric extensions of the standard model have been proposed which attempt to resolve the μ problem of the minimal supersymmetric standard model (MSSM) and the cosmological solar neutrino problem and which have strings [20]. Both these models were considered recently [21] in the context of electroweak baryogenesis from cosmic strings. Top color models [14] are another class of models where a U(1) gauge group is broken close to the electroweak scale. In our case, almost regardless of motivation, any extension of the standard model that produces strings at the electroweak scale will work. The cosmic strings need not have a special structure or satisfy any particular requirements. The effective scalar whose vacuum expectation value (VEV) causes the formation of the cosmic strings will naturally have quartic couplings with the bosons X_1 and X_2. This ensures that X_1 and X_2 will be produced from the decay of cosmic string loops.

The scenario for baryogenesis is now very similar to [22] where the authors considered emissions of heavy particles from collapsing cosmic strings. If the heavy particles are produced at a scale which is sufficiently small compared to their mass, they may become overabundant and through CP-violating decays generate baryon number. In the present model we focus on the overabundant production of X_1 and X_2 particles from strings.

Immediately after the electroweak phase transition, the space will be filled with a criss-crossing of string network that looks like a random walk in three dimensions. The initial correlation length is $\psi_{t_c} \sim 1/T_c$. Numerical simulations indicate [23] that a large fraction ($\sim$80%) of the total string length resides in the infinite strings. The rest is in the form of loops which have a scale invariant distribution

$$\frac{dn}{dR} = R^{-4}, \tag{9}$$

where R is the characteristic size of the loops. The initial network has loops which decay rapidly. The infinite segments also generate more loops by frequent intercommutations. The net result is that the correlation length increases with time and the string network enters a scaling solution when the correlation length equals the horizon size [22].

In the literature the period from the time of the phase transition (t_c) to the time $t^* = (G\mu)^{-1} t_c$ is called the friction dominated period [24]. (G is Newton's constant and μ is the mass per unit length of the strings.) Loops produced at time t with $t_c < t < t^*$ immediately shrink to the radius

$$r_f(t) = G\mu m_{\rm Pl}^{1/2} t^{3/2}, \tag{10}$$

where $m_{\rm Pl}$ is the Planck mass ($\sim 10^{19}$ GeV). Below this scale, friction effects are subdominant and the loops shrink predominantly by processes like gravitational radiation and cusp annihilation. The shrinkage rate from gravitational radiation is [25]

$$\frac{dR}{dt} = -\gamma G\mu, \tag{11}$$

where γ is numerically determined to be about 10.

The time for a loop to shrink to a size which is of the order of its thickness due to purely gravitational effects is $t_G=(\gamma G\mu)^{-1}R$. In the present case $t_G\sim 10^{31}R\sim 10^{18}s$ even for the smallest loops ($R\sim\psi$). This is a very long time compared to the cosmological times of interest and gravitational effects are therefore completely negligible in our model.

Cusp annihilations, on the other hand, can occur at a much faster rate. The rate of shrinkage in this case can be modeled by [26]

$$\frac{dR}{dt}=-\frac{\gamma_\rho}{(R\eta)^{1/3}}. \tag{12}$$

The corresponding decay time is $t_{\rm cusp}=R(R\eta)^b/\gamma_\rho$. As shown in [22] this is less than one expansion time if the time of formation $t<(\gamma_\rho^2/G\mu)^{1/2}t_c$. For $G\mu\sim 10^{-32}$ it is safe to assume that this condition is satisfied for a long time. Thus in our model, the loops formed at time t immediately shrink to $R=r_f(t)$ after which they shrink by cusp annihilation to $R\sim\mu^{-1/2}$ within one expansion time. The lifetime of a loop formed at time t is then $\tau(t)\sim t$.

When the string loops have shrunk to a radius that is comparable to their thickness $\mu^{-1/2}$ (where μ is the mass per unit length of the string), nonlinearities in the scalar field potential will cause the entire loop to decay into elementary particles. It is from this final burst process that we can expect the heavy scalars X_1 and X_2 to be produced. Since $\mu\sim m_{X_1}^2, m_{X_2}^2$; the number N of X_1 and X_2 particles that we get from each loop is ~ 1.

The baryon number produced by string decays can now be evaluated by computing the number of string decays from the time t_c of the phase transition up to the present time. There are two kinds of loops to be taken care of; those formed at the time of phase transition ($t=t_c$) and those formed after the phase transition ($t>t_c$). Also, the baryon number produced from the decay of X_1 and X_2 bosons will be washed out unless baryon number violating processes are effectively frozen after the X_1 and X_2 decays. To compute the remnant baryon asymmetry we therefore need to evaluate the number of string decays that take place after $t=t_f$, where t_f is the freeze out time for baryon number violating interactions. Two cases, thus, arise.

