Damping rate of quasiparticles in degenerate ultrarelativistic plasmas

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We compute the damping rate of a fermion in a dense relativistic plasma at zero temperature. Just above the Fermi sea, the damping rate is dominated by the exchange of soft magnetic photons (or gluons in QCD) and is proportional to $(E-\mu)$, where E is the fermion energy and μ the chemical potential. We also compute the contribution of soft electric photons and of hard photons. As in the nonrelativistic case, the contribution of longitudinal photons is proportional to $(E-\mu)^2$, and is thus nonleading in the relativistic case. [S0556-2821(97)01505-1]

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The properties of quasiparticles in ultrarelativistic (UR) plasmas have attracted much attention in the recent past [1]. A crucial property of a quasiparticle is its decay (or damping) rate: a quasiparticle which propagates in a plasma is not stable, as it undergoes collisions with the other particles of the plasma, and the very concept of a quasiparticle makes sense only if its damping rate is small enough.

The damping rate of an electron propagating in a nonrelativistic (NR) plasma was computed almost 40 years ago by Quinn and Ferrell [2,3]. At first sight, this damping rate is infinite, due to the singular behavior of the Rutherford cross section at small angles. However, Quinn and Ferrell were able to obtain a finite result because the Coulomb interaction is screened in a plasma (Debye screening), and in the case of a degenerate plasma, they showed that the damping rate is proportional to $(\varepsilon_p-\varepsilon_F)^2$, where $\varepsilon_p=p^2/2m$ is the NR kinetic energy and ε_F the Fermi energy, when ε_p is slightly larger than ε_F. The damping rate remains finite for a nonzero temperature T: only the value of the Debye screening length is modified.

It is interesting to extend the calculation of the damping rate to the case of relativistic plasmas: one may have in mind either electromagnetic (QED) plasmas, such as found in white dwarves or in the core of nascent neutron stars, or chromodynamic (QCD) plasmas such as the quark-gluon plasma which is believed to be formed for large enough values of the temperature T and/or the chemical potential μ. The NR results are not easily transposed to the relativistic case because the exchange of magnetic (or transverse) photons in QED or of magnetic gluons in QCD becomes important, while in the NR case it is suppressed by powers of $(v/c)^2$ with respect to the exchange of electric (or longitudinal) gauge bosons, and is usually neglected. These magnetic photons, or gluons, give rise to severe infrared (IR) divergences which are not easily cured because there is no static magnetic screening analogous to Debye screening in the electric case, but only a weaker dynamical screening [4]. In many cases, this dynamical screening is sufficient to remove the IR divergences [5,6], but it has been known for some time that it cannot solve easily the IR problem of the damping rate [7], at least for nonzero T. In a recent paper [8], Blaizot and Iancu were nevertheless able to derive a finite result in the $T\neq 0$, $\mu=0$ case, by using a nonperturbative approach to resum the leading divergencies. However they also discovered that the decay law is no longer exponential.

In this work, we address the problem of computing the damping rate of quasiparticles in degenerate UR plasmas. For the sake of definiteness, we treat the case of a QED plasma, but our results may be trivially extended to the QCD case by substituting to the QED coupling e the QCD coupling g, and by taking into account some color group factors. In this computation, the basic physical idea is that the collisions of the charged quasiparticle with the particles in the plasma are governed by photon exchange, and that one must take into account the fact that the photon propagator is dressed by the interactions. Actually, this approach is a particular case of the resummation method proposed by Braaten and Pisarski [9], which relies on the properties of the so-called ''hard thermal loops'' [9,10] or ''hard dense loops'' in the degenerate case [11]. Braaten and Pisarski pointed out the importance of a hierarchy of scales, based on the existence of a ''hard scale'' of order T (or μ), and a ''soft scale'' of order eT (or $e\mu$), with $e\ll 1$. When soft scales are involved, one must use dressed (or resummed) propagators and vertices instead of the bare ones in a perturbative expansion. An important feature of the resummation method is that it leads to gauge independent results, due to the gauge independence of the hard thermal (or dense) loops.

