Generalized messengers of supersymmetry breaking and the sparticle mass spectrum

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We investigate the sparticle spectrum in models of gauge-mediated supersymmetry breaking. In these models, supersymmetry is spontaneously broken at an energy scale only a few orders of magnitude above the electroweak scale. The breakdown of supersymmetry is communicated to the standard model particles and their superpartners by ''messenger'' fields through their ordinary gauge interactions. We study the effects of a messenger sector in which the supersymmetry-violating F-term contributions to messenger scalar masses are comparable to the supersymmetry-preserving ones. We also argue that it is not particularly natural to restrict attention to models in which the messenger fields lie in complete SU(5) ground unified theory multiplets, and we identify a much larger class of viable models. Remarkably, however, we find that the superpartner mass parameters in these models are still subject to many significant contraints. [S0556-2821(97)04705-X]

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I. INTRODUCTION

The masses of the superpartners of the standard model (SM) particles should not greatly exceed the TeV scale if supersymmetry is to solve the hierarchy problem associated with the ratio $M_Z/M_{\rm Planck}$. However, this fact by itself tells us surprisingly little about the scale $\Lambda_{\rm SUSY}$ at which supersymmetry is ultimately broken. It is also necessary to have an understanding of the mechanism by which supersymmetry breaking is communicated from its original source to the fields of the minimal supersymmetric standard model (MSSM). If gravitational or other Planck-suppressed interactions communicate supersymmetry breaking, then $\Lambda_{\rm SUSY}$ is perhaps 10^{11} GeV or so. While this scenario has received the most attention in the last decade, it is hardly inevitable. Another possibility [1,2] is that the ordinary gauge interactions are responsible for communicating supersymmetry breaking to the MSSM through their couplings to a messenger sector of chiral superfields, which in turn couple directly or indirectly to the fields which break supersymmetry.

In the ''minimal'' model of gauge-mediated supersymmetry breaking (GMSB) [2], all of the soft supersymmetry-breaking interactions of the MSSM are determined by just a few free parameters. Perhaps the most attractive feature of this type of model is that the masses generated for squarks and sleptons with the same $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ quantum numbers are automatically degenerate, so that flavor-changing neutral currents are suppressed without additional assumptions. This feature depends only on the fact that ordinary gauge interactions are flavor blind, and will be true in a much larger class of models than just the minimal GMSB model.

This class of models has another feature which may allow it to be dramatically confirmed at existing or currently planned collider facilities. Because local supersymmetry is spontaneously broken at a relatively low scale, the lightest supersymmetric particle is the gravitino (the spin-$\frac{3}{2}$ superpartner of the graviton), with a mass that is entirely irrelevant for collider kinematics (but not for cosmology [3]). The next-to-lightest supersymmetric particle (NLSP) can therefore decay into its SM partner and the gravitino. In the case that the lightest neutralino ($\widetilde{N}_1$) is the NLSP, one has the interesting decay [4,5] $\widetilde{N}_1\to\gamma\widetilde{G}$ as long as the photino content of $\widetilde{N}_1$ is nonzero. The decay length for this process depends on the ultimate scale of supersymmetry breaking $\Lambda_{\rm SUSY}$, according to

$$\Gamma(\widetilde{N}_1\to\gamma\widetilde{G})=\frac{\kappa_{1\gamma}}{16\pi}\frac{m_{\widetilde{N}_1}^5}{\Lambda_{\rm SUSY}^4}, \tag{1}$$

where $\kappa_{1\gamma}=|N_{11}\cos\theta_W+N_{12}\sin\theta_W|^2$ (in the notation of [6]) is the photino content of $\widetilde{N}_1$. Since in GMSB the typical F term responsible for supersymmetry breaking can correspond to $\Lambda_{\rm SUSY}$ of order 10^2 or 10^3 TeV, it is quite possible that this decay can occur (at least a significant fraction of the time) inside a typical detector, with many interesting phenomenological consequences [4,5,7–14]. If it is sufficiently heavy $\widetilde{N}_1$ can also have decays into $Z\widetilde{G}$ and $h\widetilde{G}$, with decay widths which suffer, however, from very strong kinematic suppression [12].

Recently it was pointed out [8,9] that a single $ee\gamma\gamma+\not{E}_T$ event [15] observed at the Collider Detector at Fermilab (CDF) could be naturally explained[1] by GMSB (and other theories with a light gravitino). This event had an energetic electron and positron, two energetic photons each with pseudorapidity $|\eta|<1$ and transverse energy $E_T>30$ GeV, and large missing transverse energy $\not{E}_T>50$ GeV. The SM and detector backgrounds for such events are reputed to be extremely small. This event can be explained by GMSB as either selectron pair production or chargino pair production, but only if $\widetilde{N}_1$ is the NLSP, and if $\Lambda_{\rm SUSY}$ is less than about 10^3 TeV. These are not automatic consequences of all mod-

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[1]The event can also be explained in the usual MSSM framework without a light gravitino, if parameters are chosen so that the radiative one-loop decay $\widetilde{N}_2\to\widetilde{N}_1\gamma$ dominates [9,16]. The parameter space in which this can occur will be largely but not entirely explored at the CERN e^+e^- colliders LEP 161 and LEP 190.

els, and therefore will give (if taken seriously, which clearly should not be considered mandatory) nontrivial theoretical constraints.

Moreover, the discovery signatures of supersymmetry with a prompt decay $\widetilde{N}_1 \to \gamma \widetilde{G}$ are so spectacular that it is possible to set quite strong bounds even with existing Tevatron data. In contrast to the usual supersymmetry search strategies, one can obtain a very high detection efficiency at the Fermilab Tevatron for the inclusive signal $\gamma\gamma + X + \not{E}_T$ with suitable cuts on the transverse energy and isolation of the photons, and on the total missing transverse energy. In [12] it was argued that with the present 100 pb^{-1} of data at the Tevatron, it should be possible to exclude a lightest chargino ($\widetilde{C}_1$) mass up to 125 GeV and neutralino masses up to about 70 GeV, assuming gaugino mass ''unification'' relations as in the minimal GMSB model. In this paper we will discuss other models which do not share this feature. Even when all assumptions about gaugino mass relations are abandoned, however, it was argued in [12] that one can still find a model-independent bound $m_{\widetilde{C}_1} > 100$ GeV as long as $m_{\widetilde{N}_1} > 50$ GeV (to supply energetic photons) by exploiting the inclusive $\gamma\gamma + X + \not{E}_T$ signal. These bounds are quite competitive with and somewhat complementary to what can be done at LEP upgrades. However, it should be kept in mind that these bounds all assume that the decay $\widetilde{N}_1 \to \gamma \widetilde{G}$ occurs within the detector 100% of the time. This is not necessary, even to explain the CDF $ee\gamma\gamma + \not{E}_T$ event, which only requires that some non-negligible fraction of $\widetilde{N}_1$ decays occur within the detector. If most decays occur outside the detector, then one would expect many more single photon events than diphoton events, with unfortunately a much larger SM background, and much more difficult challenges for simulation studies. Thus for example the discovery mode at LEP2 from $e^+e^- \to \widetilde{N}_1\widetilde{N}_1$ could be predominantly $\gamma \not{E}$ rather than $\gamma\gamma \not{E}$. We should also note that in a significant fraction of the models to be studied in this paper, $\widetilde{N}_1$ cannot be the NLSP anyway unless it is Higgsino-like.

