Model-independent analysis of the simultaneous mixing of gauge bosons and mixing of fermions

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We discuss the case of simultaneous mixing of gauge bosons and mixing of fermions in a model-independent way and for a variety of extra-fermion representations. In this context we analyze a class of lepton family-violating processes, namely, $Z\to e\overline{\tau}$, $Z\to\mu\overline{\tau}$, $Z\to e\overline{\mu}$, $\mu\to ee\overline{e}$, $\tau\to ee\overline{e}$, $\tau\to\mu\mu\overline{\mu}$, $\tau\to e\mu\overline{\mu}$, and $\tau\to\mu e\overline{e}$ in the presence of one extra neutral gauge boson Z' with universal, nonuniversal, or family-changing couplings. We derive bounds on the combined effect of Z-Z' mixing and ordinary–exotic lepton mixing. [S0556-2821(97)05205-3]

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I. INTRODUCTION

Tree-level *family-changing neutral current* (FCNC) interactions arise in extended models from three possible sources: (i) the exchange of family-changing neutral gauge bosons, (ii) the mixing between exotic and ordinary fermions, and (iii) the existence of neutral scalars in the Higgs sector with family-violating couplings. However, if the standard neutral Z^0 boson mixes with a boson which has a coupling which is either family changing or nonuniversal, its coupling to light (that is, the ordinary) fermions becomes family changing even in the absence of mixing between exotic and ordinary fermions.

In previous works extensive research has been performed in the context of FCNC produced by the mixing of the standard neutral gauge boson with one which does not couple universally to fermion generation [1] or by the mixing between exotic and ordinary fermions [2,3]. In this paper we show how this phenomenon arises in the general case of the simultaneous mixing of neutral gauge bosons and the mixing of ordinary fermions with exotic ones. We do not consider in this article FCNC arising from the exchange of scalars or additional indirect effects such as the shifts induced by the mixing between neutral gauge bosons in the values of the weak angle θ_w, the ρ parameter, and the Fermi coupling constant G_F [4–6], since they are irrelevant for the present analysis. We apply the formalism in a model-independent way to several lepton family-violating processes in the e-μ, μ-τ, and e-τ sectors, considering several possible exotic fermionic representations. We obtain in each case bounds for the mixing parameters including the possibility that the contribution of the neutral gauge boson mixing and that of the fermion mixing are of the same order.

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We describe in Sec. II the formalism for dealing with FCNC which arise from simultaneous mixing of gauge bosons and mixing of fermions. This formalism is applied to the leptonic sector in Sec. III. In Sec. III A we describe how the mixing effects modify the diagonal couplings of the Z. In Sec. III B we apply the formalism to the $Z\to l_i\overline{l}_j$, $l_i\to l_j l_j\overline{l}_j$, and $l_i\to l_j l_k\overline{l}_k$ decays and obtain constraints for the mixing parameters. These bounds are refined in Sec. IV, considering special types of representations for the additional fermions.

II. MIXING EFFECTS: THE GENERAL FORMALISM FOR SIMULTANEOUS MIXING OF GAUGE BOSONS AND MIXING OF FERMIONS

To discuss the mixing of the massive neutral gauge bosons of a general theory we first divide them into two classes: the standard Z^0 gauge boson which is a linear combination of the $SU(2)_L\otimes U(1)_Y$ neutral bosons and has *universal family diagonal* (UFD) couplings determined by the eigenvalues t_3 and q of the electroweak generators T_3 and Q; the extra Z_i^0 gauge bosons which can have either UFD or *nonuniversal family diagonal* (NUFD) or FC couplings. The last two types of couplings arise when the Z_i^0 gauge bosons are associated with horizontal interactions. Since the case where Z_i^0 has UFD couplings has already been discussed in the literature [3,4,7], we will concentrate our attention on the cases of NUFD [1] and FC couplings.

To discuss the general mixing of fermions, including additional ones, we follow Langacker and London [2], grouping all fermions of a given electric charge q and a given helicity $a=$ L, R in a n_a+m_a vector column of n_a ordinary (O) and m_a exotic (E) gauge eigenstates $\boldsymbol{\psi}_a^0=(\boldsymbol{\psi}_{\rm O},\boldsymbol{\psi}_{\rm E})_a^\top$. The relation between the gauge eigenstates and the corresponding light (l) and heavy (h) mass eigenstates $\boldsymbol{\psi}_a=(\boldsymbol{\psi}_l,\boldsymbol{\psi}_h)_a^\top$ is given by

$$\boldsymbol{\psi}_a^0 = \mathsf{U}_a \boldsymbol{\psi}_a , \tag{1}$$

where the unitary matrices U_a have the block form

$$\mathsf{U}_a = \begin{pmatrix} \mathsf{A}_a & \mathsf{E}_a \\ \mathsf{F}_a & \mathsf{G}_a \end{pmatrix}, \tag{2}$$

and the submatrices A_a and G_a are not unitary but satisfy the conditions

$$(\mathsf{U}^\dagger \mathsf{U})_a = \begin{pmatrix} \mathsf{A}^\dagger\mathsf{A}+\mathsf{F}^\dagger\mathsf{F} & \mathsf{A}^\dagger\mathsf{E}+\mathsf{F}^\dagger\mathsf{G} \\ \mathsf{E}^\dagger\mathsf{A}+\mathsf{G}^\dagger\mathsf{F} & \mathsf{E}^\dagger\mathsf{E}+\mathsf{G}^\dagger\mathsf{G} \end{pmatrix}_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3}$$

The term $(\mathsf{F}^\dagger\mathsf{F})_a$, second order in the small exotic–ordinary-fermion mixing, induces FC transitions in the light–light sector.

The neutral current term for the multiplet $\boldsymbol{\psi}$ of a given electric charge, for the case when both types of mixings are present, is then

$$-\mathcal{L}^{\mathrm{nc}} = \frac{e}{\mathsf{s}_{\theta_\mathrm{w}} \mathsf{c}_{\theta_\mathrm{w}}} \sum_{a=\mathrm{L,R}} \overline{\boldsymbol{\psi}_a^0} \gamma^\mu (\mathsf{D}_a, \mathsf{H}_a^1, \ldots, \mathsf{H}_a^n) \boldsymbol{\psi}_a^0 \begin{pmatrix} Z^0 \\ Z_1^0 \\ \vdots \\ Z_n^0 \end{pmatrix}_\mu \tag{4a}$$