(1) $t_f\approx t_c$. The number of decaying loops is the sum of loops produced at t_c and after t_c. The number density of heavy bosons produced from loops formed at t_c is obtained by integrating Eq. (9):

$$n_{t_c}\sim\frac{N}{\psi_{t_c}^3}\left(\frac{a_{t_c}}{a_t}\right)^3, \tag{13}$$

where the last factor takes care of the dilution of the number density due to the expansion of the universe. More cosmic string loops are formed after the phase transition as the coherence length increases with time and loops are chopped off from ''infinite'' sections of the string network. The loop production rate is related to the rate at which the coherence length increases by [22]

$$\frac{dn}{dt}=\frac{\nu}{\psi^4}\frac{d\psi}{dt}, \tag{14}$$

where ν is a constant of the order of unity. Integrating Eq. (14) we have

$$n_{t>t_c}=N\int_{\psi_{t_c}}^{\psi_t}\frac{\nu}{\psi'^4}\left(\frac{a_{t'}}{a_t}\right)^3 d\psi'\sim N\frac{\nu a_{t_c}^3}{(\psi_{t_c}a_t)^3}\approx n_{t_c}. \tag{15}$$

In order to have a large n_{t_c} we need to minimize ψ_{t_c}. Using $\psi_{t_c}\sim 1/T_c$ for strings produced at the electroweak scale, we get the ratio of the total number density of X_1 and X_2 to the entropy density to be

$$\omega_{\rm max}\sim\frac{n_{t_c}}{s}\approx\frac{N}{g_*}, \tag{16}$$

where s is the entropy density of the universe and $g_*\sim 100$ is the number of effectively massless degrees of freedom at the electroweak scale.

(2) $t_f>t_c$. Now the loops produced at $t=t_c$ decay within one expansion time and the resulting baryon number is washed out. Thus we need consider only the contribution from the loops decaying after t_f:

$$n=N\int_{\psi_{t_f}}^{\psi_t}\frac{\nu}{\psi'^4}\left(\frac{a_{t'}}{a_t}\right)^3 d\psi'\sim n_{t_c}\frac{\psi_{t_c}^3 a_{t_f}^3}{\psi_{t_f}^3 a_{t_c}^3}. \tag{17}$$

In the friction dominated period $\psi\sim t^{5/2}$ [22], therefore we have, $\omega=\omega_{\rm max}(t_c/t_f)^3$. Thus unless $t_f=t_c$, there is some damping in the production of baryons. In the next section we show that it is possible to have $t_f\approx t_c$ in our model.

V. APPROACH TO EQUILIBRIUM

The baryon asymmetry generated from X_1 and X_2 decays will evolve according to the Boltzman's equations. A large number of particles and interactions are relevant. For each species of particles one gets a Boltzman's equation. The equations are coupled integro-differential equations and for an accurate estimate of the baryon number, one must integrate them numerically. Useful analytical estimates can, however, be made by making simplifying approximations that reduce the number of degrees of freedom (and hence the number of Boltzman's equations) to a few.

The greatest simplification results from considering a single scalar boson X instead of X_1 and X_2. In the following we consider the most dominant interactions of this scalar boson. The relevant processes are (1) $X\bar{X}\to q\bar{q}, GG, ll$, etc., (2) $X\to q_4 q_4$, (3) $X\to\bar{q}_4\bar{l}_4$, (4) $q_4 q_4\to\bar{q}_4\bar{l}_4$, (5) $q_4\to q_3 W$, and (6) $l_4\to l_3 W$.

X is the generic colored scalar; q and l refer to generic

quarks of any generation while q_4 and l_4 refer to a quark and a lepton of the fourth generation. The W in interactions 5 and 6 represents the W bosons of the weak interactions. We denote gluons by G.

The processes 1 are dominant annihilation channels for the colored scalars. The processes 2,3,4 are baryon number violating processes. The processes 2,3 generate the baryon number while their inverse processes and processes 4 can wash out the produced baryon number. The processes 5 and 6 transport baryon and lepton number from the fourth generation to the third generation. We have omitted processes of the kind $q_4 \to \overline{q_4}\ \overline{l_4}\ \overline{q_4}$ to simplify the Boltzman's equations. These processes are prohibited if the quarks and leptons of the fourth generation have nearly equal masses. Inclusion of these processes does not change our main results in a significant way. The processes of baryogenesis and ''freeze out'' can now be treated as two distinct stages in the evolution of the baryon number.