While the minimal model of GMSB is quite elegant and can explain the CDF $ee\gamma\gamma + \not{E}_T$ event, it is important to consider what all the related alternatives might be, especially in setting discovery and exclusion strategies. Future phenomenological studies should therefore take into account the full richness of model-building possibilities, which undoubtedly extend far beyond the minimal GMSB model and in several different directions [17–22]. In this paper we will begin to explore a few such possibilities. In Sec. II we develop the formalism for arbitrary messenger sector field content including the effects of arbitrary masses [from scalar vacuum expectation values (VEV's) and F-term breaking] in the messenger sector. In Sec. III we will examine the discrete model space allowed by generalizing the particle content of the messenger sector to include possibilities which do not form complete grand unified theory (GUT) multiplets. We will argue that it is not particularly unnatural or even inelegant to consider such generalizations. These effects serve to considerably enlarge the available parameter space, but in Sec. IV we show that some strong model-independent statements can still be made, and the GMSB models retain a distinct character even without taking into account the possibility of discovery modes involving decays into the gravitino.

FIG. 1. Contribution to MSSM gaugino masses from messenger field loops.

II. BEYOND THE MINIMAL MODEL

In this section we consider a slightly generalized treatment of the minimal model of GMSB. The messenger sector consists of a set of chiral superfields $\Phi_i, \overline{\Phi}_i$ which transform as a vectorlike representation of the MSSM gauge group. The supersymmetry breaking mechanism is parameterized by a (perhaps not fundamental) chiral superfield S, whose auxiliary component F is assumed to acquire a VEV. The messenger fields couple to S according to the superpotential

$$W = \lambda_i S \Phi_i \overline{\Phi}_i . \tag{2}$$

(Here we have assumed that the messengers obtain their masses only from coupling to a single chiral superfield S; we will comment briefly on the effects of relaxing this assumption below. With this assumption, a possible coupling matrix $\lambda_{ij}\Phi_i\Phi_j$ can always be diagonalized as shown.) In the minimal model of GMSB [2], Φ_i and $\overline{\Phi}_i$ consist of chiral superfields transforming as a $\mathbf{5}+\overline{\mathbf{5}}$ of $\text{SU}(5) \supset \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$. This choice is sufficient to give masses to all of the MSSM scalars and gauginos.

In the following we will use the same symbol for S and F and for their VEV's. The fermionic components of Φ_i and $\overline{\Phi}_i$ obtain a Dirac mass equal to $\lambda_i S$. Their scalar partners have a $(\text{mass})^2$ matrix equal to

$$\begin{pmatrix} |\lambda_i S|^2 & \lambda_i F \\ \lambda_i^* F^* & |\lambda_i S|^2 \end{pmatrix} \tag{3}$$

with eigenvalues $|\lambda_i S|^2 \pm |\lambda_i F|$. The supersymmetry violation apparent in this spectrum is then communicated to the MSSM sector via the ordinary gauge interactions of Φ_i and $\overline{\Phi}_i$.

The gauginos of the MSSM obtain their masses at one loop from the diagram shown in Fig. 1. The particles in the loop are the messenger fields. Evaluating this graph one finds that the MSSM gaugino mass parameters induced are

$$M_a = \frac{\alpha_a}{4\pi} \frac{F}{S} \sum_i n_a(i) g(x_i) \quad (a = 1,2,3), \tag{4}$$

where

$$x_i = |F/\lambda_i S^2| \tag{5}$$

for each messenger coupling λ_i and

$$g(x) = \frac{1}{x^2}\left[(1+x)\ln(1+x) + (1-x)\ln(1-x)\right]. \tag{6}$$

In Eq. (4), $n_a(i)$ is the Dynkin index for the pair $\Phi_i, \overline{\Phi}_i$ in a normalization where $n_a = 1$ for $\mathbf{N}+\overline{\mathbf{N}}$ of $\text{SU}(N)$. We always use a GUT normalization for α_1 so that $n_1 = \frac{6}{5}Y^2$ for each

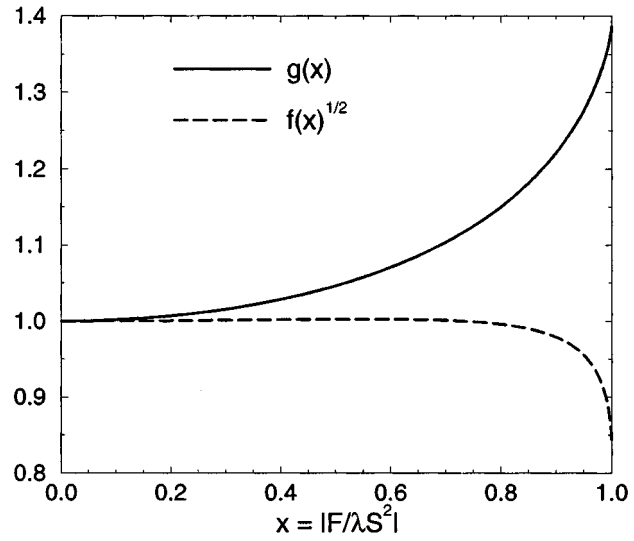

FIG. 2. The functions $g(x)$ and $\sqrt{f(x)}$ described in the text.

messenger pair with weak hypercharge $Y=Q_{\rm EM}-T_3$. The variable x_i must lie in the range $0<x_i<1$, with the upper limit coming from the requirement that the lighter scalar messenger has positive (mass)^2. The minimal $\mathbf{5}+\mathbf{\bar{5}}$ model has $\Sigma_i n_1(i)=\Sigma_i n_2(i)=\Sigma_i n_3(i)=1$. Since the function $g(x)$ obeys $g(0)=1$, in the small x_i limit one recovers the result $M_a=(\alpha_a/4\pi)F/S$ of [2] for the minimal model.

For larger x the expansion

$$g(x)=1+\frac{x^2}{6}+\frac{x^4}{15}+\frac{x^6}{28}+\cdots \tag{7}$$

gives good accuracy except near $x=1$. The function $g(x)$ is graphed in Fig. 2, and can be seen to increase monotonically with x, reaching a maximum value $g(1)=2\ln 2\approx 1.386$. It is sometimes convenient to write $M_a=(\alpha_a/4\pi)\Lambda_{Ga}$, where

$$\Lambda_{Ga}=\frac{F}{S}\sum_i n_a(i)g(x_i), \quad a=1,2,3, \tag{8}$$

parametrizes the possible effects of a nonminimal messenger sector and non-negligible x_i. In general one finds

$$\frac{F}{S}N_a\leqslant\Lambda_{Ga}\leqslant 1.386\,\frac{F}{S}N_a \tag{9}$$

depending on x_i, where

$$N_a=\sum_i n_a(i). \tag{10}$$

The scalar masses of the MSSM arise at leading order from two-loop graphs shown in Fig. 3, with messenger fields, gauge bosons, and gauginos on the internal lines. The calculation of these graphs is described in an Appendix, where we obtain the result already given by Dimopoulos, Giudice, and Pomarol [18]

$$\tilde{m}^2=2|F/S|^2\sum_a\left(\frac{\alpha_a}{4\pi}\right)^2 C_a\sum_i n_a(i)f(x_i) \tag{11}$$

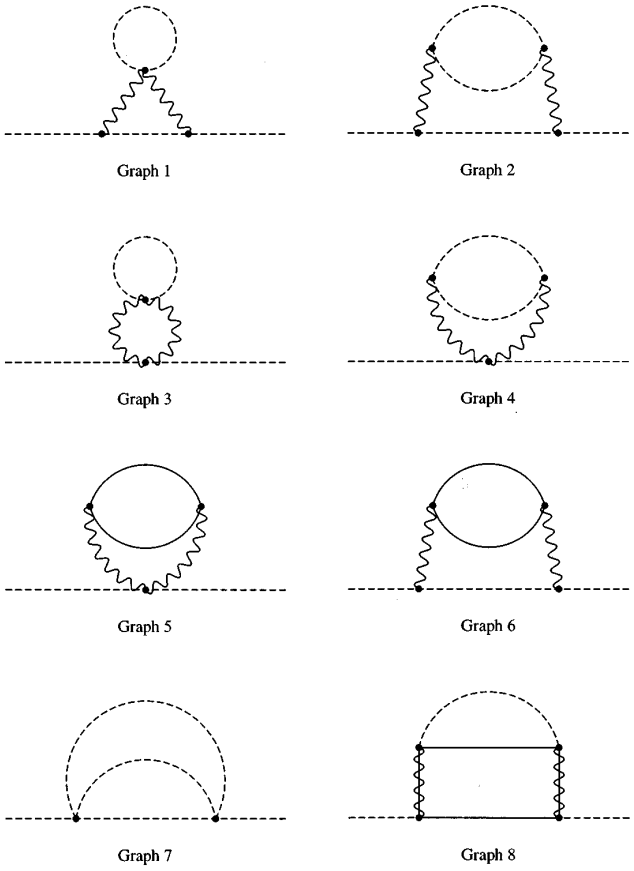

FIG. 3. Two-loop contributions to MSSM scalar masses involving messenger sector fields.