$$= \frac{e}{\mathsf{s}_{\theta_\mathrm{w}} \mathsf{c}_{\theta_\mathrm{w}}} \sum_{a=\mathrm{L,R}} \overline{\boldsymbol{\psi}_a} \gamma^\mu (\mathsf{U}_a^\dagger \mathsf{D}_a \mathsf{U}_a, \mathsf{U}_a^\dagger \mathsf{H}_a^1 \mathsf{U}_a, \ldots, \mathsf{U}_a^\dagger \mathsf{H}_a^n \mathsf{U}_a)_\mu \boldsymbol{\psi}_a \mathsf{R} \begin{pmatrix} Z \\ Z_1 \\ \vdots \\ Z_n \end{pmatrix}_\mu , \tag{4b}$$

where s_{θ_w} and c_{θ_w} are $\sin\theta_w$ and $\cos\theta_w$, respectively, θ_w is the weak mixing angle, R is the $(n+1)\times(n+1)$ orthogonal matrix that diagonalizes the neutral boson mass matrix, D_a is the $(n_a+m_a)\times(n_a+m_a)$ matrix that expresses the coupling of the Z^0 gauge boson to matter fields, and similarly H_a^i are the $(n_a+m_a)\times(n_a+m_a)$ matrices that express the coupling of the NUFD and FC gauge bosons to matter. The electromagnetic part of $\mathcal{L}^{\mathrm{nc}}$ has not been displayed since its structure is not affected by the mixing effects.

In the simple case of only one extra neutral gauge boson, the R matrix is easily parametrizable as

$$\mathsf{R} = \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix} \tag{5}$$

and the neutral current term is now

$$-\mathcal{L}^{\mathrm{nc}} = \frac{e}{\mathsf{s}_{\theta_w} \mathsf{c}_{\theta_w}} \sum_{a=\mathrm{L,R}} \overline{\boldsymbol{\psi}_a} \gamma^\mu \mathsf{U}_a^\dagger (\mathsf{D}_a \cos\Theta + \mathsf{H}_a \sin\Theta, \mathsf{H}_a \cos\Theta - \mathsf{D}_a \sin\Theta) \mathsf{U}_a \boldsymbol{\psi}_a \begin{pmatrix} Z \\ Z' \end{pmatrix}_\mu . \tag{6}$$

It should be obvious that the treatment of the general case, Eqs. (4a) and (4b), is straightforward. From now on we restrict the discussion to the case of only one extra gauge boson just to simplify the notation. In Eqs. (4a), (4b), and (6) the D_a matrices are given by

$$\mathsf{D}_a \equiv (\mathsf{T}_3 - \mathsf{Q}\mathsf{s}_{\theta_w}^2)_a . \tag{7}$$

They are diagonal by definition, and in the case, which we assume for simplicity from now on, that the exotics of a given charge and helicity have a common eigenvalue $t_{3\mathrm{E}a}$ of T_{3a}, they can be written as

$$\mathsf{D}_a = \begin{pmatrix} \mathsf{t}_{3\mathrm{O}} - \mathsf{q}_\mathrm{O}\mathsf{s}_{\theta_w}^2 & 0 \\ 0 & \mathsf{t}_{3\mathrm{E}} - \mathsf{q}_\mathrm{E}\mathsf{s}_{\theta_w}^2 \end{pmatrix}_a , \tag{8}$$

where $\mathsf{t}_{3\mathrm{O}a}$ and $\mathsf{t}_{3\mathrm{E}a}$ are square matrices of dimension n_a and m_a, respectively. They correspond to the ordinary and exotic part of the T_{3a} operator, and they are proportional to the unit matrix through the eigenvalues $t_{3\mathrm{O}a}$ and $t_{3\mathrm{E}a}$ of T_{3a}. This is the same situation for the $\mathsf{q}_{\mathrm{O}a}$ and $\mathsf{q}_{\mathrm{E}a}$ matrices in relation to the Q operator.

Contrary to the D_a matrices, the H ones are not diagonal in general, but can, however, be written as

$$\mathsf{H} = \begin{pmatrix} \mathsf{H}_\mathrm{O} & 0 \\ 0 & \mathsf{H}_\mathrm{E} \end{pmatrix}, \tag{9}$$

where H_O and H_E represent the interactions of the Z_1^0 with the ordinary and exotic fermions, respectively. The point is that there are no H_{EO} or H_{OE} terms in H (which would give rise to Z_1^0-mediated transitions between exotic and ordinary fermions) as long as the horizontal group commutes with the standard model (SM) gauge group.

In the basis where the fermions are mass eigenstates, the form of D and H is

$$(\mathsf{U}^\dagger\mathsf{D}\mathsf{U})_{\mathrm{R}}=\begin{pmatrix}\mathsf{F}^\dagger\mathsf{F} & \mathsf{F}^\dagger\mathsf{G}\\ \mathsf{G}^\dagger\mathsf{F} & \mathsf{G}^\dagger\mathsf{G}\end{pmatrix}_{\mathrm{R}} t_{3\mathrm{ER}}-\mathsf{Q}s^2_{\theta_w}, \tag{10a}$$

$$(\mathsf{U}^\dagger\mathsf{D}\mathsf{U})_{\mathrm{L}}=\begin{pmatrix}\mathsf{F}^\dagger\mathsf{F} & \mathsf{F}^\dagger\mathsf{G}\\ \mathsf{G}^\dagger\mathsf{F} & \mathsf{G}^\dagger\mathsf{G}\end{pmatrix}_{\mathrm{L}} t_{3\mathrm{EL}}+\begin{pmatrix}\mathsf{A}^\dagger\mathsf{A} & \mathsf{A}^\dagger\mathsf{E}\\ \mathsf{E}^\dagger\mathsf{A} & \mathsf{E}^\dagger\mathsf{E}\end{pmatrix}_{\mathrm{L}} t_{3\mathrm{OL}}-\mathsf{Q}s^2_{\theta_w}, \tag{10b}$$

and by using the unitarity conditions (3) we can rewrite the last equation as

$$(\mathsf{U}^\dagger\mathsf{D}\mathsf{U})_{\mathrm{L}}=\begin{pmatrix}\mathsf{F}^\dagger\mathsf{F} & -\mathsf{A}^\dagger\mathsf{E}\\ \mathsf{G}^\dagger\mathsf{F} & -\mathsf{E}^\dagger\mathsf{E}\end{pmatrix}_{\mathrm{L}} (t_{3\mathrm{EL}}-t_{3\mathrm{OL}})+\mathsf{T}_{3\mathrm{L}}-\mathsf{Q}s^2_{\theta_w}. \tag{10c}$$

From Eq. (10a) one can see that for the light fermions and in the absence of Z^0-Z_1^0 mixing, the coupling of the Z^0 to right-handed FC and *nonuniversal family diagonal neutral currents* (NUFDNC) is possible only if $t_{3\mathrm{ER}}\neq 0$. Furthermore, from Eqs. (10a) and (10c) it is easy to see that sequential fermions do not induce FC or NUFD couplings for the standard Z^0 since their contribution to these currents is canceled out by that of the ordinary fermions.

