

Threshold effects and radiative electroweak symmetry breaking in SU(5) extensions of the MSSM

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We make a complete analysis of radiative symmetry breaking in the MSSM and its SU(5) extensions including low- and high-energy threshold effects in the framework of the two-loop renormalization group. In particular, we consider *minimal* SU(5), the *missing-doublet* SU(5), a *Peccei-Quinn*-invariant version of SU(5), as well as a version with light adjoint remnants. We derive permitted ranges for the parameters of these models in relation to predicted α_s and M_G values within the present experimental accuracy. The parameter regions allowed under the constraints of radiative symmetry breaking, perturbativity, and proton stability, include the experimentally designated domain for α_s . In the case of the *minimal* SU(5), the values of α_s obtained are somewhat large in comparison with the experimental average. The *missing-doublet* SU(5), generally, predicts smaller values of α_s . In both versions of the *missing doublet*, the high-energy threshold effects on α_s operate in the opposite direction than that in the case of the minimal model, leading to small values. In the case of the *Peccei-Quinn* version, however, the presence of an extra intermediate scale allows us to achieve an excellent agreement with the experimental α_s values. Finally, the last considered version, with light remnants, exhibits unification of couplings at string scale at the expense, however, of rather large α_s values.

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I. INTRODUCTION

Supersymmetric unification, in the framework of supersymmetric grand unified theories (SUSY GUT's) [1], or superstrings [2], is in good agreement [3,4] with the low-energy values of the three gauge couplings, known to the present experimental accuracy, as well as with available bounds on the stability of the proton. The effective low-energy theory resulting from such a framework is a supersymmetric SU(3)_c × SU(2)_L × U(1)_Y model with softly broken supersymmetry. The simplest model of that class is the minimal extension of the standard model (MSSM) [1]. A most appealing feature realized in the MSSM is the breaking of the electroweak symmetry through radiative corrections [5]. The Higgs boson mass squared parameters, although positive definite at high energies, are radiatively corrected, as can be most easily studied by the use of the renormalization group, yielding a negative mass squared eigenvalue at low energies which triggers electroweak symmetry breaking [6,7].

In the present article we study the radiative breaking of electroweak symmetry in the framework of SU(5) SUSY GUT's. In addition to the standard low-energy inputs (α_{EM} , G_F , M_Z , ...) and the soft-breaking parameters (M_0 , $M_{1/2}$, A_0), we have the thresholds of the superheavy particles parametrized in terms of at least two more parameters (M_Σ , M_{H_c} , ...). Our output includes the strong cou-

pling α_s and the unification scale M_G , as well as the complete spectrum of new particles. The predicted strong coupling values can depend strongly on the high-energy thresholds. Thus, our analysis discriminates between the various GUT models.

Our basic low-energy inputs are the boundary values of the gauge couplings in the dimensional reduction with modified minimal subtraction (\overline{DR}) scheme [8] $\hat{\alpha}_1 \equiv \alpha_1(M_Z)|_{\overline{DR}}$ and $\hat{\alpha}_2 \equiv \alpha_2(M_Z)|_{\overline{DR}}$. Their values can be determined in terms of the Fermi constant $G_F = 1.16639 \times 10^5 \text{ GeV}^{-2}$, the Z-boson mass $M_Z = 91.1884 \pm 0.0022 \text{ GeV}$ [9], the electromagnetic coupling $\alpha_{EM}^{-1} = 137.036$, the bottom quark mass $m_b = 5 \text{ GeV}$, the tau mass $m_\tau = 1.777 \text{ GeV}$, and the top quark mass. We can write down the formulas

$$\hat{\alpha}_1^{-1} = \frac{3}{5} \alpha_{EM}^{-1} \cos^2 \theta \left[1 - \Delta_\gamma + \frac{\alpha_{EM}}{2\pi} \ln \left(\frac{M_S}{M_Z} \right) \right], \quad (1)$$

$$\hat{\alpha}_2^{-1} = \alpha_{EM}^{-1} \sin^2 \theta \left[1 - \Delta_\gamma + \frac{\alpha_{EM}}{2\pi} \ln \left(\frac{M_S}{M_Z} \right) \right], \quad (2)$$

where $\Delta_\gamma = 0.0682 \pm 0.0007$ [10] includes the light quark and lepton contributions, and the \overline{DR} mixing angle is given by

$$\sin^2 \theta = \frac{1}{2} \left\{ 1 - \left[1 - \frac{4\pi\alpha_{EM}}{\sqrt{2}G_F M_Z^2 (1 - \Delta r)} \right]^{1/2} \right\}. \quad (3)$$

The quantity Δr will be given below. The scale M_S appearing in Eqs. (1) and (2) is not a physical scale but a convenient parametrization for the contribution of all sparticle and heavy particles (W, t, H^+), defined as

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$$M_S = \frac{M_W^{-7} M_t^{16/9} M_{H^+}^{1/3} M_{\tilde{t}_1}^{4/9} M_{\tilde{t}_2}^{4/9} M_{\tilde{u}_{1,2}}^{8/9} M_{\tilde{u}_{1,2}^c}^{8/9} M_{\tilde{b}_1}^{1/9} M_{\tilde{b}_2}^{1/9} M_{\tilde{d}_{1,2}}^{2/9} M_{\tilde{d}_{1,2}^c}^{2/9} M_{\tilde{\tau}_1}^{1/3} M_{\tilde{\tau}_2}^{1/3} M_{\tilde{e}_{1,2}}^{2/3} M_{\tilde{e}_{1,2}^c}^{2/3} M_{\tilde{\chi}_1}^{4/3} M_{\tilde{\chi}_2}^{4/3}}{M_Z^{19/9}}. \quad (4)$$

An additional useful parametrization scale \tilde{M}_S , relevant to the strong coupling constant α_s , is

$$\tilde{M}_S = \frac{M_t^{2/3} M_{\tilde{t}_1}^{1/6} M_{\tilde{t}_2}^{1/6} M_{\tilde{u}_{1,2}}^{1/3} M_{\tilde{u}_{1,2}^c}^{1/3} M_{\tilde{b}_1}^{1/6} M_{\tilde{b}_2}^{1/6} M_{\tilde{d}_{1,2}}^{1/3} M_{\tilde{d}_{1,2}^c}^{1/3} M_{\tilde{g}}^2}{M_Z^{11/3}}. \quad (5)$$

The quantity Δr appearing in Eq. (3) can be written as [11]

$$\Delta r = \Delta_\gamma - \frac{\alpha_{\text{EM}}}{2\pi} \ln \frac{M_S}{M_Z} - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}(0)}{M_W^2} + \Delta_{\text{SM}} + \delta\rho^{\text{QCD}} + \delta\rho^{\text{Higgs}}, \quad (6)$$