A. X production and decay

We will assume that immediately after the phase transition the decays of cosmic string loops raise n_X, the number density of X, to about T_c^3. (Later we show that $n_X > 10^{-2} T_c^3$ may be sufficient for baryogenesis.) Since $M_X > T_c$, the X's are overabundant and their number will decrease rapidly through decays and annihilations.

The Boltzman's equation for X is [1]

$$\frac{dn_X}{dt} + 3Hn_X = -\int DP_{X,ij}[f_X |M(X \to ij)|^2 - f_i f_j |M(ij \to X)|^2] - \int DP_{X\overline{X},ij}[f_X f_{\overline{X}} |M(X\overline{X} \to ij)|^2 - f_i f_j |M(ij \to X\overline{X})|^2], \tag{18}$$

where

$$DP_{a_1 a_2 \ldots, b_1, b_2 \ldots} = \Pi_i \int \frac{d^3\mathbf{p}_{a_1}}{(2\pi)^3 2E_{a_i}} \Pi_j \int \frac{d^3\mathbf{p}_{b_j}}{(2\pi)^3 2E_{b_j}} \times (2\pi)^4 \delta^4(\Sigma_i p_{a_i} - \Sigma_j p_{b_j})$$

is the phase space volume element, H is the Hubble constant, f_i is the phase space density of species i and $|M(ij \ldots \to \ldots kl)|^2$ is the matrix element squared for the process $ij \ldots \to \ldots kl$. The matrix element is summed over initial and final state color, spin, and flavor degeneracies. The number density n_X is the number density of all X particles regardless of color.

When the final state particles are light, Eq. (18) reduces to [27]:

$$\frac{dn_X}{dt} + 3Hn_x = -[n_X - n_X^{\rm eq}]\langle \Gamma_X \rangle - [n_X^2 - (n_X^{\rm eq})^2] \times \langle \sigma(X\overline{X} \to ij) \rangle, \tag{19}$$

where $\langle \Gamma_X \rangle$ is the total thermally averaged decay width of X averaged over initial color degeneracies, $n_X^{\rm eq}$ is the equilibrium density of X and $\langle \sigma(XX \to ij) \rangle$ is the thermally averaged cross section for XX annihilations (averaged over initial state color degeneracies).

The decay modes for X are: $X \to q_4 q_4, \overline{q_4}\ \overline{l_4}$. The dominant annihilation channels are: $XX \to qq, GG$. Since X is very heavy we can use the zero temperature decay width for Γ_X to good approximation. The same is true for the annihilation cross section with an appropriate value for the c.m. energy. In Appendix B we have computed these rates. Our results are:

$$\langle \Gamma_X \rangle \approx \frac{1}{2\pi} |g|^2 M_X, \tag{20}$$

$$\langle \sigma(X\overline{X} \to ij) \rangle \approx \frac{(4\pi\alpha_{qcd})^2}{9\pi M_X^2}, \tag{21}$$

where g is a typical coupling of X to q_4 and l_4.

Because X is overabundant, $n_X \gg n_X^{\rm eq}$. The reduction in n_X is initially dominated by annihilation processes. The decays overtake annihilations when the X number density reaches the critical value

$$n_{X\rm crit} = \frac{9|g|^2}{2} \frac{M_X^3}{(4\pi\alpha_{\rm qcd})^2}. \tag{22}$$

From this point onwards, the annihilations are quenched out and most of the Xs decay through the CP-violating processes producing baryons.

B. Freeze out

Once a large number of X decays have taken place, there is an excess of baryons (and leptons) over antibaryons (antileptons) in the fourth generation. Baryon number violating processes like inverse decays ($q_4 q_4, \overline{q_4}\ \overline{l_4} \to X$) or $2 \to 2$ processes ($q_4 q_4 \to \overline{q_4}\ \overline{l_4}$) will tend to wash out this excess. Decays to Ws and third generation quarks and leptons will also reduce the baryon excess in the fourth generation (although preserving the total baryon excess).