On the other hand, no general statement can be made for the transformed H couplings:

$$(\mathsf{U}^\dagger\mathsf{H}\mathsf{U})_a=\begin{pmatrix}\mathsf{A}^\dagger\mathsf{H}_{\mathrm{O}}\mathsf{A}+\mathsf{F}^\dagger\mathsf{H}_{\mathrm{E}}\mathsf{F}, & \mathsf{A}^\dagger\mathsf{H}_{\mathrm{O}}\mathsf{E}+\mathsf{F}^\dagger\mathsf{H}_{\mathrm{E}}\mathsf{G}\\ \mathsf{E}^\dagger\mathsf{H}_{\mathrm{O}}\mathsf{A}+\mathsf{G}^\dagger\mathsf{H}_{\mathrm{E}}\mathsf{F}, & \mathsf{E}^\dagger\mathsf{H}_{\mathrm{O}}\mathsf{E}+\mathsf{G}^\dagger\mathsf{H}_{\mathrm{E}}\mathsf{G}\end{pmatrix}_a \equiv\begin{pmatrix}\mathsf{H}_{ll} & \mathsf{H}_{lh}\\ \mathsf{H}^\dagger_{lh} & \mathsf{H}_{hh}\end{pmatrix}_a. \tag{11}$$

From the last equation one can see that in the presence of neutral gauge boson mixing, $\Theta\neq 0$, there will be in general FC couplings of the Z in the light sector, H_{ll} nondiagonal, even in the absence of mixing between exotic and ordinary fermions, $\mathsf{F}=0$, and even if the coupling of Z_1^0 to the ordinary fermions, H_{O}, is diagonal but nonuniversal, since in general the mass and gauge eigenstates will not coincide in the light sector, $\mathsf{A}\neq 1$.

A. General neutral current Lagrangian term in the light sector

From Eqs. (10a), (10c), and (11) we obtain for the neutral-current Lagrangian in the light-light sector the expression

$$-\mathcal{L}^{\mathrm{nc}}=\frac{e}{s_{\theta_w}c_{\theta_w}}\sum_{a=\mathrm{L,R}}\overline{\boldsymbol{\psi}}_{la}\gamma^\mu(\mathsf{K}_a,\mathsf{K}'_a)\boldsymbol{\psi}_{la}\begin{pmatrix}Z\\ Z'\end{pmatrix}_\mu, \tag{12}$$

where

$$\mathsf{K}_{\mathrm{L}}=[(\mathsf{F}^\dagger\mathsf{F})_{\mathrm{L}}(t_{3\mathrm{EL}}-t_{3\mathrm{OL}})+\mathsf{t}_{3\mathrm{OL}}-\mathsf{Q}s^2_{\theta_w}]\cos\Theta+(\mathsf{H}_{ll})_{\mathrm{L}}\sin\Theta, \tag{13a}$$

$$\mathsf{K}_{\mathrm{R}}=[(\mathsf{F}^\dagger\mathsf{F})_{\mathrm{R}}t_{3\mathrm{ER}}-\mathsf{Q}s^2_{\theta_w}]\cos\Theta+(\mathsf{H}_{ll})_{\mathrm{R}}\sin\Theta, \tag{13b}$$

$$\mathsf{K}'_{\mathrm{L}}=-[(\mathsf{F}^\dagger\mathsf{F})_{\mathrm{L}}(t_{3\mathrm{EL}}-t_{3\mathrm{OL}})+\mathsf{t}_{3\mathrm{OL}}-\mathsf{Q}s^2_{\theta_w}]\sin\Theta+(\mathsf{H}_{ll})_{\mathrm{L}}\cos\Theta, \tag{13c}$$

$$\mathsf{K}'_{\mathrm{R}}=-[(\mathsf{F}^\dagger\mathsf{F})_{\mathrm{R}}t_{3\mathrm{ER}}-\mathsf{Q}s^2_{\theta_w}]\sin\Theta+(\mathsf{H}_{ll})_{\mathrm{R}}\cos\Theta. \tag{13d}$$

From these equations it is easy to see that there are two contributions to the FC couplings of the light fermions to the Z, proportional to $(\mathsf{F}^\dagger\mathsf{F})_a\cos\Theta$ and $(\mathsf{H}_{ll})_a\sin\Theta$, respectively, which may be in principle of the same order, in the limit of no mixing between exotics and ordinary fermions ($\mathsf{F}_a=0$) and no mixing between the Z and the extra gauge boson ($\Theta=0$) the SM couplings are recovered, and in the absence of mixing with the exotic fermions, the FC couplings of the ordinary fermions (of a given helicity) to the Z may still survive through the term $(\mathsf{H}_{ll})_a\sin\Theta$, provided that the family of ordinary fermions of the given helicity transforms nontrivially under the horizontal generator H_{O}.

Further details of these couplings depend on the model and on the processes under consideration and are the subject of the next sections.

We may rewrite Eqs. (13a) and (13b) as

$$\mathsf{K}_{\mathrm{L}}=(\boldsymbol{\Lambda}_{\mathrm{L}}+\mathsf{t}_{3\mathrm{OL}}-\mathsf{Q}\,s^2_{\theta_w})\cos\Theta+\boldsymbol{\Xi}_{\mathrm{L}}\sin\Theta, \tag{14a}$$

$$\mathsf{K}_{\mathrm{R}}=(\boldsymbol{\Lambda}_{\mathrm{R}}-\mathsf{Q}s^2_{\theta_w})\cos\Theta+\boldsymbol{\Xi}_{\mathrm{R}}\sin\Theta, \tag{14b}$$

where

$$\boldsymbol{\Lambda}_{\mathrm{L}}=(\mathsf{F}^\dagger\mathsf{F})_{\mathrm{L}}(t_{3\mathrm{EL}}-t_{3\mathrm{OL}}), \tag{15a}$$

$$\boldsymbol{\Lambda}_{\mathrm{R}}=(\mathsf{F}^\dagger\mathsf{F})_{\mathrm{R}}t_{3\mathrm{ER}}, \tag{15b}$$

$$\boldsymbol{\Xi}_a=(\mathsf{H}_{ll})_a, \tag{15c}$$

together with Θ, represent the physics beyond the SM.