Π_{ZZ} and Π_{WW} stand for the Z and W self-energies calculated using dimensional reduction. The quantity Δ_{SM} stands for standard model vertex+box corrections and is given by [12],

$$\Delta_{\text{SM}} = \frac{\alpha_{\text{EM}}}{4\pi \sin^2 \theta} \left\{ 6 + \frac{\text{In} \cos^2 \theta_W}{\sin^2 \theta_W} \left[\frac{7}{2} - \frac{5 \sin^2 \theta_W}{2} - \sin^2 \theta \left(5 - \frac{3 \cos^2 \theta_W}{2 \cos^2 \theta} \right) \right] \right\}. \quad (7)$$

In the last formula, by definition $\cos^2 \theta_W = M_W^2/M_Z^2$. The pole mass of W -gauge boson in Eq. (4) is related to M_Z by $M_W^2 = M_Z^2 \rho \cos^2 \theta$ where the ρ parameter is given by,

$$\rho = 1 - \frac{\Pi_{WW}(M_W^2)}{M_W^2} + \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + 2\text{-loop finite corrections}. \quad (8)$$

We have explicitly written in Eq. (6) the two-loop corrections due to QCD and the Higgs calculated in Ref. [13].

The low-energy value of the strong coupling constant considered as an output is given by the formula [14],

$$\alpha_s^{-1} \equiv \alpha_s^{-1}(M_Z)|_{\overline{\text{MS}}} = \hat{\alpha}_3^{-1}(M_Z)|_{\overline{\text{DR}}} + \frac{1}{4\pi} - \frac{1}{2\pi} \ln \frac{\tilde{M}_S}{M_Z}. \quad (9)$$

The current experimental values of α_s at Z pole extracted from QCD experiments with various methods are presented in Table I [9]. R refers to the ratio of cross sections or partial widths to hadrons versus leptons and the values of strong coupling are in modified minimal subtraction $\overline{\text{MS}}$ renormalization scheme. The average value of α_s given in Ref. [9] is $\alpha_s(M_Z) = 0.118 \pm 0.003$.

The values (1) and (2) will serve as low-energy boundary conditions for the corresponding two-loop renormalization group equations. As high-energy boundary conditions for the gauge couplings we shall impose unification at a scale M_G :

$$\hat{\alpha}_1(M_G) = \hat{\alpha}_2(M_G) = \hat{\alpha}_3(M_G) \equiv \alpha_G. \quad (10)$$

Note that when we depart from the minimal supersymmetric extension of the standard model and consider $\text{SU}(5)$ extensions of it the effect of the superheavy particles with masses around the unification scale M_G has to be taken into account in the evolution of the gauge couplings.

The soft supersymmetry breaking is represented by four parameters M_0 , $M_{1/2}$, A_0 , and B_0 of which we shall consider only the first three as input parameters and treat $B(M_Z)$, as well as the Higgs mixing parameter μ , as determined by the one-loop minimization equations

$$\sin 2\beta = -\frac{2B\mu}{\bar{m}_1^2 + \bar{m}_2^2}, \quad (11)$$

$$\frac{1}{2} [M_Z^2 + \Pi_{ZZ}(M_Z^2)] = \frac{\bar{m}_1^2 - \bar{m}_2^2 \tan^2 \beta}{\tan^2 \beta - 1}, \quad (12)$$

where $\bar{m}_i^2 \equiv m_{H_i}^2 + \mu^2 + \partial(\Delta V_{1\text{-loop}})/\partial v_i^2$. Note that cases in which the above two equations are not satisfied, and thus radiative symmetry breaking does not occur, are rejected. The parameter $\beta \equiv \tan^{-1}(v_2/v_1)$ is defined at M_Z . M_Z in Eq. (12) denotes the experimental Z -boson mass. Thus, supersymmetry breaking is parametrized with the input parameters M_0 , $M_{1/2}$, A_0 , $\beta(M_Z)$, and $\text{sgn} \mu(M_Z)$. We shall take the simplest of boundary conditions at M_G , assuming universality,

$$m_i(M_G) = M_0, \quad M_i(M_G) = M_{1/2}, \quad A_i(M_G) = A_0. \quad (13)$$

We shall employ the full coupled system of two-loop renormalization group equations [15] evolved from M_G down to low energies. Since our purpose is to study the effect of high-energy thresholds in various extensions of $\text{SU}(5)$, it will be sufficient to obtain the parameters M_S and \tilde{M}_S , appearing in the boundary values of the low-energy gauge couplings, by calculating the sparticle masses in the step approximation [16]. However, we shall introduce finite part contributions whenever they are *a priori* expected to be large [17], such as QCD corrections to the top quark and gluino masses for instance, etc.

Following the Particle Data Group [9], our basic experimental constraints on supersymmetric masses as well as Higgs boson masses are shown in Table II. These limits impose stringent bounds on the extracted values of $\alpha_s(M_Z)$ and on heavy high-energy masses as we will see later.

The purpose of the present paper is to analyze the effect of high-energy thresholds on unification predictions in the various $\text{SU}(5)$ extensions of the MSSM. Our goal is to complete previous existing analyses which either have not incorporated the constraints imposed by radiative symmetry breaking or have not considered the full range allowed for

TABLE I. Values of $\alpha_s(M_Z)$ extracted from QCD experiments.

Process	$\alpha_s(M_Z)$
Deep inelastic scattering	0.112 ± 0.006
R in τ lepton decay	0.122 ± 0.005
R in Y decay	0.108 ± 0.010
Event shapes in e^+e^- annihilation	0.122 ± 0.007
$Q\bar{Q}$ lattice	0.115 ± 0.003
Fragmentation	0.122 ± 0.012
Jets at HERA	0.121 ± 0.012
R in Z^0 decay (LEP and SLC)	0.123 ± 0.006
Deep inelastic at HERA	0.120 ± 0.014

