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Particle production and symmetry restoration in collisions of vacuum bubbles

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In first-order inflation a phase transition is completed by the collisions of expanding true-vacuum bubbles. If bubble collisions produce large numbers of soft scalar particles carrying quantum numbers associated with a spontaneously broken symmetry, then symmetry restoration may occur in a ''preheating'' phase in a manner similar to symmetry restoration in the preheating phase of slow-roll inflation. Since bubble collisions lead to inhomogeneities, there is the possibility of inhomogeneous symmetry restoration where restoration occurs only in the regions of wall collisions. [S0556-2821(97)06606-X]

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To a very good approximation the universe was in local thermodynamic equilibrium (LTE) for nearly all of its early development. However, there should have been brief, but important, departures from LTE. These excursions from equilibrium left an imprint on the universe. Examples of non-LTE phenomena include baryogenesis, nucleosynthesis, freeze-out of a massive particle species, decoupling of matter and radiation, production of topological or nontopological defects in cosmological phase transitions, inflation, and reheating after inflation. In fact, it may be argued that nearly all of early-universe cosmology is the study of departures from LTE. It is commonly believed that many of the current issues in cosmology require an understanding of the nontrivial dynamics in the approach to equilibrium in the early universe. Nevertheless, despite its immense relevance, only very recently has substantial effort been devoted to a detailed understanding of nonequilibrium phenomena in the early universe.

The nonequilibrium process of interest in this study is the phenomenon of particle production after inflation. There are many varieties of inflation models, but all have an early period of rapid expansion of the universe where the Robertson-Walker scale factor ''accelerates'' (i.e., $\ddot{a}>0$). At the end of the accelerated-expansion phase the radiation density of the universe is effectively zero, and the universe must be ''reheated.''[1]

In ''slow-roll'' (sometimes referred to as ''chaotic'') inflation models [1], the universe after inflation was dominated by the energy density contained in the coherent motion of a scalar field known as the *inflaton*, whose potential energy density was responsible for the accelerated expansion. Reheating in slow-roll inflation involves conversion of this coherent scalar-field energy density into into a thermal distribution of radiation. In a simple scenario of reheating, the inflaton field coherently oscillated about the minimum of its potential until the age of the universe was equal to the lifetime of the inflaton, then the inflaton decayed, and the decay products thermalized.

Recent investigations into the nonlinear quantum dynamics of scalar fields have implications for reheating after slow-roll inflaton [1]. It was first pointed out by Linde, Kofman, and Starobinski [2] (see also [3]) that the scenario by which the energy density in coherent oscillations of the inflaton field is converted to radiation may differ significantly from the above picture, which considered only the linear evolution in time of the inflation field. Quantum nonlinear effects may lead to an extremely effective dissipational dynamics and explosive particle production in even the simplest self-interacting theory where single particle decay is kinematically forbidden. It is possible that almost all of the energy stored in the form of coherent inflaton oscillations at the end of inflation is released after only a few oscillation periods. The energy is released in the form of inflaton decay products, whose occupation number is extremely large, and have energies much smaller than the temperature that would have been obtained by an instantaneous conversion of the inflaton energy density into radiation.

Since it requires several scattering times for the low-energy decay products to form a thermal distribution, it is rather reasonable to consider the period in which most of the energy density of the universe was in the form of the nonthermal quanta produced by inflaton decay as a separate cosmological era. This is generally referred to as the ''preheating'' epoch [2].

The phenomenon of symmetry restoration during the preheating era has been first investigated by Kofman, Linde, and Starobinski [4] and, subsequently, by Tkachev [5] in the framework of typical chaotic inflationary models. Symmetry restoration processes during the nonequilibrium stage of preheating may be very efficient with important implications for baryogenesis in grand unified theories (GUT's) [6] and supersymmetry breaking [7].

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[1]Of course ''reheated'' may be somewhat of a misnomer since there is no guarantee that the universe was hot before inflation.