1. Charged fermions

Since for the light charged fermions the dimension of $\boldsymbol{\psi}_{l\mathrm{L}}$ and $\boldsymbol{\psi}_{l\mathrm{R}}$ are the same (there is an equal number of left- and right-handed fermions), we can rewrite the general Lagrangian (12) as

$$-\mathcal{L}^{\mathrm{nc}}=\frac{e}{2s_{\theta_w}c_{\theta_w}}\overline{\boldsymbol{\psi}}_l\gamma^\mu(\mathsf{g}_{\mathrm{V}}-\mathsf{g}_{\mathrm{A}}\gamma^5,\mathsf{g}'_{\mathrm{V}}-\mathsf{g}'_{\mathrm{A}}\gamma^5)\boldsymbol{\psi}_l\begin{pmatrix}Z\\ Z'\end{pmatrix}_\mu, \tag{16}$$

where

$$\mathsf{g}_{\mathrm{V}}=\mathsf{K}_{\mathrm{L}}+\mathsf{K}_{\mathrm{R}}, \tag{17a}$$

$$\mathsf{g}_{\mathrm{A}}=\mathsf{K}_{\mathrm{L}}-\mathsf{K}_{\mathrm{R}}. \tag{17b}$$

III. APPLICATIONS TO THE LEPTONIC SECTOR

A. Constraints from the lepton family diagonal processes $Z\to l_i\overline{l_i}$

The effects of mixing between ordinary and exotic fermions on the diagonal process $Z\to l_i\overline{l_i}$ have been analyzed previously [2–4]. Likewise separate effect of mixing between the standard Z and a new one were discussed in Refs. [4,5]. When both effects are present, the branching ratio $\mathrm{B}(Z\to l_i\overline{l_i})$, in the $M_Z \gg m_{l_i}$ approximation, is given by

$$\mathrm{B}(Z\to l_i\overline{l_i}) \simeq \frac{1}{\Gamma_{\mathrm{tot}}}\frac{\mathrm{G_F}M_Z^3}{6\sqrt{2}\pi}(|g_{\mathrm{V}}^{ii}|^2+|g_{\mathrm{A}}^{ii}|^2) \tag{18a}$$

$$=\frac{1}{\Gamma_{\mathrm{tot}}}\frac{\mathrm{G_F}M_Z^3}{3\sqrt{2}\pi}(|\Lambda_{\mathrm{L}}^{ii}+\Xi_{\mathrm{L}}^{ii}\Theta-\tfrac{1}{2}+\mathrm{s}_{\theta_w}^2|^2 + |\Lambda_{\mathrm{R}}^{ii}+\Xi_{\mathrm{R}}^{ii}\Theta+\mathrm{s}_{\theta_w}^2|^2)+O(\Theta^2). \tag{18b}$$

Since the agreement of the SM predictions with the experimental data for these processes is better than 0.1% [the experimental value of $\Gamma(Z\to l\overline{l})$ is 83.83 ± 0.27 [8] against the theoretical one equal to 83.97 ± 0.07], the quantities $\Lambda_a^{ii}+\Xi_a^{ii}\Theta$ are bounded practically by the experimental uncertainty in the data [8]:

$$\mathrm{B}_{l_i\overline{l_i}}\equiv \mathrm{B}(Z\to l_i\overline{l_i}) = \begin{cases} \mathrm{B}_{e\overline{e}}=(3.366\pm0.008)\times10^{-2} \\ \mathrm{B}_{\mu\overline{\mu}}=(3.367\pm0.013)\times10^{-2}, \\ \mathrm{B}_{\tau\overline{\tau}}=(3.360\pm0.015)\times10^{-2}. \end{cases} \tag{19}$$

We may also write

$$|\Lambda_{\mathrm{L}}^{ii}+\Xi_{\mathrm{L}}^{ii}\Theta-\tfrac{1}{2}+\mathrm{s}_{\theta_w}^2|^2+|\Lambda_{\mathrm{R}}^{ii}+\Xi_{\mathrm{R}}^{ii}\Theta+\mathrm{s}_{\theta_w}^2|^2 = c\mathrm{B}_{l_i\overline{l_i}}, \tag{20}$$

where

$$c^{-1}=\left(\frac{1}{\Gamma_{\mathrm{tot}}}\frac{\mathrm{G_F}M_Z^3}{3\sqrt{2}\pi}\right)=0.2675\pm0.0005$$

and from which we obtain, in a neighborhood of $|\Lambda_a^{ii}+\Xi_a^{ii}\Theta|=0$ and with $\mathrm{s}_{\theta_w}^2=0.2237\pm0.0010$, the bounds

$$|\Lambda_a^{ii}+\Xi_a^{ii}\Theta|<\mathrm{few}\times10^{-3}. \tag{21}$$

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B. Constraints from lepton family-violating processes

1. Constraints from $Z\to l_i\overline{l_j}$

With the approximation $M_Z \gg m_{l_i}, m_{l_j}$ and taking into account that experimental limits exist only for the sum of the charge states of particles and antiparticles states, we should consider, for $i\neq j$,

$$\mathrm{B}(Z\to l_i\overline{l_j}+\overline{l_i}l_j)\simeq 2\frac{\mathrm{B}(Z\to l\overline{l})}{|g_{\mathrm{V}}|^2+|g_{\mathrm{A}}|^2}(|g_{\mathrm{V}}^{ij}|^2+|g_{\mathrm{A}}^{ij}|^2) \tag{22a}$$

$$\simeq 4\frac{\mathrm{B}(Z\to l\overline{l})}{|g_{\mathrm{V}}|^2+|g_{\mathrm{A}}|^2}(|\Lambda_{\mathrm{L}}^{ij}+\Xi_{\mathrm{L}}^{ij}\Theta|^2 + |\Lambda_{\mathrm{R}}^{ij}+\Xi_{\mathrm{R}}^{ij}\Theta|^2)+O(\Theta^2). \tag{22b}$$

It then follows that

$$|\Lambda_{\mathrm{L}}^{ij}+\Xi_{\mathrm{L}}^{ij}\Theta|^2+|\Lambda_{\mathrm{R}}^{ij}+\Xi_{\mathrm{R}}^{ij}\Theta|^2<c\widetilde{\mathrm{B}}_{l_i\overline{l_j}}, \tag{23}$$

where

$$c^{-1}=\left(4\frac{\mathrm{B}(Z\to l\overline{l})}{|g_{\mathrm{V}}|^2+|g_{\mathrm{A}}|^2}\right)=0.536$$

(using the conventional SM branching ratio 0.0337 for $\mathrm{B}_{l\overline{l}}$ and the standard values for g_{V} and g_{A}) and where

$$\mathrm{B}_{l_i\overline{l_j}}\equiv \mathrm{B}(Z\to l_i\overline{l_j}+\overline{l_i}l_j) = \begin{cases} \mathrm{B}_{e\overline{\mu}}<1.7\times10^{-6}\equiv\widetilde{\mathrm{B}}_{e\overline{\mu}}, \\ \mathrm{B}_{e\overline{\tau}}<7.3\times10^{-6}\equiv\widetilde{B}_{e\overline{\tau}} \\ \mathrm{B}_{\mu\overline{\tau}}<1.0\times10^{-5}\equiv\widetilde{\mathrm{B}}_{\mu\overline{\tau}}, \end{cases} \tag{24}$$

according to the experimental limits [9,10]. This means that the fermion mixing parameters Λ_a^{ij} are bound to lie in a circular region centered at $(-\Xi_{\mathrm{L}}^{ij}\Theta, -\Xi_{\mathrm{R}}^{ij}\Theta)$ and of radius $\sim10^{-3}$.