GUT parameters. In our consideration of supersymmetric versions of SU(5) we have to take into account the constraints imposed by proton decay into $K^+ \bar{\nu}_\mu$ through dimension-5 operators. Assuming that this is the dominant process, the proton lifetime is [18–20]

$$\begin{aligned} \tau(p \rightarrow K^+ \bar{\nu}_\mu) &= 6.9 \times 10^{31} \text{ yr} \\ &\times \left| \frac{0.003 \text{ GeV}^3 0.67 \sin(2\beta) M_{H_c} \text{ TeV}^{-1}}{\beta_n A_S (1+y^{tK}) 10^{17} \text{ GeV} [f(\tilde{u}, \tilde{d}) + f(\tilde{u}, \tilde{e})]} \right|^2, \end{aligned} \quad (14)$$

where M_{H_c} is the effective color triplet mass in GeV, $\beta_n \sim (0.003-0.03) \text{ GeV}^3$, $|1+y^{tK}| \geq 0.4$, and $A_S = [\alpha_1(M_Z)/\alpha_5(M_G)]^{7/99} [\alpha_3(M_Z)/\alpha_5(M_{\text{GUT}})]^{-4/9}$. The function $f(x,y)$ is defined as

$$f(x,y) = m_{\tilde{w}} \frac{1}{x^2 - y^2} \left[\frac{x^2}{x^2 - m_{\tilde{w}}^2} \ln \left(\frac{x^2}{m_{\tilde{w}}^2} \right) - \frac{y^2}{y^2 - m_{\tilde{w}}^2} \ln \left(\frac{y^2}{m_{\tilde{w}}^2} \right) \right] \quad (15)$$

and masses are in GeV. The current experimental limit is

$$\tau(p \rightarrow K^+ \bar{\nu}_\mu) \geq 10^{32} \text{ yr}. \quad (16)$$

In what follows, we choose the most conservative values of β_n and $|1+y^{tK}|$ which are 0.003 GeV^3 and 0.4 , respectively.

An additional constraint that will be imposed is the absence of Landau poles on the dimensionless couplings of the theory, or equivalently, the validity of perturbation theory (perturbativity) of those couplings above M_G and up to the Planck scale. Although this is, in general, not a severe constraint, it should be taken into account in the cases of extended versions of SU(5) because of the existence of a large number of massless particles above M_G [see, for instance, Fig. 3(c)]. This constraint is implemented through the numerical solution of the one-loop renormalization group (RG) equations for the Yukawa couplings of SU(5) above M_G (see Appendix).

TABLE II. Experimental bounds on supersymmetric particles and Higgs Bosons.

Particle	Bound (GeV)
Neutralinos	
$m_{\chi_1^0}$ (LSP)	> 23
$m_{\chi_2^0}$	> 52
$m_{\chi_3^0}$	> 84
$m_{\chi_4^0}$	> 127
Charginos	
$m_{\chi_1^c}$	> 45.2
$m_{\chi_2^c}$	> 99
Sneutrino	
$m_{\tilde{\nu}}$	> 41.8
Charged sleptons	
$m_{\tilde{e}, \tilde{\mu}, \tilde{\tau}}$	> 45
Squarks	
$m_{\tilde{q}}$	> 224
Gluino	
$m_{\tilde{g}}$	> 154
Higgs bosons	
m_h	> 44
m_A^0	> 24.3
m_{H^\pm}	> 40

II. MINIMAL SU(5)

The standard superpotential [21] in the minimal SU(5) is

$$\begin{aligned} \mathcal{W} = & \frac{1}{2} M_1 \text{Tr}(\Sigma^2) + \frac{1}{3} \lambda_1 \text{Tr}(\Sigma^3) + M_2 \bar{H} H + \lambda_2 \bar{H} \Sigma H \\ & + \sqrt{2} Y_{(d)}^{ij} \Psi_i \phi_j \bar{H} + \frac{1}{4} Y_{(u)}^{ij} \Psi_i \Psi_j H, \quad i, j = 1, \dots, 3. \end{aligned} \quad (17)$$

SU(5) is spontaneously broken to $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ when the adjoint Higgs boson Σ develops a vacuum expectation value (VEV) in the direction $\langle \Sigma \rangle \equiv \text{VDiag}(2, 2, 2, -3, -3)$. The resulting superheavy masses are

$$M_V = 5g_5 V, \quad M_{H_c} = 5\lambda_2 V, \quad M_\Sigma = 5\lambda_1 V. \quad (18)$$

We have imposed the usual fine-tuning condition $M_2 = 3\lambda_2 V$ that gives massless isodoublets of H, \bar{H} . The mass M_Σ stands for the mass of the surviving color octet and isotriplet parts of Σ . The leading contribution of these masses to the β function coefficients of the three gauge couplings $dg_i/dt = g_i^3(b_i + \Delta b_i)/16\pi^2$, $i = 1, \dots, 3$, $t = \ln(Q/M_G)$, $(b_3, b_2, b_1) = (-3, 1, 33/5)$ is

$$\Delta b_3 = 3\theta(Q^2 - M_\Sigma^2) + \theta(Q^2 - M_{H_c}^2) - 4\theta(Q^2 - M_V^2), \quad (19)$$

$$\Delta b_2 = 2\theta(Q^2 - M_\Sigma^2) - 6\theta(Q^2 - M_V^2), \quad (20)$$

$$\Delta b_1 = \frac{2}{5}\theta(Q^2 - M_{H_c}^2) - 10\theta(Q^2 - M_V^2). \quad (21)$$

Demanding perturbativity up to $M_P/\sqrt{8\pi}\approx 2.4\times 10^{18}$ GeV for the couplings appearing in Eq. (17), we are led after numerically integrating the coupled system of SU(5) renormalization group equations, to the inequalities

$$\lambda_1 < 1.4, \quad \lambda_2 < 1.5, \quad Y_t < 1.5, \quad Y_b < 1.4 \quad (22)$$

at M_G . Note that although, in general,

$$M_G = \max\{M_V, M_{H_c}, M_\Sigma\}, \quad (23)$$

only the case $M_G = M_V$ can be realized under the combined constraints in our analyses. The case in which all superheavy masses are equal, although allowed by the bounds given in Eq. (22), is the well-studied case of the MSSM. In the context of the SU(5) this requires the couplings $\lambda_{1,2}, g_5$ to be fine tuned according to Eq. (18).

Our standard outputs are the values of the strong coupling $\alpha_s(M_Z)$ and the scale M_G where the couplings meet. The input values of M_0 , $M_{1/2}$, and A_0 are always taken to be smaller than 1 TeV. It must be noted that in the figures we have chosen input values such that the acceptable region in the $(\alpha_s - M_G)$ plane is the optimum one in the following sense: We vary one parameter at a time while we keep the others constant. This is done for every input parameter, until we reach the maximum acceptable area. In the figures shown, we have adopted for the mass of the top quark an average value of the Collider Detector at Fermilab (CDF) and D0 [22] experimental results, $m_t = 180$ GeV. Note also that variation of ± 5 GeV in $m_t = 180$ GeV results in ± 0.0005 and $\pm 0.08 \times 10^{16}$ GeV on α_s and M_G , respectively. In addition, a variation of ± 0.0007 in the central value of Δ_γ gives variation of ∓ 0.001 and $\mp 0.5 \times 10^{16}$ GeV on α_s and M_G , respectively.

