QCD sum rules for heavy baryons at next-to-leading order in α_S

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We derive QCD sum rules for heavy baryons at leading order in $1/m_Q$ and at next-to-leading order in α_S. The calculation involves the evaluation of four different perturbative three-loop diagrams which determine the α_S corrections to the Wilson coefficients of the leading term in the operator product expansion. From the sum rules we obtain estimates for the masses and the residues of the heavy baryons Λ_Q and Σ_Q. The perturbative $O(\alpha_S)$ corrections to the leading order spectral function amount to about 100%, and they shift the calculated values for the baryon masses slightly upward. The residues are shifted upward by about 20–50%. For the bound state energy $\bar{\Lambda}$ given by the difference of the heavy baryon mass and the pole mass of the heavy quark m_Q we obtain $m_{\Lambda_Q}-m_Q \cong 780$ MeV and $m_{\Sigma_Q}-m_Q \cong 950$ MeV. For the residues we find $|F_\Lambda| \cong 0.028\ \mathrm{GeV}^3$ and $|F_\Sigma| \cong 0.039\ \mathrm{GeV}^3$. [S0556-2821(97)04405-6]

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I. INTRODUCTION

There has been a great deal of interest in the physics of heavy hadrons containing one heavy quark. The heavy quark effective theory (HQET) allows one to study the properties of the heavy hadrons in a systematic $1/m_Q$ expansion. The leading term of the expansion gives rise to the spin-flavor symmetry of heavy quark symmetry (HQS). The corrections to the leading HQS results are determined by the small expansion parameter $\Lambda_{\mathrm{QCD}}/m_Q$, where $\Lambda_{\mathrm{QCD}} \approx 300$ MeV is the scale of low-energy physics (for a review of HQET see [1], for a review of HQS and the sum rule approach for heavy mesons see [2]).

Among the well-known predictions of HQS are, e.g., the relations between different heavy hadron transition form factors. Take, for example, the $\Lambda_b \rightarrow \Lambda_c$ electroweak transitions. The six form factors describing this transition are reduced to one universal Isgur-Wise function in the HQS limit [3,4,5]. Even then one still remains with many nonperturbative parameters characterizing the process and the heavy baryons participating in it. These concern the functional behavior of the Isgur-Wise function itself, the masses and residues of the heavy baryons and, at next-to-leading order in the heavy mass expansion, the average kinetic and chromomagnetic energy of the heavy quark in the heavy baryon. All these nonperturbative parameters can be determined by using nonperturbative methods as, e.g., lattice calculations, QCD sum rule methods [6] or, in a less fundamental approach, by using potential models.

In the present paper we study the correlator of two heavy baryon currents in the HQS limit when $m_Q \rightarrow \infty$. Using the QCD sum rule method we calculate the masses and residues of the heavy baryons associated with the heavy baryon currents. In its original form the QCD sum rule method was suggested by Shifman *et al.* [6] as a tool to investigate the properties of light meson systems. Later on the method was extended to the case of light baryons in [7–10]. The QCD sum rule approach has proven itself to be a very powerful nonperturbative QCD-based tool which takes into account the properties of the QCD vacuum. It allows one to obtain reliable estimates for hadron masses, their residues and their elastic as well as their transition form factors.

In the heavy-light sector the first leading order analysis (leading both in $1/m_Q$ as well as in α_S) of heavy meson properties within the QCD sum rule approach was performed in [11]. Later on the heavy meson sum rule calculation was extended to include next-to-leading order radiative corrections. The next-to-leading order corrections proved to be rather important [12,13,14]. QCD sum rules for baryons with large but finite masses m_Q were first studied in [15,16]. Later on the methods of HQET were incorporated in the sum rule analysis. The leading order QCD sum rules for heavy baryons were first considered in [11,17,18], again to leading order both in $1/m_Q$ as well as in α_S. Finite mass corrections to these sum rules were discussed in [19].

In order to improve on the accuracy of the existing QCD sum rule analysis of heavy baryons one needs to avail of the next-to-leading order radiative corrections to the sum rules. This forms the subject of the present paper. We calculate the QCD radiative corrections to the leading perturbative term in the operator product expansion (OPE) and, from these, we derive next-to-leading order QCD sum rules for heavy baryons in the HQS limit. We then go on to analyze the sum rules and compute the values of the masses and the residues of the heavy baryons at next-to-leading order accuracy.

The paper is organized as follows. In Sec. II we introduce heavy baryon currents as interpolating fields for the heavy ground state baryons. In Sec. III we construct the correlator of two heavy baryon currents by means of the OPE and

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define the spectral density. In Sec. IV we present our results on the radiative corrections to the perturbative part of the spectral density and construct renormalization group invariant QCD sum rules by recapitulating some known results on the one- and two-loop anomalous dimensions of the currents. Section V contains the results of our numerical analysis. Section VI, finally, contains our summary and our conclusions. In Appendix A we provide a detailed collection of results on the calculation of the two- and three-loop contributions to the correlator of two heavy baryon currents. These results are quite general in that they are given for general space-time dimensions and for a general baryonic current structure.

II. BARYONIC CURRENTS

The currents of the heavy baryon Λ_Q and the heavy quark spin baryon doublet $\{\Sigma_Q,\Sigma_Q^*\}$ are associated with the spin-parity quantum numbers $j^P=0^+$ and $j^P=1^+$ for the light diquark system with antisymmetric and symmetric flavor structure, respectively. Adding the heavy quark to the light quark system, one obtains $j^P=\frac{1}{2}^+$ for the Λ_Q baryon and the pair of degenerate states $j^P=\frac{1}{2}^+$ and $j^P=\frac{3}{2}^+$ for the baryons Σ_Q and Σ_Q^*. The general structure of the heavy baryon currents has the form (see, e.g., [17], and Refs. therein)

$$J=[q^{iT}C\Gamma\tau q^j]\Gamma' Q^k\epsilon_{ijk}. \tag{1}$$

Here the index T means transposition, C is the charge conjugation matrix with the properties $C\gamma_\mu^T C^{-1}=-\gamma_\mu$ and $C\gamma_5^T C^{-1}=\gamma_5$, i,j,k are color indices, and τ is a matrix in flavor space. The effective static field of the heavy quark is denoted by Q. For each of the ground-state baryons there are two independent interpolating currents J_1 and J_2 which both have the appropriate quantum numbers to interpolate to the respective ground-state baryons. They are given by [11,17]

$$J_{\Lambda 1}=[q^{iT}C\tau\gamma_5 q^j]Q^k\varepsilon_{ijk}, \quad J_{\Lambda 2}=[q^{iT}C\tau\gamma_5\gamma_0 q^j]Q^k\varepsilon_{ijk},$$

$$J_{\Sigma 1}=[q^{iT}C\tau\vec{\gamma}q^j]\cdot\vec{\gamma}\gamma_5 Q^k\varepsilon_{ijk},$$

$$J_{\Sigma 2}=[q^{iT}C\tau\gamma_0\vec{\gamma}q^j]\cdot\vec{\gamma}\gamma_5 Q^k\epsilon_{ijk},$$

$$\vec{J}_{\Sigma *1}=[q^{iT}C\tau\vec{\gamma}q^j]Q^k\varepsilon_{ijk}+\tfrac{1}{3}\;\vec{\gamma}[q^{iT}C\tau\vec{\gamma}q^j]\cdot\vec{\gamma}Q^k\varepsilon_{ijk},$$

$$\vec{J}_{\Sigma *2}=[q^{iT}C\tau\gamma_0\vec{\gamma}q^j]Q^k\varepsilon_{ijk}+\tfrac{1}{3}\;\vec{\gamma}[q^{iT}C\gamma_0\vec{\gamma}q^j]\cdot\vec{\gamma}Q^k\varepsilon_{ijk}, \tag{2}$$

where $\vec{J}_{\Sigma *1}$ and $\vec{J}_{\Sigma *2}$ satisfy the spin-$\frac{3}{2}$ condition $\vec{\gamma}\vec{J}_{\Sigma * i}=0$ (i=1,2). The flavor matrix τ is antisymmetric for Λ_Q and symmetric for the heavy quark spin doublet $\{\Sigma_Q,\Sigma_Q^*\}$. The currents written down in Eq. (2) are rest frame currents. The corresponding expressions in a general frame moving with velocity four-vector v^μ can be obtained by the substitutions $\gamma_0\to\not{v}$ and $\vec{\gamma}\to\gamma_\perp^\mu=\gamma^\mu-\not{v}v^\mu$. In the following analysis we shall be using both of these equivalent descriptions alternatively, i.e., we shall either use the static description with v^μ=(1,0,0,0) or a moving frame description with $v^\mu=(1,\vec{v})$ and $\vec{v}\neq 0$.