It is evident that the contribution of Θ in the analysis of Λ_a^{ij} is non-negligible when

$$\Xi_a^{ij}\Theta \gtrsim \sqrt{\mathrm{c}\widetilde{\mathrm{B}}_{l_i\overline{l_j}}}. \tag{25}$$

This may be a common situation, since in general $\Xi_a^{ij}=O(1)$ and the upper bounds for Θ, which are model dependent, are of the order of 10^{-1}–10^{-3}. Taking the limit $\Theta\to0$ could lead to wrong conclusions: A contribution of $\Xi_a^{ij}\Theta\sim5\times10^{-3}$ is enough to give a completely new region of solutions for Λ_a^{ij}. The results of this section are resumed in Table I.

2. Constraints from $l_i\to l_j l_j\overline{l_j}$

Assuming $m_{l_i}\gg m_{l_j}$ and ignoring possible contributions from scalars, the branching ratio $\mathrm{B}(l_i\to l_j l_j\overline{l_j})$ for $i\neq j$ is

$$\frac{\mathrm{B}(l_i\to l_j l_j\overline{l_j})}{\mathrm{B}(l_i\to l_j\overline{\nu_{l_j}}\nu_{l_i})}=\frac{1}{2}[3(|g_{\mathrm{V}}^{jj}|^2+|g_{\mathrm{A}}^{jj}|^2)(|g_{\mathrm{V}}^{ij}|^2+|g_{\mathrm{A}}^{ij}|^2)+2\mathrm{Re}(g_{\mathrm{V}}^{jj}g_{\mathrm{A}}^{jj*})2\mathrm{Re}(g_{\mathrm{V}}^{ij}g_{\mathrm{A}}^{ij*})]$$
$$+\frac{M_Z^2}{M_{Z'}^2}\mathrm{Re}[3(g_{\mathrm{V}}^{jj}g_{\mathrm{V}}'^{jj*}+g_{\mathrm{A}}^{jj}g_{\mathrm{A}}'^{jj*})(g_{\mathrm{V}}^{ij}g_{\mathrm{V}}'^{ij*}+g_{\mathrm{A}}^{ij}g_{\mathrm{A}}'^{ij*})+(g_{\mathrm{V}}^{jj}g_{\mathrm{A}}'^{jj*}+g_{\mathrm{A}}^{jj}g_{\mathrm{V}}'^{jj*})(g_{\mathrm{V}}^{ij}g_{\mathrm{A}}'^{jj*}+g_{\mathrm{A}}^{jj}g_{\mathrm{V}}'^{jj*})]$$
$$+\frac{1}{2}\frac{M_Z^4}{M_{Z'}^4}[3(|g_{\mathrm{V}}'^{jj}|^2+|g_{\mathrm{A}}'^{jj}|^2)(|g_{\mathrm{V}}'^{ij}|^2+|g_{\mathrm{A}}'^{ij}|^2)+2\mathrm{Re}(g_{\mathrm{V}}'^{jj}g_{\mathrm{A}}'^{jj*})2\mathrm{Re}(g_{\mathrm{V}}'^{ij}g_{\mathrm{A}}'^{ij*})] \tag{26a}$$
$$\simeq 4[(2|-\tfrac{1}{2}+s_{\theta_w}^2|^2+|s_{\theta_w}^2|^2)|\Lambda_{\mathrm{L}}^{ij}+\Xi_{\mathrm{L}}^{ij}\Theta|^2+(|-\tfrac{1}{2}+s_{\theta_w}^2|^2+2|s_{\theta_w}^2|^2)|\Lambda_{\mathrm{R}}^{ij}+\Xi_{\mathrm{R}}^{ij}\Theta|^2]+\mathrm{O}(\Theta^2), \tag{26b}$$

where we have assumed $(M_Z/M_{Z'})^2\sim\Theta$, $\Lambda_a^{ij}\lesssim\Theta$ (remember that Λ_a^{ij} is second order in the ordinary-exotic mixing) and we have taken into account the stringent limits obtained in Eq. (21) from which

$$|\Lambda_{\mathrm{L}}^{jj}+\Xi_{\mathrm{L}}^{jj}\Theta-\tfrac{1}{2}+s_{\theta_w}^2|\simeq|-\tfrac{1}{2}+s_{\theta_w}^2|, \tag{27a}$$
$$|\Lambda_{\mathrm{R}}^{jj}+\Xi_{\mathrm{R}}^{jj}\Theta+s_{\theta_w}^2|\simeq|s_{\theta_w}^2|. \tag{27b}$$

Using the experimental bounds [11–13]

$$\mathrm{B}_{l_il_jl_j\overline{l_j}}\equiv\mathrm{B}(l_i\to l_jl_j\overline{l_j})=\begin{cases}\mathrm{B}_{\mu ee\bar{e}}<1.0\times10^{-12}\equiv\widetilde{\mathrm{B}}_{\mu ee\bar{e}},\\ \mathrm{B}_{\tau ee\bar{e}}<3.3\times10^{-6}\equiv\widetilde{\mathrm{B}}_{\tau ee\bar{e}},\\ \mathrm{B}_{\tau\mu\mu\bar{\mu}}<1.9\times10^{-6}\equiv\widetilde{\mathrm{B}}_{\tau\mu\mu\bar{\mu}},\end{cases} \tag{28}$$

and $s_{\theta_w}^2=0.2237$, the constraints on the mixing parameters are

$$0.203|\Lambda_{\mathrm{L}}^{ij}+\Xi_{\mathrm{L}}^{ij}\Theta|^2+0.176|\Lambda_{\mathrm{R}}^{ij}+\Xi_{\mathrm{R}}^{ij}\Theta|^2<c_{l_i}\widetilde{\mathrm{B}}_{l_il_jl_j\overline{l_j}}, \tag{29}$$

where $c_{l_i}=[4\mathrm{B}(l_i\to l_j\overline{\nu_{l_j}}\nu_{l_i})]^{-1}$ and

$$\mathrm{B}(l_i\to l_j\overline{\nu_{l_j}}\nu_{l_i})=\begin{cases}\mathrm{B}_{\mu e\bar{\nu}_e\nu_\mu}\approx1.00,\\ \mathrm{B}_{\tau\mu\bar{\nu}_\mu\nu_\tau}=0.1735\pm0.0014,\\ \mathrm{B}_{\tau e\bar{\nu}_e\nu_\tau}=0.1783\pm0.0008.\end{cases} \tag{30}$$