with

$$f(x)=\frac{1+x}{x^2}\left[\ln(1+x)-2\mathrm{Li}_2(x/[1+x])\right.$$
$$\left.+\tfrac{1}{2}\mathrm{Li}_2(2x/[1+x])\right]+(x\to -x). \tag{12}$$

In Eq. (11), C_a is the quadratic Casimir invariant of the MSSM scalar field in question, in a normalization where $C_3=\frac{4}{3}$ for color triplets, $C_2=\frac{3}{4}$ for $\mathrm{SU}(2)_L$ doublets, and $C_1=\frac{3}{5}Y^2$. It is convenient to write $\tilde{m}^2=2\Sigma_a(\alpha_a/4\pi)^2C_a\Lambda_{Sa}^2$ with

$$\Lambda_{Sa}^2=|F/S|^2\sum_i n_a(i)f(x_i). \tag{13}$$

In this way the six quantities Λ_{Ga} and Λ_{Sa} parametrize the effects of a nonminimal messenger sector and non-negligible x_i on the masses of MSSM gauginos and scalars respectively. In the limit $|F/\lambda_i S^2|\ll 1$, one recovers the result $\Lambda_{Ga}=\Lambda_{Sa}=F/S$ for the minimal model of [2], since $f(0)=1$. In order to illustrate the relative effects of non-negligible x_i on gaugino and sfermion masses, we graph in Fig. 2 the function $\sqrt{f(x)}$ to compare with $g(x)$. When x is not very close to 1, one finds excellent precision from the expansion

$$f(x)=1+\frac{x^2}{36}-\frac{11}{450}x^4-\frac{319}{11760}x^6+\cdots . \tag{14}$$

The function $f(x)$ is nearly constant for x not near 1, and falls sharply near $x=1$ to a minimum value of $f(1)=2\ln 2+2\ln^2 2-\pi^2/6\approx 0.702$ or $\sqrt{f(1)}\approx 0.838$. Note that $\sqrt{f(x)}$ is always within one per cent of unity for $x<0.85$. Thus as long as $|F/\lambda_i S^2|\lesssim 0.85$ for all messenger fields, one has simply

$$\Lambda_{Sa}^2=|F/S|^2 N_a \tag{15}$$

to a very good approximation. More generally, one finds

$$0.838\sqrt{N_a}|F/S|\leqslant\Lambda_{Sa}\leqslant\sqrt{N_a}|F/S|. \tag{16}$$

By combining the bounds on $g(x)$ and $\sqrt{f(x)}$ we obtain the result

$$\sqrt{N_a}\leqslant\frac{\Lambda_{Ga}}{\Lambda_{Sa}}\leqslant 1.65\sqrt{N_a} \tag{17}$$

in *any* model in which all messenger fields obtain their masses only from a single chiral superfield S and its F term. The effect of non-negligible x_i is always to lower the masses of squarks and sleptons compared to the gaugino mass parameters. With some rather mild restrictions, the range (17) can be significantly tightened. For example, if all $x_i<0.85$, one can replace the value 1.65 by 1.19. With the further restriction that all $x_i<0.5$, the same number becomes only 1.044, so that the scales entering the gaugino and scalar mass formulas differ only at the few % level. The 1% accuracy level (to which higher-loop corrections are probably comparable anyway) for $\Lambda_{Ga}\approx\sqrt{N_a}\Lambda_{Sa}$ is reached if all $x_i<0.25$.

The masses predicted by Eqs. (4) and (11) are given at the messenger mass scale(s) and must be renormalized down to the scale of MSSM sparticles. Decoupling each set of messengers Φ_i,Φ_i at the appropriate $\lambda_i S$, one obtains running dimensional reduction with modified minimal subtraction ($\overline{\rm DR}$) gaugino mass parameters

$$M_a(Q)=\frac{\alpha_a(Q)}{4\pi}\frac{F}{S}\sum_{\lambda_i S>Q} n_a(i)g(x_i). \tag{18}$$

Below the lightest messenger scale this reduces simply to

$$M_a(Q)=\frac{\alpha_a(Q)}{4\pi}\Lambda_{Ga} \tag{19}$$

up to small two-loop corrections [23,24,25].

The scalar (mass)^2 parameters obtain renormalization group corrections proportional to gaugino masses squared, with the result

$$\tilde m^2(Q)=2\sum_a C_a\left(\left|\frac{F}{S}\right|^2\sum_i\left[\frac{\alpha_a(\lambda_i S)}{4\pi}\right]^2 n_a(i)f(x_i)+\int_{\ln Q}^{\ln\lambda_i S} d(\ln Q')\,\frac{\alpha_a(Q')}{\pi}M_a^2(Q')\right) \tag{20}$$

with $M_a(Q)$ given by Eq. (18), and $\alpha_a(Q)$ by a similar step-function decoupling of messengers. As long as the couplings λ_i do not feature large hierachies, one may safely neglect messenger-scale threshold contributions of order $\delta\tilde m^2\sim 2C_a\ln(\lambda_i/\lambda_j)M_\alpha^2\alpha_a/\pi$ by choosing a representative messenger scale $Q_0\approx\lambda_i S$. In this approximation one finds

$$\tilde m^2(Q)=2\sum_a\left(\frac{\alpha_a(Q)}{4\pi}\right)^2 C_a\left[r_a\Lambda_{Sa}^2+\frac{1}{b_a}(1-r_a)\Lambda_{Ga}^2\right], \tag{21}$$

where $(b_1,b_2,b_3)=(-33/5,\,-1,3)$ and

$$r_a(Q)=[\alpha_a(Q_0)/\alpha_a(Q)]^2=[1+(b_a\alpha_a(Q)/2\pi)\ln(Q_0/Q)]^{-2}. \tag{22}$$

In the case that all x_i are small and not too different, the running scalar and gaugino masses and running gauge couplings can be directly related at any scale by

$$\tilde m^2=2\sum_a C_a M_a^2\left(\frac{r_a}{N_a}+\frac{1}{b_a}(1-r_a)\right), \tag{23}$$

while more generally one finds

$$\tilde m^2=2\sum_a C_a M_a^2\left(r_a\frac{\Lambda_{Sa}^2}{\Lambda_{Ga}^2}+\frac{1}{b_a}(1-r_a)\right), \tag{24}$$

with the ratio $\Lambda_{Sa}^2/\Lambda_{Ga}^2$ bounded by $0.366/N_a$ and $1/N_a$ according to Eq. (17). These equations hold at the one-loop level (with Yukawa couplings and trilinear scalar couplings neglected) in a nondecoupling $\overline{\rm DR}$ scheme, which means that MSSM sparticles and Higgs fields are not decoupled at their mass thresholds. In order to make precise predictions about the sparticle masses, these parameters must be related to the physical masses of the particles. The necessary equations have been given for the gluino and first and second family squarks in [23], and in general for all of the MSSM particles in [26].

