Photon-meson transition form factors $\gamma\pi^0$, $\gamma\eta$, and $\gamma\eta'$ at low and moderately high Q^2

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We study photon-meson transition form factors $\gamma^*(Q^2)\gamma\to\pi^0,\eta,\eta'$ at low and moderately high Q^2 assuming a nontrivial hadronlike $q\overline{q}$ structure of the photon in the soft region. In the hard region, the $q\overline{q}$ photon wave function contains both a perturbative tail of the soft wave function such as the wave function of an ordinary hadron and a standard QED pointlike $q\overline{q}$ component. The latter provides the $1/Q^2$ asymptotical behavior of the transition form factor in accordance with QCD. The data on the $\gamma\pi^0$ form factor are used for fixing the soft photon wave function which is found to have the same structure as the soft wave function of the pion reconstructed from the elastic pion form factor. Assuming universality of the ground-state pseudoscalar meson wave functions we calculate the $\gamma\eta$, $\gamma\eta'$ transition form factors and $\eta\to\gamma\gamma$, $\eta'\to\gamma\gamma$ partial widths and found them to be in perfect agreement with the data. [S0556-2821(97)01905-X]

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I. INTRODUCTION

In this paper we continue a study of the transition regime from the soft nonperturbative physics to the physics of hard processes described by perturbative QCD (PQCD). In Ref. [1] we have proposed a method for considering a form factor in a broad range of momentum transfers starting with the nonperturbative region of small Q^2 and moving to moderately large Q^2 by representing the form factor as a series in α_s and taking into account the nonperturbative term and $O(\alpha_s)$ corrections. This approach allows a continuous transition from small to asymptotically large momentum transfers. Using this procedure, we have determined the pion light-cone wave function by describing the pion elastic form factor in the range $0\le Q^2\le 10$ GeV^2.

Recent experiments on pseudoscalar meson production in e^+e^- collisions [2] provide new data on the transition form factors $\gamma^*(Q^2)\gamma\to\pi^0,\eta,\eta'$ in the region $0\le Q^2\le 20$ GeV^2. These results open a new possibility for studying the onset of the asymptotical PQCD regime.

Theoretical investigation of the photon-meson transition processes has given two important results on the photon-pion transition form factor.

(1) The Adler-Bell-Jackiw axial anomaly [3] yields a nonvanishing transition form factor of the pion into two real photons in the chiral limit of vanishing quark masses:

$$F_{\gamma^*\gamma^*\pi}(Q_1^2=0,Q_2^2=0)=\frac{1}{2\sqrt{2}\pi^2 f_\pi},\qquad f_\pi=130\ \text{MeV},\tag{1}$$

where the photon-pion transition form factor is defined as $(Q_1^2=-q_1^2,Q_2^2=-q_2^2)$

$$\langle\pi(P)|T|\gamma(q_1,\mu)\gamma(q_2,\nu)\rangle=-e^2\varepsilon_{\mu\nu\alpha_1\alpha_2}q_1^{\alpha_1}q_2^{\alpha_2}F_{\gamma^*\gamma^*\pi}(-q_1^2,-q_2^2).\tag{2}$$

(2) In the kinematical region where at least one of the photon virtualities is large, PQCD gives the following prediction for the behavior of the transition form factor [4]:

$$F_{\gamma^*\gamma^*\pi}(Q_1^2,Q_2^2)=\frac{\sqrt{2}}{3}\int_0^1\frac{\phi_\pi(x)dx}{xQ_1^2+(1-x)Q_2^2},\tag{3}$$

where $\phi_\pi(x)$ is the leading twist wave function (distribution amplitude) which describes the longitudinal momentum distribution of valence quark-antiquark pair in the pion. PQCD also predicts the asymptotic behavior of the pion distribution amplitude in the form [5]

$$\phi_\pi^{\rm as}(x)=6f_\pi x(1-x).\tag{4}$$

For $Q_2^2\to 0$ and large $Q_1^2\equiv Q^2$ which correspond to the kinematics of the experiments [2], Eq. (3) gives

$$F_{\gamma\pi}(Q^2)=\frac{\sqrt{2}}{3}\int_0^1\frac{\phi_\pi(x)dx}{xQ^2}[1+O(\alpha_s(Q^2))]+O\left(\frac{1}{Q^4}\right),\tag{5}$$

where $F_{\gamma\pi}(Q^2)\equiv F_{\gamma^*\gamma^*\pi}(Q^2,0)$.

At asymptotically large Q^2 one can use the asymptotic pion distribution amplitude to find the leading behavior of the transition form factor:

$$F_{\gamma\pi}(Q^2)=\frac{\sqrt{2}f_\pi}{Q^2}[1+O(\alpha_s(Q^2))]+O\left(\frac{1}{Q^4}\right).\tag{6}$$

The leading term corresponds to the diagram of Fig. 1(b) with pointlike vertices of the quark-photon interaction.