To see if a freeze out can occur, we look at the Boltzman's equation for q_4:

$$\frac{dn_{q_4}}{dt}+3Hn_{q_4}=-3\int DP_{q_4q_4,\overline{q_4}\,\overline{l_4}}[f_{q_4}f_{q_4}|M'(qq\to\overline{q_4}\,\overline{l_4})|^2-f_{\overline{q_4}}f_{\overline{l_4}}|M'(\overline{q_4}\,\overline{l_4}\to qq)|^2]$$
$$+2\int DP_{X,q_4q_4}[f_X|M(X\to q_4q_4)|^2-f_{q_4}f_{q_4}|M(q_4q_4\to X)|^2]$$
$$+\int DP_{\overline{X},q_4l_4}[f_{\overline{X}}|M(\overline{X}\to q_4l_4)|^2-f_{q_4}f_{L_4}|M(q_4l_4\to\overline{X})|^2]$$
$$-\int DP_{q_4,q_3W}[f_{q_4}|M(q_4\to q_3W)|^2-f_{q_3}f_W|M(q_3W\to q_4)|^2]. \tag{23}$$

$|M'(q_4q_4\to\overline{q_4}\,\overline{l_4})|^2$ and $|M'(\overline{q_4}\,\overline{l_4}\to q_4q_4)|^2$ have primes on them to indicate that the matrix elements do not include **s** channel contributions in which the intermediate X is on shell (a physical particle), since these contributions have already been included in the decay and the reverse decay terms. The full matrix element (squared), $|M(q_4q_4\to q_4l_4)|^2$, is related to $|M'(q_4q_4\to q_4l_4)|^2$ by

$$|M(q_4q_4\to\overline{q_4}\,\overline{l_4})|^2=|M'(q_4q_4\to\overline{q_4}\,\overline{l_4})|^2+\frac{\pi}{M_X\Gamma_X}\times\delta[p_{q_4}^2(1)+p_{q_4}^2(2)-m_X^2]\times|M(q_4q_4\to X)|^2|M(X\to\overline{q_4}\,\overline{l_4})|^2. \tag{24}$$

Following [27] we will simplify Eq. (23) by parametrizing the CP violation of the system in the following manner. We define the matrix elements M_0 and the numbers η and $\overline{\eta}$ by

$$|M(X\to q_4q_4)|^2=|M_0|^2(1+\eta)/2=|M(\overline{q_4}\,\overline{q_4}\to\overline{X})|^2,$$
$$|M(X\to\overline{q_4}\,\overline{l_4})|^2=|M_0|^2(1-\eta)/2=|M(q_4l_4\to\overline{X})|^2,$$
$$|M(\overline{X}\to\overline{q_4}\,\overline{q_4})|^2=|M_0|^2(1+\overline{\eta})/2=|M(q_4q_4\to X)|^2,$$
$$|M(\overline{X}\to q_4l_4)|^2=|M_0|^2(1-\overline{\eta})/2=|M(\overline{q_4}\,\overline{l_4}\to X)|^2. \tag{25}$$

We have used CPT and unitarity to relate the squared matrix elements. Note that all matrix elements are summed over initial and final state spin and color degeneracies. The CP violation parameters η and $\overline{\eta}$ are related to ΔB by the relation

$$\Delta B=(\eta-\overline{\eta})/4. \tag{26}$$

Since all the quarks and leptons are in thermal equilibrium we have

$$f_{q_4}(p)=e^{-E/T+\mu_1/T}\approx e^{-E/T}\left(1+\frac{b}{2}\right),$$
$$f_{\overline{q_4}}(p)=e^{-E/T-\mu_1/T}\approx e^{-E/T}\left(1-\frac{b}{2}\right),$$
$$f_{L_4}(p)=e^{-E/T+\mu_2/T}\approx e^{-E/T}\left(1+\frac{l}{2}\right),$$
$$f_{\overline{L_4}}(p)=e^{-E/T-\mu_2/T}\approx e^{-E/T}\left(1-\frac{l}{2}\right), \tag{27}$$

where μ_1 and μ_2 are chemical potentials related to the (approximately) conserved baryon and lepton numbers in the fourth generation. In expanding the exponents we have used the fact that baryon and lepton excesses are small. From Eq. (27) we obtain

$$\sum_{g_i=1}^{2}\sum_{s=1}^{2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}f_{q_4}(p)f_{\overline{q_4}}(p)=B_4,$$
$$\sum_{g_i=1}^{2}\sum_{s=1}^{2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}f_{l_4}(p)f_{\overline{l_4}}(p)=L_4, \tag{28}$$

which relate b and l to the density of excess baryons and leptons, respectively. The sums are over the flavor and spin indices in the fourth generation.