The shaded areas of Figs. 1(a) and 1(b) represent the allowed parameter space for the outputs $\alpha_s(M_Z)$ and M_G . The results do not depend significantly on M_0 and A_0 which have been chosen to have the representative values shown. The allowed area shrinks with increasing $\tan\beta$ because of the proton decay bound. For smaller values the allowed area shrinks because of the perturbativity of $Y_t(M_G)$ and the constraint from radiative symmetry breaking. In Fig. 1(a) we have chosen a characteristic value for M_Σ while we have varied M_{H_c} through its allowed range of values $(1.8-4.6) \times 10^{16}$ GeV. Analogously, in Fig. 1(b) we have taken $M_{H_c} = 3 \times 10^{16}$ GeV, while we have varied M_Σ through the range of values $(0.1-2) \times 10^{16}$ GeV. The values of α_s obtained are, in general, too large in comparison with the average experimental value. Making a parameter search, we conclude that the lowest possible value that we can reach in this model is close to ≈ 0.130 . Nevertheless, there are processes (Table I) whose determined α_s agrees in a limiting sense with the smallest values in Fig. 1. It should be noted that the effect of high-energy thresholds has made the access to the smaller values of α_s worse than that in the case of the MSSM. The general dependence of α_s is that it increases with increasing M_{H_c} , while it decreases with increasing M_Σ .

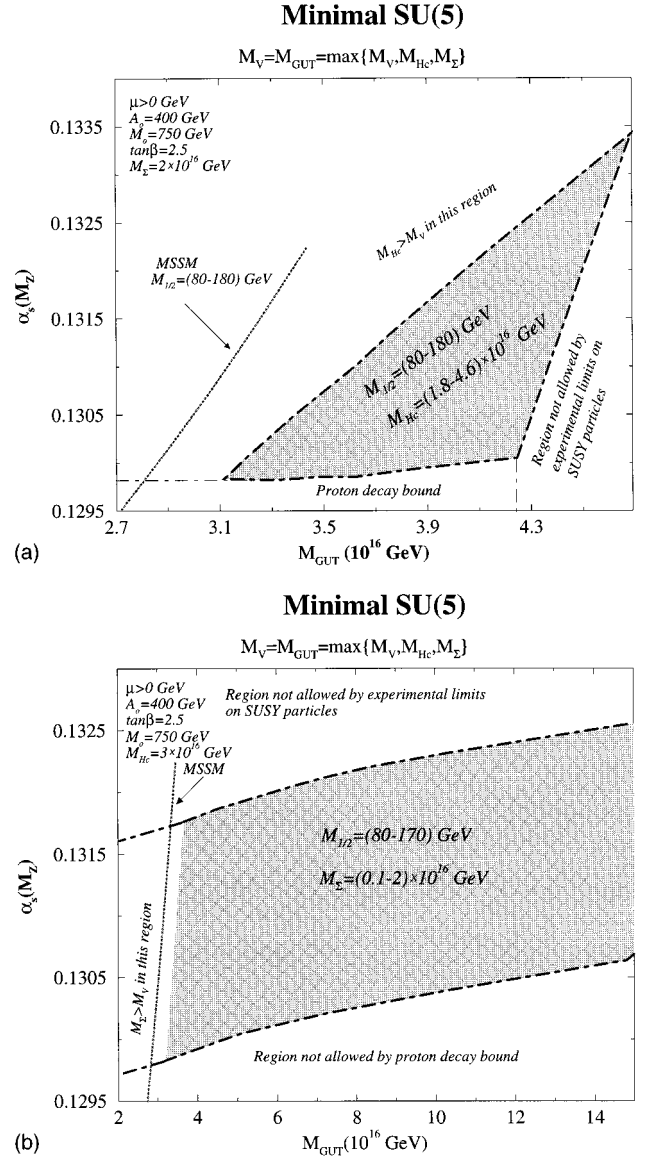


FIG. 1. (a), (b) $\alpha_s(M_Z)$ versus M_G when M_{H_c} , M_Σ , respectively, are varied.

III. MISSING DOUBLET MODEL

Let us now consider an extended version of the SU(5) known as the missing doublet model. This model [23], constructed in order to avoid the numerical fine tuning required in the minimal SU(5), has instead of the adjoint GUT Higgs field, a Higgs field in the **75** representation as well as an extra pair of Higgs doublets in the $\mathbf{50} + \overline{\mathbf{50}}$ representation. The superpotential is

$$\begin{aligned} \mathcal{W} = & M_1 \text{Tr}(\Sigma^2) + \frac{1}{3} \lambda_1 \text{Tr}(\Sigma^3) + \lambda_2 H \Sigma \Theta + \bar{\lambda}_2 \bar{H} \bar{\Sigma} \bar{\Theta} \\ & + M_2 \bar{\Theta} \Theta + \frac{1}{4} Y_{(u)}^{ij} \Psi_i \Psi_j H + \sqrt{2} Y_{(d)}^{ij} \Psi_i \phi_j \bar{H} \end{aligned} \quad (24)$$

$$i, j = 1, \dots, 3. \quad (24)$$

Since the $\Theta, \bar{\Theta}$ do not contain any isodoublets, only the colored triplets obtain masses while the isodoublets in H, \bar{H} stay massless. The vacuum expectation value (VEV)

$$\Sigma_{AB}^{CD} = V \left[(\delta_c)_A^C (\delta_c)_B^D + 2(\delta_w)_A^C (\delta_w)_B^D - \frac{1}{2} \delta_A^C \delta_B^D - (C \leftrightarrow D) \right] \quad (25)$$

leads to the masses

$$M(\mathbf{3}, \mathbf{1}, \frac{5}{3}) = \frac{4}{5} M_\Sigma, \quad M(\mathbf{8}, \mathbf{1}, 0) = \frac{M_\Sigma}{5}, \quad M(\mathbf{8}, \mathbf{3}, 0) = M_\Sigma,$$

$$M(\mathbf{6}, \mathbf{2}, \frac{5}{6}) = \frac{2}{5} M_\Sigma, \quad M(\mathbf{1}, \mathbf{1}, 0) = \frac{2}{5} M_\Sigma$$

for the remnants of **75**. The assignment of the quantum numbers refers to the group $SU(3) \times SU(2) \times U(1)$. We shall assume that the parameter M_2 is larger than the GUT scale, possibly of the order of the Planck mass. Otherwise, perturbativity, as can be easily seen, cannot be satisfied. The charge $-\frac{1}{3}$ color triplets in H, \bar{H} and $\Theta, \bar{\Theta}$ will give one supermassive combination of mass $M_{H_c} = O(M_2)$ and a light combination of mass

$$M_{H_c} \simeq \frac{9}{100} \left(\frac{32\lambda_2 \bar{\lambda}_2}{\lambda_1^2} \right) \left(\frac{M_\Sigma^2}{M_2} \right). \quad (26)$$