Our main result is that, in the case $T=0$, $\mu\neq 0$, dynamical screening is able to cure the IR divergences of the damping rate due to magnetic photon exchange in UR plasmas; however, in contrast to the NR case, the damping rate is dominated by magnetic exchange and is proportional to $(E-\mu)$, where E is the relativistic energy of the quasiparticle, while electric photon exchange gives a contribution proportional to $(E-\mu)^2$, as in the NR case, which may be, in fact, obtained as a low velocity limit of the relativistic one. Note, however, that, by convention, energies and chemical potentials differ by the rest mass of the particle in the NR

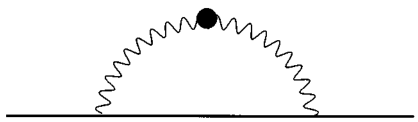

FIG. 1. Resummed one-loop self-energy of the fermion.

and relativistic cases. Note also that we use a system of units where $\hbar = c = k_B = 1$, and that we follow closely the notation of [1].

Let us now proceed to the derivation of our result. We assume that the quasiparticle energy E is hard (this is automatically ensured in the case of a degenerate plasma). The damping rate $\gamma(E)$ is given by the imaginary part of the quasiparticle self-energy Σ [12]; more precisely:

$$\gamma(E) = -\frac{1}{4E}\mathrm{Tr}[\mathrm{Im}\Sigma(p_0 + i\eta, \mathbf{p})(\not{P} + m)]\Big|_{p_0=E}, \tag{1}$$

where m is the electron mass, $E = (p^2 + m^2)^{1/2}$, $\eta \to 0^+$, and we have used the, by now, standard notation $P_\mu = (p_0, \mathbf{p})$; the lowest order graph for Σ is drawn in Fig. 1. We are mainly interested in the contribution of soft photons, so that the electron-photon vertex and the electron propagator may be replaced by the bare ones [1,9]: only the photon propagator need be dressed.

We perform the calculation of Σ in the imaginary time formalism; then the (free) electron propagator is given by

$$S_f(i\omega_n, \mathbf{k}) = \int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\frac{(\not{K} + m)\rho_f(K)}{k_0 - i\omega_n - \mu} \tag{2}$$

with

$$\rho_f(K) = 2\pi\varepsilon(k_0)\,\delta(k_0^2 - E_k^2). \tag{3}$$

In Eq. (2), $\omega_n = \pi(2n+1)T$ is a fermionic Matsubara frequency and $\varepsilon(k_0) = k_0/|k_0|$. The (resummed) photon propagator $\Delta_{\mu\nu}(Q)$ is written in the Coulomb gauge

$$\Delta_{\mu\nu}(Q) = \delta_{\mu 0}\delta_{\nu 0}\Delta_L(Q) + (\delta_{ij} - \hat{q}_i\hat{q}_j)\Delta_T(Q), \tag{4}$$

where the spectral representations of Δ_T and Δ_L read

$$\Delta_L(i\omega_s, q) = \int_{-\infty}^{\infty}\frac{dq_0}{2\pi}\frac{\rho_L(q_0, q)}{q_0 - i\omega_s} - \frac{1}{q^2}, \tag{5a}$$

$$\Delta_T(i\omega_s, q) = \int_{-\infty}^{\infty}\frac{dq_0}{2\pi}\frac{\rho_T(q_0, q)}{q_0 - i\omega_s}. \tag{5b}$$

In Eqs. (4) and (5), $\hat{q}_i = \mathbf{q}_i/|\mathbf{q}|$ and $\omega_s = 2\pi sT$ is a bosonic Matsubara frequency. The explicit expressions of the longitudinal and transverse spectral functions ρ_L and ρ_T are found by taking the imaginary parts of Δ_L and Δ_T; this gives, for massless electrons,

$$\rho_L(q_0, q) = 2\,\mathrm{Im}\Delta_L(q_0 + i\eta, q) = 2\,\mathrm{Im}\frac{-1}{q^2 + 3\omega_P^2[1 - (x/2)\ln(x+1)/(x-1)]}, \tag{6a}$$

$$\rho_T(q_0, q) = 2\,\mathrm{Im}\Delta_T(q_0 + i\eta, q) = 2\,\mathrm{Im}\frac{-1}{(q_0 + i\eta)^2 - q^2 - (3/2)\omega_P^2\{x^2 + [x(1-x^2)/2]\ln(x+1)/(x-1)\}}, \tag{6b}$$