In first-order inflation models (generally, any model in which inflation is completed by a strongly first-order phase transition, e.g., the extended inflationary scenario proposed by La and Steinhardt [8]; for a review of first-order inflation models, see [9]) the universe was dominated by scalar-field vacuum energy as in slow-roll inflation, but inflation was terminated by the nucleation of true vacuum bubbles. At the end of first-order inflation most of the energy density of the universe was contained in the bubble walls. Reheating in first-order inflation does not involve the coherent oscillations of the inflation field, but is instigated by the collisions of bubble walls, which converted the bubble-wall tension into individual quanta of the scalar field, which then decayed into normal particles, which eventually scattered and formed a thermal distribution.

The aim of the present paper is to suggest another situation in which symmetry restoration can occur efficiently out of equilibrium, namely during the preheating era subsequent to first-order inflation.

As discussed above, the basic idea of reheating in first order is similar in some respects to chaotic inflation: energy initially stored in a coherent scalar field must be converted into radiation. However, in first-order inflation this releasing of energy takes place through a number of steps involving both classical and quantum processes, and a rich phenomenology associated with these scenarios can arise. For example, it has been suggested that gravitational waves [10,12], black holes [11,13], and the baryon asymmetry [14] may have been produced during the phase transition. Whether or not such phenomena actually occur depends in part on the details of reheating. For instance, in the baryogenesis scenario of Ref. [14] it is important to know if the only source of heavy grand unified theory (GUT) bosons is from primary particles produced in the bubble wall collisions which, in turn, depends crucially whether the GUT symmetry is restored after bubble collisions, i.e., on the value of the reheat temperature, $T_{\rm RH}$.[2]

We shall show, however, that similar to what occurs in the chaotic inflationary scenarios, the details of symmetry restoration may turn out to be rather independent of $T_{\rm RH}$, and may in fact be quite complicated, with the symmetry restored in some regions of the universe, but not others.

In order to keep the discussion as general as possible, we will not specify any particular first-order inflaton model, but describe the salient features of the inflaton potential in terms of three parameters (λ_σ, σ_0, and ϵ). We denote the inflaton field by σ, which has a potential of the general form suitable to provide for a first-order phase transition. (Table I lists the fields and their interactions.) The potential will be described in terms of a dimensionless coupling constant λ_σ, a dimensionless constant ϵ that determines the splitting between false-vacuum and true-vacuum potential energy densities, and a mass scale σ_0, which also plays the role of the vacuum expectation value when the symmetry is broken. The mass of the field will be $\lambda_\sigma^{1/2}\sigma_0$, and the difference in energy density between the false and true vacuum states will be denoted as $\Delta V = \epsilon\lambda_\sigma\sigma_0^4$. The parameter ϵ must be less than unity for sufficient inflation to occur. This also implies that the bubbles of true vacuum formed in the transition will be ''thin-wall'' bubbles, with wall thickness much smaller than the radius.

TABLE I. Three fields are involved in our consideration: the inflaton field σ; the field χ into which the domain walls disperse; and ϕ, a field whose spontaneously broken symmetry may be restored by the χ background. In some models χ and σ may be the same field.

Interaction	Potential term
Inflaton self-interaction:	$V_0(\sigma)=\lambda_\sigma(\sigma^2-\sigma_0^2)^2$
Inflaton false-vacuum energy density:	$\Delta V=\epsilon\lambda_\sigma\sigma_0^4$
χ-ϕ interaction:	$V_{\chi\phi}=g\phi^2\chi^2$
ϕ self-interaction:	$V_0(\phi)=\lambda_\phi(\phi^2-\phi_0^2)^2$

From the few parameters λ_σ, ϵ, and σ_0, one can find all the information required about the bubbles formed in the phase transition. For instance, in the thin-wall approximation, the size of a nucleated bubble is given by $R_c \sim (\epsilon\lambda_\sigma^{1/2}\sigma_0)^{-1}$. Bubbles with a radius smaller than this critical size will not grow, whereas bubbles larger than the critical size are exponentially disfavored. Another crucial parameter is the thickness of the wall separating the true-vacuum region inside from the false-vacuum region outside the bubble: $\Delta \sim (\lambda_\sigma^{1/2}\sigma_0)^{-1}$. The ratio of the bubble-wall thickness to its size is $\Delta/R_C \sim \epsilon$, which is much lees than unity if the thin-wall approximation is adopted. Finally, the energy per unit area of the bubble wall is $\eta \sim \lambda_\sigma^{1/2}\sigma_0^3$.