For a general analysis it proves to be convenient to represent the general light-side Dirac structure of the currents in Eq. (2) by an antisymmetrized product of n Dirac matrices $\Gamma=\gamma^{[\mu_1}\cdots\gamma^{\mu_n]}$. When calculating the one- and two-loop vertex corrections one encounters γ contractions of the form $\gamma_\alpha\Gamma\gamma^\alpha$ and $\gamma_0\Gamma\gamma_0$. The γ_α contraction leads to an n dependence according to

$$\gamma_\alpha\Gamma\gamma^\alpha=h\Gamma=(-1)^n(D-2n)\Gamma. \tag{3}$$

The γ_0 contraction depends in addition on an additional parameter s which takes the value ($s=+1$) and ($s=-1$) for an even or odd number of γ_0's in Γ, respectively. The γ_0 contraction reads

$$\gamma_0\Gamma\gamma_0=(-1)^n s\Gamma. \tag{4}$$

In order to facilitate the use of Eqs. (3) and (4) we have compiled a table of the (n,s) values relevant for the heavy baryon currents treated in this paper (see Table I).

TABLE I. Specific values of the parameter pair (n,s) for particular cases of the light-side Dirac structure Γ. γ_5^{AC} refers to the naive γ_5-scheme with an anticommuting γ_5 [20] and γ_5^{HV} to the γ_5-scheme due to Breitenlohner, Maison, 't Hooft, and Veltman [21].

Γ	n	s	Particles
γ_5^{AC}	0	+1	Λ_1
$\gamma_5^{\mathrm{AC}}\gamma_0$	1	−1	Λ_2
$\vec{\gamma}$	1	+1	Σ_1,Σ_1^*
$\gamma_0\vec{\gamma}$	2	−1	Σ_2,Σ_2^*
γ_5^{HV}	4	−1	Λ_1
$\gamma_5^{\mathrm{HV}}\gamma_0$	3	+1	Λ_2

III. CORRELATOR OF TWO BARYONIC CURRENTS

In this section we describe the steps needed for the evaluation of baryonic QCD sum rules. One starts with the correlator of two baryonic currents:

$$\Pi(\omega=k\cdot v)=i\int d^4x e^{ikx}\langle 0|T\{J(x),\overline{J(0)}\}|0\rangle, \tag{5}$$

where k_μ and p_μ are the residual and full momentum of the heavy quark and v_μ is the four velocity using the momentum expansion $p_\mu=m_Q v_\mu+k_\mu$. As was mentioned before, there are two possible choices of interpolating currents for each of the heavy baryon states, given by Γ_1 and $\Gamma_2=\Gamma_1\not{v}$. Thus one may consider correlators of the same currents (diagonal correlators) or of different currents (nondiagonal correlators). In the general case, one may even consider correlators built from a linear combination $J=J_1+bJ_2$ of these currents with an arbitrary coefficient b. We mention that the choice $b=1$ corresponds to a constituent quark model current which has maximal overlap with the ground state baryons in the constituent quark model picture. In this paper we limit our attention to diagonal correlators only.

The correlator in Eq. (5) depends only on the energy variable $\omega=k\cdot v$ because of the static nature of the heavy propagator. It can be factorized into a spinor dependent piece and a scalar correlator function $P(\omega)$ according to

$$\Pi(\omega)=\Gamma' \frac{1+\not{v}}{2} \overline{\Gamma}' \frac{1}{4} \mathrm{Tr}(\Gamma\overline{\Gamma})2 \ \mathrm{Tr}(\tau\tau^{\dagger})P(\omega). \tag{6}$$

Following the standard QCD sum rule method [6], the correlator is calculated in the region $-\omega\approx 1-2$ GeV, including perturbative and nonperturbative contributions, where the nonperturbative contributions can in general be quite large. The nonperturbative effects are taken into account by employing an operator product expansion (OPE) for the time-ordered product of currents in Eq. (5). One then has

$$T\{J(x),\overline{J(0)}\}=\sum_d C_d(x^2)O_d, \tag{7}$$

where the operators O_d are local operators with a given dimension d, $O_0=\hat{1}$, $O_3=\langle\overline{q}q\rangle$, $O_4=\langle GG\rangle, \ldots$, and the expansion coefficients $C_d(x^2)$ are the corresponding coefficient functions or Wilson coefficients of the OPE.

A straightforward dimensional analysis shows that the OPE of the diagonal correlator contains only even-dimensional terms. We take into account the perturbative term for $d=0$, the gluon condensate term for $d=4$ and a condensate term with four quark fields for $d=6$. The four-quark operator will be factorized into a product of two two-quark operators, $\langle\overline{q(0)}q(x)\rangle^2$ [6]. Accordingly the Fourier transform of the scalar correlator function $P(\omega)$ reads

$$P(t)=P_{\mathrm{OPE}}(t)=i\theta(t)N_c!\left(\frac{1}{\pi^4t^6}+\frac{c\alpha_S\langle GG\rangle}{32N_c(N_c-1)\pi^3t^2}-\frac{1}{4N_c^2}\langle\overline{q(0)}q(t)\rangle^2\right), \tag{8}$$

where $c=1$ for Λ_Q, $c=-\frac{1}{3}$ for $\{\Sigma_Q,\Sigma_Q^*\}$ and N_c is the number of colors. For the nonlocal quark condensate $\langle\overline{q(0)}q(t)\rangle$ one may use the OPE about $\langle\overline{q}q\rangle:=\langle\overline{q(0)}q(0)\rangle$, namely

$$\langle\overline{q(0)}q(t)\rangle=\langle\overline{q}q\rangle\left(1+\tfrac{1}{16}m_0^2t^2+\pi\alpha_S\langle GG\rangle\frac{t^4}{96N_c}+\cdots\right), \tag{9}$$

where the parameter m_0 is defined in Eq. (11). Alternatively one may use the Gaussian ansatz [22]

$$\langle\overline{q(0)}q(t)\rangle=\langle\overline{q}q\rangle\ \exp(\tfrac{1}{16}\ m_0^2t^2). \tag{10}$$

When expanding the Gaussian ansatz one sees that the two forms agree up to the term linear in t^2. Thus the two representations of the nonlocal quark condensate are quite similar to one another for small values of t. In our sum rule analysis we shall make use of the Gaussian ansatz because it provides for better stability of the sum rules.

