Strange and charmed quarks in the nucleon

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We discuss the general method of the calculation of the nucleon matrix elements of an operator associated with nonvalence quarks. The method is based on the QCD sum rules and low energy theorems. As an application of these considerations, we calculate the strange quark matrix element as well as the momentum distribution of the strangeness in the nucleon. We also calculate the singlet axial constant associated with η' meson as well as an axial constant associated with heavy quarks. [S0556-2821(97)06205-X]

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I. INTRODUCTION

For a long time it was widely believed that the admixture of the pairs of strange quarks in the nucleons is small. The main justification of this picture was the constituent quark model where there is no room for strange quark in the nucleon. It has been known for a while that this picture is not quite true: In scalar and pseudoscalar channels one can expect a noticeable deviation from this naive prediction. This is because these channels are very unique in a sense that they are tightly connected to the QCD-vacuum fluctuations with $0^+,0^-$ singlet quantum numbers. Manifestation of the uniqueness can be seen, in particular, in the existence of the axial anomaly (0^- channel) and the trace anomaly (0^+ channel). Nontrivial QCD vacuum structure tells us that one could expect some unusual properties when we deal with those quantum numbers.

As we now know, this is indeed the case. In particular, we know that the strange quark matrix element $\langle N|\overline{s}s|N\rangle$ does not vanish and has the same order of magnitude as $\langle N|\overline{d}d|N\rangle$. This information can be obtained from analysis of the so-called σ term [1,2]. Similarly analysis of the ''proton spin crisis'' essentially teaches us that the spin which is carried by the strange quark in the nucleon is not small as naively one could expect; see, e.g., the recent review in [3].

Another phenomenological manifestation of the same kind is the very old observation that in the scalar and pseudoscalar channels the Zweig rule is badly broken and there is substantial admixture of s quarks in the scalar mesons $f_0(980)$ (was S^*), and $a_0(980)$ (was δ), and $f_0(1300)$ (was ϵ), as well as in the pseudoscalar mesons η and η'. At the same time, in the vector channel the Zweig rule works well. Phenomenologically it is evident in, e.g., the smallness of the ϕ-ω mixing. In terms of QCD such a smallness corresponds to the numerical suppression of the nondiagonal correlation function $\int dx\langle 0|T\{\overline{s}\gamma_\mu s(x),\overline{u}\gamma_\nu u(0)\}|0\rangle$ in comparison with the diagonal one $\int dx\langle 0|T\{\overline{u}\gamma_\mu u(x),\overline{u}\gamma_\nu u(0)\}|0\rangle$. In the scalar and pseudoscalar channels diagonal and nondiagonal channels have the same order of magnitude. We believe that analysis of such kinds of correlation functions is an appropriate method for a QCD-based explanation of the unusual hadronic properties mentioned above.

In this paper we present some general methods and ideas for analysis of the nucleon matrix elements from a nonvalence operator. The ideology and methods (unitarity, dispersion relations, duality, low-energy theorems) we use are motivated by QCD sum rules. However, we do not use the QCD sum rules in the common sense. Instead, we reduce one complicated problem (the calculation of nonvalence nucleon matrix elements) to another one (the behavior of some vacuum correlation functions at low momentum transfer). One could think that such a reducing of one problem to another one (which may be even more complicated) does not improve our understanding of the phenomenon. However, this is not quite true: Analysis of the vacuum correlation functions with vacuum quantum numbers, certainly, is a very difficult problem. However, some nonperturbative information based on low energy theorems is available for such a correlation function. This gives some chance to estimate some interesting quantities.

II. STRANGENESS IN THE NUCLEON 0^+ CHANNEL

A. First estimations

We start by calculating the strange scalar matrix elements over the nucleon, assuming an octet nature of SU(3) symmetry breaking. We follow Ref. [4] (see also [5] for a review) in our calculations [6], but with a small difference in details. We present these results for completeness of the paper.

The results of the fit to the data on πN scattering presented in [2] lead to the following estimates for the so-called σ term [7]:

$$\frac{m_u+m_d}{2}\langle p|\overline{u}u+\overline{d}d|p\rangle\simeq 45\ \text{MeV}. \tag{1}$$

(Here and in what follows we omit kinematical structures such as $\overline{p}p$ in expressions for matrix elements.) Taking the values of the quark masses to be $m_u=5.1\pm0.9$ MeV, $m_d=9.3\pm1.4$ MeV, and $m_s=175\pm25$ MeV [8], from Eq. (1) we have

$$\langle p|\overline{u}u+\overline{d}d|p\rangle\simeq 6.2. \tag{2}$$

Further, assuming octet-type SU(3) breaking to be responsible for the mass splitting in the baryon octet, we find

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$$\langle p|\overline{u}u-\overline{d}d|p\rangle=\frac{m_\Xi-m_\Sigma}{m_s}=0.7, \quad (3)$$

$$\langle p|\overline{u}u+\overline{d}d-2\overline{s}s|p\rangle=3\frac{m_\Xi-m_\Lambda}{m_s}=3.4. \quad (4)$$

Here m_Ξ, m_Σ, and m_Λ are masses of Ξ, Σ, and Λ hyperons, respectively. The values (3) and (4) are quite reasonable: The former is close to the difference of the number of u and d quarks in a proton (should be 1), and the latter is close to the total number of valence quarks u and d in a nucleon (should be 3). From Eqs. (2)–(4) one obtains

$$\langle p|\overline{u}u|p\rangle\simeq 3.5, \quad (5)$$

$$\langle p|\overline{d}d|p\rangle\simeq 2.8, \quad (6)$$

$$\langle p|\overline{s}s|p\rangle\simeq 1.4. \quad (7)$$

We should mention that the accuracy of these equations is not very high. For example, the error in the value of the σ term already leads to a large error in each matrix element discussed above. In addition to that, chiral perturbation corrections also give a noticeable contribution the matrix elements (5)–(7); see [7]. However, the analysis of possible errors in Eqs. (5)–(7) is not the goal of this paper. Rather, we wanted to demonstrate that these very simple calculations explicitly show that the strange matrix element is not small. Recent lattice calculations [9] also support a large magnitude for the strange matrix element.