When a bubble wall forms, false-vacuum energy is transformed into bubble-wall energy, with the wall energy initially in the form of static surface energy. As the bubbles expand converting false vacuum to true vacuum, more and more of the wall energy becomes kinetic as the walls become highly relativistic. Numerical simulations [11,12,] demonstrate that during collisions the walls oscillate through each other, dispersing the kinetic energy at a rate determined by the frequency of these oscillations.

Although the particles produced in the initial collisions of the walls may play an interesting role in preheating and reheating, in the following we will concentrate on the implications of the particles produced by the potential energy density of the bubble walls. Indeed, when they have slowed after a few oscillations, the bubble wall surface energy is dissipated by bubble collisions into particles of typical energy determined by the wall thickness.

Bubble walls can be envisaged as coherent states of inflaton particles, so that the typical energy of the products of their decays is simply the mass of the inflaton. This energy scale is just equal to the inverse thickness of the wall. Note that by the time the walls actually disperse, most of the kinetic energy has been radiated away, so the walls are probably no longer relativistic [11,12].

Let us envision the collision of two plane-parallel domain walls. The potential energy per unit area of the bubble walls is given by $\eta \sim \lambda_\sigma^{1/2}\sigma_0^3$. Taking the mean energy of the particles produced in the bubble wall collisions to be of order of the inverse thickness of the wall, $E \sim \Delta^{-1}$, the mean number per area of particles produced from the potential energy in

[2]The reheating temperature $T_{\rm RH}$ is usually defined as the temperature of the universe when the thermal spectrum of radiation was first obtained after inflation.

the collisions is $N \simeq \eta/E \sim \lambda_\sigma^{1/2}\sigma_0^3\Delta$.

Let us now assume that the particles are spread out a distance d from the region of the wall collision. If we approximate the particle density as uniform out to a distance d, then the particle number density within the region is simply

$$n = N/d \sim \lambda_\sigma^{1/2}\sigma_0^3\Delta/d \sim \sigma_0^2/d. \tag{1}$$

In the limit that the walls are spherical with radius R and the collision products instantly fill the bubble interior, then the factor of d in Eq. (1) should be replaced by R.

Eventually the products of bubble-wall collisions will be redistributed throughout the bubble interior and thermalized. If we assume that thermalization is instantaneous, the reheating temperature is found by imposing $\rho_R = (g_* \pi^2/30) T_{\rm RH}^4 = \Delta V$, where g_* is the effective number of degrees of freedom in all the species of particles formed in the thermalization processes. Using $\Delta V = \epsilon \lambda_\sigma \sigma_0^4$ results in a reheat temperature of $T_{\rm RH} \sim g_*^{-1/4}\epsilon^{1/4}\lambda_\sigma^{1/4}\sigma_0$. Let us now assume that the typical energy of the particles produced through bubble collisions is smaller than $T_{\rm RH}$, i.e., $\Delta^{-1} \lesssim T_{\rm RH}$, which translates into the condition (taking $g_* \sim 100$) $\lambda_\sigma \lesssim 10^{-1}\epsilon$. If this condition is satisfied, then a period is required for equilibration, namely for particles to scatter from energies approximately equal to Δ^{-1} to a thermal distribution of temperature $T_{\rm RH}$. In addition, since the bubbles were originally empty, homogenation is not instantaneous, and requires a time at least as long as the light travel time across a bubble. If either of these two time scales is sufficiently long, we may consider the time interval during which particles do not have a homogeneous thermal distribution function as a separate epoch: the preheating era.

As a first approximation, during the preheating period the distribution function of the created particles can be chosen of the form [5]

$$f(\omega) = A\,\delta(\omega - E), \tag{2}$$

where $E = \Delta^{-1}$ and the constant A may be fixed by computing the number density of particles, $n = (2\pi)^{-3}\int d^3p\; f(p)$, and setting it equal to the estimate given in Eq. (1). Of course, A has mass dimension one.