For the condensates we use the numerical values

$$\langle\overline{q}q\rangle=-(0.23\ \ \mathrm{GeV})^3,$$

$$\alpha_S\langle GG\rangle=0.04\ \ \mathrm{GeV}^4,$$

$$g_S\langle\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle=m_0^2\langle\overline{q}q\rangle \quad \text{with}\ \ m_0^2=0.8\ \ \mathrm{GeV}^2. \tag{11}$$

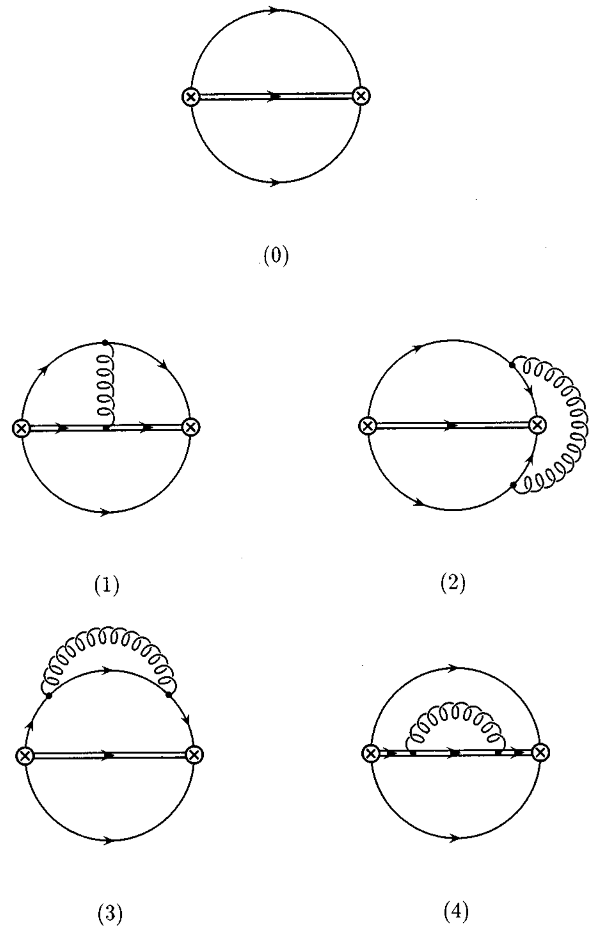

FIG. 1. Two-loop and three-loop contributions to the correlator of two heavy baryon currents. (0) two-loop lowest order contribution, (1)–(4) three-loop $O(\alpha_S)$ contributions.

With these condensate values one sees that the OPE in Eq. (8) with Euclidian time $\tau=it$ converges nicely for $1/\tau>0.3$ GeV. In this region one may thus safely truncate the OPE series after the second term. At $1/\tau=0.3$ GeV the contribution of the first term is two times larger than the last quark condensate term. Its contribution grows rapidly with $1/\tau$. When $1/\tau$ is further increased we see that the correlator becomes dominated by the perturbative contribution. For example, at $1/\tau=0.6$ GeV the perturbative term is two orders of magnitude larger than the contribution of the condensate terms. Note, however, that at $1/\tau=0.4$ GeV the contribution of the ground state to the correlator is ten times smaller than the contribution of the excited states and the continuum. This would imply that if the theoretical and phenomenological continuum contributions differ by about 10% (and are not equal to each other as assumed here), this difference would induce a 100% change in the contribution of the ground state. Thus the sum rules can only be trusted at values $1/\tau<0.4$ GeV (see also the discussions of the numerical results of the sum rules). In the next section we will show that the perturbative corrections become even more important at small Euclidian distances in comparison to the nonperturbative condensate contributions.

As a next step one determines the spectral density using the coordinate space representation $P(t)$ of the current correlator. The simplest way to proceed is as follows. The scalar correlator function $P_{\mathrm{OPE}}(\omega)$ satisfies a dispersion relation

$$P_{\mathrm{OPE}}(\omega)=P(\omega)=\int_0^\infty \frac{\rho(\omega')d\omega'}{\omega'-\omega-i0}+P'(\omega), \qquad (12)$$

where $\rho(\omega)=\mathrm{Im}[P(\omega)]/\pi$ is the spectral density and $P'(\omega)$ is a polynomial in ω, which takes into account possible subtractions in the dispersion representation. The Fourier transform of the polynomial $P'(\omega)$ consists of the δ function $\delta(t)$ and derivatives $\delta^{(n)}(t)$ of the δ function. A comparison with Eq. (8) shows that one does not in fact need any subtractions. We therefore set $P'(\omega)=0$. Taking the Fourier transform of Eq. (12) according to

$$P(t)=\int \frac{d\omega}{2\pi} e^{-i\omega t}P(\omega), \qquad (13)$$

we obtain

$$P(t)=i\theta(t)\int_0^\infty \rho(\omega)e^{-i\omega t}d\omega=i\theta(t)\widetilde{P}(t). \qquad (14)$$

Then we analytically continue $P(t)$ from $t>0$ to imaginary times by introducing the Euclidean time $\tau=it$. After this transformation, Eq. (14) becomes the well-known Laplace transformation. One may thus use an inverse Laplace transformation in order to obtain an Euclidean time representation of the spectral density:

$$\rho(\omega)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\widetilde{P}(-i\tau)e^{\omega\tau}d\tau, \qquad (15)$$

where c is to be chosen as a real constant to the right of all singularities of $P(t)$. It is then easy to check that the form $P(t)=i\theta(t)/t^{n+1}$ gives the spectral density $\rho(\omega)=i^{n+1}\theta(\omega)\omega^n/n!$, whereas $P(t)=i\theta(t)t^n$ results in $\rho(\omega)=(-i)^n\delta^{(n)}(\omega+0)$ for $n\geqslant 0$. Following the argumentation in [8] we do not include forms of the second kind into the spectral density ρ. So the leading order perturbative contribution and the next-to-leading order contribution of the gluon condensate to the spectral density are given by

$$\rho(\omega)=\rho_0(\omega)+\rho_4(\omega), \qquad (16)$$

where

$$\rho_0(\omega)=\frac{\omega^5}{20\pi^4} \quad \text{and} \quad \rho_4(\omega)=c\,\frac{\alpha_S\langle GG\rangle}{32\pi^3}\,\omega.$$

IV. RADIATIVE CORRECTION TO THE PERTURBATIVE TERM

Next we consider radiative corrections to the leading order spectral density in Eq. (16). There are altogether four different three-loop graphs that contribute to the correlator of two baryonic currents, which are shown in Fig. 1. Contrary to the experience in the two-loop case, the most convenient way to calculate the three-loop contributions is to evaluate them in momentum space. The fact that all graphs in Fig. 1 have two-point two-loop subgraphs greatly simplifies the calculational task. One can first evaluate the respective subgraphs such that one remains with a one-loop integration. The subgraph two-loop integration can be performed by using the algebraic methods described in [23]. It is important to note that the results of the two-loop integration can be expressed in terms of a polynomial function of the external momentum that flows into the subgraph. Hence, the remaining integration is a one-loop-type integration, where the power of one of the propagators has become a noninteger number due to the use of dimensional integration. The upshot of this is that all steps of the three-loop integration can be reduced to purely algebraic manipulations.

We present the results of calculating the two-loop and three-loop contributions to the correlator in the form

$$-\widetilde{\Gamma}_0 P(\omega)=\lambda_0 C_0 B_0+\lambda_1\sum_{i=1}^4 C_i B_i, \qquad (17)$$

where we have used the abbreviations $\lambda_0=(-2\omega/\mu)^{(2D-3)}$, $\lambda_1=g_S^2/(4\pi)^{D/2}(-2\omega/\mu)^{(3D-7)}$, and where $D=4-2\epsilon$ is the space-time dimension. Concerning the color structure we have defined the color factors C_i $(i=0,\ldots,4)$ according to the labeling of the graphs in Fig. 1. Their values are given by $C_0=N_c!$, $C_1=C_2=-N_c!C_B$ and $C_3=C_4=N_c!C_F$, where $C_F=(N_c^2-1)/2N_c$ and $C_B=(N_c+1)/2N_c$. Values for the scalar coefficients B_i defined in Eq. (17) are listed in Appendix A.