We would like to rewrite relations (5)–(7) to separate the vacuum contribution to the nucleon matrix element from the valence contribution. In order to do so, let us define

$$\langle p|\overline{q}q|p\rangle\equiv\langle p|\overline{q}q|p\rangle_0+\langle p|\overline{q}q|p\rangle_1, \quad (8)$$

where index 0 labels a (sea) vacuum contribution and index 1 a valence contribution for a quark q. We assume that the vacuum contribution which is related to the sea quarks is the same for all light quarks u, d, and s. Thus, the nonzero magnitude for the strange matrix elements comes exclusively from the vacuum fluctuations. At the same time, the matrix elements related to the valence contributions are equal to

$$\langle p|\overline{u}u|p\rangle_1\simeq(3.5-1.4)\simeq 2.1, \quad (9)$$

$$\langle p|\overline{d}d|p\rangle_1\simeq(2.8-1.4)\simeq 1.4. \quad (10)$$

These values are in remarkable agreement with the numbers 2 and 1, which one could expect from the naive picture of a nonrelativistic constituent quark model. In spite of the very rough estimations presented above, we believe we have presented arguments that should convince the reader that (a) the magnitude of the nucleon matrix element for $\overline{s}s$ is not small, (b) the large magnitude for this matrix element is due to the nontrivial QCD vacuum structure where vacuum expectation values of u, d, and s quarks are developed and they are almost the same in magnitude: $\langle 0|\overline{d}d|0\rangle\simeq\langle 0|\overline{u}u|0\rangle\simeq\langle 0|\overline{s}s|0\rangle$.

Once we realize that the phenomenon under discussion is related to the nontrivial vacuum structure, it is clear that the best way to understand such a phenomenon is to use some method where QCD vacuum fluctuations and hadronic properties are strongly interrelated. We believe that the most powerful analytical nonperturbative method which exhibits these features is the QCD sum rule approach [10,11].

In what follows we use the QCD sum rule method in order to relate hadronic matrix elements and vacuum characteristics. Let me emphasize from the very beginning that we do not use the QCD sum rules in the standard way: We do not fit them to extract any information about lowest resonance (as is usually done in this approach), and we do not use any numerical approximation or implicit assumptions about higher states. Instead, we concentrate on the qualitative relations between hadronic properties and QCD vacuum structure. We try to explain in a qualitative way some magnitudes for the nucleon matrix elements which may look very unexpected from the naive point of view. At the same time those matrix elements can be easily understood in terms of the QCD vacuum structure.

We close this section with the formulation of the following question: *What is the QCD explanation of the unusual properties mentioned above* (in particular, the large magnitude for the strange nucleon matrix element, the special role of the scalar and pseudoscalar channels, etc.)?

Our answer on this question is as follows: *Hadronic matrix elements with $0^\pm$ quantum numbers are singled out because of the special role they play in the QCD vacuum structure.* The next section changes this answer from a qualitative remark to a quantitative description.

B. Strangeness in the nucleon and vacuum structure

To study the problem of calculation $\langle N|\overline{s}s|N\rangle$ using the QCD sum rule approach, we consider the vacuum correlation function [6]

$$T(q^2)=\int e^{iqx}dxdy\langle 0|T\{\eta(x),\overline{s}s(y),\overline{\eta}(0)\}|0\rangle \quad (11)$$

at $-q^2\to\infty$. Here η is an arbitrary current with nucleon quantum numbers. In particular, this current may be chosen in the standard form $\eta=\epsilon^{abc}\gamma_\mu d^a(u^bC\gamma_\mu u^c)$. Note, however, that the results obtained below do not imply such a concretization. For future convenience we consider the unit matrix kinematical structure in Eq. (11).

This is the standard first step of any calculation of such a kind: Instead of a direct calculation of a matrix element, we reduce the problem to the computation of some correlation function. As the next step, we use the duality and dispersion relations to relate a physical matrix element to the QCD-based formula for the corresponding correlation function. This is essentially the basic idea of the QCD sum rules.

In our specific case (11), as a result of the absence of the s-quark field in the nucleon current η, any substantial contribution to $T(q^2)$ is connected only to nonperturbative, so-called induced vacuum condensates; see Fig. 1. Such a contribution arises from the region when some distances are large: $(y-0)^2\sim(y-x)^2\gg(x-0)^2$. Thus, it cannot be directly calculated in perturbative theory; instead, we code the corresponding large-distance information in the form of a bilocal operator

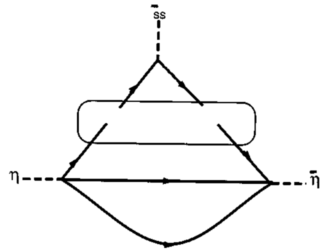

FIG. 1. Bilocal contribution (12) into the correlator (11).

$$K=i\int dy\langle 0|T\{\overline{s}s(y),\overline{u}u(0)\}|0\rangle. \tag{12}$$

see Fig. 1. Similar contributions were considered for the first time in Ref. [13], but in quite different context. Besides that, the corresponding discussions were based on the specific instanton calculations as an example of nonperturbative fluctuations. In a more general framework similar contributions were discussed in Ref. [14]. For different applications of this approach when bilocal operators play essential role, see also Ref. [15].

Along with consideration of the three-point correlation function (11), we would like to consider the standard two-point correlator

$$P(q^2)=\int e^{iqx}dx\langle 0|T\{\eta(x),\overline{\eta(0)}\}|0\rangle, \tag{13}$$

see Fig. 2. The correlator (13) is determined by the nucleon residues $\langle 0|\eta|N\rangle$ and some duality interval S_0. At the same time the correlator (11) includes the information on the nucleon matrix element $\langle N|\overline{s}s|N\rangle$ also. Comparing Eq. (11) with Eq. (13) at $-q^2\to\infty$, we arrive at the relation

$$\langle N|\overline{s}s|N\rangle\simeq\frac{-m}{\langle\overline{q}q\rangle}K, \tag{14}$$

where m is the nucleon mass. We would like to note here that the relation between the matrix element of nonvalence quark bilinears in the nucleon and the corresponding vacuum structure is not new. Such a relation has been discussed many times. In the context of the present paper this connection was discussed in [16] and [6].

The main assumptions which have been made in the derivation of this relation are the following. First, we made the standard assumption about local duality for the nucleon. In other words, we assumed that a nucleon saturates both correlation functions with duality interval S_0. The second assumption is that typical scales (or, what is the same, duality intervals) in the limit $-q^2\to\infty$ in the corresponding sum rules Eqs. (11) and (13) are not much different in magnitude from each other. In this case the dependence on residues $\langle 0|\eta|N\rangle$ is canceled out in the ratio and we are left with the matrix element $\langle N|\overline{s}s|N\rangle$, Eq. (14), we are interested in.

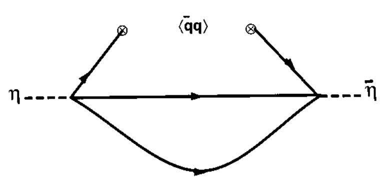

FIG. 2. Condensate contribution into the correlator (13).