The mass parameter M_Σ is related to the vector boson mass through

$$\frac{M_\Sigma}{M_V} = \frac{5M_1}{2\sqrt{15}g_5 V} = \frac{(10/3)\lambda_1}{2\sqrt{15}g_5} = \frac{1}{3} \sqrt{\frac{5}{3}} \left(\frac{\lambda_1}{g_5} \right). \quad (27)$$

The modifications in the β function coefficients are,

$$\begin{aligned} \Delta b_3 = & -4\theta(Q^2 - M_V^2) + 9\theta(Q^2 - M_\Sigma^2) + \theta(Q^2 - 0.8^2 M_\Sigma^2) \\ & + 10\theta(Q^2 - 0.4^2 M_\Sigma^2) + 3\theta(Q^2 - 0.2^2 M_\Sigma^2) \\ & + \theta(Q^2 - M_{H_c}^2) + \theta(Q^2 - M_{H_c'}^2) + 34\theta(Q^2 - M_2^2), \end{aligned} \quad (28)$$

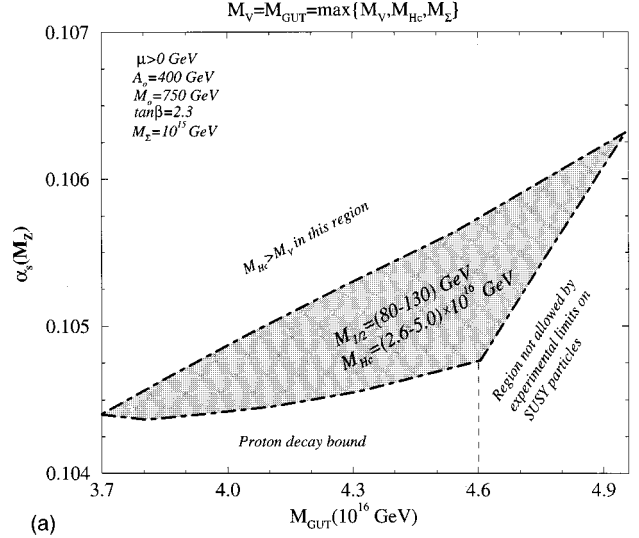
$$\begin{aligned} \Delta b_2 = & -6\theta(Q^2 - M_V^2) + 16\theta(Q^2 - M_\Sigma^2) \\ & + 6\theta(Q^2 - 0.4^2 M_\Sigma^2) + 35\theta(Q^2 - M_2^2), \end{aligned} \quad (29)$$

$$\begin{aligned} \Delta b_1 = & -10\theta(Q^2 - M_V^2) + 10\theta(Q^2 - 0.8^2 M_\Sigma^2) \\ & + 10\theta(Q^2 - 0.4^2 M_\Sigma^2) + \frac{2}{5}\theta(Q^2 - M_{H_c}^2) \\ & + \frac{2}{5}\theta(Q^2 - M_{H_c'}^2) + \frac{173}{5}\theta(Q^2 - M_2^2). \end{aligned} \quad (30)$$

Perturbativity above M_G , as in the case of minimal $SU(5)$, leads us to the extra constraints at M_G

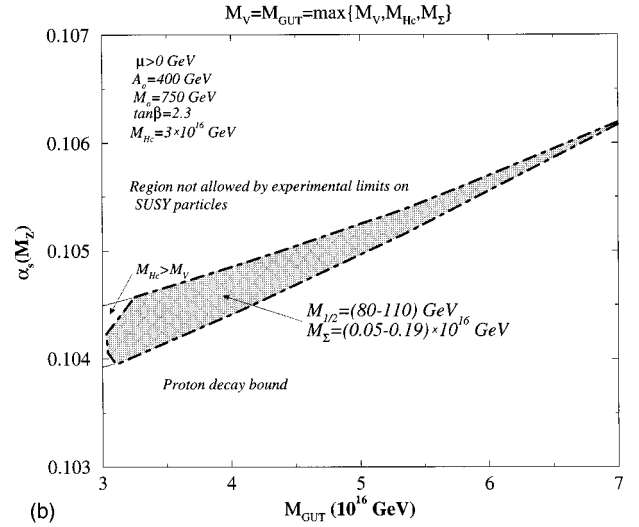
$$\lambda_1 < 0.18, \quad Y_t < 1.6, \quad Y_b < 1.5. \quad (31)$$

Missing Doublet Model



(a)

Missing Doublet Model



(b)

FIG. 2. (a), (b) $\alpha_s(M_Z)$ versus M_G when M_{H_c} , M_Σ , respectively, are varied.

In this model, we obtain a stronger constraint on λ_1 and, consequently, on M_Σ , because of the fact that now we have a larger-dimensional representation (**75**).

Note that the model as it stands does not contain any μ term. We assume, however, that a μ term is generated through an independent mechanism [24].

In Figs. 2(a) and 2(b), it can be seen that the values of α_s obtained are much smaller than the average experimental value. We should note that α_s is pushed towards smaller values because of the splittings within **75** that give a large correction in the opposite direction than that in the case of the minimal model [23]. The maximum value of α_s that we are able to obtain in the *missing doublet* model is approximately ≈ 0.106 . As we can see from Table I, there are still QCD processes where the values of α_s are in agreement with the results of the missing doublet model. The heavy masses M_{H_c} and M_Σ are constrained to be in the regions $(2.6-5.0) \times 10^{16}$ GeV and $(0.05-0.19) \times 10^{16}$ GeV, respec-

tively. Because of the fact that the extracted values of α_s in MSSM are greater than 0.125 [for the inputs of Figs. 2(a) and 2(b)], we do not display in Figs. 2(a) and 2(b) the corresponding MSSM plane. Experimental limits on LSP (see Table II) puts a lower bound on the universal soft gaugino masses such that $M_{1/2} \geq 80$ GeV.