Putting everything together, the two-loop and three-loop scalar correlation factor $P(\omega)$ defined in Eq. (17) is given by

$$P(\omega)=-\frac{32\omega^5}{(4\pi)^4}\Bigg[\left(\frac{-2\omega}{\mu}\right)^{-4\epsilon}\frac{1}{40}\left(\frac{1}{\epsilon}+\frac{107}{15}\right)+\frac{\alpha_S}{4\pi}\left(\frac{-2\omega}{\mu}\right)^{-6\epsilon}\Bigg(\frac{n^2-4n+6}{45\epsilon^2}+\frac{40\zeta(2)+61n^2-234n+396}{225\epsilon}+\frac{(n-2)s}{90}$$
$$+\frac{5(195n^2-780n+1946)\zeta(2)-2200\zeta(3)+4907n^2-18\,408n+34\,352}{2250}\Bigg)\Bigg]. \qquad (18)$$

The scalar correlation function $P(\omega)$ is renormalized by the square of the renormalization factor Z_J of the baryonic current derived in [17]. Accordingly one has

$$P(\omega)=Z_J^2 P^{\mathrm{ren}}(\omega)$$

with

$$Z_J=1+\frac{\alpha_S C_B}{4\pi\epsilon}(n^2-4n+6). \qquad (19)$$

The multiplication of $P(\omega)$ in Eq. (18) with Z_J^2 results in the cancellation of the second power in $1/\epsilon$. The remaining $1/\epsilon$ singularity is purely real and hence does not contribute to the spectral density. Since the renormalized spectral density

$\rho^{\rm ren}(\omega)=\mathrm{Im}[P^{\rm ren}(\omega)]/\pi$ has to be finite, this provides a check on our calculation. The spectral density can be read off from Eq. (18) and is given by

$$\rho^{\rm ren}(\omega,\mu)=\rho_0(\omega)\left[1+\frac{\alpha_S}{4\pi}\,r(\omega/\mu)\right],$$

where

$$\rho_0(\omega)=\frac{\omega^5}{20\pi^4}$$

and

$$r(\omega/\mu)=r_1\ln\left(\frac{\mu}{2\omega}\right)+r_2$$

with

$$r_1:=\tfrac{8}{3}(n^2-4n+6)$$

and

$$r_2:=\tfrac{8}{45}\;(60\zeta(2)+38n^2-137n+273). \qquad (20)$$

The coefficient r_1 of the logarithmic term in Eq. (20) coincides with twice the one-loop anomalous dimension given in Eq. (36), as expected. The reason is that the evolution of $\rho(\omega,\mu)$ is controlled by the renormalization group equation and that the anomalous dimension of $\langle JJ\rangle$ and $\rho(\omega,\mu)$ coincide.

The α_S correction can be seen to depend on the properties of the light-side Dirac matrix Γ in the heavy baryon current, as specified in Table I. As an explicit result we list representations of the $r(\omega/\mu)$ functions of the four baryon currents in the naively anticommuting (AC) γ_5 scheme. They read

$$\begin{aligned} r_{\Lambda 1}(\omega/\mu) &= 16\ln\left(\frac{\mu}{2\omega}\right)+\underbrace{\frac{8\big(20\zeta(2)+91\big)}{15}}_{\approx 66.04},\\ r_{\Lambda 2,\Sigma 1}(\omega/\mu) &= 8\ln\left(\frac{\mu}{2\omega}\right)+\underbrace{\frac{16\big(10\zeta(2)+29\big)}{15}}_{\approx 48.48},\\ r_{\Sigma 2}(\omega/\mu) &= \frac{8}{3}\ln\left(\frac{\mu}{2\omega}\right)+\underbrace{\frac{8\big(60\zeta(2)+151\big)}{45}}_{\approx 44.40}. \end{aligned} \qquad (21)$$

The results for the two different baryon currents Λ_1 and Λ_2 in the 't Hooft–Veltman (HV) γ_5 scheme differ from those presented above. It is well known that currents in different γ_5 schemes are connected by a finite renormalization factor Z such that

$$J_{\rm AC}=ZJ_{\rm HV}. \qquad (22)$$

These finite factors also appear in the calculation of two-loop anomalous dimensions of baryonic currents [24]. From the results of [24] one has

$$Z_{\Lambda 1}=1-\frac{4\alpha_S}{3\pi}$$

and

$$Z_{\Lambda 2}=1-\frac{2\alpha_S}{3\pi}. \qquad (23)$$

Using these finite renormalization factors one may convert the results in the naively anticommuting γ_5 scheme given in Eq. (21) to the corresponding results in the 't Hooft–Veltman scheme. Least the reader worry that we do not list the corresponding Σ-type conversion factors we remind him that the γ_5 in the Σ-type currents act on the heavy quark line and thus there are no γ_5 ambiguities. Nevertheless, the 't Hooft–Velman γ_5 scheme needs some counter terms to satisfy some kind of Ward identities. To avoid this complication, we will henceforth concentrate on the naively anticommuting γ_5 scheme, where such counterterms are not necessary at all. We only mention that the finite renormalization in Eq. (22) will bring the results of the two γ_5 schemes in line.

In order to allow for a quick appraisal of the importance of the perturbative corrections we have exhibited the numerical values of the second terms in Eq. (21). For α_S we use the running coupling constant, which we normalized to the value of $\alpha_S(m_Z)=0.118$ at the mass of the Z boson for $N_f=5$ active flavors. By doing so one has $\alpha_S(\mu)=0.333$ at $\mu=1$ GeV for $N_f=3$ active flavors. Using this value for $\alpha_S(\mu=1$ GeV), the above results show that the perturbative α_S corrections to the spectral density amount to about 100%. This highlights the importance of perturbative QCD radiative corrections in QCD sum rule applications. The same observation was made in the heavy meson sector [12,13,14]. As in the heavy meson sector on remains with several unsettled questions: (1) Are there any special reasons for such big QCD "corrections"? (2) Can we trust the QCD sum rule predictions and the α_S expansion when the α_S corrections are so big? (3) How big are the α_S^2 corrections? Is it possible to estimate them? These questions should be clarified in the near future.

A. Residues and QCD sum rules

To proceed with the usual QCD sum rules analysis, we evaluate the scalar correlator function $P(\omega)$ using the theoretical result $P_{\rm OPE}(t)$ given in Eq. (8) and equate this to the dispersion integral over the contributions of hadron states. These consist of the lowest-lying ground state with bound state energy Λ plus the excited states and the continuum. To leading order in $1/m_Q$ the bound state energy of the ground state is defined by

$$m_{\rm baryon}=m_Q+\bar{\Lambda}, \qquad (24)$$

where m_Q is the pole mass of the heavy quark. Note that the leading order sum rules do not depend on m_Q at all since the heavy mass dependence has been eliminated by employing the heavy mass expansion.

We assume that the continuum is given by the OPE expression above a certain threshold energy E_C [6]. For the hadron-side (HS) contribution to the spectral density we thus write

$$\rho_{\rm HS}(\omega)=\rho_{\rm GS}(\omega)+\rho_{\rm cont}(\omega), \tag{25}$$

where the contribution of the lowest-lying ground state (GS) baryon is contained in $\rho_{\rm GS}$ and is given by

$$\rho_{\rm GS}(\omega)=\tfrac{1}{2}F^2\delta(\omega-\overline{\Lambda}). \tag{26}$$

In this expression F is the absolute value of one of the residues F_i $(i=\Lambda,\Sigma,\Sigma^*)$ of the baryonic currents defined by

$$\langle 0|J|\Lambda_Q\rangle=F_\Lambda u,\quad \langle 0|J|\Sigma_Q\rangle=F_\Sigma u$$

and

$$\langle 0|J_\nu|\Sigma_Q^*\rangle=\frac{1}{\sqrt{3}}F_{\Sigma*}u_\nu, \tag{27}$$

where u and u_ν are the usual spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ spinors. Note that $F_{\Sigma*}$ coincides with F_Σ in the lowest order of the heavy quark mass expansion that we are working in.