One can now use Eqs. (24) and (27) in Eq. (23) and express products such as $f_{q_4}(p_1)f_{q_4}(p_2)$ as $f_X^{\rm eq}(p_1+p_2)(1+2b)$ in decay and inverse decay terms. Subtracting Boltzman's equation for the antiquarks from the equation for the quarks one gets the equation for the baryon number in the fourth generation:

$$\frac{dB_4}{dt}+3HB_4\approx\frac{1}{2}(n_X-n_X^{\rm eq})(\eta-\overline{\eta})\langle\Gamma(X\to q_4q_4,\overline{q_4}\,\overline{l_4})\rangle-\frac{1}{4}(3B_4+L_4)n_\gamma\langle\sigma(q_4q_4\to\overline{q_4}\,\overline{l_4})\rangle-\frac{1}{6}\frac{n_X^{\rm eq}}{n_\gamma}\left[\frac{1}{2}(B_4+L_4)\langle\Gamma(X\to q_4l_4)\rangle+B_4\langle\Gamma(X\to q_4q_4)\rangle\right]-\frac{1}{2}(B_4-B_3)\times\langle\Gamma(q_4\to q_3W)\rangle, \tag{29}$$

where $\langle\Gamma(X\to\overline{q_4}\,\overline{l_4})\rangle$ and $\langle\Gamma(X\to q_4q_4)\rangle$ are averaged over initial state degeneracies and summed over final state degeneracies, $\langle\sigma(q_4q_4\to\overline{q_4}\,\overline{l_4})\rangle$ is summed over both initial and final state degeneracies and $n_\gamma\approx T_c^3$ is the photon

number density. The correct sign for the term $-n_X^{\rm eq}(\eta-\overline{\eta})\langle\Gamma(X\to q_4q_4,\overline{q_4}\,\overline{l_4})\rangle$ is obtained only after including the CP-violating part of $|M'(q_4q_4\to\overline{q_4}\,\overline{l_4})|^2-|M'(q_4q_l\to q_4q_4)|^2$ [27].

The various terms in the r.h.s. of Eq. (29) are readily interpreted. The first term is the driving term for baryogenesis. It becomes small as $n_X\to n_X^{eq}$ and plays no role in freeze out. The second term comes from inverse decays and the third term comes from $2\to2$ baryon number violating processes. These two terms can potentially cause a washout. The last term is the rate at which baryon number is drained out of the fourth generation into the third generation. We have ignored similar drainage terms to other generations because the mixing angles are smaller by one or two orders of magnitude.

The ''washout terms'' can be ineffective only if they are smaller than the Hubble dilution term $3HB_4$. We must, therefore, have (using $L_4\approx B_4$)

$$3HB_4>B_4n_\gamma\langle\sigma(q_4q_4\to\overline{q_4}\,\overline{l_4})\rangle,$$

$$3HB_4>\frac{1}{6}\frac{n_X^{\rm eq}}{n_\gamma}B_4\langle\Gamma(X\to q_4q_4,\overline{q_4}\,\overline{l_4})\rangle. \tag{30}$$

C. Range of parameters

At the weak phase transition $H\sim3\times10^{-16}T_c$. Using our estimates of the decay widths and cross sections from Appendix B we can reduce the conditions (30) to

$$10^{-16}>\frac{4|g|^4}{3\pi k^4},$$

$$10^{-16}>\frac{|g|^2}{36\pi}k^{5/2}e^{-k}, \tag{31}$$

where $k=m_X/T_c$.

When these inequalities are satisfied, there is no significant washout of the baryon number and the net baryon to entropy ratio is

$$\eta_B=\frac{n_{X\rm crit}\Delta B}{g_*T_c^3}\sim10^{-4}|g|^4k^3\epsilon. \tag{32}$$

We have taken $(4\pi\alpha_{qcd})^2=2$ and $g_*=100$. Two parameters in Eq. (32) are bounded from above. The maximum value of $\epsilon\sim1$ and the maximum value of $n_{X\rm crit}\sim T_c^3$. When these bounds are taken into consideration, the inequalities (31) and (1) yield the following range of values for k and $|g|^2$:

$$25<k<80,$$

$$10^{-6}<|g|^2<10^{-3.5}. \tag{33}$$

Picking some value for k further constraints the range for $|g|^2$ and vice versa. Realistic values for η_B can be obtained with these values. For instance, taking $k=30$, $|g|^2=10^{-5}$ we obtain

$$\eta_B\sim2.7\times10^{-10}\epsilon. \tag{34}$$

For ϵ close to 1, this falls within the range given by Eq. (1). Note that for $k\sim25$, $m_X\sim6.25$ TeV. This value is to be compared with the mass of the smallest string loops. Indeed if the mass per unit length of the strings is μ, a string loop of size $R\sim1/T_c$ has a mass of about $\beta R\mu$, where β is a numerical factor that takes into account the fact that loops are not exactly circular. Numerical simulations indicate that $\beta\sim9$ [28]. If $\mu\sim(\text{TeV})^2$ then the mass of the smallest loops is about 36 TeV. Also note that η_B is insensitive to the number N of Xs produced per string loop as long as $NT_c^3>n_{X\rm crit}$. For $|g|^2=10^{-5}$, $k=30$ we have $n_{X\rm crit}\sim10^{-1}T_c^3$.