As in Sec. III B 1, the contribution of Θ in Eq. (29) is important when

$$\Xi_{\mathrm{L}}^{ij}\Theta\gtrsim\sqrt{\frac{c_{l_i}\widetilde{\mathrm{B}}_{l_il_il_i\overline{l_j}}}{0.203}}\quad\text{and}\quad\Xi_{\mathrm{R}}^{ij}\Theta\gtrsim\sqrt{\frac{c_{l_i}\widetilde{\mathrm{B}}_{l_il_il_i\overline{l_j}}}{0.176}}. \tag{31}$$

The bounds for $\Lambda_a^{\tau e}+\Xi_a^{\tau e}\Theta$ and $\Lambda_a^{\tau\mu}+\Xi_a^{\tau\mu}\Theta$ obtained from Eq. (29) are similar to those obtained from Eq. (23). For the μe case Eq. (29) is more stringent than Eq. (23). The results of this section are resumed in Table II.

3. Constraints from $l_i\to l_jl_k\overline{l_k}$

Assuming $m_{l_i}\gg m_{l_j},m_{l_k}$, ignoring possible contributions from scalars, and neglecting the tree-level diagrams which involve simultaneously two FCNC vertices, the branching ratio $\mathrm{B}(l_i\to l_jl_k\overline{l_k})$ is

$$\frac{\mathrm{B}(l_i\to l_jl_k\overline{l_k})}{\mathrm{B}(l_i\to l_j\overline{\nu_{l_j}}\nu_{l_i})}=(|g_{\mathrm{V}}^{kk}|^2+|g_{\mathrm{A}}^{kk}|^2)(|g_{\mathrm{V}}^{ij}|^2+|g_{\mathrm{A}}^{ij}|^2)+\frac{M_Z^2}{M_{Z'}^2}2\mathrm{Re}(g_{\mathrm{V}}^{kk}g_{\mathrm{V}}'^{kk*}+g_{\mathrm{A}}^{kk}g_{\mathrm{A}}'^{kk*})(g_{\mathrm{V}}^{ij}g_{\mathrm{V}}'^{ij*}+g_{\mathrm{A}}^{ij}g_{\mathrm{A}}'^{ij*})$$
$$+\frac{M_Z^4}{M_{Z'}^4}(|g_{\mathrm{V}}'^{kk}|^2+|g_{\mathrm{A}}'^{kk}|^2)(|g_{\mathrm{V}}'^{ij}|^2+|g_{\mathrm{A}}'^{ij}|^2) \tag{32a}$$
$$\simeq 4(|-\tfrac{1}{2}+s_{\theta_w}^2|^2+|s_{\theta_w}^2|^2)(|\Lambda_{\mathrm{L}}^{ij}+\Xi_{\mathrm{L}}^{ij}\Theta|^2+|\Lambda_{\mathrm{R}}^{ij}+\Xi_{\mathrm{R}}^{ij}\Theta|^2)+O(\Theta^2), \tag{32b}$$

where we made the same assumptions as in Sec. III B 2. Using the experimental limits [12]

$$\mathrm{B}_{l_il_jl_k\overline{l_k}}\equiv\mathrm{B}(l_i\to l_jl_k\overline{l_k})=\begin{cases}\mathrm{B}_{\tau e\mu\bar{\mu}}<3.6\times10^{-6}\equiv\widetilde{\mathrm{B}}_{\tau e\mu\bar{\mu}},\\ \mathrm{B}_{\tau\mu e\bar{e}}<3.4\times10^{-6}\equiv\widetilde{\mathrm{B}}_{\tau\mu e\bar{e}},\end{cases} \tag{33}$$

the constraints on the mixing parameters are

$$0.126(|\Lambda_{\mathrm{L}}^{ij}+\Xi_{\mathrm{L}}^{ij}\Theta|^2+|\Lambda_{\mathrm{R}}^{ij}+\Xi_{\mathrm{R}}^{ij}\Theta|^2)<c_{l_i}\widetilde{\mathrm{B}}_{l_il_jl_k\overline{l_k}}, \tag{34}$$

which are not of interest in our analysis since they are somewhat weaker than those of Eq. (29).

TABLE I. Bounds from the process $Z \to l_i \overline{l_j}$.

Limits from $Z \to l_i \overline{l_j}$

$$|\Lambda_L^{ij} + \Xi_L^{ij}\Theta|^2 + |\Lambda_R^{ij} + \Xi_R^{ij}\Theta|^2 < 1.87 \widetilde{B}_{l_i \overline{l_j}} = 1.87 \times \begin{cases} 1.7\times 10^{-6} \equiv \widetilde{B}_{e\overline{\mu}} \\ 7.3\times 10^{-6} \equiv \widetilde{B}_{e\overline{\tau}} \\ 1.0\times 10^{-5} \equiv \widetilde{B}_{\mu\overline{\tau}}, \end{cases}$$

Mixing e-μ

$$|\Lambda_L^{e\mu} + \Xi_L^{e\mu}\Theta|^2 + |\Lambda_R^{e\mu} + \Xi_R^{e\mu}\Theta|^2 < 3.2\times 10^{-6} \Rightarrow |\Lambda_a^{e\mu} + \Xi_a^{e\mu}\Theta| < 1.8\times 10^{-3}$$

Mixing e-τ

$$|\Lambda_L^{e\tau} + \Xi_L^{e\tau}\Theta|^2 + |\Lambda_R^{e\tau} + \Xi_R^{e\tau}\Theta|^2 < 1.4\times 10^{-5} \Rightarrow |\Lambda_a^{e\tau} + \Xi_a^{e\tau}\Theta| < 3.7\times 10^{-3}$$

Mixing μ-τ

$$|\Lambda_L^{\mu\tau} + \Xi_L^{\mu\tau}\Theta|^2 + |\Lambda_R^{\mu\tau} + \Xi_R^{\mu\tau}\Theta|^2 < 1.9\times 10^{-5} \Rightarrow |\Lambda_a^{\mu\tau} + \Xi_a^{\mu\tau}\Theta| < 4.3\times 10^{-3}$$

IV. SOME $SU(2)_L$ REPRESENTATIONS FOR ADDITIONAL FERMIONS

Some improvement on the above-derived bounds for the mixing parameters may be obtained with information about the $SU(2)_L$ transformation properties of the additional fermions and for this reason we analyze here a few simple $SU(2)_L$ representations in which new additional charged leptons may appear. In this analysis we will not consider any particular case for the Ξ_a^{ij} parameters, but we will assume that they are of $O(1)$. What follows is valid for one or more additional families, independently of whether the extra families are fundamental or excited leptons in the context of composite models.