As is usual we assume hadron-parton duality for the contribution of excited states and continuum contributions and take $\rho_{\rm cont}(\omega)=\theta(\omega-E_C)\rho(\omega)$, where ρ is the result of the OPE calculations given in Eqs. (8) and (16). With these assumptions we arrive at the sum rule

$$P_{\rm OPE}(\omega)=\frac{(1/2)F^2}{\overline{\Lambda}-\omega-i0}+\int_{E_C}^{\infty}\frac{\rho(\omega')d\omega'}{\omega'-\omega-i0} \tag{28}$$

or

$$\frac{(1/2)F^2}{\overline{\Lambda}-\omega-i0}=\int_0^{E_C}\frac{\rho(\omega')d\omega'}{\omega'-\omega-i0}+P_{\rm PC}(\omega), \tag{29}$$

where the power counting part $P_{\rm PC}(\omega)$ is defined as the Fourier transform of that part of the correlator function $P(t)$ which contains non-negative powers $(t^2)^n$ $(n\geqslant 0)$. Finally we apply the Borel transformation

$$\hat{B}_T=\lim\frac{\omega^n}{\Gamma(n)}\left(-\frac{d}{d\omega}\right)^n\quad n,-\omega\to\infty\quad (T=-\omega/n\ \text{ fixed}) \tag{30}$$

to the sum rule in Eq. (29). Using $\hat{B}_T[1/(\omega-\omega')]=\exp(-\omega'/T)/T$ we obtain the Borel sum rule

$$\begin{aligned}\tfrac{1}{2}F^2(\mu)e^{-\overline{\Lambda}/T}&=\int_0^{E_C}\rho(\omega',\mu)e^{-\omega'/T}d\omega'+\hat{B}P_{\rm PC}(T)\\&=:K(E_C,T,\mu),\end{aligned} \tag{31}$$

where we reintroduced the μ dependence of the spectral density, which causes a μ dependence for the residue. The Borel-transformed $\hat{B}P_{\rm PC}(T)$ can be obtained directly from $P_{\rm PC}(t)$ by the substitution $t\to -i/T$ (see the discussion in [17]). Note that the bound state energy $\overline{\Lambda}$ can be obtained from the sum rule in Eq. (31) by taking the logarithmic derivative with respect to the inverse Borel parameter according to

$$\overline{\Lambda}=-\frac{d\ \ln[K(E_C,T,\mu)]}{dT^{-1}}. \tag{32}$$

Returning to the sum rule in Eq. (31), one has

$$\begin{aligned}&\tfrac{1}{2}F^2(\mu)e^{-\overline{\Lambda}/T}\\&=\frac{N_c!}{\pi^4}\left[T^6\left(f_5(x_C)+\frac{\alpha_S}{4\pi}\left\{r_1\left[\ln\left(\frac{\mu}{2T}\right)f_5(x_C)-f_5^l(x_C)\right]\right.\right.\right.\\&\left.\left.\left.+r_2f_5(x_c)\right\}\right)+cE_G^4T^2f_1(x_C)+E_Q^6\ \exp\left(-\frac{2E_0^2}{T^2}\right)\right]\end{aligned} \tag{33}$$

with the polynomials r_1 and r_2 presented in Eq. (20) and the functions

$$f_n(x):=\int_0^x\frac{x'^n}{n!}e^{-x'}dx'=1-e^{-x}\sum_{m=0}^{n}\frac{x^m}{m!},$$

$$f_n^l(x):=\int_0^x\frac{x'^n}{n!}\ln x' e^{-x'}dx'. \tag{34}$$

In order to simplify the notation we have introduced the abbreviations

$$x_C:=\frac{E_C}{T},\quad E_0:=\frac{m_0}{4},\quad (E_Q)^3:=-\frac{\pi^2}{2N_c}\langle\overline{q}q\rangle$$

and

$$(E_G)^4:=\frac{\pi\alpha_S\langle GG\rangle}{32N_c(N_c-1)}. \tag{35}$$

The numerical analysis of the Borel sum rule is the subject of Sec. V.

B. Anomalous dimensions

The one-loop renormalization of the effective heavy baryon currents was considered in [17], the two-loop case was studied in [24]. In general they differ from those in conventional QCD. The one-loop anomalous dimension of baryonic currents, namely the first coefficient in the expansion $\gamma=\Sigma_k(\alpha_S/4\pi)^k\gamma_k$, only depends on n and is given by [17,24]

$$\gamma_1=-\tfrac{4}{3}[(n-2)^2+2]. \tag{36}$$

The general (n,s)-dependent formula for the two-loop anomalous dimension case is rather lengthy and can be found in [24]. As an illustration we list explicit values for the two-loop anomalous dimensions as calculated in the minimal subtraction (MS) scheme using the naive γ_5 scheme. One has (with explicit values given for $N_f=3$)

$$\gamma_{\Lambda 1} = -8\left(\frac{\alpha_S}{4\pi}\right) + \underbrace{\frac{1}{9}\big(16\zeta(2)+40N_f-796\big)}_{\approx -72.19}\left(\frac{\alpha_S}{4\pi}\right)^2, \tag{37}$$

$$\gamma_{\Lambda 2} = -4\left(\frac{\alpha_S}{4\pi}\right) + \underbrace{\frac{1}{9}\big(16\zeta(2)+20N_f-322\big)}_{\approx -26.19}\left(\frac{\alpha_S}{4\pi}\right)^2, \tag{38}$$

$$\gamma_{\Sigma 1} = -4\left(\frac{\alpha_S}{4\pi}\right) + \underbrace{\frac{1}{9}\big(16\zeta(2)+20N_f-290\big)}_{\approx -22.63}\left(\frac{\alpha_S}{4\pi}\right)^2, \tag{39}$$

$$\gamma_{\Sigma 2} = -\frac{8}{3}\left(\frac{\alpha_S}{4\pi}\right) + \underbrace{\frac{1}{27}\big(48\zeta(2)+8N_f+324\big)}_{\approx 15.81}\left(\frac{\alpha_S}{4\pi}\right)^2. \tag{40}$$

C. Renormalization group invariant sum rules

It is clear that the currents $J(\mu)$ depend on the renormalization scale μ. This dependence can be expressed by the renormalization group equation

$$\left(\mu\frac{d}{d\mu}+\gamma\right)J(\mu)=0, \quad \gamma := \frac{d\ln Z_J}{d\ln\mu}, \tag{41}$$

arising from the scale independence of the bare current $J_0 = Z_J(\mu)J(\mu)$, where γ is the anomalous dimension of the current discussed in the preceding subsection. To construct a renormalization group invariant quantity $J_{\rm inv}$, the renormalized current $J(\mu)$ has to be multiplied with the appropriate Wilson coefficient $C[\alpha_S(\mu)]$, i.e., $J_{\rm inv}=J(\mu)C[\alpha_S(\mu)]$. Accordingly the Wilson coefficients satisfy the renormalization group equation (see also [14,16])

$$\left(\mu\frac{d}{d\mu}-\gamma\right)C[\alpha_S(\mu)]=0$$

$$\Leftrightarrow \quad \left[\alpha_S\beta(\alpha_S)\frac{\partial}{\partial\alpha_S}-\gamma(\alpha_S)\right]C(\alpha_S)=0, \tag{42}$$

where $\beta := d\ln\alpha_S/d\ln\mu = \Sigma_k(\alpha_S/4\pi)^k\beta_k$ is the β function of QCD with

$$\beta_1 = -2(11-\tfrac{2}{3}N_f)$$

and

$$\beta_2 = -4(51-\tfrac{19}{3}N_f). \tag{43}$$

The formal solution of Eq. (42) is given by

$$C[\alpha_S(\mu)] = \exp\left(\int^{\alpha_S(\mu)}\frac{d\alpha}{\alpha}\frac{\gamma(\alpha)}{\beta(\alpha)}\right). \tag{44}$$

Finally, when the perturbative expansion of the β function and the anomalous dimension is inserted to second order in α_S one arrives at

$$C[\alpha_S(\mu)] = \alpha_S(\mu)^{\gamma_1/\beta_1}\left[1+\frac{\alpha_S(\mu)}{4\pi}\frac{\gamma_1}{\beta_1}\left(\frac{\gamma_2}{\gamma_1}-\frac{\beta_2}{\beta_1}\right)\right]. \tag{45}$$

The first factor in Eq. (45) is the result of resumming the leading logarithmic terms $(\alpha_S\ln\mu)^n$, where the result is valid only in the logarithmic approximation. As Eq. (45) shows, one needs to know also the two-loop anomalous dimension of the baryon current in order to obtain the evolution at next-to-leading log accuracy, e.g., in the order $\alpha_S(\alpha_S\ln\mu)^n$.