So far we have assumed that the messengers all obtain their masses entirely through coupling to a single chiral superfield S. If this assumption is relaxed, one clearly obtains a much more general set of models with a concomitant loss of predictive power. However, the assumption that only one field S plays a significant role is perhaps sufficiently compelling that the alternatives can be considered disfavored. For example, the existence of only one S field successfully addresses the supersymmetric CP problem, since all phases in the theory are proportional to the phase of F/S, and can be rotated away. This need not be so if there is more than one field S. The simplest model of this type is perhaps the obvious extension of the minimal model of GMSB, i.e., with messenger fields $D+\bar D$ and $L+\bar L$, and the superpotential $W=\lambda_3 S_3 D\bar D+\lambda_2 S_2 L\bar L$. The gaugino masses obtained from this model are given by, in the small $|F_i/\lambda_i S_i^2|$ limit,

$$M_3=\frac{\alpha_3}{4\pi}\frac{F_3}{S_3};\quad M_2=\frac{\alpha_2}{4\pi}\frac{F_2}{S_2};\quad M_1=\frac{\alpha_1}{4\pi}\left(\frac{2F_3}{5S_3}+\frac{3F_2}{5S_2}\right). \tag{25}$$

If the phases of the VEV's are not aligned, this gives rise to an observable CP-violating phase $\arg(M_1M_2^*)$ which could potentially feed into an electric dipole moment for the neutron or electron. On the other hand, if squark and slepton masses are very large, such new phases could be tolerable,

and the interference in Eq. (25) could allow M_1 to be somewhat suppressed relative to M_2 and M_3 and the slepton masses. However, we will not consider such possibilities further here.

III. VARIATIONS IN THE MESSENGER SECTOR

One of the outstanding features of the minimal model of GMSB is its predictive power, since the values of the soft supersymmetry-breaking MSSM parameters are determined by only a few parameters in the messenger sector. However, one can also entertain the possibility of different field contents in the messenger sector. The original choice of messenger fields in $\mathbf{5}+\overline{\mathbf{5}}$ of SU(5) is motivated by the fact that it is the simplest one which simultaneously provides for plausible MSSM masses and maintains the apparent unification of gauge coupling observed at LEP. It is well known that the latter feature is shared by any set of chiral superfields which lie in complete SU(5) GUT multiplets. The number of such fields which can be used as messenger fields is then limited by the requirement that the MSSM gauge couplings should stay perturbative up to the GUT scale $M_U \approx 2\times 10^{16}$ GeV, which amounts to the statement that there can be at most four $\mathbf{5}+\overline{\mathbf{5}}$ sets or one $\mathbf{5}+\overline{\mathbf{5}}$ and one $\mathbf{10}+\overline{\mathbf{10}}$.

While maintaining the apparent unification of gauge coupling is a fine goal, it is not clear how much this really should tell us about the messenger sector. First, it is sufficient but not necessary to have complete $\mathbf{5}+\overline{\mathbf{5}}$ and $\mathbf{10}+\overline{\mathbf{10}}$ multiplets of SU(5) in order to maintain perturbative gauge coupling unification. A counterexample with $N_1 = N_2 = N_3 = 3$ is a messenger sector transforming under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ as

$$(\mathbf{3},\mathbf{2},\tfrac{1}{6}) + (\overline{\mathbf{3}},\mathbf{1},\tfrac{1}{3}) + 2\times(\mathbf{1},\mathbf{1},1) + \text{conj.} \tag{26}$$

which by itself (or with additional gauge singlets) does not happen to form any combination of irreducible representations of any simple GUT group. Furthermore, it is not necessary that all TeV or messenger scale vectorlike chiral superfields must obtain their masses primarily by coupling to the field S. Those that do not can still participate in ensuring perturbative gauge coupling unification, but may not act as messenger fields and in particular can have little or no effect on the masses of MSSM sparticles. (There is, after all, a precedent already in the MSSM of chiral superfields in vectorlike, non-GUT, representations of the MSSM gauge group without very large masses, namely the Higgs fields.) Finally, a skeptic might point out that the apparent unification of gauge couplings could be partially or wholly accidental, so that it is prudent to consider equally all alternatives rather than trust the detailed results of extrapolating coupling constant relationships over 13 orders of magnitude in energy.

Therefore we will consider here the effects of a somewhat less constrained messenger sector. We will maintain the constraint that messenger fields should occupy the same representations as MSSM chiral superfields. This is motivated by the fact that stable messenger particles with exotic $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ quantum numbers are probably a disaster for cosmology. In fact it should be noted that in any case the lightest and the lightest color nonsinglet members of the messenger sector must be stable insofar as they do not couple to MSSM fields through nongauge interactions. (There is an interesting possibility that a stable neutral messenger might make up the cold dark matter, however [18].) Fortunately, small mixings between nonexotic messengers and their MSSM counterparts can allow them to decay; the necessary couplings may or may not [21] significantly affect the predictions of GMSB. So we consider five possible types of messenger fields,

$$Q+\overline{Q} = (\mathbf{3},\mathbf{2},\tfrac{1}{6}) + \text{conj.}; \quad U+\overline{U} = (\overline{\mathbf{3}},\mathbf{1},-\tfrac{2}{3}) + \text{conj.}; \tag{27}$$

$$D+\overline{D} = (\overline{\mathbf{3}},\mathbf{1},\tfrac{1}{3}) + \text{conj.}; \quad L+\overline{L} = (\mathbf{1},\mathbf{2},-\tfrac{1}{2}) + \text{conj.}; \tag{28}$$

$$E+\overline{E} = (\mathbf{1},\mathbf{1},1) + \text{conj.}, \tag{29}$$

with multiplicities denoted $(n_Q, n_U, n_D, n_L, n_E)$ respectively. Thus the particle content of the messenger sector is specified by a five-tuple of integers, given our assumptions.

[Actually, as long as we are only using the numbers $(n_Q, n_U, n_D, n_L, n_E)$ to parameterize our ignorance of non-MSSM physics, we can set $n_U = 0$. This is because the gauge interactions of any $U+U$-type messengers can always be replaced by messengers in the representations $D+D+E+E$, as far as the MSSM sector is concerned, since they have the same index for each group. Global features of the theory do depend on n_U, of course. One could also consider messenger sectors which include single adjoint representations $(\mathbf{8},\mathbf{1},0)$ or $(\mathbf{1},\mathbf{3},0)$, but we will neglect those possibilities here.]

The number of chiral superfields is limited by requiring that gauge couplings remain perturbative. However, we do not require that the messenger fields by themselves maintain gauge coupling unification, for the reasons mentioned above. Instead, we require as our first criterion only that the messenger fields should be a subset of some set of fields that maintains perturbative gauge coupling unification. Assuming that no messenger field mass greatly exceeds 10^4 TeV, the perturbativity part of the requirement ($\alpha_a \lesssim 0.2$ at M_{Planck}) amounts to

$$N_1 = \tfrac{1}{5}(n_Q + 8n_U + 2n_D + 3n_L + 6n_E) \leqslant 4, \tag{30}$$

$$N_2 = 3n_Q + n_L \leqslant 4, \tag{31}$$

$$N_3 = 2n_Q + n_U + n_D \leqslant 4, \tag{32}$$

while the full requirement can be written as

$$\begin{aligned}(n_Q, n_U, n_D, n_L, n_E) \leqslant\ & (1,0,2,1,2) \\ & \text{or } (1,1,1,1,1) \text{ or } (1,2,0,1,0) \\ & \text{or } (0,0,4,4,0).\end{aligned} \tag{33}$$

It is possible that the requirements (30)–(32) can be weakened, but only slightly, by allowing the extrapolated gauge couplings to diverge between M_U and M_{Planck} or by enlarging the MSSM gauge group below M_U. (Additional gauge bosons can contribute negatively to the β functions for $\alpha_{1,2,3}$, but this effect is limited by constraints on proton decay, and by the fact that additional chiral superfields which contribute positively to the β functions must also be introduced to break