IV. PECCEI-QUINN SYMMETRIC MISSING DOUBLET MODEL

The problem of proton decay through $D=5$ operators, that is present in the minimal SU(5) model, provided strong motivation to construct versions of SU(5) with a *Peccei-Quinn* symmetry [25] that naturally suppresses these operators by a factor proportional to the ratio of the *Peccei-Quinn*-breaking scale over the GUT scale [26]. A *Peccei-Quinn* version of the missing doublet SU(5) model requires the doubling of $\mathbf{5} + \bar{\mathbf{5}}$ and $\mathbf{50} + \bar{\mathbf{50}}$ representations. The relevant superpotential terms which must be added to the previous superpotential are

$$\begin{aligned} & \lambda_2 H \Sigma \Theta + \bar{\lambda}_2 \bar{H} \bar{\Sigma} \bar{\Theta} + \lambda_2' H' \Sigma \Theta' + \bar{\lambda}_2' \bar{H}' \bar{\Sigma} \bar{\Theta}' + M_2 \Theta \bar{\Theta}' \\ & + M_2' \Theta' \bar{\Theta} + \lambda_3 P \bar{H}' H'. \end{aligned} \quad (32)$$

P stands for an extra gauge singlet superfield. The charges under the *Peccei-Quinn* symmetry are $\Psi(\alpha/2)$, $\phi(\beta/2)$, $H(-\alpha)$, $\bar{H}[-(\alpha+\beta)/2]$, $\Theta(\alpha)$, $\bar{\Theta}[(\alpha+\beta)/2]$, $\Theta'(-\alpha)$, $\bar{\Theta}'[-(\alpha+\beta)/2]$, $H'[(\alpha+\beta)/2]$, $\bar{H}'(\alpha)$, $P[-(3\alpha+\beta)]$. The breaking of the *Peccei-Quinn* symmetry can be achieved with a suitable gauge singlet system at an intermediate energy $\langle P \rangle \equiv M_{H_f} / \lambda_3 \sim 10^{10} - 10^{12}$ GeV. Assuming M_2, M_2' to be of the order of the Planck scale, we obtain two massive pairs of colored triplets with masses

$$M_{H_c} \simeq 32 \lambda_2 \bar{\lambda}_2' \frac{V^2}{M_2}, \quad M_{\bar{H}_c} \simeq 32 \bar{\lambda}_2 \lambda_2' \frac{V^2}{M_2'}, \quad (33)$$

somewhat below the GUT scale. Note that $M_V = 2\sqrt{15}g_5 V$. In addition, we have two pairs of isodoublets, one of which is massless while the other pair receives the intermediate mass M_{H_f} . The modifications in the renormalization group β function coefficients are

$$\begin{aligned} \Delta b_3 = & -4\theta(Q^2 - M_V^2) + 9\theta(Q^2 - M_\Sigma^2) + \theta(Q^2 - 0.8^2 M_\Sigma^2) \\ & + 10\theta(Q^2 - 0.4^2 M_\Sigma^2) + 3\theta(Q^2 - 0.2^2 M_\Sigma^2) \\ & + \theta(Q^2 - M_{H_c}^2) + \theta(Q^2 - M_{\bar{H}_c}^2), \end{aligned} \quad (34)$$

$$\begin{aligned} \Delta b_2 = & -6\theta(Q^2 - M_V^2) + 16\theta(Q^2 - M_\Sigma^2) \\ & + 6\theta(Q^2 - 0.4^2 M_\Sigma^2) + \theta(Q^2 - M_{H_f}^2), \end{aligned} \quad (35)$$

$$\begin{aligned} \Delta b_1 = & -10\theta(Q^2 - M_V^2) + 10\theta(Q^2 - 0.8^2 M_\Sigma^2) \\ & + 10\theta(Q^2 - 0.4^2 M_\Sigma^2) + \frac{2}{5}\theta(Q^2 - M_{H_c}^2) \\ & + \frac{2}{5}\theta(Q^2 - M_{\bar{H}_c}^2) + \frac{3}{5}\theta(Q^2 - M_{H_f}^2). \end{aligned} \quad (36)$$

Bounds, which come from perturbativity of couplings in this model, are numerically similar to those of the previous one (see Appendix). The extra coupling λ_3 obeys the constraint $\lambda_3 < 2.7$ at M_G .

It is evident from Fig. 3 that the values of α_s obtained for the case of this model are in excellent agreement with the experiment. The range of these values (0.107–0.140) covers the experimental average value of α_s and lies between the gap of the *minimal* SU(5) model and the *missing doublet* model. This model possesses an additional parameter, the intermediate scale M_{H_f} which increases the values of α_s when it takes lower values. The allowed range of values for $M_{1/2}$ has been increased now to 730 GeV or 900 GeV in Figs. 3(a) and 3(b), respectively. For this model the allowed range for M_0 can be extended to lower values. Figures 3(a) and 3(b) have been obtained for $M_0 = 300$ GeV. Note, however, that still α_s does not depend significantly on M_0 . In addition, $\tan\beta$ can practically now take much larger values, as large as $\tan\beta \sim 40$. Figures 3(a), 3(b), and 3(c) have been obtained for an intermediate value $\tan\beta = 10$. The allowed range of values for the intermediate scale M_{H_f} is $(1-450) \times 10^{10}$ GeV. Similarly, $5 \times 10^{14} < M_{H_c} < M_G$ and $7 \times 10^{14} < M_\Sigma < 15 \times 10^{14}$ GeV. Note, finally, that the grand unification scale can take values as large as the ‘‘string scale’’ for rather large but not excluded values of α_s .

V. A VERSION OF SU(5) WITH LIGHT REMNANTS

Recently, there has been some activity around models with gauge group $G \times G$ with intent to bypass the known problem of $k=1$ superstring constructions where no adjoint Higgs field can appear in the massless spectrum [27]. Vector-vector Higgs doublets present in the spectrum of $G \times G$ can break it into G_{diag} . An $SU(5) \times SU(5)$ model with Higgs representations $Z(\mathbf{5}, \bar{\mathbf{5}}) + \bar{Z}(\bar{\mathbf{5}}, \mathbf{5})$ can have renormalizable couplings only of the type $\Phi Z_j^i \bar{Z}_i^j = \Phi \text{Tr}(Z \bar{Z})$ to a singlet Φ , in addition to self-couplings of singlets. Thus, we could construct an analogue SU(5) GUT with superpotential:

$$\mathcal{W} = \frac{\lambda_1}{2} \Phi_1 \text{Tr}(\Sigma^2) + \frac{\lambda_2}{2} \Phi_1 \Phi_2^2 + \frac{\lambda_3}{3} \Phi_1^3, \quad (37)$$

where Σ is the adjoint and Φ_1, Φ_2 are singlets. This superpotential is invariant under $\Phi_2 \rightarrow -\Phi_2$. The F -flatness conditions give, apart from $\langle \Sigma \rangle = V \text{Diag}(2, 2, 2, -3, -3)$,

$$\Phi_2^2 = -\frac{\lambda_1}{\lambda_2} (30V^2), \quad \Phi_1 = 0. \quad (38)$$