The usage of the invariance property of $J_{\rm inv}$ also provides a connection between currents at different renormalization scales:

$$J(\mu_2)C[\alpha_S(\mu_2)] = J(\mu_1)C[\alpha_S(\mu_1)],$$

thus

$$J(\mu_2) = J(\mu_1)C[\alpha_S(\mu_1)]C[\alpha_S(\mu_2)]^{-1} =: J(\mu_1)U(\mu_1,\mu_2), \tag{46}$$

$$\begin{aligned} U(\mu_1,\mu_2) &= \exp\left(\int_{\alpha_S(\mu_2)}^{\alpha_S(\mu_1)}\frac{d\alpha}{\alpha}\frac{\gamma(\alpha)}{\beta(\alpha)}\right) \\ &= \left(\frac{\alpha_S(\mu_1)}{\alpha_S(\mu_2)}\right)^{\gamma_1/\beta_1}\left[1+\frac{\alpha_S(\mu_1)-\alpha_S(\mu_2)}{4\pi}\right. \\ &\quad \left.\times\frac{\gamma_1}{\beta_1}\left(\frac{\gamma_2}{\gamma_1}-\frac{\beta_2}{\beta_1}\right)\right], \end{aligned} \tag{47}$$

where $U(\mu_1,\mu_2)$ is perturbatively evaluated up to next-to-leading order in α_S (see also the discussion in [2,25,26]).

As is evident from Eq. (27), also the residues are functions of the renormalization scale parameter μ. The functional form of this dependence is the same as for the currents. Thus one can construct the renormalization group invariant quantity $F_{\rm inv}=F(\mu)C[\alpha_S(\mu)]$ by means of the same Wilson coefficient. A renormalization group invariant sum rule can then be written down by considering the expression

$$\tfrac{1}{2}F_{\rm inv}^2\exp(-\bar{\Lambda}/T) = K(E_C,T,\mu)C[\alpha_S(\mu)]^2 =: K_{\rm inv}(E_C,T). \tag{48}$$

The theoretical part of the sum rule $K(E_C,T,\mu)$ depends on the renormalization scale μ through the QCD perturbative corrections which contain the logarithmic factor $\ln(\mu)$. On the other hand, the left-hand side of Eq. (48) is independent of the renormalization scale μ by construction, and thus the right-hand side must also be renormalization scale independent. It is easy to check this to first order in α_S by introducing a second scale μ' such that one has

$$\frac{\alpha_S(\mu)}{\alpha_S(\mu')} = 1-\frac{\alpha_S(\mu)}{8\pi}\beta_1\ln\left(\frac{\mu'^2}{\mu^2}\right). \tag{49}$$

Remembering that $\rho(\omega,\mu)$ in Eq. (20) appears as an integrand of $K(E_C,T,\mu)$, one obtains cancellations (to first order of α_S) of the logarithmic factors $\ln(\mu)$ in $\rho(\omega,\mu)C[\alpha_S(\mu)]^2$ and thereby in $K(E_C,T,\mu)C[\alpha_S(\mu)]^2$. The cancellation occurs because of the relation $r_1=-2\gamma_1$. In this paper we will make no use of the renormalization group invariant sum rule in Eq. (48). Instead we analyze the sum rule (31) at some fixed

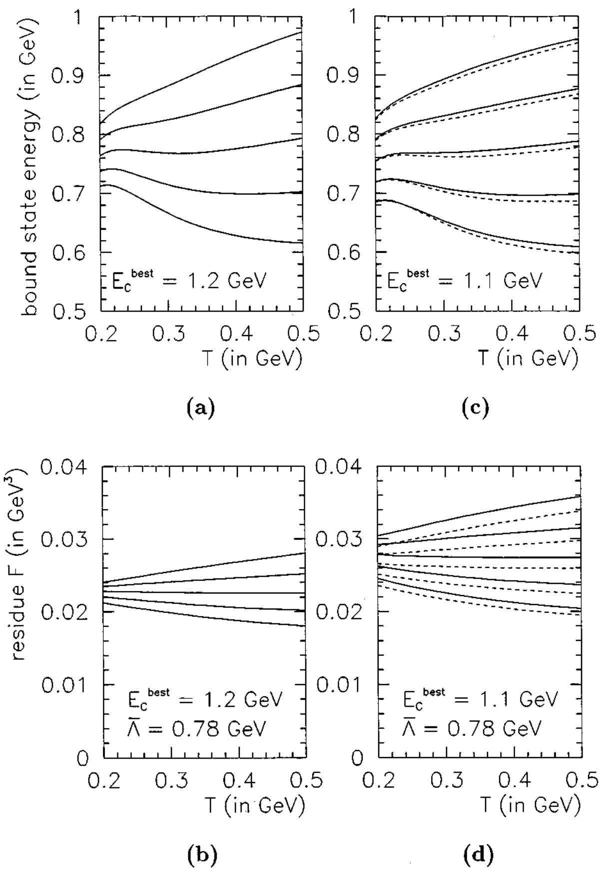

FIG. 2. Sum rule results on the nonperturbative parameters of the Λ_Q as functions of the Borel parameter T. Shown are five curves for five different values of the threshold energy E_C spaced by 100 MeV around the central value $E_C = E_C^{\rm best}$. E_C grows from bottom to top. These are in detail (a) lowest order sum rule results for the bound state energy $\bar{\Lambda}(\Lambda)$; (b) lowest order sum rule results for the absolute value of the residue F_Λ; (c) $O(\alpha_S)$ sum rule results for the bound state energy $\bar{\Lambda}(\Lambda)$; for the currents $J_{\Lambda 1}$ (solid) and $J_{\Lambda 2}$ (dashed); (d) $O(\alpha_S)$ sum rule results for the absolute value of the residue F_Λ for the currents $J_{\Lambda 1}$ (solid) and $J_{\Lambda 2}$ (dashed).