Note that both these assumptions are very likely to be satisfied because we know that in most cases the lowest state (nucleon) does saturate the sum rules. If it does, then the typical scale (which in variety of sum rules is one and the same and of order of 1 GeV^2) guarantees that the duality intervals are likely to be very close to each other. Anyway, quantitative analysis of the corresponding sum rules is possible; however, it is not our main goal. Rather, we want to demonstrate the relation between matrix elements such as $\langle N|\overline{s}s|N\rangle$ and the corresponding vacuum properties which are hidden in the correlator K, Eq. (12). In principle one could analyze the sensitivity of the corresponding QCD sum rules to the lowest state nucleon. Once it is demonstrated, we believe that the accuracy of our formula (14) is of order $20\%-30\%$ which is a typical error for the sum rule approach.

Thus, the calculation of $\langle N|\overline{s}s|N\rangle$ reduces to the evaluation of the vacuum correlator K. Fortunately, sufficient information about the latter comes from the low energy theorems. We note also that this method of reducing the nucleon matrix elements to that of the vacuum correlator is directly generalized to cover the arbitrary scalar O_S or pseudoscalar O_P operator[1]:

$$\langle N|O_S|N\rangle\simeq\frac{-m\overline{N}N}{\langle\overline{q}q\rangle}i\int dy\langle 0|T\{O_S,\overline{u}u(0)\}|0\rangle, \tag{15}$$

$$\langle N|O_P|N\rangle\simeq\frac{-m\overline{N}i\gamma_5N}{\langle\overline{q}q\rangle}i\int dy\langle 0|T\{O_P,\overline{u}i\gamma_5u(0)\}|0\rangle. \tag{16}$$

The estimation of the nonperturbative correlator K can be done by using some low energy theorems. In this case K is expressed in terms of some vacuum condensates [6]:

$$K=i\int dy\langle 0|T\{\overline{s}s(y),\overline{u}u(0)\}|0\rangle\simeq\frac{18}{b}\frac{\langle\overline{q}q\rangle^2}{\langle(\alpha_s/\pi)G_{\mu\nu}^2\rangle}$$
$$\simeq 0.04\ \text{GeV}^2, \tag{17}$$

where $b=\frac{11}{3}N_c-\frac{2}{3}N_f=9$ and we use the standard values for the vacuum condensates [10]:

$$\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^2\right\rangle\simeq 1.2\times10^{-2}\ \text{GeV}^4,\quad \langle\overline{q}q\rangle\simeq-(250\ \text{MeV})^3.$$

With the estimation (17) for K, our formula (14) gives the following expression for the nucleon expectation value for $\overline{s}s$:

$$\langle p|\overline{s}s|p\rangle\simeq -m\frac{18}{b}\frac{\langle\overline{q}q\rangle}{\langle(\alpha_s/\pi)G_{\mu\nu}^2\rangle}\simeq 2.4, \tag{18}$$

[1]We assume of course that these operators do not contain u and d quarks. Otherwise, an additional contribution which comes from the small distances must be also included.

which is not far away from the naive estimation (7).

We believe that the main uncertainty in the formula (18) is related to our lack of knowledge of the nonperturbative correlation function K, Eq. (17). Therefore, for our estimates which follow we prefer to use formula (14) in order to extract the corresponding value for K from the experimental data instead of using our rough estimation (17). In this case K is a little smaller:

$$K\simeq -\frac{1}{m}\langle p|\overline{s}s|p\rangle\langle 0|\overline{u}u|0\rangle\sim 0.025 \ \ \mathrm{GeV}^2. \tag{19}$$

Let us stress that we are not pretending to have made a reliable calculation of the matrix element $\langle p|\overline{s}s|p\rangle$ here. Rather, we wanted to emphasize the qualitative picture which demonstrates the close relation between nonvalence matrix elements and QCD vacuum structure.

We close this section by noting that the method presented above gives a very simple physical explanation of why the Zweig rule in the scalar and pseudoscalar channels is badly broken and, at the same time, in the vector channel the Zweig rule works well. In particular, the matrix element $\langle N|\overline{s}\gamma_\mu s|N\rangle$ is expected to be very small as well as the corresponding coupling constant $g_{\phi NN}$. In terms of QCD such a smallness corresponds to the numerical suppression (10^{-2}–10^{-3}) of the nondiagonal correlation function $\int dx\langle 0|T\{\overline{s}\gamma_\mu s(x),\overline{u}\gamma_\nu u(0)\}|0\rangle$ in comparison with the diagonal one $\int dx\langle 0|T\{\overline{u}\gamma_\mu u(x),\overline{u}\gamma_\nu u(0)\}|0\rangle$; see the QCD estimation in [10]. In the scalar and pseudoscalar channels the diagonal and nondiagonal correlators have the same order of magnitude; therefore, no suppression occurs. This is the fundamental explanation of the phenomenon we are discussing in this paper. Specifically, the magnitude of correlator K is not changing much if we replace an s quark with a u quark in formula (12). Of course, it is in contradiction with the large-N_c (number of colors) counting rule where a nondiagonal correlator should be suppressed. The fact that the naive counting of powers of N_c fails in channels with total spin 0 is well known: Quantities small in the limit $N_c\to\infty$ turn out to be large and vice versa. This is a manifestation of the phenomenon discovered in Ref. [12]: Not all hadrons in the real world are equal to each other.

One may ask the same question regarding the axial matrix element as measured in polarized deep inelastic scattering. As is known, the corresponding measurement shows a large mixing. We believe that this phenomenon is related to the divergence part of the axial vector current. Therefore one can treat the corresponding large mixing as if it were a pseudoscalar matrix element (with an anomaly piece included in the singlet case). See Sec. III for details. We have nothing like that for the vector current. Therefore one could expect a small nonvalence matrix elements in the vector case.

In the next few sections we discuss some applications of the obtained results.

C. In the world where s quark is massless

We would like to look at formula (12) from a different side. Namely, we note that K not only enters expression (14), but also determines the variation of the condensate $\langle\overline{u}u\rangle$ with s-quark mass:

$$\frac{d}{dm_s}\langle\overline{u}u\rangle = -i\int dy\langle 0|T\{\overline{s}s(y),\overline{u}u\}|0\rangle = -K\simeq -0.025 \ \ \mathrm{GeV}^2. \tag{20}$$

To understand how large this number is and in order to make some rough estimations, we assume that this behavior can be extrapolated from the physical value $m_s\simeq 175$ MeV until $m_s=0$. In this case we estimate that

$$\left|\frac{\langle\overline{u}u\rangle_{m_s=175}-\langle\overline{u}u\rangle_{m_s=0}}{\langle\overline{u}u\rangle_{m_s=175}}\right|\simeq 0.3. \tag{21}$$

Such a decrease of $|\langle\overline{u}u\rangle|$ by a 30% as m_s varies from $m_s\simeq 175$ MeV to $m_s=0$ is a very important consequence of the previous discussions: Once we accept the relatively large magnitude for the nucleon matrix element $\langle p|\overline{s}s|p\rangle\simeq 1.4$, we are forced to accept the relatively large variation of the light quark condensate as well. This statement is the direct consequence of QCD; see Eq. (21).