A. No additional fermions

Equations (23) and (29) are valid even if no additional charged leptons are present in the extended theory. In this case $\mathsf{F} = \mathbf{\Lambda}_L = \mathbf{\Lambda}_R = 0$ and

$$\Xi_a = (\mathsf{A}^\dagger \mathsf{H}_O \mathsf{A})_a . \tag{35}$$

There are three subcases.

(1) H_O is of the UFD type. Then Z does not couple to FCNC since $(\mathsf{A}^\dagger \mathsf{A})_a = 1$ and therefore

$$\Xi_L^{ij} = \Xi_R^{ij} = 0 \quad \text{for } i \neq j. \tag{36}$$

(2) H_O is of the NUFD type. Then there are two possibilities. (a) $\mathsf{A} = 1$ (no mixing among the ordinary leptons). Then Z does not couple to FCNC. (b) $\mathsf{A} \neq 1$. Then Z couples to FCNC.

(3) H_O is of the FCNC type. Then Z couples to FCNC. Therefore, when FCNC exist, Eqs. (23) and (29) read

$$|\Xi_L^{ij}\Theta|^2 + |\Xi_R^{ij}\Theta|^2 < c \widetilde{B}_{l_i \overline{l_j}} = \begin{cases} c\widetilde{B}_{e\overline{\mu}} = 3.2\times 10^{-6}, \\ c\widetilde{B}_{e\overline{\tau}} = 1.4\times 10^{-5}, \\ c\widetilde{B}_{\mu\overline{\tau}} = 1.9\times 10^{-5}, \end{cases} \tag{37}$$

TABLE II. Bounds from the process $l_i \to l_j l_j \overline{l_j}$.

Limits from $l_i \to l_j l_j \overline{l_j}$

$$0.203|\Lambda_L^{ij} + \Xi_L^{ij}\Theta|^2 + 0.176|\Lambda_R^{ij} + \Xi_R^{ij}\Theta|^2 < c_{l_i} \widetilde{B}_{l_i l_j l_j \overline{l_j}} = c_{l_i} \times \begin{cases} 1.0\times 10^{-12} \equiv \widetilde{B}_{\mu e e \overline{e}} \\ 3.3\times 10^{-6} \equiv \widetilde{B}_{\tau e e \overline{e}} \\ 1.9\times 10^{-6} \equiv \widetilde{B}_{\tau\mu\mu\overline{\mu}} \end{cases}$$

Mixing e-μ

$$0.203|\Lambda_L^{e\mu} + \Xi_L^{e\mu}\Theta|^2 + 0.176|\Lambda_R^{e\mu} + \Xi_R^{e\mu}\Theta|^2 < 0.25\times 10^{-12}$$

$$|\Lambda_L^{e\mu} + \Xi_L^{e\mu}\Theta| < 1.1\times 10^{-6} \qquad |\Lambda_R^{e\mu} + \Xi_R^{e\mu}\Theta| < 1.2\times 10^{-6}$$

Mixing $e-\tau$

$$0.203|\Lambda_L^{e\tau} + \Xi_L^{e\tau}\Theta|^2 + 0.176|\Lambda_R^{e\tau} + \Xi_R^{e\tau}\Theta|^2 < 4.8\times 10^{-6}$$

$$|\Lambda_L^{e\tau} + \Xi_L^{e\tau}\Theta| < 4.9\times 10^{-3} \qquad |\Lambda_R^{e\tau} + \Xi_R^{e\tau}\Theta| < 5.2\times 10^{-3}$$

Mixing $\mu-\tau$

$$0.203|\Lambda_L^{\mu\tau} + \Xi_L^{\mu\tau}\Theta|^2 + 0.176|\Lambda_R^{\mu\tau} + \Xi_R^{\mu\tau}\Theta|^2 < 2.7\times 10^{-6}$$

$$|\Lambda_L^{\mu\tau} + \Xi_L^{\mu\tau}\Theta| < 3.6\times 10^{-3} \qquad |\Lambda_R^{\mu\tau} + \Xi_R^{\mu\tau}\Theta| < 3.9\times 10^{-3}$$

and

$$0.203|\Xi_L^{ij}\Theta|^2+0.176|\Xi_R^{ij}\Theta|^2 = c_{l_i}\mathrm{B}_{l_i l_j l_j \overline{l_j}} < \begin{cases} c_\mu \widetilde{\mathrm{B}}_{\mu e e \overline{e}} = 0.25\times 10^{-12}, \\ c_\tau \widetilde{\mathrm{B}}_{\tau e e \overline{e}} = 4.8\times 10^{-6}, \\ c_\tau \widetilde{\mathrm{B}}_{\tau \mu\mu \overline{\mu}} = 2.7\times 10^{-6}, \end{cases} \quad (38)$$

respectively. All the constraints are for the product $\Xi_a^{ij}\Theta$ of the couplings of the light fermions to the Z' and the Z-Z' mixing angle. In particular, $\Xi_a^{e\mu}\Theta<10^{-6}$.

B. Sequential fermions

$$\left.\begin{matrix} t_{3EL}=-\frac{1}{2} \\ t_{3ER}=0 \end{matrix}\right\} \Rightarrow \mathbf{\Lambda}_L=\mathbf{\Lambda}_R=0, \quad (39a)$$

$$\mathbf{\Xi}_a=(\mathsf{A}^\dagger \mathsf{H}_O \mathsf{A}+\mathsf{F}^\dagger \mathsf{H}_E \mathsf{F})_a . \quad (39b)$$

The situation is the same as that of no additional fermions. Equations (37) and (38) hold with the only difference that when $\mathsf{F}\neq 0$, $\mathbf{\Xi}_a$ contains the $(\mathsf{F}^\dagger \mathsf{H}_E \mathsf{F})_a$ contribution. As in the previous case the strongest constraint is for $\Xi_a^{e\mu}\Theta<10^{-6}$.