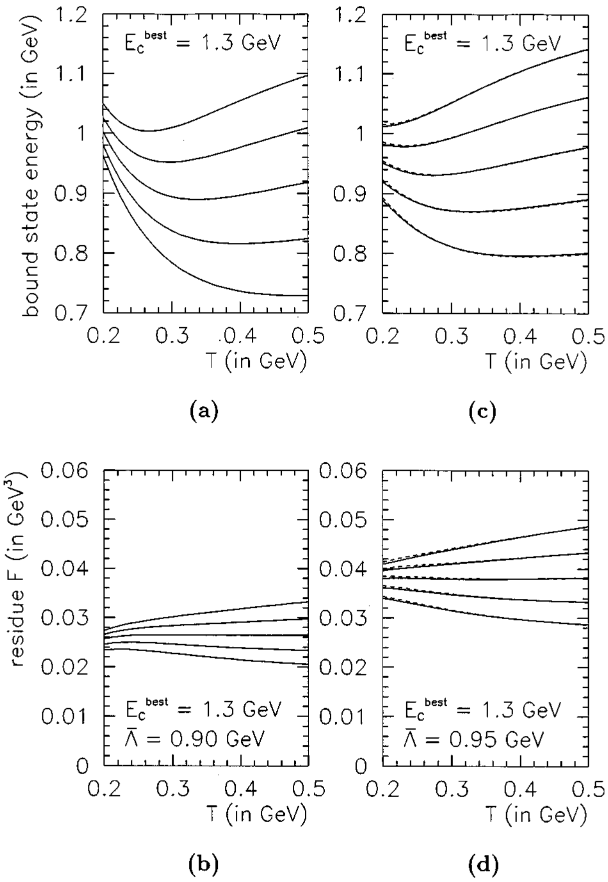

FIG. 3. Sum rule results on the nonperturbative parameters of the Σ_Q as functions of the Borel parameter T. Shown are five curves for five different values of the threshold energy E_C spaced by 100 MeV around the central value $E_C = E_C^{\rm best}$. E_C grows from bottom to top. These are in detail (a) lowest order sum rule results for the bound state energy $\bar{\Lambda}(\Sigma)$; (b) lowest order sum rule results for the absolute value of the residue F_Σ; (c) $O(\alpha_S)$ sum rule results for the bound state energy $\bar{\Lambda}(\Sigma)$; for the current doublets $\{J_{\Sigma 1}, J_{\Sigma * 1}\}$ (solid) and $\{J_{\Sigma 2}, J_{\Sigma * 2}\}$ (dashed); (d) $O(\alpha_S)$ sum rule results for the absolute value of the residue F_Σ for the corrent doublets $\{J_{\Sigma 1}, J_{\Sigma * 1}\}$ (solid) and $\{J_{\Sigma 2}, J_{\Sigma * 2}\}$ (dashed).

point $\mu' = 1$ GeV in order to estimate the bound state energy $\bar{\Lambda}$ and the residue $F(\mu')$. The value of the residue $F(\mu)$ at other scales can then be obtained by using the evolution function $U(\mu', \mu)$, while the μ-independent function $F_{\rm inv}$ can be immediately obtained by multiplying with $C[\alpha_S(\mu)]$.

V. NUMERICAL RESULTS

Let us present the sum rule analysis in some detail. We start by discussing the sum rules without radiative corrections and perform the analysis in consecutive steps. As Eq. (33) shows, the analysis of the lowest order sum rules does not depend on which of the two different current cases are being discussed. First, we analyze the dependence of the bound state energy $\bar{\Lambda}$ on the threshold parameter E_C and the Borel parameter T in a large window of parameter space. The aim is to try and find regions of stability in T and E_C. By looking at three-dimensional plots for $\bar{\Lambda}$ as functions of T and E_C we found a stability region for the sum rules only in the case of the exponential ansatz for the nonlocal operator $\langle \overline{q(0)} q(x) \rangle$ [see Eq. (10)]. The stability region lies in the acceptable range 0.3 GeV$<T<$0.4 GeV [see discussion after Eq. (8)]. We mention that the range of acceptable values for T extends down to $T>0.2$ GeV when radiative corrections are included, since these enlarge the perturbative contributions. This, however, does not bring in a new region of stability.

Returning to the analysis of the lowest order sum rules for the Λ_Q baryon, we find areas of stability around $E_C = 1.2$ GeV in the window 0.3 GeV$<T<$0.4 GeV. Acceptable stability is found in the range 1.0 GeV$<E_C<$1.4 GeV. Therefore in Fig. 2(a) we show plots for five values of E_C around $E_C^{\rm best} = 1.2$ GeV, namely for $E_C = E_C^{\rm best}$, $E_C = E_C^{\rm best} \pm 0.1$ GeV and $E_C^{\rm best} \pm 0.2$ GeV. From these curves we then read off values for E_C and $\bar{\Lambda}$ with good sum rule stability, namely

$$\bar{\Lambda}(\Lambda_Q) = 0.78 \pm 0.05 \ \text{GeV}$$

TABLE II. Sum rule results on nonperturbative and sum rule parameters of heavy ground state baryons. The continuum threshold parameter E_C, the bound state energy $\bar{\Lambda}$, and the difference between the two bound state energies are given in GeV, whereas the residues are listed in units of 10^{-2} GeV^3. The value of the Borel parameter is $T=0.35$ GeV.

	[17]	[19]	[28]	L.O.	N.L.O.
$E_C(\Lambda)$	1.20	1.20±0.15	1.2±0.1	1.2±0.1	1.1±0.1
$E_C(\Sigma)$	1.46	1.30±0.15	1.4±0.1	1.3±0.1	1.3±0.1
$\bar{\Lambda}(\Lambda)$	0.78	0.79±0.05	0.9±0.1	0.78±0.05	0.78±0.05
$\bar{\Lambda}(\Sigma)$	0.99	0.96±0.05		0.90±0.05	0.95±0.05
$\bar{\Lambda}(\Sigma)-\bar{\Lambda}(\Lambda)$	0.21	0.17		0.12	0.17
$\lvert F_\Lambda\rvert$	2.3±0.5	1.7±0.6	2.5±0.5	2.3±0.1	2.8±0.2
$\lvert F_\Sigma\rvert$	3.5±0.6	4.1±0.6	4.0±0.5	2.6±0.2	3.9±0.3

in the range

$$E_C(\Lambda_Q)=1.2\pm0.1 \text{ GeV}, \tag{50}$$

where the quoted errors present rough error estimates taken from Fig. 2(a) according to the range for E_C specified in Eq. (50).

Next we estimate the value of the residue. The sum rules now depend on the three parameters $\bar{\Lambda}$, E_C, and T. In Fig. 2(b) we plot $|F_\Lambda|$ for a fixed bound state energy $\bar{\Lambda}(\Lambda_Q)=0.78$ GeV in the indicated window for the Borel parameter T. The five different curves again correspond to the above five different values of E_C. Sum rule stability is found at

$$|F_\Lambda|=0.023\pm0.002 \text{ GeV}^3, \tag{51}$$

where the errors again represent rough error estimates taken from Fig. 2(b).

Next we take into account the α_S correction to the spectral density. As is evident from Eq. (20), the sum rule analysis now depends on which of the two types of baryonic currents are used. The results for the bound state energy for both cases are displayed in Fig. 2(c). However, as this figure shows, both diagonal sum rules lead to compatible values for $\bar{\Lambda}(\Lambda_Q)$ and E_C. We therefore combine the two results to a single value, since the differences lie within the error range. Using the same analysis as for Fig. 2(a) we obtain

$$\bar{\Lambda}(\Lambda_Q)=0.78\pm0.05 \text{ GeV}$$

in the range

$$E_C(\Lambda_Q)=1.1\pm0.1 \text{ GeV}. \tag{52}$$

As Figs. 2(a) and 2(c) show, the α_S-corrected sum rules show more stability, albeit at the lower value $E_C=1.1$ GeV. However, the prediction for the bound state energy $\bar{\Lambda}$ remains the same as in the leading order analysis.

Using the central value $\bar{\Lambda}(\Lambda_Q)=0.78$ GeV one can then obtain values for the residue looking at Fig. 2(d), which give rise to

$$|F_\Lambda|=0.028\pm0.002 \text{ GeV}^3. \tag{53}$$

Doing the same leading order analysis for the Σ baryon, we obtain

$$\bar{\Lambda}(\Sigma_Q)=0.90\pm0.05 \text{ GeV}, \quad E_C(\Sigma_Q)=1.3\pm0.1 \text{ GeV}$$

and

$$|F_\Sigma|=0.026\pm0.002 \text{ GeV}^3. \tag{54}$$

Including the α_S radiative corrections we have

$$\bar{\Lambda}(\Sigma_Q)=0.95\pm0.05 \text{ GeV}, \quad E_C(\Sigma_Q)=1.3\pm0.1 \text{ GeV}$$

and

$$|F_\Sigma|=0.039\pm0.003 \text{ GeV}^3, \tag{55}$$

where again the results for different currents are combined into single values. The results are displayed graphically in Figs. 3(a) and 3(b) and in Figs. 3(c) and 3(d), respectively.