We note that this result does not seem very surprising since other vacuum condensates, e.g., $\langle(\alpha_s/\pi)G^2_{\mu\nu}\rangle$, possess analogous properties [12]. From the microscopic point of view, a decrease of the absolute values of the vacuum matrix elements with a decrease of the s-quark mass is expected since any topologically nontrivial vacuum configurations, e.g., instantons, are suppressed by light quarks. The corresponding numerical calculation is very difficult to perform; however, a qualitative picture of the QCD vacuum structure definitely supports this idea [17].

D. s-quark and nucleon mass

We would like to discuss here one more fundamental characteristic of the hadron world: the nucleon mass and its dependence on the strange quark. We start our discussion from the following well-known result: The nucleon mass is determined by the trace of the energy-momentum tensor $\theta_{\mu\mu}$ and in the chiral limit $m_u=m_d=m_s=0$ the nonzero result comes exclusively from the strong interacting gluon fields:

$$m=-\frac{b}{8}\left\langle N\left|\frac{\alpha_s}{\pi}G^2_{\mu\nu}\right|N\right\rangle,\quad m_u=m_d=m_s=0. \tag{22}$$

However, as we know, in our world the strange quark is not massless, but rather it requires some (large enough) mass ($\sim$175 MeV). As we have seen, Eq. (21), the nonzero mass of an s quark considerably changes the vacuum properties of the world. Thus, we would expect that it might have a strong influence on the nucleon mass as well. The main argument which supports this point of view is the same as before and is based on our general philosophy that the nucleon matrix elements and vacuum properties are tightly related. So, if the strange quark has a strong influence on the vacuum properties, then its impact on the nucleon mass should also be strong.

In order to check these reasons it would be useful to calculate the strange quark contribution to the nucleon mass directly and independently from the gluon contribution (22). Fortunately, it can be easily done by using our previous estimation (18) for the nucleon matrix element and exact expression for the trace of the energy-momentum tensor taking into account nonzero quark masses:

$$m=+\left\langle N\left|\sum_q m_q\overline{q}q\right|N\right\rangle-\frac{b}{8}\left\langle N\left|\frac{\alpha_s}{\pi}G^2_{\mu\nu}\right|N\right\rangle, \qquad (23)$$

where the sum over q is the sum over all light quarks u, d, and s. One can easily see from Eq. (1) that the u,d contribution into the nucleon mass does not exceed 7%; thus, we can safely neglect this. At the same time, adopting the values (7) and (18) for $\langle p|\overline{s}s|p\rangle$ and $m_s\simeq 175$ MeV [8], one can conclude that a noticeable part of the nucleon mass (about 20%) is due to the strange quark. In this case the gluon contribution into the nucleon mass is far away from the chiral SU(3) prediction (22) and approximately equals

$$-\frac{b}{8}\left\langle N\left|\frac{\alpha_s}{\pi}G^2_{\mu\nu}\right|N\right\rangle\sim 700\ \text{MeV}. \qquad (24)$$

This rough estimation confirms our argument that a variation of the strange quark mass from its physical value to zero may considerably change some vacuum characteristics as well as nucleon matrix elements.

The simple consequence of this result is the observation that the *quenched approximation* in the lattice calculations is not justified simply because such a calculation clearly does not account for the fluctuations of the strange (nonvalence) quark as well as vacuum fluctuations of u and d quarks. As we argued above, the nucleon mass undergoes some influence from the s quark.

How one can understand these results within the framework of the QCD sum rules? Let us recall that in the QCD sum rule approach information about any dimensional parameter is contained in the vacuum condensates $\langle\overline{u}u\rangle,\langle G^2_{\mu\nu}\rangle,\ldots$. As we discussed previously all these condensates varying with m_s considerably. It is important that this variation certainly proceed in the right direction: Absolute values of condensates decrease with decreasing m_s. This leads to a smaller scale in the sum rules, and finally, to the decrease of all dimensional parameters such as m. However, it is difficult to make any reliable calculations because of a large number of factors playing an essential role in such a calculation.

E. Momentum distribution of the strangeness in the nucleon

We continue our study of the role of the strange quark in the nucleon with the following remark. We found out earlier that the matrix element $\langle N|\overline{s}s|N\rangle$ is not small; we interpreted this result as a result of strong vacuum fluctuations which penetrate into the nucleon matrix element. Now, we would like to ask the following question: What is the mean value of the momentum (denoted as $\langle k_\perp^2\rangle_s$) of the s quark inside of a nucleon? Let us note that this question is not a purely academic one. Rather, the answer on the question might be important for the construction of a more sophisticated quark model which would incorporate the strange context into the nucleon wave function.

First of all, let us try to formulate this question in terms of QCD. We *define* the mean value $\langle k_\perp^2\rangle_s$ of the momentum carried by the strange quark in a nucleon by the matrix element

$$\langle k_\perp^2\rangle_s\langle N|\overline{s}s|N\rangle\equiv\langle|N\overline{s}(i\vec{D}_\perp)^2s|N\rangle, \qquad (25)$$

where $i\vec{D}_\mu\equiv i\vec{\partial}_\mu+gA^a_\mu\lambda^a/2$ is the covariant derivative and A^a_μ is a gluon field. The arrow shows the quark whose momentum is under discussion.

We assume the nucleon to be moving rapidly in the z direction. We are interested in the momentum distribution in the direction which is perpendicular to its motion. Precisely this characteristic has a dynamical origin. Indeed, as we shall see in a moment, while we are studying a nucleon matrix element $\langle k_\perp^2\rangle_s$, we are actually probing the QCD vacuum properties. The nucleon motion as a whole system with arbitrary velocity does not affect this characteristic. Thus, essentially, what we discuss is the so-called light cone wave function. Apart from the reasons mentioned above, there are a few more motives to do so: First of all, the light cone wave function (WF) with a minimal number of constituents is a good starting point. As is known such a function gives parametrically leading contributions to hard exclusive processes. Higher Fock states are also well defined in this approach and can be considered separately. The second reason to work with a light cone wave function is the existence of the nice relation between that WF and the structure function measured in the deep inelastic scattering. We refer to the review paper of [18] for the introduction into the subject. The relation to the standard quark model wave functions (see, e.g., [19]) is also worked out. The relevant discussions can be found in Ref. [20]. In addition to these, we have one more reason to work with the light cone WF: We believe that this is the direction where a valence quark model can be understood and formulated in QCD terms [21].