The range of allowed values for the mixing angles is much wider. From Eqs. (8) and (33) we can see that

$$|\Theta_{q_4q_1}^3\Theta_{l_4l_3}|\leqslant10^{-22}. \tag{35}$$

Now consider the decay of the fourth generation baryons and leptons. The decay widths are $\sim(|g_W\Theta_{l_4l_3}|^2T_c)/4\pi$ (where g_W is the weak gauge coupling). A lower bound on the mixing angles is obtained by requiring that the decays happen before nucleosynthesis. This means that the decay time should at most be $1s$. The corresponding limit on the mixing angles is: $\Theta_{l_4l_3},\Theta_{q_4q_3}\geqslant10^{-13}$. If we also require that $\Theta_{l_4l_3}\approx\Theta_{q_4q_3}\approx10\Theta_{q_4q_2}\approx100\Theta_{q_4q_1}$, then we have

$$10^{-13}\leqslant\Theta_{q_4q_3}\leqslant10^{-4}. \tag{36}$$

VI. CONCLUSIONS

We have shown that baryogenesis from the decay of light scalar bosons is viable even at energies as low as the electroweak scale. This is interesting, since perturbative violation of baryon number at low energies seems incompatible with the observed stability of the proton. However the minimal model of B, C, and CP violation involving light scalar bosons can naturally have very small mixings between new heavy quarks with B violating interactions and the lighter quarks which shields the proton from B-violating effects. Other phenomenological and cosmological constraints are shown to be satisfied. In particular the small mixings between the fourth family and the other families is shown to be sufficient for quarks in the fourth generation to decay into quarks of the lighter generations in a cosmologically acceptable time.

Some members of the fourth family can be as light as 100 GeV. They can also be relatively long lived (decay time $\sim10^{-5}s$). It would be interesting to explore signatures of their existence in future experiments. In particular, if the lighter quark in the fourth family, the b', happens to be lighter than the top quark then its dominant decay mode is the FCNC mode $b'\to b+Z$. One expects this decay mode to be explored in collider experiments of the near future. Signatures associated with new quarks with small mixing angles ($\Theta_{q_4q_3}\leqslant10^{-4}$) as predicted by this model, would seem to point strongly toward a mechanism of baryogenesis through perturbative B violation at the electroweak phase transition.

The scalar bosons must be at least 25 times heavier than the electroweak scale. In our model they are produced copiously from the decay of loops of cosmic strings immediately after the electroweak phase transition. We show in Appendix

A that it is possible to extend the standard model so that the smallness of the new mixing angles and the presence of the cosmic strings are justified simultaneously.

Variations of this model can be conceived. The only necessary ingredients are topological defects like cosmic strings and heavy baryons. The model has all the advantages of baryogenesis models where baryogenesis occurs *after* the electroweak phase transition including compatibility with the usual models of inflation. It is also viable as a baryogenesis model even if the electroweak phase transition is a second order phase transition.

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APPENDIX A

The model described in Sec. II has an anomaly-free global $(B-L)_4$ symmetry in addition to the usual $B-L$ when the mixing angles between the fourth and lighter generation are put equal to zero and there is no coupling between the lighter generation quarks and leptons and the colored scalars X_1 and X_2. When the mixings and couplings mentioned above are nonzero but small, $(B-L)_4$ is broken weakly. The smallness of these parameters is, therefore, not unnatural in the technical sense. Below, we describe a way of explicitly realizing this scenario as an effective low energy limit of a theory where the small $(B-L)_4$ breaking terms come from operators of dimension 5 or higher and are suppressed by a large mass scale.

We first define two U(1) symmetries:

$$U=\sum_{i=1}^{4}(B-L)_i=B-L; \quad V=(B-L)_4-\sum_{i=1}^{3}(B-L)_i. \tag{A1}$$

The first is the usual $B-L$ which we keep as a global unbroken symmetry of our theory. At a scale much higher than the electroweak scale one can conceive of a theory where V is a gauged symmetry. Consider for instance extending the model in Sec. III by first taking away all terms violating V and then gauging V. In order that V be realized as a weakly broken symmetry in the low energy theory we can introduce a scalar field X_3 which is a singlet under all gauge symmetries except V. A VEV for X_3 breaks V.