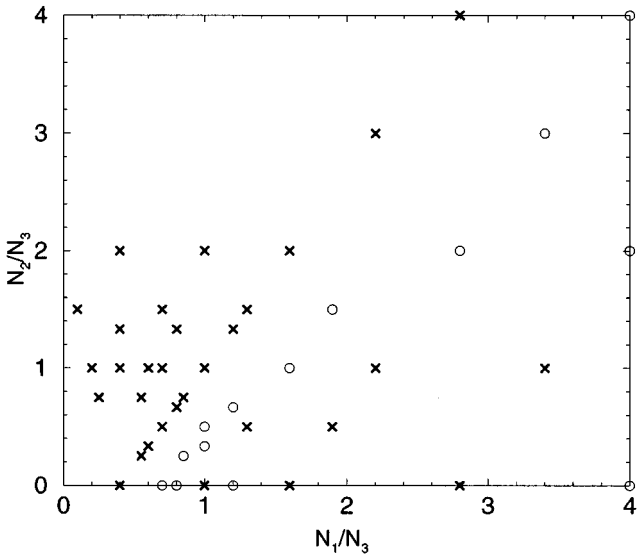

FIG. 4. A scatterplot of the quantities N_1/N_3 and N_2/N_3 for all messenger models satisfying the perturbativity constraints (30)–(32) in the text. Models which do (do not) also satisfy the ''unification'' criteria (33) are plotted as X's (circles). In the regime $|F/\lambda_i S^2| \ll 1$, the axis quantities are equal to the renormalization group scale-independent ratios $(M_1/\alpha_1)/(M_3/\alpha_3)$ and $(M_2/\alpha_2)/(M_3/\alpha_3)$ respectively.

the additional gauge interactions). The requirements that the gluino and the right-handed selectron not be massless at leading order imply $N_3 \geq 1$ and $N_1 \geq 1/5$ respectively. The possibility $N_2=0$ may not be ruled out yet [19], if $\tan\beta$ is very small, but it should be decisively confronted at LEP2 since it requires a chargino mass smaller than M_W. Furthermore, it should be possible to exclude these models with existing Tevatron data if the decay $\widetilde{N}_1 \to \gamma \widetilde{G}$ is prompt, and perhaps even if it is not.

There are 66 distinct five-tuples $(n_Q, n_U, n_D, n_L, n_E)$ which satisfy these criteria, of which 53 have $N_2 \neq 0$. The number of distinct combinations (N_1, N_2, N_3) arising from these models is 40. The ones with $N_2 \neq 0$ are, in ascending order of N_1: $(\frac{1}{5},3,2)$; $(\frac{3}{5},3,3)$; $(\frac{4}{5},4,2)$; (1,1,1); (1,3,4); $(\frac{6}{5},4,3)$; $(\frac{7}{5},1,2)$; $(\frac{7}{5},3,2)$; $(\frac{8}{5},2,1)$; $(\frac{8}{5},4,4)$; $(\frac{9}{5},1,3)$; $(\frac{9}{5},3,3)$; (2,2,2); (2,4,2); $(\frac{11}{5},1,1)$; $(\frac{11}{5},1,4)$; $(\frac{11}{5},3,1)$; $(\frac{11}{5},3,4)$; $(\frac{12}{5},2,3)$; $(\frac{12}{5},4,3)$; $(\frac{13}{5},1,2)$; $(\frac{13}{5},3,2)$; $(\frac{14}{5},2,4)$; $(\frac{14}{5},4,1)$; $(\frac{14}{5},4,4)$; (3,3,3); $(\frac{16}{5},4,2)$; $(\frac{17}{5},1,1)$; $(\frac{17}{5},3,4)$; $(\frac{18}{5},4,3)$; $(\frac{19}{5},1,2)$; and (4,4,4). The ones with $N_2=0$ and therefore $M_2=0$ at the one loop level are: $(\frac{2}{5},0,1)$; $(\frac{4}{5},0,2)$; $(\frac{6}{5},0,3)$; $(\frac{8}{5},0,1)$; $(\frac{8}{5},0,4)$; $(\frac{14}{5},0,1)$; (2,0,2); and $(\frac{16}{5},0,2)$.

Some indication of the variety which can be obtained is illustrated in Fig. 4, which shows a scatterplot of N_1/N_3 vs N_2/N_3 for the 66 models (each shown as an X) which fit the criteria (30)–(33). For $x_i \ll 1$, these quantities are equal to the scale-independent gaugino mass ratios $(M_1/\alpha_1)/(M_3/\alpha_3)$ and $(M_2/\alpha_2)/(M_3/\alpha_3)$ respectively. Some of the points on this plot are occupied by several models. We have also indicated by circles the presence of 33 models which fit the perturbativity requirements (30)–(32), but for which Eq. (33) is not satisfied, so that the particle content cannot be embedded into a set which allows perturbative unification of the gauge couplings unless additional fields with masses far above the messenger scale are invoked.

The $n\times(\mathbf{5}+\overline{\mathbf{5}})$ and $\mathbf{10}+\overline{\mathbf{10}}$ models [and the model in Eq. (26)] all occupy the point $N_1/N_3=N_2/N_3=1$, but there are other models which give quite distinctive and interesting predictions. The models on the $N_2/N_3=0$ axis are the ones with $n_Q=n_L=0$, which must have small $\tan\beta$ and a chargino lighter than the W boson; we will omit them from the discussions to follow. The models close to the $N_1/N_3=0$ axis have a very large hierarchy $m_{\tilde{e}_R} \ll m_{\tilde{q}}$, and so may be strongly disfavored by naturalness criteria. (We will not attempt here a complete analysis of electroweak symmetry breaking requirements). The most ''extreme'' such model has

$$(n_Q, n_U, n_D, n_L, n_E) = (1,0,0,0,0); \quad (N_1, N_2, N_3) = (\tfrac{1}{5},3,2) \tag{34}$$

with M_1 less than gluino and squark masses by perhaps a factor of 50, depending on α_3. As can be seen already from Fig. 4, there can be quite a wide variety in the mass hierarchies between squarks and gluinos and the sleptons and electroweak gauginos.

IV. CONSTRAINTS ON SPARTICLE MASSES

In this section we will study some features of the sparticle mass spectrum which follow from the 53 models which satisfy the constraints (30)–(33) and $N_a>0$ as discussed in the previous section. We will consider here only the gaugino mass parameters M_1, M_2, M_3, and the squark and slepton masses of the first two families, for which Yukawa interactions can be neglected. We will also not concern ourselves with the possible origins or role of the μ and $B\mu$ terms or scalar trilinear terms. Constraints following from requiring correct electroweak symmetry breaking (with viable models for the origins of such terms) will only further tighten the constraints we will derive. In the following we assume $F/S < 250$ TeV and $0.01 < x_i < 0.99$. Taken together these imply that the messenger mass scales are bounded by $\lambda_i S < 2.5\times 10^4$ TeV.

It is perhaps easiest to understand the impact of variations in the messenger sector by first considering the case that x_i is small for each messenger pair. In that case the quantities $\Sigma_i n_a(i) g(x_i)$ and $\Sigma_i n_a(i) f(x_i)$ are each equal to N_a, so that the MSSM masses are approximately determined by just the parameters $F/S, N_1, N_2, N_3$. Using the values listed above one can then place some bounds on the ratios of gaugino mass parameters as

$$0.067 \frac{\alpha_1}{\alpha_2} \leq \frac{M_1}{M_2} \leq 3.8 \frac{\alpha_1}{\alpha_2}, \tag{35}$$

$$0.1 \frac{\alpha_1}{\alpha_3} \leq \frac{M_1}{M_3} \leq 3.4 \frac{\alpha_1}{\alpha_3}, \tag{36}$$

$$0.25 \frac{\alpha_2}{\alpha_3} \leq \frac{M_2}{M_3} \leq 4 \frac{\alpha_2}{\alpha_3}. \tag{37}$$

Although the gaugino masses run with scale, the veracity of the inequalities (35)–(37) is renormalization group scale independent at one loop. [It is not completely accurate, however, to replace $\alpha_1, \alpha_2, \alpha_3$ by their measured values at LEP here, since Eqs. (35)–(37) hold in the nondecoupling $\overline{\text{DR}}$

scheme.] The lower bounds in Eqs. (35) and (36) are set by the ''extreme'' model in Eq. (34). If this model is discounted, the values 0.067 and 0.1 are each raised to 0.2.