Anyhow, formula (25) with the derivatives taken in the direction perpendicular to the nucleon momentum, $p_\mu=(E,0_\perp,p_z)$, is a very natural definition for the mean square of the quark transverse momentum. Of course it is different from a naive, gauge-dependent definition such as $\langle N|\overline{s}\partial_\perp^2 s|N\rangle$, because the physical transverse gluon is a participant of this definition. However, expression (25) is the only possible way to define the $\langle k_\perp^2\rangle_s$ in a gauge theory such as QCD. We believe that such a definition is a useful generalization of the transverse momentum conception for the interactive quark system. Let us note that the Lorentz transformation in the z direction does not affect the transverse directions. Thus, the transverse momentum $\langle k_\perp^2\rangle_s$ as calculated from Eq. (25) remains unchanged while we pass from the light cone system to the rest frame system where a quark model is supposed to be formulated.

Now, let us come back to our definition (25) for $\langle k_\perp^2\rangle_s$. In order to calculate this matrix element, we use the same trick as before: We reduce our original problem of calculating the nucleon matrix element to the problem of computing the corresponding vacuum correlation function (15):

$$\langle N|\overline{s(i\vec{D}_\perp)}^2 s|N\rangle \simeq \frac{-m\overline{N}N}{\langle\overline{q}q\rangle} i\int dy\langle 0|T\{\overline{s(i\vec{D}_\perp)}^2 s,\overline{u}u(0)\}|0\rangle. \quad (26)$$

To estimate the right-hand side of Eq. (26) we introduce an auxiliary vacuum correlation function

$$i\int dy\langle 0|T\{\overline{s(i\vec{D}_\mu i\vec{D}_\nu)}s,\overline{u}u(0)\}|0\rangle = Cg_{\mu\nu}, \quad (27)$$

where C is a constant. From the definition (26) it is clear that the correlator we are interested in can be expressed in terms of the constant C:

$$i\int dy\langle 0|T\{\overline{s(i\vec{D}_\perp)}^2 s,\overline{u}u(0)\}|0\rangle = -2C. \quad (28)$$

At the same time the constant C is given by the correlation function which contains $G^a_{\mu\nu}$ and not a covariant derivative D_μ:

$$C = \frac{1}{4} i\int dy\langle 0|T\{\overline{s(i\vec{D}_\mu i\vec{D}_\mu)}s,\overline{u}u(0)\}|0\rangle = -\frac{1}{8} i\int dy\langle 0|T\{\overline{s}igG^a_{\mu\nu}(\lambda^a/2)\sigma_{\mu\nu}s,\overline{u}u(0)\}|0\rangle, \quad (29)$$

where we have used the equation of motion and identity[2]:

$$D_\mu D_\nu g_{\mu\nu}s = \gamma_\mu\gamma_\nu D_\mu D_\nu s - \sigma_{\mu\nu}\tfrac{1}{2}[D_\mu, D_\nu]s = -m_s^2 s + \frac{ig}{2}\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}s. \quad (30)$$

Now we can estimate the unknown vacuum correlator (29) exactly in the same way as we have done before for the correlation function K; see Eq. (17). Collecting all formulas (25)–(30) together, we arrive at the following final result for the mean value of the momentum carried by the strange quark in a nucleon:

$$\langle k_\perp^2\rangle_s \equiv \frac{\langle N|\overline{s(i\vec{D}_\perp)}^2 s|N\rangle}{\langle N|\overline{s}s|N\rangle} \simeq \frac{\langle N|\overline{s}ig\sigma_{\mu\nu}G^a_{\mu\nu}(\lambda^a/2)\, s|N\rangle}{4\langle N|\overline{s}s|N\rangle} \simeq \frac{\langle\overline{s}ig\sigma_{\mu\nu}G^a_{\mu\nu}(\lambda^a/2)s\rangle}{4\langle\overline{s}s\rangle}\,\frac{d^{\overline{s}ig\sigma_{\mu\nu}G^a_{\mu\nu}(\lambda^a/2)s}}{d^{\overline{s}s}} \simeq \frac{1}{4}(0.8\ \mathrm{GeV}^2)\frac{5}{3} \sim 0.33\ \mathrm{GeV}^2, \quad (31)$$

where d^O denotes the dimension of the operator O. For a numerical estimation we use the standard magnitude for the mixed vacuum condensate $\langle\overline{s}ig\sigma_{\mu\nu}G^a_{\mu\nu}(\lambda^a/2)s\rangle = 0.8\ \mathrm{GeV}^2\ \langle\overline{s}s\rangle$. The obtained numerical value (31) for $\langle k_\perp^2\rangle_s$ looks very reasonable from the phenomenological point of view. We believe that the main uncertainty in our estimation of the nonperturbative correlation function (29) is canceled out when we consider the ratio (31) of similar objects. Therefore, we believe that the accuracy in formula (31) is much better than in formula (17) where we estimated an absolute value of the corresponding correlator.

We close this section with a few remarks. First, the nonvalence nucleon matrix elements can be expressed in terms of vacuum condensates in a very nice way. All numerical results obtained in such a way look very reasonable. As the second remark, we emphasize that a study of nonvalence nucleon matrix elements and an analysis of the QCD vacuum structure is one and the same problem. We would like to note, also, that the nucleon matrix element (31) might be very important in the analysis of neutron dipole moments. This observation is based on the fact that the so-called chromoelectric dipole moment of the s quark, related to the operator $\overline{s}g\gamma_5\sigma_{\mu\nu}G^a_{\mu\nu}(\lambda^a/2)s$, in many models gets a large factor $\sim m_s/m_q \sim 20$ in comparison with a similar d quark contribution [22,6,23]. At the same time, as we can see from Eq. (31) there is no any suppression due to the presence of the s quark in the corresponding nucleon matrix elements.

III. STRANGENESS IN THE NUCLEON 0^- CHANNEL

A. Singlet axial constant g_A^0

In this section we discuss the contribution of the strange quarks to nucleon matrix elements similar to Eq. (23), with the only difference that we switch the scalar channel $\overline{s}s$ into the pseudoscalar one $\overline{s}i\gamma_5 s$. In our previous study of the scalar channel we concluded that a considerable part of the nucleon mass (about 40%) is due to the strange quark.[3] We made this estimation by using the two following facts: First, we knew the mass of the nucleon [left-hand side of Eq. (23)], which is considered as experimental data. Second, we calculated independently the matrix element $\langle N|\overline{s}s|N\rangle$. Comparing this theoretical result (18) with the (23), we have made the aforementioned conclusion about a serious deviation from the chiral SU(3) limit.

[2]We neglect the term proportional to m_s^2 in Eq. (29). It can be justified by using the estimation (17).

[3]In the chiral limit $m_s \to 0$, the corresponding contribution is zero, of course.

We want to repeat all these steps for the pseudoscalar channel also. In this case an equation analogous to Eq. (23) looks as follows:

$$2mg_A^0\overline{p}i\gamma_5 p = +\left\langle N\left|\sum_q 2m_q\overline{q}i\gamma_5 q\right|N\right\rangle + \frac{3}{4}\left\langle N\left|\frac{\alpha_s}{\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}\right|N\right\rangle, \qquad (32)$$

where the sum over q is the sum over all light quarks u, d, and s and g_A^0 is the nucleon axial constant in the flavor-singlet channel. The world average is $g_A^0 = 0.27\pm 0.04$ [3].