The effective action for this theory can have a dimension $4+n$ operator of the kind $(1/M^n)\overline{q_4}hq_3X_3^n$ if the V charge of X_3 is $2/3n$. M is a large mass suppressing this operator (h is the standard Higgs boson). When X_3 gets a VEV, V is broken and we get the dimension 4 mixing term $(\langle X_3\rangle/M)^n\overline{q_4}hq_3$. The smallness of the mixing results from suppression due to small coupling constants and factors of $1/4\pi$ as well as the ratio $\langle X_3\rangle/M$.

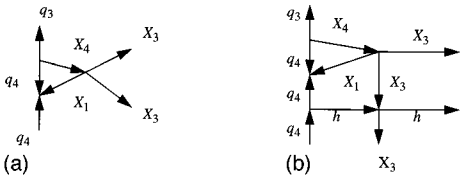

FIG. 3. (a) Dim. 6 operator $\overline{q_4}Dq_3X_3$. (b) Dim. 6 operator $\overline{q_4}hq_3X_3X_3$

To actually get this operator we must introduce new fields and interactions that couple the fourth and the third generations. Since we are not interested in solving the hierarchy problem, scalars are cheap. We can introduce three more, X_4, X_5, and X_6 with the following Yukawa couplings

$$L_{\text{Yukawa}}=g'(q_4q_3X_4+\overline{q_4}\,\overline{l_3}X_5+\overline{q_3}\,\overline{l_4}X_6), \tag{A2}$$

where q_i and l_i denote quarks and leptons of the ith generation. We have chosen the same coupling g' for all the terms. The V charges of X_4, X_5, and X_6 are 0 and 4/3 and $-4/3$, respectively. All are color triplets and $SU(2)_W$ singlets. By suppressing the flavor and helicity indices we imply that all possible gauge invariant couplings are included in (A2).

Now suppose X_4, X_5, and X_6 have masses of the order of M. Integrating them out one may obtain the dimension 6 operator $q_3q_3q_3l_3$. This operator, a potentially dangerous candidate for proton decay, is not induced by renormalization at one loop and must be suppressed by at least a factor of $(g^2g'^4)/(16\pi^3M^2)$, where g is one of the couplings in Eq. (2). If we choose $|g|'^2<|g|^2\approx10^{-5}$, the proton decay problem is avoided for $M^2>10^9$ GeV^2.

The mixing between the fourth and third generations can occur through the operators $\overline{q_4}Dq_3X_3^2$ (where D is the gauge-covariant derivative) or $\overline{q_4}hq_3X_3^2$. Figures 3(a) and 3(b) show typical leading order contributions to these operators. Clearly the V charge of X_3 is $\frac{1}{3}$. The mixing is suppressed by the small number $[|g||g'|/(4\pi)^2](\langle X_3\rangle/M)^2$. For $\langle X_3\rangle\approx10^3$ GeV, $M\approx10^{4.5}$ GeV and $|g||g'|\approx10^{-5}$, the mixing is $\Theta_{q_4q_3}\approx10^{-10}$. A similar mixing is obtained in the lepton sector. Mixings of this order are certainly small enough for the viability of our model. The mixings are also large enough for the baryon number in the fourth generation to be transported to the lighter generations in a cosmologically acceptable time. Indeed with this mixing the decay time of the fourth generation quarks and leptons is $\sim10^{-5}$ s which corresponds to a temperature of about 1 GeV. By choosing $\langle X_3\rangle\sim10^3$ GeV we also get the much needed cosmic strings at the electroweak scale as a bonus.

APPENDIX B

The dominant annihilation modes of X are $X\overline{X}\rightarrow GG,q\overline{q}$. Annihilations to leptons, W's, Z's, Higgs bosons, and photons have much smaller rates because they are down by small coupling constants while the annihilation rates to quarks and gluons are enhanced markedly by large color factors. We estimate the dominant annihilation processes in perturbation theory. The lowest order Feynman diagram contributing to the annihilation to quarks is shown in Fig. 4(a). There is a single gluon exchange. The squared amplitude, after sum-