where $x = (q_0 + i\eta)/q$, $\omega_P = M/\sqrt{3}$ is the plasma frequency which is related to the Debye mass M given by

$$M^2 = \frac{e^2}{\pi^2}\left(\mu^2 + \frac{\pi^2 T^2}{3}\right). \tag{7}$$

The diagram in Fig. 1 is now evaluated in the imaginary time formalism $[P = (i\omega_n, \mathbf{p})]$

$$\Sigma(P) = e^2 T\sum_s \int\frac{d^3q}{(2\pi)^3}\gamma_\mu S_f[i(\omega_n - \omega_s), \mathbf{p} - \mathbf{q}] \times \gamma_\nu \Delta_{\mu\nu}(i\omega_s, \mathbf{q}). \tag{8}$$

The sum over Matsubara frequencies is easily performed when one plugs in Eq. (8) the spectral representations (2) and (5) of the propagators. Taking the imaginary part of Σ after the analytical continuation $i\omega_n + \mu \to p_0 + i\eta$ to Minkowski space, and taking the trace in Eq. (1), one finds, for the damping rate, with $\mathbf{k} = \mathbf{p} - \mathbf{q}$,

$$\gamma(E) = \frac{\pi e^2}{E}\int\frac{d^3q}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{dk_0}{2\pi}\rho_f(k_0)\int_{-\infty}^{\infty}\frac{dq_0}{2\pi}[1 + n(q_0) - \tilde{n}(k_0)]\,\delta(E - k_0 - q_0)\{[p_0k_0 + \mathbf{p}\cdot\mathbf{k} + m^2]\rho_L(q_0, q) + 2[p_0k_0 - (\mathbf{p}\cdot\hat{\mathbf{q}})(\mathbf{k}\cdot\hat{q}) - m^2]\rho_T(q_0, q)\}. \tag{9}$$

In Eq. (9), n and $\tilde{n}$ are Bose-Einstein and Fermi-Dirac distribution functions ($\beta = 1/T$):

$$n(q_0) = \frac{1}{e^{\beta q_0} - 1}, \quad \tilde{n}(k_0) = \frac{1}{e^{\beta(k_0 - \mu)} + 1}. \tag{10}$$

Equation (9) could have also been derived from kinetic theory [14], using standard identities between the distribution functions (10). The case $k_0 < 0$ [see Eq. (3)] corresponds in kinetic theory to $e^+ - e^-$ annihilation, which is not IR singular and is even absent in the $T = 0$ case. We thus concentrate on the $k_0>0$ case, which corresponds in kinetic theory to $e^- - e^-$ scattering. Equation (9) holds for any value of m, T, and μ. For the sake of simplicity, we shall restrict ourselves to the $m=0$ and $T=0$ case. In the $T=0$ limit, $[1+n(q_0)]=\Theta(q_0)$ and $\tilde{n}(k_0)=\Theta(\mu-E+q_0)$, where Θ is the step function. The q_0 integration is then limited by

$$0\leqslant q_0\leqslant E-\mu. \tag{11}$$

This is also easily seen in kinetic theory, since, due to Pauli blocking, the quasiparticle can only scatter into states with energy $E_{|\mathbf{p}-\mathbf{q}|}$ such that $E_{|\mathbf{p}-\mathbf{q}|}\leqslant E$ and, furthermore, the particle on which it scatters must leave the Fermi sea, so that $E_{|\mathbf{p}-\mathbf{q}|}\geqslant\mu$. Note also that the exchanged photon must be spacelike: $Q^2<0$, so that the pole part of $\rho_{L,T}$ [1] does not contribute.