Now, we would like to repeat all steps which would bring us to a conclusion similar to Eq. (23) for the pseudoscalar case. We shall try to answer the following question: What is the strange quark contribution in formula (32)? Let us recall that in the chiral limit $m_u = m_d = m_s = 0$ the nonzero contribution comes exclusively from the gluon term, in close analogy to formula (22):

$$2mg_A^0\overline{p}i\gamma_5 p = \frac{3}{4}\left\langle N\left|\frac{\alpha_s}{\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}\right|N\right\rangle, \quad m_u = m_d = m_s = 0. \qquad (33)$$

Thus, in order to answer the question formulated above, we have to estimate the matrix element

$$\langle p|2m_s\overline{s}i\gamma_5 s|p\rangle \qquad (34)$$

in a somewhat independent way.[4] First of all, the relevant contribution with octet quantum numbers (η) can be easily evaluated by standard technics. One should take the derivative from the octet, anomaly-free, current $\sim\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d - 2\overline{s}\gamma_\mu\gamma_5 s$. The result is

$$\langle p|2m_s\overline{s}i\gamma_5 s|p\rangle_\eta = -m(3F-D)\overline{p}i\gamma_5 p, \qquad (35)$$

where $D\simeq 0.63g_A$ and $F\simeq 0.37g_A$ are the standard SU(3) parameters. One could expect that a similar contribution with singlet quantum numbers (η') is also large, although it is zero in the chiral limit where $m_s = 0$.

We shall estimate the corresponding contribution with η' quantum numbers by using our previous trick Eq. (16). Namely, we reduce our original problem of the calculation of the nucleon matrix element to the problem of the computation of certain vacuum correlation functions where we should limit ourselves by calculating the contribution with singlet quantum numbers only:

$$\langle p|2m_s\overline{s}i\gamma_5 s|p\rangle \simeq \frac{-m\overline{p}i\gamma_5 p}{\langle\overline{q}q\rangle} i\int dy\langle 0|T\{2m_s\overline{s}i\gamma_5 s, \overline{u}i\gamma_5 u(0)\}|0\rangle. \qquad (36)$$

In order to make the corresponding estimations we need to know the η'-matrix elements $\langle 0|\overline{s}i\gamma_5 s|\eta'\rangle$ and $\langle 0|\overline{u}i\gamma_5 u|\eta'\rangle$. The PCAC does not provide us with the corresponding information; however, a quark model prejudice suggests that

$$\left\langle 0\left|\frac{1}{\sqrt{2}}(\overline{u}i\gamma_5 u - \overline{d}i\gamma_5 d)\right|\pi\right\rangle \simeq \left\langle 0\left|\frac{1}{\sqrt{6}}(\overline{u}i\gamma_5 u + \overline{d}i\gamma_5 d - 2\overline{s}i\gamma_5 s)\right|\eta\right\rangle \simeq \langle 0|\overline{u}i\gamma_5 s|K\rangle \simeq \left\langle 0\left|\frac{1}{\sqrt{3}}(\overline{u}i\gamma_5 u + \overline{d}i\gamma_5 d + \overline{s}i\gamma_5 s)\right|\eta'\right\rangle \simeq -\frac{2\langle\overline{q}q\rangle}{f_\pi}. \qquad (37)$$

Strong support in favor that relations (37) are correct comes from the analysis of the two-photon decays of π, η, and η'; see, e.g., [5]. All of these decay amplitudes have the same Lorentz structure and are determined by the matrix elements (37); therefore, the quark model prediction is found to work surprisingly well in this particular case. Combining formulas (35)–(37) we arrive at the estimation

$$\langle p|2m_s\overline{s}i\gamma_5 s|p\rangle = 2m\overline{p}i\gamma_5 p\left\{-\frac{1}{2}(3F-D) - \frac{4m_s\langle\overline{q}q\rangle}{3f_\pi^2 m_{\eta'}^2}\right\} \simeq (-0.3+0.16)2m\overline{p}i\gamma_5 p \simeq (-0.14)2m\overline{p}i\gamma_5 p, \qquad (38)$$

where we used $3F-D\simeq 0.6$ for the numerical estimation. The two terms in this formula are the octet and singlet contributions correspondingly. One should note that in spite of the fact that the singlet term is parametrically suppressed in the limit $m_s = 0$, this contribution numerically is not small. It is only by a factor of 2 less than the parametrically leading term.

Now, let us come back to Eq. (32). We would like to answer the previously formulated question: What is the s-quark contribution to formula (32)? From our estimation (38) we suggest the following pattern of saturation of the experimental data for $g_A^0 = 0.27\pm 0.04$:

$$2m\overline{p}i\gamma_5 p(0.27\pm 0.04) = +\langle p|2m_s\overline{s}i\gamma_5 s|p\rangle + \frac{3}{4}\left\langle p\left|\frac{\alpha_s}{\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}\right|p\right\rangle \simeq (-0.14)2m\overline{p}i\gamma_5 p + (+0.41)2m\overline{p}i\gamma_5 p. \qquad (39)$$

Here the first term is due to the strange quark contribution which we calculated directly. It equals (-0.14); see Eq. (38). The second part is due to the gluon contribution. We assign an average number $0.41 = 0.27-(-0.14)$ for this contribution in order to match the experimental data for g_A^0.

We would like to comment here that the gluon contribution in this formula and a gluon contribution in the standard expression for the axial-vector matrix element are very different in nature. Therefore, the interpretation is also different. In particular, the standard contribution to the axial vector current from u and d quarks is large. In our formula the corresponding large contribution is hidden in the gluon term,

[4]We neglect u and d quark contributions to formula (32) for the obvious reasons.

because the corresponding pseudoscalar current comes with a small factor m_u or m_d. Indeed, in the standard analysis of the light quark contribution into g_A^0 one deals with the matrix elements of the axial vector currents such as $\langle p|\overline{q}\gamma_\mu\gamma_5 q|p\rangle$, which are not small. However, we prefer to present the same formula using the operator identity:

$$\partial_\mu(\overline{q}\gamma_\mu\gamma_5 q)=2m_q\overline{q}i\gamma_5 q+\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}. \qquad (40)$$

In this case the large contribution of the light quark q is not related to the light quark operator $2m_q\overline{q}i\gamma_5 q$ (which is zero in the chiral limit $m_q=0$), but rather is related to the gluon operator $(\alpha_s/4\pi)G_{\mu\nu}\widetilde{G}_{\mu\nu}$.