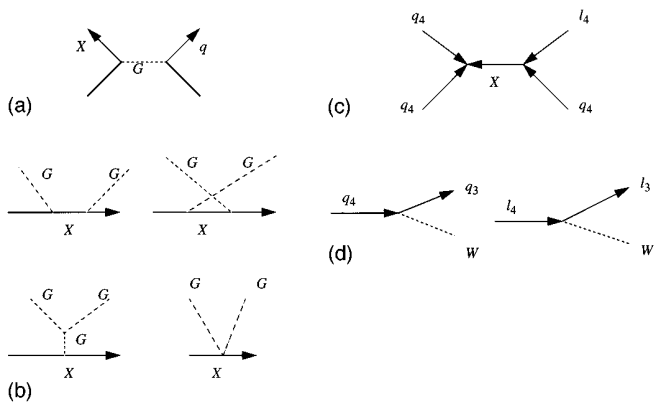

FIG. 4. (a) X annihilation to quarks. (b) X annihilation to gluons. (c) 2 to 2 baryon number violating process. (d) Decays of q_4 and l_4.

ming over final state (spin, color, and flavor) degeneracies and averaging over initial state degeneracies, is

$$|M(X\to q\overline{q})|^2=\frac{32}{9}\,(4\pi\alpha_{qcd})^2\,\frac{|\vec{p}|^2}{p_0^2}\,\sin^2\theta, \tag{B1}$$

where p is the 4 momentum of the X and θ is the scattering angle in the c.m. frame. We have taken all the quarks to be massless. Feynman diagrams corresponding to annihilation to gluons are shown in Fig. 4(b). The invariant squared amplitude is

$$|M(X\overline{X}\to GG)|^2=\frac{(4\pi\alpha_{qcd})^2}{9}\left[12\left(\frac{\vec{p}^2}{p_0^2}\right)^2+6\right](1+\cos^2\phi), \tag{B2}$$

where ϕ is the angle between the two final state gluons in the rest frame of one of the incoming particles and p is the four-momentum of the X in the c.m. frame. The large numerical factors in (B1) and (B2) come from color and flavor sums. Note that we have four generations of quarks now.

The thermally averaged cross section can be approximated by the zero temperature cross section with $|\vec{p}|\approx p_0\approx M_X$. We then obtain

$$\langle\sigma(X\overline{X}\to GG,q\overline{q})\rangle\approx\frac{(4\pi\alpha_{qcd})^2}{9\pi M_X^2}. \tag{B3}$$

The $X\overline{X}$ annihilation rate is to be compared with the baryon number violating decay rates of X's. The lowest order Feynman diagrams for these decays are shown in Fig. 4(c). The squared amplitude for the decay to two quarks (after summing over final state degeneracies and averaging over initial state degeneracies) is

$$|M(X\to q_4q_4)|^2\approx 4|g|^2M_X^2. \tag{B4}$$

Once again we have taken the final state particles to be massless. The corresponding rate for a decay to an antiquark and an antilepton is exactly the same (larger flavor factor compensates for the smaller color factor):

$$|M(X\to\overline{q_4}\,\overline{l_4})|^2\approx 4|g|^2M_X^2. \tag{B5}$$

Approximating the thermally averaged decay width by the zero temperature decay width we get

$$\langle\Gamma(X\to q_4q_4,\overline{q_4}\,\overline{l_4})\rangle\approx\frac{1}{2\pi}\,|g|^2M_X\,. \tag{B6}$$

Figure 4(c) shows the Feynman diagram corresponding to the leading order contribution to the process $q_4q_4\to\overline{q_4}\,\overline{l_4}$. The invariant squared amplitude for a typical process is

$$|M(q_4q_4\to\overline{q_4}\,\overline{l_4})|^2\approx 96|g|^4\,\frac{k_0^2}{M_X^2}, \tag{B7}$$

where k_0 is the c.m. energy of a q_4 in the initial state. The thermally averaged cross section (summed over all initial and final state degeneracies) is approximated by a zero temperature cross section with k_0 set equal to T_c. The result is

$$\langle\sigma(q_4q_4\to\overline{q_4}\,\overline{l_4})\rangle\approx\frac{12|g|^4T_c^2}{\pi M_X^4}. \tag{B8}$$

Finally the $q_4\to q_3W$ and $l_4\to l_3W$ decays [Fig. 4(d)] have the widths

$$\langle\Gamma(q_4\to q_3W)\rangle=\frac{1}{8\pi}\,\Theta^2_{q_4q_3}|g_W|^2T_c\,,$$

$$\langle\Gamma(l_4\to l_3W)\rangle=\frac{1}{8\pi}\,\Theta^2_{l_4l_3}|g_W|^2T_c\,, \tag{B8}$$

where we have made the approximation that the final state particles are much lighter than the decaying particle and averaged over initial state degeneracies. We have also approximated the thermal averaging by taking all masses and momenta in the final expression to be of order T_c. Even with the limitations of the above approximations, the expressions in Eq. (B8) are useful as order of magnitude estimates of these decay rates.

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