Let us now imagine that particles χ are produced in the bubble wall collisions and are charged under some symmetry group, so that their mass m_χ depends upon some scalar field ϕ as $m_\chi^2(\phi) = m_0^2 + g\phi^2$.[3] Here, g represents a combination of numerical factors and a coupling constant. As a simple example we might assume that the ϕ-dependent mass originates from a potential term of the form $V_{\chi\phi} = g\phi^2\chi^2$.

As opposed to large-angle scattering processes, forward-scattering processes do not alter the distribution function of the particles traversing a gas of quanta, but simply modify the dispersion relation. This remains true also in the case of a nonequilibrium system. Forward scattering is manifest, for example, as ensemble and scalar background corrections to the particle masses. Since the forward scattering rate is usually larger than the large-angle scattering rate responsible for establishing a thermal distribution, the nonequilibrium ensemble and scalar background corrections are present even before the initial distribution function, Eq. (2), relaxes to its thermal value. These considerations allow us to impose $\omega^2 = \mathbf{p}^2 + m_\chi^2(\phi)$ as the dispersion relation for the particles created by bubble collisions.

We cannot use the imaginary-time formalism to determine the effective potential for the scalar field ϕ during the nonequilibrium preheating period since in the nonequilibrium case there is no relation between the density matrix of the system and the time evolution operator, which is of essential importance in the formalism. There is, however, the real-time formalism of thermofield dynamics, which suits our purposes [15]. The contribution of the particles created by bubble collisions to the one-loop effective potential of the scalar field ϕ can be written as

$$\Delta V(\phi) = \int \frac{d^3p}{(2\pi)^3}\int_\infty^{\omega_{\mathbf{p}}(\phi)} d\omega\; f(\omega). \tag{3}$$

The first integration in ω must be done treating ω as a free parameter and setting $\omega_{\mathbf{p}}(\phi) = \sqrt{\mathbf{p}^2 + m_\chi^2(\phi)}$. By making use of Eq. (2), one obtains

$$\Delta V(\phi) = -A\int \frac{d^3p}{(2\pi)^3}\,\theta[E - \omega_{\mathbf{p}}(\phi)] \simeq \frac{n}{E}\,[m_\chi^2(\phi) - E^2]. \tag{4}$$

Since we are interested in the ϕ-dependent part of the potential, we can ignore the nE term and the factor of m_0^2 in $m_\chi^2(\phi)$, and write the potential for the nonequilibrium configuration as $\Delta V(\phi) = B_{\rm NE}\phi^2$, where $B_{\rm NE} = gn/E$. A similar expression was obtained by Tkachev in Ref. [5], using the definition of the effective potential as (the negative of) the pressure of the system, and assuming that the number of particles does not change on time scales of interest as the field ϕ evolves.

We now use $n = \sigma_0^2/d$ from Eq. (1), and $E \sim \Delta^{-1}$, to obtain $B_{\rm NE} \sim g\sigma_0^2\Delta/d$. Of course d will depend upon the details of the model and the complexities involved in the completion of the phase transition. But it is reasonable to expect, at least initially, that d is of order Δ, so let us write $d = \xi\Delta$. Of course as the bosons diffuse into the bubble interior ξ will change in time, so we expect ξ to grow and eventually to become much greater than unity. But initially, at least, ξ should not be too much larger than unity. In terms of ξ, we may express $B_{\rm NE}$ as $B_{\rm NE} \sim g\sigma_0^2/\xi$.

Now there are two things left to do. First, we will determine the conditions under which the nonequilibrium contributions to the effective potential can restore the symmetry, and then determine the criterion for the nonequilibrium effects to be more important than the equilibrium effects obtained after reheating.

Let us take the ϕ tree-level self-interaction potential to be of the form $V_0(\phi) = \lambda_\phi(\phi^2 - \phi_0^2)^2$. The symmetry will be restored (i.e., $\phi = 0$ will be a stable minimum) if $d^2V/d\phi^2$ evaluated at $\phi = 0$ is positive, where now V includes the sum of the tree-level potential and the one-loop correction, $V = V_0 + \Delta V$. Symmetry restoration will occur due to nonequilibrium effects if $-\lambda_\phi\phi_0^2 + B_{\rm NE} > 0$. This translates into a bound on ξ for symmetry restoration:

[3] Of course by ϕ^2 and χ^2 we mean the appropriate sum over the members of the group representation.

$$\frac{g}{\lambda_\phi}\frac{\sigma_0^2}{\phi_0^2} > \xi. \tag{5}$$

We can imagine three interesting limits depending upon the magnitude of the left-hand side (LHS) of this inequality. Since we expect ξ always to be greater than one, if the LHS is less than unity we would expect nonequilibrium effects *never* to cause symmetry restoration. If the LHS is greater than one but not very large, then one might expect temporary restoration of symmetry around the regions of bubble collisions. Then as ξ starts to grow as the bubble interior is filled, the symmetry will be broken when the inequality is violated. Finally, the LHS may be so much greater than unity that the symmetry is restored even after the bubble interiors are filled.

Of course the symmetry may also be broken after reheating by thermal effects. This can be seen by calculating the ϕ-dependent term in the one-loop effective potential obtained by assuming that the system is in LTE at temperature $T_{\rm RH}$. Including $V_{\chi\phi} = g\phi^2\chi^2$, in the high-temperature limit the one-loop thermal corrections lead to $\Delta V(\phi,T) \sim gT^2\phi^2 + \lambda_\phi T^2\phi^2$. If we write $\Delta V(\phi,T_{\rm RH}) = B_{\rm EQ}\phi^2$, with $B_{\rm EQ} = (g+\lambda_\phi)T_{\rm RH}^2$, then $B_{\rm NE}$ plays a role in nonequilibrium transitions similar to that played by $B_{\rm EQ}$ for thermal transitions.[4]

Symmetry will be restored after reheating if $-\lambda_\phi\phi_0^2 + B_{\rm EQ} > 0$, or expressing this as a limit to $T_{\rm RH}$: $T_{\rm RH}^2 > \lambda_\phi\phi_0^2/(g+\lambda_\phi)$. Now we know $T_{\rm RH}$ in terms of the parameters of the inflation potential, so we may express the criterion for symmetry restoration after reheating as

[4]Thus we see that so far as symmetry restoration is concerned, in the presence of the soft bosons left behind in the debris of wall collisions, a scalar field behaves as if it was in LTE at an effective temperature $T_{\rm eff}^2 = B_{\rm NE}/(g+\lambda_\phi) \sim \sigma_0^2 g/[\xi(g+\lambda_\phi)]$.

$$\frac{g+\lambda_\phi}{\lambda_\phi}\frac{\sigma_0^2}{\phi_0^2} > \sqrt{g_*/\epsilon\lambda_\sigma}. \tag{6}$$

The condition for symmetry restoration in preheating, Eq. (5), and the condition for symmetry restoration in reheating, Eq. (6), are most easily contrasted in the limit $g > \lambda_\phi$. In that limit

$$\frac{g}{\lambda_\phi}\frac{\sigma_0^2}{\phi_0^2} > \begin{cases} \xi & \text{(symmetry restoration during preheating),} \\ \sqrt{g_*/\epsilon\lambda_\sigma} & \text{(symmetry restoration during reheating).} \end{cases} \tag{7}$$

Depending upon the parameters, it is possible to have restoration during *both* preheating and reheating, during *neither* preheating or reheating, or during one and not the other. Of particular interest might be the case where restoration occurs only during preheating when ξ is not too large. Then the effects of inhomogeneous symmetry restoration will not be erased during reheating.

In conclusion, symmetry restoration may well occur in the preheating phase following first-order inflation. Unlike symmetry restoration in the preheating phase of chaotic inflation, the restoration may be inhomogeneous after first-order inflation. The basic point is that the phase-space density of bosons created in wall collisions is greatest in regions of wall interactions. One may imagine situations where restoration occurs among the debris of wall collisions, but not in the initially empty interior of the bubbles. In such a case, the subsequent symmetry breaking restoration might result in creation of topological defects if the region of wall interactions is large enough to contain these defects.

Cosmological implications of this possibility require further study.

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[1] A. Linde, Phys. Lett. **129B**, 177 (1983).