Therefore, we understand the u and d quark contribution in formula (39) as a contribution of u and d quarks *together* with their anomaly parts:

$$g_A^{u+d}2m\overline{p}i\gamma_5 p=\left\langle p\left|2m_u\overline{u}i\gamma_5 u+2m_d\overline{d}i\gamma_5 d+\frac{2\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}\right|p\right\rangle \simeq(\tfrac{2}{3}0.41)2m\overline{p}i\gamma_5 p. \qquad (41)$$

The same interpretation is also true for s quarks. In this case g_A^s is compatible with zero as can be seen from similar estimations:

$$g_A^s 2m\overline{p}i\gamma_5 p=\left\langle p\left|2m_s\overline{s}i\gamma_5 s+\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}\right|p\right\rangle \simeq(-0.14+\tfrac{1}{3}0.41)2m\overline{p}i\gamma_5 p, \quad g_A^s\simeq 0. \qquad (42)$$

Such an interpretation is in very good agreement with the valence quark model philosophy, where the s quark does not play any essential role. Let us note that this interpretation is very different from the old simplest assumption of the spin of the strange quark in the nucleon; see, e.g., [3,5]. In our interpretation we understand the strange quark contribution as a joined contribution of s fields as well as its regulator field (or, what is the same, its anomalous contribution).

Two remarks are in order. First, the strange quark and gluon terms contribute with opposite signs to g_A^0. In the formula for mass, Eq. (23), similar terms interfere constructively with the same signs. It is very easy to understand the difference: In the pseudoscalar channel we have the Goldstone boson η whose total contribution is zero to the sum (32) because of the octet origin of the η meson. However, the η-meson contributions to the matrix elements $\langle p|2m_s\overline{s}i\gamma_5 s|p\rangle$ and to the gluon operator, taken separately, are not zero. Moreover, its contribution to the $\langle p|2m_s\overline{s}i\gamma_5 s|p\rangle$ has the opposite sign to the η' contribution [because of the difference in the quark context; see Eq. (37)]. Even more, it has a parametrical enhancement. We have nothing like that in the scalar channel (23), where the flavor-singlet states dominate.

The second remark is the observation that, like in the scalar channel, the strange quark operator gives a noticeable contribution into the final formula (39) in spite of the fact that in the chiral limit the corresponding contribution is zero as we mentioned earlier Eq. (33).

B. Singlet axial constant of heavy particles

In this section we try to answer the following question: How one can check the estimation (39) about a noticeable contribution of the strange quark? A more specific question we would like to know is, is it possible to measure a nucleon matrix element where some independent combination of those operators enters? If the answer were ''yes,'' we would be able to find the contribution of each term separately.

To answer this question we suggest considering the weak neutral current containing an isoscalar axial component associated with nonvalence quarks [24]:

$$\langle p|\overline{c}\gamma_\mu\gamma_5 c-\overline{s}\gamma_\mu\gamma_5 s+\overline{t}\gamma_\mu\gamma_5 t-\overline{b}\gamma_\mu\gamma_5 b|p\rangle \equiv g_A^{\text{heavy}}\overline{p}\gamma_\mu\gamma_5 p. \qquad (43)$$

Before going into details, let us mention that on the quantum level the current divergence of the massive quark field has the form

$$\partial_\mu\overline{Q}\gamma_\mu\gamma_5 Q=2m_Q\overline{Q}i\gamma_5 Q+\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}, \qquad (44)$$

where the first term is the standard one and the second term is due to the anomaly. There are many ways to understand the origin of the anomaly; basically, it arises from the necessity of the ultraviolet regularization of the theory. In the heavy quark mass limit, one can expand the $2m_Q Qi\gamma_5 Q$ term in Eq. (44) with the result [10,11]:

$$2m_Q\overline{Q}i\gamma_5 Q|_{m_Q\to\infty}=-\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}+c\,\frac{G\widetilde{G}G}{m_Q^2}+O\left(\frac{1}{m_Q^4}\right)+\ldots, \qquad (45)$$

where all coefficients, in principle, can be calculated. We are interested, however, in the leading term $\sim G_{\mu\nu}\widetilde{G}_{\mu\nu}$ only.

One can easily note that the leading term in the expansion (45) has the same structure as an anomaly term (44) and it goes with the opposite sign.[5] Thus, the terms which do not depend on mass are canceled out and we are left with a term $\sim G\widetilde{G}G/m_Q^2$ which vanishes in the limit $m_Q\to\infty$. Such a vanishing of the heavy quark contribution to the nucleon matrix element is in perfect agreement with the physical in-

[5]The opposite signs of those contributions can be easily understood in terms of the Pauli-Villars regulator fields with mass $M_{\text{PV}}\to\infty$. As is known these fields are introduced into the theory for regularization purposes and they play a crucial role in the calculation of the anomaly (44). A regulator contribution is obtained, by definition, by a replacement $m_Q\to M_{\text{PV}}$ in the corresponding formula. It goes, by definition, with a relative sign minus. From such a calculation it is clear that the leading terms which do not depend on mass, are canceled out, in full agreement with the explicit formulas (44) and (45).

tuition that a nucleon does not contain any heavy quark fields, at least in the limit $m_Q \to \infty$.

The situation with strange s quarks in formula (43) is much more complicated. This quark is not heavy enough to apply the arguments given above. Thus, we should keep all operators in formula (44) for the current divergence in the original form:

$$\left\langle p \left| 2m_s \overline{si}\gamma_5 s + \frac{\alpha_s}{4\pi} G_{\mu\nu}\widetilde{G}_{\mu\nu} \right| p \right\rangle = -g_A^{\text{heavy}} 2m \overline{pi}\gamma_5 p, \tag{46}$$

where we have neglected the terms $\sim 1/m_Q^2$ for c, b, and t quarks in accordance with our previous discussion.[6]

As we already mentioned, the measurement of the constant g_A^{heavy}, Eq. (46) gives independent information complementary to the singlet axial constant measurement g_A^0, Eq. (39). If we knew those constants with high enough precision, we would be able to find out both nucleon matrix elements $\langle p|2m_s \overline{si}\gamma_5 s|p\rangle$ and $\langle p|(\alpha_s/4\pi)G_{\mu\nu}\widetilde{G}_{\mu\nu}|p\rangle$. At the moment the experimental errors for $g_A^{\text{heavy}} = 0.15 \pm 0.09$ [25] are large: The result is only two standard deviations from zero.