The bounds from Eqs. (35)–(37) can be strongly modified by different couplings λ_i for messenger fields with different gauge quantum numbers. However, some general rules can still be found. For example, we find numerically that M_1/M_3 is always less than 1 at the gluino mass scale, with rough bounds

$$0.12\,\frac{N_1}{N_3} \lesssim \frac{M_1}{M_3} \lesssim 0.3\,\frac{N_1}{N_3}. \tag{38}$$

(Note that $N_1/N_3 \leqslant 3.4$ in these models.) Also, M_2 can only exceed M_3 at the gluino mass scale if $N_2/N_3 \geqslant 2$, and we always find

$$0.21\,\frac{N_2}{N_3} \lesssim \frac{M_2}{M_3} \lesssim 0.6\,\frac{N_2}{N_3}. \tag{39}$$

Since N_2/N_3 has a maximum value of 4 in these models, the overall upper limit is $M_2/M_3 \lesssim 2.4$. Similarly, M_1/M_2 can be as large as about 2.7 at the electroweak scale, when the x_i are chosen appropriately and N_1/N_2 is large. Numerically we find

$$0.35\,\frac{N_1}{N_2} \lesssim \frac{M_1}{M_2} \lesssim 0.72\,\frac{N_1}{N_2}. \tag{40}$$

It is interesting to consider the ordering between the mass of the lightest slepton and the bino mass parameter M_1, since if $|\mu|$ is large, this will give an indication whether a slepton or a neutralino is the NLSP. Using the approximation of Eq. (24) one finds that

$$m^2_{\tilde{e}_R} = \frac{6}{5}\,M_1^2\left[r_1\,\frac{\Lambda^2_{S1}}{\Lambda^2_{G1}} - \frac{5}{33}\,(1-r_1)\right] \tag{41}$$

for $\overline{\rm DR}$ parameters $m^2_{\tilde{e}_R}$, M_1 and r_1. Since $(\Lambda^2_{S1}/\Lambda^2_{G1}) \leqslant 1/N_1$, one finds that $m_{\tilde{e}_R} > M_1$ can occur only if $N_1 < 66 r_1(65-10r_1)$. Now, r_1 depends on both the messenger scale and the scale at which we evaluate the running mass parameters. But a reasonable estimate for the upper bound is $r_1 \lesssim 1.7$ [in the regime of validity of Eq. (24)], from which we learn that $m_{\tilde{e}_R} > M_1$ can only occur if $N_1 \leqslant 2.2$. This result still applies in more general situations when Eq. (20) must be applied. Only 21 models which fit the criteria of the previous section can satisfy this constraint. The maximum values of the ratio $m_{\tilde{e}_R}/M_1$ in these models are approximately 3.0, 1.7, 1.5, and 1.35 for $(n_Q, n_U, n_D, n_L, n_E)$ equal to, respectively, (1,0,0,0,0); (1,0,1,0,0); (1,0,0,1,0); and (0,0,1,1,0) (the minimal model). Of course the effect of nonzero x_i can only be to diminish the ratio $m_{\tilde{e}_R}/M_1$, but the electroweak D-term corrections to $m_{\tilde{e}_R}$ can raise this ratio slightly if M_1 is not too large. There is also a possibility that M_2 can be less than both $m_{\tilde{e}_R}$ and M_1, if $N_1 > N_2$. However, even taking into account the effects of nonzero x_i, we find that this only occurs for a few models with

$$(N_1,N_2,N_3) = (\tfrac{11}{5},1,1);\quad (\tfrac{11}{5},1,4);\quad (\tfrac{13}{5},1,2);\quad (\tfrac{17}{5},1,1)$$
$$\text{and}\quad (\tfrac{19}{5},1,2). \tag{42}$$

These are the models for which a line drawn to the origin on Fig. 4 makes the smallest angle with the $N_2/N_3 = 0$ axis.

If $M_1, M_2 > m_{\tilde{e}_R}$, it is still possible that a neutralino is the NLSP if $|\mu|$ is not large. This typically means that $\tilde{N}_1$ has a rather large Higgsino content, and $\tilde{N}_1 \to \gamma\tilde{G}$ can be suppressed. However, the competing decays $\tilde{N}_1 \to h\tilde{G}$ and $\tilde{N}_1 \to Z\tilde{G}$ may be kinematically forbidden, and in any case are subject to very strong kinematic suppressions $(1 - m_h^2/m^2_{\tilde{N}_1})^4$ and $(1-m_Z^2/m^2_{\tilde{N}_1})^4$ respectively [12]. Therefore if $\sqrt{F} < 10^3$ TeV it is still possible to explain the CDF $ee\gamma\gamma + \not{E}_T$ event with small $|\mu|$. This may be particularly plausible in the chargino interpretation [12] in which the event is due to $p\bar{p} \to \tilde{C}_1\tilde{C}_1$ with allowed two-body decays $\tilde{C}_1 \to \tilde{\nu}e$ and $\tilde{\nu} \to \nu\tilde{N}_1$ or $\tilde{C}_1 \to \tilde{e}_L\nu$ and $\tilde{e}_L \to e\tilde{N}_1$ followed by $\tilde{N}_1 \to \gamma\tilde{G}$ in each case. Since the production cross section for chargino pairs at the Tevatron remains large even for $m_{\tilde{C}_1} \approx 200$ GeV, it is sensible to suppose that the two-photon event could have been seen even if the decay length of $\tilde{N}_1$ is increased by a smaller photino component of $\tilde{N}_1$.

The models in Eq. (42) are also interesting because they minimize the ratio of left-handed to right-handed slepton masses. In the regime that all $x_i \ll 1$, we find that the running mass parameters satisfy $m_{\tilde{e}_L}/m_{\tilde{e}_R} \gtrsim 1.1$ for all of the models which fit our criteria (with $N_2 \geqslant 1$). The modification of this ratio due to electroweak D terms happens to be extremely small because of the numerical accident $\sin^2\theta_W \approx 1/4$. However, with appropriately chosen x_i, it is possible to obtain $m_{\tilde{e}_L} \approx m_{\tilde{e}_R}$ in the last two models of Eq. (42). In all other cases, the hierarchy $m_{\tilde{e}_L} > m_{\tilde{e}_R}$ holds.