Our predictions for the bound state energy $\bar{\Lambda}$ combined with the experimental charm and bottom baryon masses may be taken to calculate the charm and bottom quark pole masses m_Q. Taking into account the experimental results as given by the Particle Data Group [27], namely $m(\Lambda_c)=2284.9\pm0.6$ MeV, $m(\Sigma_c^+)=2453.5\pm0.9$ MeV, and $m(\Lambda_b)=5641\pm50$ MeV, we obtain the pole masses $m_c\approx1500$ MeV and $m_b\approx4860$ MeV for the heavy quarks. The experimental difference of $m(\Lambda_c)-m(\Sigma_c)\approx167$ MeV [27] is quite close to our prediction $m(\Lambda_Q)-m(\Sigma_Q)\approx170$ MeV. Here we present only central values. As was discussed above, the accuracy of our predictions is connected with the internal accuracy of the QCD sum rules method (mainly because of the dependence on the energy threshold of the continuum, E_C) and is probably not better than 20%.

All the results are summarized in Table II, where we compare our leading order and next-to-leading order results with the leading order results obtained in [17,19,28]. This concludes our analysis.

VI. CONCLUSIONS

We have considered the operator product expansion of the correlator of two static heavy baryon currents at small Euclidian distances and determined the α_S radiative corrections to the first Wilson coefficient in the expansion. Based on this expansion we formulated and analyzed heavy baryon sum rules for the Λ-type and Σ-type heavy baryons using two different types of interpolating fields for the baryons in each case. We have discussed in some detail the scale independence of the α_S sum rules which requires the consideration of the anomalous dimensions of the heavy baryon currents at the two-loop level.

Similar to the case of heavy mesons the QCD radiative correction to the first term in the OPE is quite large and amounts to a 100% change in the perturbative contribution. The radiative correction to the perturbative term increase the calculated sum rule values for the baryon masses by about 10% and the residues by about 20–50 % relative to the corresponding lowest order values. The sum rule results do not depend very much on which of the two possible interpolating fields is used in each case. The sum rule analysis is, however, quite sensitive to changes in the assumed threshold energy of the continuum. This sensitivity is the main source of uncertainty in our results and is partly due to the use of diagonal correlators in the sum rules for the following reason. QCD sum rules based on the diagonal correlators feature a leading order spectral density which grows rapidly as $\rho(\omega)\approx\omega^5$. This rapid growth introduces a strong dependence of the sum rule results on the assumed continuum threshold.

We have not considered nondiagonal sum rules which come in when one considers correlators between two different currents with the same quantum numbers. These nondiagonal sum rules bring in some new features such as a more ''normal'' behavior of the spectral density $\rho(\omega)\approx\langle\overline{q}q\rangle\omega^2$ which reduces the dependence of the sum rule analysis on the continuum threshold E_C. One can therefore expect more moderate QCD corrections to the spectral density. On the other hand, the leading term for nondiagonal sum rules is proportional to the quark condensate, whose value $\langle\overline{q}q\rangle=(-0.23\pm0.02\ \text{GeV})^3$ is known only with an accuracy of 10%. This is an additional source of uncertainty for the nondiagonal sum rules. The analysis of the nondiagonal sum rules forms the subject of a subsequent paper.

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APPENDIX: DIAGRAMMATIC CONTRIBUTIONS

In this Appendix we collect together results on the calculation of the two-loop and three-loop contributions to the diagonal correlators of the two heavy baryon currents. We start with the two-loop contribution depicted in Fig. 1 ($i=0$), where one has

$$B_0=\frac{(D-2)E_2}{4(2D-7)(2D-5)(2D-3)E_1}\,\tilde{b}_0\mathrm{Tr}(\overline{\Gamma}\not{v}\Gamma\not{v})$$

with

$$\tilde{b}_0=\frac{E_1}{(D-4)(D-3)}. \tag{A1}$$

We have introduced the abbreviation $E_n=\Gamma(1-\epsilon)^n\Gamma(1+n\epsilon)$ (with natural numbers $n=1,2,3,\ldots$) which is also used in the subsequent presentation of the three-loop results.

For the three-loop contributions $i=1$, 2, and 4 depicted in Fig. 1 one has

$$B_i=\frac{8(D-2)(2D-7)E_3}{9(3D-11)(3D-10)(3D-8)(3D-7)E_2}\times\tilde{b}_i\mathrm{Tr}(\overline{\Gamma}\not{v}\Gamma\not{v})$$

with

$$\tilde{b}_1=\frac{4(D-2)E_1^2}{(D-4)^3(D-3)^2}-\frac{2(D-2)(3D-10)E_2}{(D-4)^3(D-3)^2(2D-7)},$$

$$\tilde{b}_3=\frac{(D-2)E_2}{(D-4)^2(D-3)(2D-7)},$$

$$\tilde{b}_4=\frac{-(D-2)E_2}{(D-4)^2(D-3)^2(2D-7)} \tag{A2}$$

[note the combinatorical factor of 2 for the diagrams (1) and (3)]. The contribution of diagram (2) in Fig. 1 is the most involved one. In order to be able to write the results in a compact form we introduce the abbreviations

$$Q_1=\Gamma(1-\epsilon)^2\Gamma(1+\epsilon)/\Gamma(1-2\epsilon)$$

and

$$Q_2=\Gamma(1-\epsilon)^3\Gamma(1+2\epsilon)/\Gamma(1-3\epsilon). \tag{A3}$$

In terms of the basic structure terms

$$\tilde{\Gamma}_0=\mathrm{Tr}(\overline{\Gamma}\not{v}\Gamma\not{v}),\quad \tilde{\Gamma}_1=\mathrm{Tr}(\overline{\Gamma}\gamma_\mu\Gamma\gamma^\mu),$$

and

$$\tilde{\Gamma}_2=\mathrm{Tr}(\overline{\Gamma}\gamma_\mu\gamma_\nu\not{v}\Gamma\not{v}\gamma^\nu\gamma^\mu), \tag{A4}$$

one obtains

$$B_2=\frac{E_3}{9(D-3)(3D-11)(3D-7)Q_2}\sum_{j=0}^{2}\tilde{b}_{2,j}\tilde{\Gamma}_j$$

with

$$\tilde{b}_{2,0}=\frac{12(D-2)^2Q_1^2}{(D-4)^3(D-3)^2(D-1)}-\frac{24D(D-2)^2Q_2}{(D-4)^3(D-1)(3D-10)(3D-8)},$$

$$\tilde{b}_{2,1}=\frac{(D^2-7D+16)Q_1^2}{(D-4)^2(D-3)^2(D-1)}-\frac{4(D^2-4D+8)Q_2}{(D-4)^2(D-1)(3D-10)(3D-8)},$$

$$\tilde{b}_{2,2}=\frac{3Q_1^2}{(D-4)^2(D-3)(D-1)}-\frac{4Q_2}{(D-4)(D-1)(3D-10)(3D-8)}. \tag{A5}$$

[1] See, for example, M. B. Wise, in *Particle Physics-The Factory Era*, Proceedings of the 6th Lake Louise Winter Institute, Lake Louise, Canada, 1991, edited by B. A. Campbell, A. N. Kitching, and F. G. Khanna (World Scientific, Singapore, 1991), p. 222.
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