Now, the IR singular contribution comes from small values of the photon momentum q; in order to isolate this kinematical region, we follow Braaten and Yuan [13] and introduce an intermediate cutoff q^* such that $e\mu \ll q^* \ll \mu$. The ''soft'' region is defined by $q<q^*$, the ''hard'' one by $q>q^*$: in this latter region we may take the $M^2=0$ limit in the denominators of the spectral functions $\rho_{L,T}$ in Eq. (9) [1]. Let us concentrate on the soft region, where we can make the approximation

$$E_{|\mathbf{p}-\mathbf{q}|}=E-q_0\simeq E-\hat{\mathbf{p}}\cdot\mathbf{q}. \tag{12}$$

Keeping only the leading terms in Eq. (9), we find the contribution from the soft region to $\gamma(E)$:

$$\gamma_{\rm soft}(E)\simeq\frac{e^2}{2}\int\frac{d^3q}{(2\pi)^3}[\Theta(q_0)-\Theta(\mu-E+q_0)]\Theta(q^*-q)\times\{\rho_L(q_0,q)+(1-\cos^2\theta)\rho_T(q_0,q)\}, \tag{13}$$

with $q_0=\hat{\mathbf{p}}\cdot\mathbf{q}=q\cos\theta$. It is convenient to use as integration variables q_0 and q, the integration domain D being

$$D:\{0\leqslant q_0\leqslant E-\mu;\;\; q_0\leqslant q\leqslant q^*\}. \tag{14}$$

Then Eq. (13) becomes ($x=q_0/q$)

$$\gamma_{\rm soft}(E)\simeq\frac{e^2M^2}{4\pi}\int_D dq_0dq\left\{\frac{q_0}{2[q^2+M^2Q_1(x)]^2+M^4\pi^2x^2/2}+\frac{q_0}{[2\,q^2+M^2Q_2(x)]^2+M^4\pi^2x^2/4}\right\}, \tag{15}$$

where

$$Q_1(x)=1-\frac{x}{2}\ln\frac{1+x}{1-x},\quad Q_2(x)=-Q_1(x)+\frac{1}{1-x^2}. \tag{16}$$

Note that in the absence of screening [namely, by setting $M=0$ in the denominators of Eq. (15)], one would get IR divergent integrals. In general, the integrals in Eq. (15) must be computed numerically. Fortunately, it is possible to derive an accurate analytical result in the physically interesting case $(E-\mu)\ll M$. Indeed, it is easy to check that in this region one may expand the denominators in Eq. (15) in powers of q_0. Keeping only the leading terms, the first denominator in Eq. (15), corresponding to longitudinal photon exchange, may be replaced by (q^2+M^2), which leads to Debye screening. The second denominator in Eq. (15), corresponding to transverse photon exchange, may be replaced by

$$4\,q^4+\frac{\pi^2M^4x^2}{4}+8M^2q^2x^2. \tag{17}$$

It can be shown that the last term in Eq. (17) gives a subdominant contribution, while the second term leads to the usual form of dynamical screening [4–6]. Computing separately the longitudinal and transverse contributions, we find, with $u^*=(q^*/M)^2$,

$$\gamma^L_{\rm soft}(E)\simeq\frac{e^2(E-\mu)^2}{32\pi M}\int_0^{u^*}\frac{du}{\sqrt{u}(u+1)^2}\simeq\frac{e^2M^2}{16\pi}(E-\mu)^2\left(\frac{\pi}{4M^3}-\frac{1}{3q^{*3}}\right), \tag{18}$$

$$\gamma^T_{\rm soft}(E)\simeq\frac{e^2M}{4\pi^3}\int_0^{u^*}du\sqrt{u}\ln\left(1+\frac{\pi^2(E-\mu)^2}{16M^2u^3}\right)\simeq\frac{e^2}{24\pi}(E-\mu)+\frac{e^2M^2}{32\pi}(E-\mu)^2\left(-\frac{1}{3q^{*3}}\right), \tag{19}$$

where we have only kept the leading terms in $(E-\mu)$ and $1/q^*$.

The total contribution of the soft region to the decay rate is obtained by adding the longitudinal and transverse contributions to get

$$\gamma_{\rm soft}(E)\simeq\frac{e^2}{24\pi}(E-\mu)+\frac{e^2M^2}{32\pi}(E-\mu)^2\left(\frac{\pi}{2M^3}-\frac{1}{q^{*3}}\right). \tag{20}$$

The transverse contribution dominates over the longitudinal one for small values of $(E-\mu)$.