One can similarly analyze the possible ranges for the ratios of squark and gluino masses. It is easiest to consider the particular ratio $M_{\tilde{d}_R}/M_{\tilde{g}}$, since this is least sensitive to electroweak effects. Neglecting the quite small effects of $U(1)_Y$, one finds for the running mass parameters

$$\frac{m_{\tilde{d}_R}}{M_3} = \frac{2\sqrt{2}}{3}\,[3(\Lambda_{S3}/\Lambda_{G3})^2 r_3 + (1-r_3)]^{1/2} \tag{43}$$

in the approximation of Eq. (24). This ratio is maximized when $N_3 = 1$ and all $x_i \approx 0$, and is minimized when $N_3 = 4$ and all $x_i \approx 1$. Thus we find

$$\frac{2\sqrt{2}}{3}\,[1-0.67r_3]^{1/2} \lesssim \frac{m_{\tilde{d}_R}}{M_3} \lesssim \frac{2\sqrt{2}}{3}\,[1+2r_3]^{1/2}. \tag{44}$$

It is now clear that both the upper and lower limits are saturated when r_3 is as large as possible, corresponding to a low messenger mass scale. Taking estimated bounds $r_3 \lesssim 0.72$ for $N_3 = 1$ and $r_3 \lesssim 0.78$ for $N_3 = 4$, we obtain

$$0.65 \lesssim \frac{m_{\tilde{d}_R}}{M_3} \lesssim 1.48 \quad (\text{running masses at } Q = m_{\tilde{d}_R}). \tag{45}$$

Now, the running masses can be converted into pole masses using the formulas in [23,26], yielding a slightly modified estimate for the bounds on the ratio of the physical pole masses

$$0.66 \lesssim \frac{M_{\tilde{d}_R}}{M_{\tilde{g}}} \lesssim 1.36 \quad \text{(pole masses)}. \tag{46}$$

Repeating this type of argument for each value of N_3 separately and taking into account Eq. (20) one finds

$$(0.91,0.76,0.70,0.65) \lesssim \frac{M_{\tilde{d}_R}}{M_{\tilde{g}}} \lesssim (1.36,1.07,0.95,0.90), \tag{47}$$

for the physical mass ratios with $N_3=(1,2,3,4)$. The upper limits in each case correspond to small x_i and small values of $\lambda_i S$. The case of left-handed first and second family squarks is slightly different, especially when N_2 is relatively large. Numerically, we find

$$(0.93,0.76,0.70,0.65) \lesssim \frac{M_{\tilde{d}_L}}{M_{\tilde{g}}} \lesssim (1.74,1.23,1.03,0.95) \tag{48}$$

for the physical mass ratios with $N_3=(1,2,3,4)$.

The masses of $SU(2)_L$-singlet squarks are never very different from each other in the models of Sec. III. Taking into account the effects of nonzero x_i, we still find a quite narrow range

$$1 < m_{\tilde{u}_R}/m_{\tilde{d}_R} \lesssim 1.04. \tag{49}$$

This is not surprising since the $U(1)_Y$ effects are relatively small even for larger N_1. The left-handed squarks are always heavier than $m_{\tilde{u}_R}, m_{\tilde{d}_R}$. Numerically we find

$$1 < m_{\tilde{d}_L}/m_{\tilde{d}_R} \lesssim (1.1,1.2,1.3,1.4) \tag{50}$$

for $N_2=(1,2,3,4)$. The squark masses also quite generally exceed slepton masses even for models with relatively small N_3. Numerically we estimate the bounds

$$(1.0,1.5,2.0,2.4) \lesssim m_{\tilde{d}_R}/m_{\tilde{e}_L} \lesssim (4,6,8,10) \tag{51}$$

for $N_3=(1,2,3,4)$. The situation $m_{\tilde{d}_R} \approx m_{\tilde{e}_L}$ only can occur for $(N_1,N_2,N_3)=(\frac{14}{5},4,1)$, the highest point in the plot of Fig. 4.

V. DISCUSSION

In this paper we have examined some of the possibilities for generalized models of the messenger sector of low-energy supersymmetry breaking. Despite the large number of discrete model choices and the freedom to vary the $x_i=|F/\lambda_i S^2|$, the parameters of the MSSM are constrained in interesting ways. For example, we note the following.

(i) The usual hierarchy $m_{\tilde{e}_R} \lesssim m_{\tilde{e}_L} \lesssim m_{\tilde{d}_R} \approx m_{\tilde{u}_R} \lesssim m_{\tilde{u}_L} \approx m_{\tilde{d}_L}$ is always preserved, with numerical bounds given by Eqs. (46)–(51).

(ii) The masses of the right-handed squarks $\tilde{u}_R$, $\tilde{d}_R$, $\tilde{s}_R$, and $\tilde{c}_R$ all lie in a narrow band, and in a window within about $\pm 35\%$ of the physical gluino mass. The upper limit on $m_{\tilde{q}_R}/m_{\tilde{g}}$ is (nearly) saturated for the minimal $\mathbf{5}+\bar{\mathbf{5}}$ model with small x_i. The masses of the corresponding left-handed squarks can be significantly larger in some models, up to about $1.75 m_{\tilde{g}}$.

(iii) The ratios of gaugino mass parameters M_1, M_2, M_3 can vary quite significantly from the predictions of the minimal model, with $M_2 > M_3$ and $M_1 > M_2$ both possible at the TeV scale. However, M_1/M_3 is always $\lesssim 1$.

(iv) Only six parameters Λ_{Ga} and Λ_{Sa} [plus the overall messenger scale(s)] enter into the definition of the gaugino mass parameters and the first and second family squark and slepton masses. As long as $x_i=|F/\lambda_i S^2|$ is less than about 0.5 (0.25) for all messenger fields, then there are only four parameters $F/S, N_1, N_2, N_3$ at the 4% (1%) accuracy level, besides a logarithmic dependence on the messenger mass scale(s) $\lambda_i S$.

Let us close by noting a slightly different way to express the constraints on squark and slepton masses which follow from the GMSB framework. One can see from the form of Eq. (24) that three parameters suffice to determine all of the scalar masses for which Yukawa interactions can be neglected. This means that for the seven scalar masses $m_{\tilde{e}_R}, m_{\tilde{\nu}}, m_{\tilde{e}_L}, m_{\tilde{d}_R}, m_{\tilde{u}_R}, m_{\tilde{u}_L}, m_{\tilde{d}_L}$ there must be four sum rules which do not depend on the input parameters. Two of these sum rules are completely model independent and should hold in *any* supersymmetric model (up to small radiative corrections [27]):

$$m^2_{\tilde{d}_L} - m^2_{\tilde{u}_L} = M_W^2 |\cos 2\beta|; \tag{52}$$

$$m^2_{\tilde{e}_L} - m^2_{\tilde{\nu}} = M_W^2 |\cos 2\beta|. \tag{53}$$

(We assume $\tan\beta > 1$.) The other two sum rules can be written as

$$m^2_{\tilde{u}_R} = m^2_{\tilde{d}_R} + \tfrac{1}{3} m^2_{\tilde{e}_R} - \tfrac{4}{3} M_Z^2 \sin^2\theta_W |\cos 2\beta|; \tag{54}$$

$$m^2_{\tilde{d}_L} = m^2_{\tilde{d}_R} + m^2_{\tilde{e}_L} - \tfrac{1}{3} m^2_{\tilde{e}_R} + \tfrac{2}{3} M_Z^2 \sin^2\theta_W |\cos 2\beta|. \tag{55}$$

These sum rules are not model independent. It is interesting to compare with the case of models with ''supergravity-inspired'' boundary conditions featuring a common m_0^2 for scalars and a common $m_{1/2}$ for gauginos at the GUT or Planck scale. In those models, one finds [28] a sum rule which is a particular linear combination of Eqs. (54) and (55):

$$2m^2_{\tilde{u}_R} - m^2_{\tilde{d}_R} - m^2_{\tilde{d}_L} + m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R} = -\frac{10}{3} M_Z^2 \sin^2\theta_W |\cos 2\beta| \tag{56}$$

This sum rule tests the assumption of a common m_0^2. But in GMSB models, one effectively has the further bit of information that $m_0^2 = 0$ (i.e., all contributions to scalar masses are proportional to the quadratic Casimir invariants; there is no group-independent piece). This leads to the presence of one additional sum rule, which can be taken to be either Eq. (54) or (55).