With the above superpotential the remnants of Σ , a color octet and an isotriplet, stay massless. Nevertheless, nonrenormalizable terms such as

$$\Delta \mathcal{W} = \frac{\lambda_4}{M} \text{Tr}(\Sigma^4) + \frac{\lambda_5}{M} [\text{Tr}(\Sigma^2)]^2 \quad (39)$$

can, in principle, induce a mass of order $O(V^2/M) = 10^{14}$ GeV or smaller, depending on the actual values of λ_4 and λ_5 . A higher order nonrenormalizable term would induce an even smaller mass $V^3/M^2 \sim 10^{12}$ GeV. The

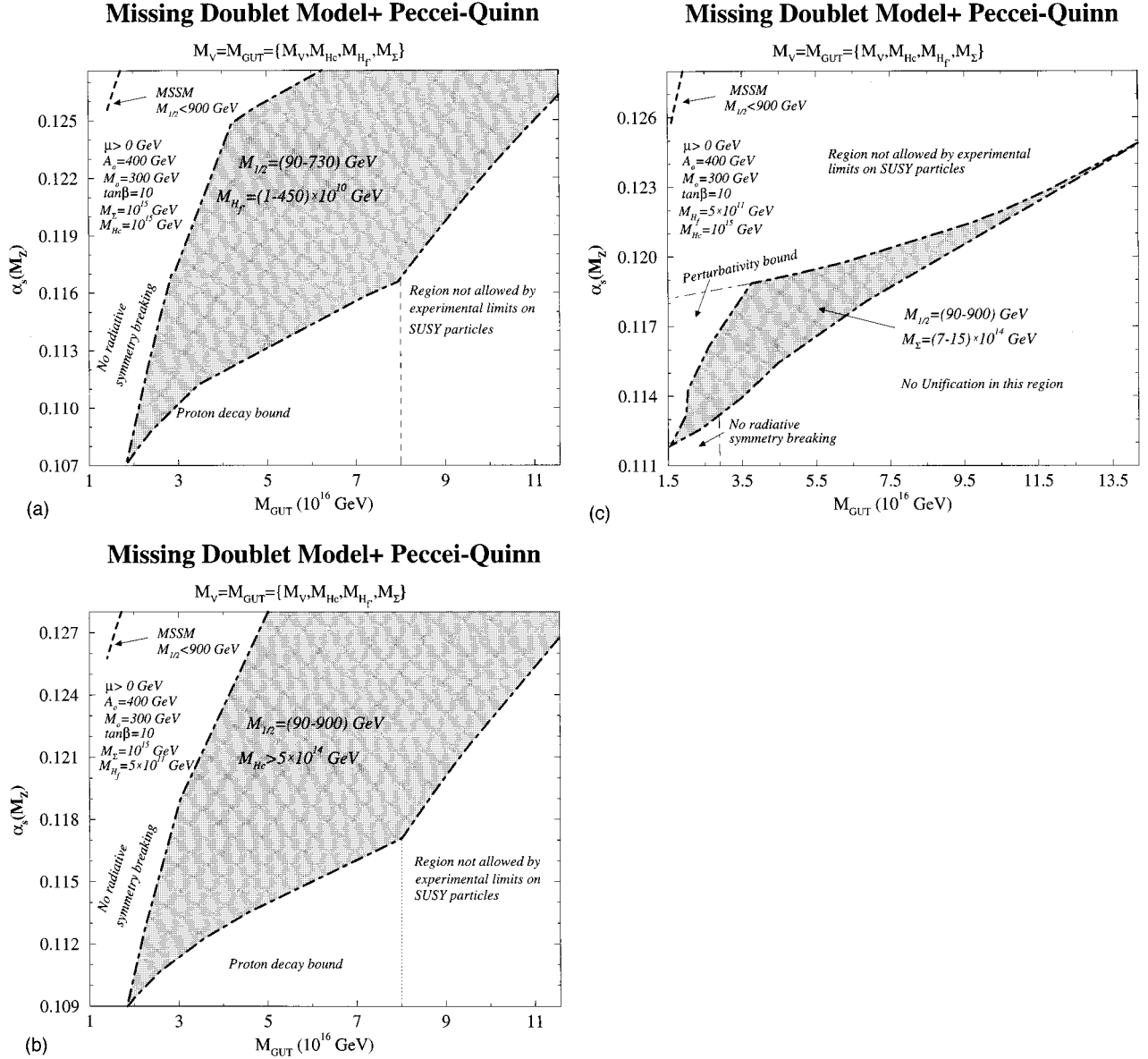


FIG. 3. (a), (b), (c) $\alpha_s(M_Z)$ versus M_G when $M_{H'_c}$, M_{H_c} , and M_Σ , are varied.

light Σ in this model could allow for a large M_G close to a string unification scale and thus in such a model there would be no string unification mismatch.

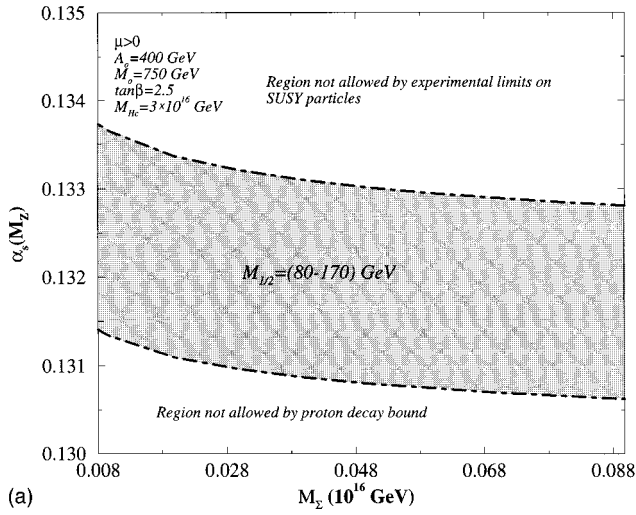
We shall, therefore, investigate the effects of small M_Σ on α_s and M_G . In order to obtain acceptably small values of α_s , we choose as input value of M_{H_c} the smallest acceptable one as it is shown in Figs. 4(a) and 4(b), since an increasing M_{H_c} tends to increase α_s . When we vary M_Σ from 10^{15} down to 10^{13} GeV, we can achieve unification at $M_G = 5 \times 10^{17}$ GeV = $O(M_{\text{string}})$. Decreasing M_Σ further towards the intermediate scale leads to even larger values of α_s . However, the values of α_s are still rather large (> 0.131). This excludes this particular version at least in this simple form.

VI. CONCLUSIONS

In this article we have studied various supersymmetric extensions of the standard model based on the group SU(5).