We finally evaluate the contribution from the hard region. Since we are only interested in extracting the leading dependence in the fermionic energy of the decay rate, we will use a simple approach to compute the hard contribution. It is possible to recover bare or unresummed perturbation theory to order e^4 by using the spectral densities (6) neglecting M^2 in the denominators [1]. This is only valid in the momentum transfer region $q>q^*$. Therefore one finds for the hard contribution to the decay rate

$$\gamma_{\rm hard}(E)\simeq\frac{e^2M^2}{8\pi}\int_{q^*}^{q_{\rm max}}dq\int_0^{E-\mu}dq_0\left\{\frac{q_0}{q^4}+\frac{q_0}{2q^4}\right\}. \tag{21}$$

After a straightforward computation one finds

$$\gamma_{\rm hard}(E)\simeq\frac{e^2M^2}{32\pi}(E-\mu)^2\left(\frac{1}{q^{*3}}-\frac{1}{q^3_{\rm max}}\right), \tag{22}$$

where $q_{\rm max}\simeq\mu$ is the maximum momentum transfer that it is allowed by kinematics.

The total decay rate is found just by adding the soft and hard contributions. Then one finds that the dependence on the scale q^* cancels, as it should. The result is

$$\gamma(E)\simeq\frac{e^2}{24\pi}(E-\mu)+\frac{e^2M^2}{32\pi}(E-\mu)^2\left(\frac{\pi}{2M^3}-\frac{1}{q_{\rm max}^3}\right). \tag{23}$$

In conclusion, we have been able to compute the damping rate of a quasiparticle in a degenerate ultrarelativistic plasma, when the fermion energy E is just above the Fermi energy μ. This damping rate is dominated by transverse photon (or gluon) exchange and proportional to $(E-\mu)$. This behavior arises from the combined effect of dynamical screening and phase space restrictions due to Pauli blocking. The lifetime τ is related to γ by $\tau\sim 1/\gamma$, and therefore the lifetime becomes infinite as the fermion energy approaches the Fermi energy, so that the Fermi sea is stable.

After this work was completed, we learned that our results have been obtained independently by J.-Y. Ollitrault and B. Vanderheyden; it was also pointed out to us by Ph. Nozières that the $(E-\mu)$ behavior of the damping rate was obtained in a different context by Holstein, Pincus and Norton [15]. C.M. wants to thank C. Lucchesi, N. Rius, A. Ramos, and J. Soto for useful discussions. She is also thankful to the Institut Non Linéaire de Nice, and to the Institut de Physique de l' Université de Neuchâtel for hospitality. This work was supported in part by funds provided by the European Contract No. CHRX-CT93-0357, ''Physics at High Energy Accelerators,'' by the Swiss National Science Foundation, by the CICYT Contract No. AEN95-0590, and by the CIRIT Contract No. GRQ93-1047.

[1] For a review, see, e.g., M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996).
[2] J. J. Quinn and R. A. Ferrell, Phys. Rev. **112**, 812 (1958).
[3] For a review of the NR case, see, e.g., D. Pines and P. Nozières, *The Theory of Quantum Liquids. Normal Fermi Liquids* (Addison-Wesley, Redwood City, 1988), Vol. 1.
[4] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).
[5] E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 1298 (1991).
[6] G. Baym, H. Monien, C. J. Pethick, and D. G. Ravenhall, Phys. Rev. Lett. **64**, 1867 (1990).
[7] For a review, see R. D. Pisarski, Phys. Rev. D **47**, 5589 (1993).
[8] J.-P. Blaizot and E. Iancu, Phys. Rev. Lett. **76**, 3080 (1996); Phys. Rev. D **55**, 973 (1997).
[9] R. D. Pisarski, Phys. Rev. Lett. **63**, 1129 (1989); E. Braaten and R. Pisarski, Nucl. Phys. **B337**, 569 (1990).
[10] J. Frenkel and J. C. Taylor, Nucl. Phys. **B334**, 199 (1990).
[11] C. Manuel, Phys. Rev. D **53**, 5866 (1996).
[12] H. A. Weldon, Phys. Rev. D **28**, 2007 (1983).
[13] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **66**, 2183 (1991).
[14] J. M. Luttinger, Phys. Rev. **121**, 942 (1961).
[15] T. Holstein, R. Norton, and P. Pincus, Phys. Rev. B **8**, 2649 (1973).