It will be an interesting challenge to see to what accuracy these sum rules can be tested at future colliders. Perhaps the most interesting possibility is that the sum rules will turn out to be violated in some gross way; this would force us to reexamine our assumptions about the origin of supersymmetry breaking. As an example, suppose that the messenger sector has some feature which causes additional unequal supersymmetry-breaking contributions to the diagonal entries in the mass matrix (3). This would lead, through a one-loop graph, to a Fayet-Iliopoulos D term proportional to weak hypercharge manifesting itself in the squark and slepton masses [2]. Since such a contribution comes in at one loop earlier in the loop expansion than the contributions from the F term, it is constrained to be quite small in order to avoid negative squared masses for some squarks and sleptons. Conversely, even tiny such contributions to the matrix (3) will be magnified in relative importance, and will therefore quite possibly be observable in the sparticle mass spectrum! The impact will be to modify each of the sum rules (54), (55), and (56) by adding contributions $-4D_Y/3$, $2D_Y/3$, and $-10D_Y/3$ respectively to the right-hand sides.

In general, we find it remarkable that the models discussed here make such a variety of testable predictions. In addition to the possibly dramatic collider signatures coming from decays of the NLSP into the gravitino, the sparticle spectrum has a rather distinct character. Future model building developments will surely tell us even more about what to expect for the parameters of the MSSM in the low-energy supersymmetry breaking framework.

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APPENDIX

In this appendix we give details of the calculation of the masses of MSSM scalars which arise at leading order from two-loop diagrams involving messenger fields. We employ the component field formalism. There are eight Feynman diagrams which contribute at two loop order, as shown in Fig. 3. We compute these graphs in the Feynman gauge; then one finds that graph six vanishes. Each of the other graphs is separately divergent but the sum is finite. It is important to compute all γ-matrix algebra in four dimensions before computing the momentum integrals with scalar integrands in $4-2\epsilon$ dimensions, in order to avoid a spurious nonsupersymmetric mismatch between the numbers of gaugino and gauge boson degrees of freedom. By straightforward methods one finds that the contributions for each pair of messenger fields Φ,Φ are given in terms of the messenger fermion mass $m_f=|\lambda_i S|$ and the two messenger scalar masses $m_\pm^2=|\lambda_i S|^2\pm|\lambda_i F|$ by

$$\Delta\tilde{m}^2=\sum_a g_a^4 C_a S_a(\Phi)\ \text{(sum of graphs)}, \tag{A1}$$

where C_a is the quadratic Casimir invariant [normalized to $(N^2-1)/2N$ for a fundamental representation of SU(N)] of the MSSM scalar, and $S_a(\Phi)$ is the Dynkin index of the messenger field Φ [normalized to $\frac{1}{2}$ for a fundamental of SU(N)]. The contributions to the ''sum of graphs'' are given by

$$\text{graph } 1=-\text{graph } 2=-\tfrac{1}{4}\text{graph } 3 = 2\langle m_+\rangle\langle 0,0\rangle+2\langle m_-\rangle\langle 0,0\rangle; \tag{A2}$$

$$\text{graph } 4=4\langle m_+\rangle\langle 0,0\rangle+4\langle m_-\rangle\langle 0,0\rangle-\langle m_+|m_+|0\rangle -\langle m_-|m_-|0\rangle-4m_+^2\langle m_+|m_+|0,0\rangle -4m_-^2\langle m_-|m_-|0,0\rangle; \tag{A3}$$

$$\text{graph } 5=8\langle m_f\rangle\langle 0,0\rangle-4\langle m_f|m_f|0\rangle+8m_f^2\langle m_f|m_f|0,0\rangle; \tag{A4}$$

$$\text{graph } 6=0; \quad \text{graph } 7=-2\langle m_+|m_-|0\rangle; \tag{A5}$$

$$\text{graph } 8=4\langle m_+\rangle\langle 0,0\rangle+4\langle m_-\rangle\langle 0,0\rangle-8\langle m_f\rangle\langle 0,0\rangle +4\langle m_+|m_f|0\rangle+4\langle m_-|m_f|0\rangle +4(m_+^2-m_f^2)\langle m_+|m_f|0,0\rangle +4(m_-^2-m_f^2)\langle m_-|m_f|0,0\rangle. \tag{A6}$$

Here we have used the following notation for Euclidean momentum integrals in $n=4-2\epsilon$ dimensions (omitting in each case a factor $\mu^{2\epsilon}$):

$$\langle m\rangle=\int\frac{d^nq}{(2\pi)^n}\frac{1}{(q^2+m^2)}, \tag{A7}$$

$$\langle m,m\rangle=\int\frac{d^nq}{(2\pi)^n}\frac{1}{(q^2+m^2)^2}, \tag{A8}$$

$$\langle m_1|m_2|m_3\rangle=\int\frac{d^nq}{(2\pi)^n}\int\frac{d^nk}{(2\pi)^n} \times\frac{1}{(q^2+m_1^2)(k^2+m_2^2)([k-q]^2+m_3^2)}, \tag{A9}$$

$$\langle m_1|m_2|m_3,m_3\rangle =\int\frac{d^nq}{(2\pi)^n}\int\frac{d^nk}{(2\pi)^n} \times\frac{1}{(q^2+m_1^2)(k^2+m_2^2)([k-q]^2+m_3^2)^2}. \tag{A10}$$

(In the quantities $\langle 0,0\rangle$ and $\langle m_1|m_2|0,0\rangle$, it is necessary to keep a finite infrared regulator mass m_ϵ which will cancel from physical quantities.)

The terms of the form $\langle m\rangle\langle 0,0\rangle$ are easily seen to cancel between the various graphs 1–8, by the magic of supersymmetry. The remaining two-loop integrals can be evaluated by standard Feynman parameter techniques. First it is convenient to use the identity

$$(-1+2\epsilon)\langle m_1|m_2|0\rangle = m_1^2\langle m_2|0|m_1,m_1\rangle + m_2^2\langle m_1|0|m_2,m_2\rangle \quad \text{(A11)}$$

to express everything in terms of dimension 8 integrands. Then one finds, following, e.g., the methods of [29]:

$$\langle m_1|m_2|0,0\rangle = \frac{\Gamma(1+2\epsilon)}{2(4\pi)^n}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}(1-2\ln m_\epsilon^2)+1-\pi^2/6 - F_2(m_1^2,m_2^2)-2F_3(m_1^2,m_2^2) + [-2+2F_1(m_1^2,m_2^2)]\ln m_\epsilon^2+\ln^2 m_\epsilon^2\right] + O(\epsilon) \quad \text{(A12)}$$

and

$$\langle m_1|0|m_2,m_2\rangle = \frac{\Gamma(1+2\epsilon)}{2(4\pi)^n}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}(1-2\ln m_2^2)+1 - \pi^2/6-2\ln m_2^2+\ln^2 m_2^2-\ln^2 m_1^2 + 2\ln m_1^2\ln m_2^2-2\mathrm{Li}_2(1-m_2^2/m_1^2)\right] + O(\epsilon) \quad \text{(A13)}$$

where we have introduced yet more notation:

$$F_1(a,b) = (a\ \ln a - b\ \ln b)/(a-b), \quad \text{(A14)}$$

$$F_2(a,b) = (a\ \ln^2 a - b\ \ln^2 b)/(a-b), \quad \text{(A15)}$$

$$F_3(a,b) = [a\mathrm{Li}_2(1-b/a) - b\mathrm{Li}_2(1-a/b)]/(a-b) \quad \text{(A16)}$$

when $a \neq b$, and

$$F_1(a,a) = 1+\ln a, \quad \text{(A17)}$$

$$F_2(a,a) = 2\ln a + \ln^2 a, \quad \text{(A18)}$$

$$F_3(a,a) = 2. \quad \text{(A19)}$$

The dilogarithm or Spence function is defined by $\mathrm{Li}_2(x) = -\int_0^1 (dt/t)\ln(1-xt)$. Now it is straightforward to add all the contributions to the ''sum of graphs.'' In particular, it is easy to show that the ultraviolet and infrared divergent terms cancel. The resulting expression can be simplified further using standard dilogarithm identities [30], finally yielding the expression given in [18], and in Eq. (12) of the present paper.

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