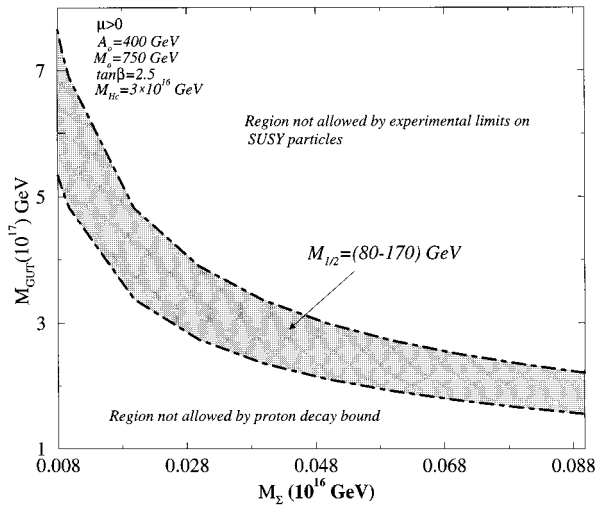
The low-energy precision data in conjunction with the existing experimental bounds on sparticle masses are known to impose strong constraints if radiative breaking of the electroweak symmetry is assumed. There exist several detailed studies in the literature in the framework of radiative symmetry breaking [6,7,10,11,16,17], which, however, have not considered in detail the effect of the superheavy degrees of freedom included in unified schemes. In SUSY GUT's there are additional constraints one has to deal with such as the experimental bound on proton's lifetime, the absence of Landau poles beyond the unification scale, and the appearance of heavy thresholds which influence the evolution of the couplings involved. All these affect the low-energy predictions. The existing analyses in this direction on the other hand [18,19,23,20,26,27], have not systematically taken into account the effect of the low-energy thresholds at the level of accuracy required by low energy precision data as was done in the previous references. Our analysis combines both and takes into account high- and low-energy thresholds at the

SU(5) with light remnants



(a)

SU(5) with light remnants



(b)

FIG. 4. (a), (b) α_s and M_G versus M_{Σ} in SU(5) model with light remnants.

accuracy required by precision experiments. In particular, we have focused our attention on the extracted value of the strong coupling constant $\alpha_s(M_Z)$, the value of the unification scale M_G , as well as the restrictions imposed on the heavy masses in some unifying schemes based on the SU(5). Sample results of our findings have been displayed in Figs. 1–4.

In the case of the minimal SU(5) we found that the values of the strong coupling constant obtained are somewhat larger as compared to the average experimental value of $\alpha_s(M_Z)$. Also, the unification scale M_G differs from the string unification scale by an order of magnitude if the lower values of $\alpha_s(M_Z)$ obtained are assumed. Access to small values of the strong coupling constant is more difficult than that in the MSSM exhibiting the influence of the superheavy degrees of freedom in a clear manner. The range of M_{H_c} allowed in this model is somewhat limited. The *missing doublet model* seems to favor small values of $\alpha_s(M_Z)$, in contrast with

those obtained in the MSSM and the minimal SU(5) version. At the same time, when M_G increases the allowed parameter space shrinks considerably. Proton decay along with perturbativity requirements seem to put a stringent constraint on both minimal SU(5) and missing doublet model (MDM). In the MDM the large splittings within **75** give a high-energy threshold effect on $\alpha_s(M_Z)$ in the opposite direction than that in the case of the minimal model leading to small values. This is also the case for the *Peccei-Quinn* version of the MDM. However, the presence of an extra intermediate scale ameliorates the situation allowing one to achieve an excellent agreement with the experimental values of $\alpha_s(M_Z)$. The grand unification scale can take values as large as the string scale at the expense, however, of having rather large values of the strong coupling constant not favored by all experiments. If we consider the range for allowed values of $\alpha_s(M_Z)$, the minimal SU(5) lies always above 0.130 while MDM lies below 0.106. The intermediate range between the two can be covered by the *Peccei-Quinn* version of the MDM and coincides with the allowed experimental range. Finally, the last considered version, with light Σ , exhibits unification of couplings at string scale but the values of α_s obtained are rather large, although within the errors of some experiments.

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APPENDIX: RGE’S ABOVE M_G

The renormalization group equations (RGE’s) for the gauge and Yukawa couplings from M_{GUT} to $M_P/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV in the case of minimal SU(5) are [19,20]

$$\frac{dg_5}{dt} = -\frac{3}{(4\pi)^2} g_5^3, \quad (\text{A1})$$

$$\frac{d\lambda_1}{dt} = \frac{\lambda_1}{(4\pi)^2} \left(\frac{63}{5} \lambda_1^2 + 3\lambda_2^2 - 30g_5^2 \right), \quad (\text{A2})$$

$$\frac{d\lambda_2}{dt} = \frac{\lambda_2}{(4\pi)^2} \left(\frac{21}{5} \lambda_1^2 + \frac{53}{5} \lambda_2^2 - \frac{98}{5} g_5^2 + 3Y_t^2 + 4Y_b^2 \right), \quad (\text{A3})$$

$$\frac{dY_t}{dt} = \frac{Y_t}{(4\pi)^2} \left(\frac{24}{5} \lambda_2^2 + 9Y_t^2 + 4Y_b^2 - \frac{96}{5} g_5^2 \right), \quad (\text{A4})$$

$$\frac{dY_b}{dt} = \frac{Y_b}{(4\pi)^2} \left(\frac{24}{5} \lambda_2^2 + 3Y_t^2 + 10Y_b^2 - \frac{84}{5} g_5^2 \right), \quad (\text{A5})$$

where $t = \ln[Q/(M_P/\sqrt{8\pi})]$.

The modification of the above system in the case of the missing doublet model is

$$\frac{dg_5}{dt} = \frac{17}{(4\pi)^2} g_5^3, \quad (\text{A6})$$

$$\frac{d\lambda_1}{dt} = \frac{\lambda_1}{(4\pi)^2} \left(\frac{50}{3} (8\lambda_1)^2 - 48g_5^2 \right). \quad (\text{A7})$$

The other two equations for the top and bottom Yukawa coupling are derived if we set $\lambda_2=0$ in the Eqs. (A3) and (A4) of the minimal model. Because of the fact that we have an extra pair of Higgs $\bar{\mathbf{5}}$ and $\mathbf{\bar{5}}$ in the Peccei-Quinn version of the missing doublet model, the only equation that changes

compared to the missing doublet model is the one for the gauge coupling which takes the form

$$\frac{dg_5}{dt} = \frac{18}{(4\pi)^2} g_5^3. \quad (\text{A8})$$

In this model we have an extra coupling λ_3 whose running is given by the following renormalization group equation:

$$\frac{d\lambda_3}{dt} = \frac{\lambda_3}{(4\pi)^2} \left(3\lambda_3^2 - \frac{48}{5} g_5^2 \right). \quad (\text{A9})$$

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