The best we can do at the moment is to estimate g_A^{heavy} from our previous calculations (39). If we literally take the values -0.14 and 0.41 from the formula (39), we get the result for g_A^{heavy} which is compatible with zero in the same way as in Eq. (42):

$$-g_A^{\text{heavy}} 2m\overline{pi}\gamma_5 p = \left\langle p \left| 2m_s \overline{si}\gamma_5 s + \frac{\alpha_s}{4\pi} G_{\mu\nu}\widetilde{G}_{\mu\nu} \right| p \right\rangle \simeq (-0.14 + \tfrac{1}{3}0.41) 2m\overline{pi}\gamma_5 p, \quad g_A^{\text{heavy}} \simeq 0. \tag{47}$$

From our point of view this is an interesting observation which essentially says that a heavy quark operator *together with its anomalous part* gives nearly *vanishing* contribution into the nucleon matrix element. As we discussed earlier, this is certainly true for a really heavy quark. What is a less trivial fact is the observation, Eqs. (42) and (47), which apperently says that this is true for s quarks also (which by no means can be considered as a heavy quark). If we accept this point, we should interpret a nonzero magnitude of g_A^0, Eq. (39), as a contribution coming exclusively from the light u and d quarks and their anomalous parts.

IV. CHARMED QUARK IN THE NUCLEON

In this section we would like to extend our analysis for the c quark. The reason to do so is twofold: First, the c quark is heavy enough to use the standard $1/m_c$ expansion similar to Eq. (45). Second, the charmed quark is light enough to get a reasonably large effect from this expansion.

We start from the pseudoscalar channel and keep only the first term in the heavy quark expansion (45):

$$\langle p|\overline{ci}\gamma_5 c|p\rangle \simeq -\left\langle p \left| \frac{\alpha_s}{8m_c\pi} G_{\mu\nu}\widetilde{G}_{\mu\nu} \right| p \right\rangle \sim \frac{-0.41}{6m_c} 2m\overline{pi}\gamma_5 p \sim -0.1\overline{pi}\gamma_5 p, \tag{48}$$

where, for the numerical estimate, we use the value (39) for the gluon matrix element over the nucleon and $m_c \simeq 1.3$ GeV for the charmed quark mass.[7] This value should be compared to a similar matrix element of the strange quark over the nucleon:

$$\langle p|\overline{si}\gamma_5 s|p\rangle \sim \frac{-0.14}{2m_s} 2m\overline{pi}\gamma_5 p \sim -0.8\overline{pi}\gamma_5 p, \tag{49}$$

where we use formula (38) for the numerical estimation of the matrix element $\langle p|\overline{si}\gamma_5 s|p\rangle$. The ratio of these values is in remarkable agreement with the ratio of their mass: $m_c/m_s \sim 1.3$ GeV/0.175 GeV~ 7.5.

Our next example is the scalar matrix element. In this case one can use a heavy quark expansion similar to formula (45), but for the scalar channel [11]:

$$\langle p|\overline{cc}|p\rangle \simeq -\left\langle p \left| \frac{\alpha_s}{12m_c\pi} G^a_{\mu\nu}G^a_{\mu\nu} \right| p \right\rangle \sim \frac{2\times 700 \text{ MeV}}{27m_c} \sim 0.04. \tag{50}$$

For the numerical estimation in this formula we adopted the value (24) for the gluon matrix element over the nucleon. The magnitude (50) for the charmed quark is approximately 30 times less than the corresponding matrix element for the strange quark (7):

$$\langle p|\overline{ss}|p\rangle \simeq 1.4. \tag{51}$$

This is in large contrast with the pseudoscalar channel, where the corresponding ratio was about a factor of 4 larger.

We conclude this section with a few remarks. First of all, the matrix elements (48) and (50) for the charmed quark are expressed in terms of the gluon operators. For the heavy quark this is an exact consequence of QCD. Corrections to these formulas can be easily estimated. One can show that they are small for the c quark. The problem of the evaluation of the gluon matrix elements over the nucleon is a different problem. However, we believe that from the measurements of g_A^0, Eq. (32), and from πN scattering, Eq. (1), we know those matrix elements with a reasonable accuracy. Thus, we expect the same accuracy for the matrix elements $\langle p|\overline{cc}|p\rangle$ and $\langle p|\overline{ci}\gamma_5 c|p\rangle$.

Our next remark is the observation that the results (48) and (50) essentially give a normalization for the intrinsic charm quark component in the proton. This is a very important characteristic of the nucleon. It might play an essential role in the explanation of a discrepancy between charm ha-

[6]The corresponding estimations even for the lightest heavy c quark support this viewpoint; see the next section.

[7]Let us note that this value for mass corresponds to the high enough normalization point of order of m_c. In principle, one should renormalize this value to the low normalization point. We neglect this small logarithmic effect in this paper.

droproduction and perturbative QCD calculations. We refer to the original paper[8] [27] on this subject where the hypothesis of intrinsic charm quarks in the proton was introduced. The experimental fit in the framework of this paper [27] suggests that the probability to have an intrinsic charm in the proton is about $\sim 0.3\%$ [28] and $(0.86\pm 0.60)\%$ [29]. These numbers cannot be related directly to the matrix element (50) we calculated. However, they give some general scale of this phenomenon. We hope that in the future some more sophisticated, QCD-based methods, will lead us to a deeper understanding of the effects related to the intrinsic charm component in the nucleon.

V. CONCLUSION

We believe that the main result of the present analysis is the observation that nonvalence quarks play an important role in the physics of nucleons. However, we should stress that such an interpretation does not contradict the bag model [5], where the nucleon matrix element

$$\langle N|\overline{s}s|N\rangle \simeq -\langle \overline{s}s\rangle V \tag{52}$$

is related to some vacuum characteristics (such as the volume of the bag V) of the model. It is clear that the chiral vacuum condensate of the strange quark is large. So there is no reason to expect that the corresponding nucleon matrix element is small. The same argument can be applied for arbitrary mixed vacuum condensates also (they are presumably not zero [21]). This information can be translated, in according to Eq. (31), into knowledge about the transverse momentum distribution of the strange quark in a nucleon.

[8]See also a recent paper in [26].

In our approach this relation, vacuum ⇔ nucleon, is clearly seen. Thus, by studying the vacuum properties of QCD, we essentially study some interesting nucleon matrix elements which can be experimentally measured.

We also note that the s-quark contribution by itself to the nucleon matrix element is not small. However, if we interpret the s-quark contribution together with its anomaly part,

$$g_A^s 2m\overline{p}i\gamma_5 p = \left\langle p\left|2m_s\overline{s}i\gamma_5 s+\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}\right|p\right\rangle \simeq(-0.14+\tfrac{1}{3}0.41)2m\overline{p}i\gamma_5 p, \quad g_A^s\simeq 0. \tag{53}$$

Such an interpretation is in very good agreement with the valence quark model philosophy, where the s quark does not play any essential role. Let us note that this interpretation is very different from the old simplest assumption of the spin of the strange quark in the nucleon; see, e.g., [3,5]. In our interpretation we understand the strange quark contribution as a joined contribution of s fields as well as its regulator field (or, what is the same, its anomalous contribution).

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