[2] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); A. D. Linde, in *Current Topics in Astrofundamental Physics, The Early Universe*, Proceedings of the Chalonge Erice School, edited by N. Sánchez and A. Zichichi, NATO ASI Series C, Vol. 467, 1995 (Kluwer Academic, Norwell, MA, 1995); in *Particles, Strings and Cosmology*, Proceedings of the Johns Hopkins Workshop on Current Problems in Particle Theory, Baltimore, Maryland, 1995, edited by J. Bagger (World Scientific, Singapore, 1996), Report No. gr-qc 19508019 (unpublished); L. A. Kofman, in *Relativistic Astrophysics: A Conference in Honor of Igor Novikov's 60th Birthday*, edited by B. Jones and D. Markovic (Cambridge University Press, Cambridge, England, in press).

[3] D. Boyanovsky and H. J. de Vega, Phys. Rev. D **47**, 2343 (1993); D. Boyanovsky, D.-S. Lee, and A. Singh, *ibid.* **48**, 800 (1993); D. Boyanovsky, H. J. de Vega, and R. Holman, *ibid.* **49**, 2769 (1994); in *Proceedings of the Second Paris Cosmology Colloquium*, Observatoire de Paris, June 1994, edited by H. J. de Vega and N. Sanchez (World Scientific, Singapore, 1995), pp. 125–215; Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D **51**, 5438 (1995); A. Dolgov and K. Freese, *ibid.* **51**, 2693 (1995); D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee, and A. Singh, *ibid.* **51**, 4419 (1995); D. Boyanovsky, M. D'Attanasio, H. J. de Vega, R. Holman, and D.-S. Lee, and A. Singh, Report No. hep-ph/9505220 (unpublished); D. Boyanovsky, M. D'Attanasio, H. J. de Vega, R. Holman, and D.-S. Lee, Phys. Rev. D **52**, 6805 (1995); M. Yoshimura, Prog. Theor. Phys. **94**, 873 (1995); H. Fujisaki, K. Kumekawa, M. Yamaguchi, and M. Yoshimura, Phys. Rev. D **53**, 6805 (1996); S. Yu. Khlebnikov and I. Tkachev, Phys. Rev. Lett. **77**, 219 (1996); D. Boyanovsky, H. J. de Vega, R. Holman, and J. F. J. Salgado, Phys. Rev. D **54**, 7570 (1996); L. A. Kofman, A. D. Linde, and A. A. Starobinsky, Report No. SU-ITP-96-39, hep-ph/9608341 (unpublished); S. Yu. Khle-

bnikov and I. Tkachev, Report No. PURD-TH-96-06, hep-ph/9608458 (unpublished).
[4] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **76**, 1011 (1996).
[5] I. Tkachev, Phys. Lett. B **376**, 35 (1996).
[6] E. W. Kolb, A. D. Linde, and A. Riotto, Phys. Rev. Lett. **77**, 4290 (1996).
[7] G. W. Anderson, A. D. Linde, and A. Riotto, Phys. Rev. Lett. **77**, 3716 (1996); G. Dvali and A. Riotto, Phys. Lett. B **388**, 247 (1996).
[8] D. La and P. J. Steinhardt, Phys. Rev. Lett. **62**, 376 (1989).
[9] E. W. Kolb, in *The Birth and Early Evolution of our Universe*, Nobel Symposium 79, edited by J. S. Nilsson, B. Gustafsson, and B.-S. Skagerstam (World Scientific, Singapore, 1991), p. 199.
[10] M. S. Turner and F. Wilczek, Phys. Rev. Lett. **65**, 3080 (1990).
[11] S. W. Hawking, I. G. Moss, and J. M. Stewart, Phys. Rev. D **26**, 2681 (1982).
[12] R. Watkins and L. Widrow, Nucl. Phys. **B374**, 446 (1992).
[13] J. D. Barrow, E. J. Copeland, E. W. Kolb, and A. R. Liddle, Phys. Rev. D **43**, 984 (1991).
[14] J. D. Barrow, E. J. Copeland, E. W. Kolb, and A. R. Liddle, Phys. Rev. D **43**, 977 (1991).
[15] H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermo Field Dynamics and Condensed States* (North-Holland, Amsterdam, 1982); See also P. Elmfors, K. Enqvist, and I. Vilja, Phys. Lett. B **326**, 37 (1994).