C. Vector singlets

Here

$$\left.\begin{matrix} t_{3EL}=0 \\ t_{3ER}=0 \end{matrix}\right\} \Rightarrow \mathbf{\Lambda}_R=0 \mathbf{\Lambda}_L=\tfrac{1}{2}(\mathsf{F}^\dagger \mathsf{F})_L , \quad (40a)$$

$$\mathbf{\Xi}_a=(\mathsf{A}^\dagger \mathsf{H}_O \mathsf{A}+\mathsf{F}^\dagger \mathsf{H}_E \mathsf{F})_a . \quad (40b)$$

Therefore Eqs. (23) and (29) now read

$$|\Lambda_L^{ij}+\Xi_L^{ij}\Theta|^2+|\Xi_R^{ij}\Theta|^2<c\widetilde{\mathrm{B}}_{l_i \overline{l_j}}, \quad (41)$$

and

$$0.203|\Lambda_L^{ij}+\Xi_L^{ij}\Theta|^2+0.176|\Xi_R^{ij}\Theta|^2<c_{l_i}\widetilde{\mathrm{B}}_{l_i l_j l_j \overline{l_j}}, \quad (42)$$

respectively. The contribution to FCNC from the ordinary–exotic-fermion mixing is only left handed. If $\Xi_a^{ij}\sim O(1)$, then the stringent bounds on Θ, a consequence of $\Xi_R^{e\mu}\Theta<10^{-6}$, imply an equally stringent bound on $\Lambda_L^{e\mu}$.

D. Vector doublets (homodoublets)

Here

$$\left.\begin{matrix} t_{3EL}=-\frac{1}{2} \\ t_{3ER}=-\frac{1}{2} \end{matrix}\right\} \Rightarrow \mathbf{\Lambda}_L=0 \quad \mathbf{\Lambda}_R=-\tfrac{1}{2}(\mathsf{F}^\dagger \mathsf{F})_R , \quad (43a)$$

$$\mathbf{\Xi}_a=(\mathsf{A}^\dagger \mathsf{H}_O \mathsf{A}+\mathsf{F}^\dagger \mathsf{H}_E \mathsf{F})_a . \quad (43b)$$

Thus Eqs. (23) and (29) now read

$$|\Xi_L^{ij}\Theta|^2+|\Lambda_R^{ij}+\Xi_R^{ij}\Theta|^2<c\widetilde{\mathrm{B}}_{l_i \overline{l_j}} \quad (44)$$

and

$$0.203|\Xi_L^{ij}\Theta|^2+0.176|\Lambda_R^{ij}+\Xi_R^{ij}\Theta|^2<c_{l_i}\widetilde{\mathrm{B}}_{l_i l_j l_j \overline{l_j}}. \quad (45)$$

The contribution to FCNC from the ordinary–exotic-fermion mixing is only right handed. The conclusions are the same as in the previous case with L ↔ R.

E. Mirror fermions

Here

$$\left.\begin{matrix} t_{3EL}=0 \\ t_{3ER}=-\frac{1}{2} \end{matrix}\right\} \Rightarrow \mathbf{\Lambda}_L=\tfrac{1}{2}(\mathsf{F}^\dagger \mathsf{F})_L \quad \mathbf{\Lambda}_R=-\tfrac{1}{2}(\mathsf{F}^\dagger \mathsf{F})_R , \quad (46a)$$

$$\mathbf{\Xi}_a=(\mathsf{A}^\dagger \mathsf{H}_O \mathsf{A}+\mathsf{F}^\dagger \mathsf{H}_E \mathsf{F})_a . \quad (46b)$$

Hence Eqs. (23) and (29) are unchanged:

$$|\Lambda_L^{ij}+\Xi_L^{ij}\Theta|^2+|\Lambda_R^{ij}+\Xi_R^{ij}\Theta|^2<c\widetilde{\mathrm{B}}_{l_i \overline{l_j}} \quad (47)$$

and

$$0.203|\Lambda_L^{ij}+\Xi_L^{ij}\Theta|^2+0.176|\Lambda_R^{ij}+\Xi_L^{ij}\Theta|^2<c_{l_i}\widetilde{\mathrm{B}}_{l_i l_j l_j \overline{l_j}}. \quad (48)$$

The contribution to FCNC from the ordinary-exotic-fermions mixing is both left and right handed. As a consequence there are no stringent bounds on Θ and the limits on Λ_a^{ij} and Θ are strongly correlated.

F. Self-conjugated triplets

Here

$$\left.\begin{matrix} t_{3EL}=-1 \\ t_{3ER}=-1 \end{matrix}\right\} \Rightarrow \mathbf{\Lambda}_L=-\tfrac{1}{2}(\mathsf{F}^\dagger \mathsf{F})_L \quad \mathbf{\Lambda}_R=-(\mathsf{F}^\dagger \mathsf{F})_R , \quad (49a)$$

$$\mathbf{\Xi}_a=(\mathsf{A}^\dagger \mathsf{H}_O \mathsf{A}+\mathsf{F}^\dagger \mathsf{H}_E \mathsf{F})_a . \quad (49b)$$

Hence Eqs. (23) and (29) are unchanged:

$$|\Lambda_L^{ij}+\Xi_L^{ij}\Theta|^2+|\Lambda_R^{ij}+\Xi_R^{ij}\Theta|^2<c\widetilde{\mathrm{B}}_{l_i \overline{l_j}} \quad (50)$$

and

$$0.203|\Lambda_L^{ij}+\Xi_L^{ij}\Theta|^2+0.176|\Lambda_R^{ij}+\Xi_R^{ij}\Theta|^2<c_{l_i}\widetilde{\mathrm{B}}_{l_i l_j l_j \overline{l_j}}. \quad (51)$$

As in the previous case the contribution to FCNC from the ordinary–exotic-fermion mixing is both left and right handed. As a consequence there are no stringent bounds on Θ and the limits on Λ_a^{ij} and Θ are strongly correlated.

V. CONCLUSIONS

In a model-independent way we obtained bounds for the strength of the FCNC, $(\mathbf{\Lambda}+\mathbf{\Xi}\Theta)_a$, in the ordinary charged-lepton sector, produced both by the ordinary–exotic-fermion mixing Λ_a^{ij} and by the Z-Z' mixing Θ. Given that the experimental bounds on the decay $\mu\to ee\overline{e}$ are more stringent than those for the FC decays of the τ into three charged leptons and of the Z into two charged leptons, the bounds on the μ-e coupling of the Z are stronger than those on the τ-e and τ-μ couplings. We have shown also that in some cases, when the $SU(2)_L$ representation of the additional fermions is relatively simple, the bounds may be refined. In other cases there may be a strong correlation between Θ and Λ_a^{ij} and then it is not safe to take the limit $\Theta \rightarrow 0$. In the same way, if one consider specific extended models, e.g., [1,14–21], some additional statements may be drawn on the Ξ_a^{ij}. In this work we have concentrated our attention to LFV (lepton family violation) in decay processes. On the other hand, there may be LFV processes of a different type [22] which will certainly put additional constraints on the LFV parameters.

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