A problem of applicability of PQCD in the region of few GeV^2 and determining the scale at which the leading PQCD term starts to dominate the form factor has been extensively discussed in connection with the pion form factor. At large Q^2 the hard scattering mechanism of PQCD [5] gives the

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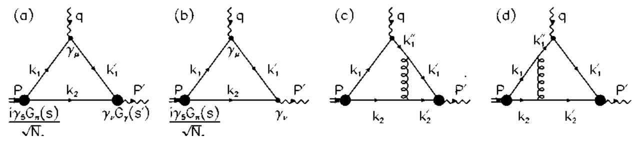

FIG. 1. Diagrams relevant to the description of the transition form factor at low and moderately high Q^2.

dominant contribution. To consider the region of low Q^2 Chernyak and Zhitnitsky [6] assumed that the hard scattering picture still remains valid but the distribution amplitude is far from its asymptotic form such that the hard contribution saturates the form factor. However, as first underlined by Isgur and Llewellyn-Smith [9], at low Q^2 a bulk of the form factor given by the perturbative hard scattering formula comes from the nonperturbative low-x region and thus the hard scattering approach at low Q^2 is not self-consistent. Moreover, this suggests a considerable soft contribution to the form factor in this region. A modified hard scattering approach which takes into account Sudakov effects [7] and transverse motion [8] makes the perturbative consideration self-consistent at few GeV^2 but considerably decreases the hard contribution to the form factor, thus leaving room for the soft contribution. A qualitative picture of the process looks as follows [9,10]: The hard scattering mechanism saturates the form factor at asymptotically large Q^2 whereas the soft contribution is dominant at low Q^2 and is still important in the region $Q^2 \simeq 10-20$ GeV^2. At the same time, the distribution amplitude is very close to its asymptotical form of Eq. (4). This picture has been supported by recent application of QCD sum rules [11] where the distribution amplitude has been found to be close to its asymptotic form. A quantitative analysis of the pion form factor within the quark model [1], based on isolating the soft and the hard contributions, also confirms the picture of Ref. [9].

The procedure of Ref. [1] is the following: the pion light-cone wave function Ψ^π is divided into the soft and hard components, Ψ_S^π and Ψ_H^π, such that Ψ_S^π is large at $s=(m^2+k_\perp^2)/[x(1-x)]<s_0$, while Ψ_H^π prevails at $s>s_0$. The parameter s_0 is a boundary of the soft and the hard regions and is expected to have the value of several GeV^2. We performed the decomposition of the wave function into the soft and the hard components using a simple step-function ansatz

$$\Psi^\pi=\Psi_S^\pi\theta(s_0-s)+\Psi_H^\pi\theta(s-s_0). \tag{7}$$

Representing the hard component Ψ_H^π as a series in α_s, one comes to the following expansion of the elastic pion form factor:

$$F_\pi=F_\pi^{SS}+2F_\pi^{SH}+O(\alpha_s^2), \tag{8}$$

where F_π^{SS} is a truly nonperturbative part of the form factor, and F_π^{SH} is an $O(\alpha_s)$ term with one-gluon exchange. The first term dominates the pion form factor at small Q^2. The second term gives a minor contribution at small Q^2 but provides the leading $\alpha_s(Q^2)/Q^2$ behavior of the elastic form factor at asymptotically large Q^2. So, the truly nonperturbative and $O(\alpha_s)$ terms accumulate the leading behavior in the two limits of small and large Q^2 and are expected to provide a realistic description also in the region of intermediate momentum transfers.

The hard component Ψ_H^π is represented as a convolution of the one-gluon exchange kernel V^{α_s} with Ψ_S:

$$\Psi_H^\pi=V^{\alpha_s}\otimes\Psi_S^\pi. \tag{9}$$

Thus, the soft wave function Ψ_S^π is responsible for the pion form factor behavior both at small and moderately large Q^2.

The value of s_0 and the pion soft wave function have been variational parameters of our consideration. From the numerical analysis we have found $s_0=9$ GeV^2 to provide the best description of the data on the pion elastic form factor. This value corresponds to an extended soft region and thus we relate a large portion of the pion form factor to the soft contribution. Although particular values of the soft and hard contributions to the form factor are model-dependent quantities, we found that a good description of the form factor at small Q^2 yields a substantial soft contribution to the form factor at $Q^2\simeq 10-20$ GeV^2.

In this paper a similar strategy is applied to the description of the photon-pion transition form factor. Namely, we introduce the photon $q\bar{q}$ wave function and decompose it into the soft and hard components as

$$\Psi^\gamma=\Psi_S^\gamma\theta(s_0-s)+\Psi_H^\gamma\theta(s-s_0). \tag{10}$$

The soft component describes a hadronic $q\bar{q}$ structure of the soft photon just in the spirit of vector meson dominance and can be expected to have the same structure as the soft wave function of a meson.

However, the hard component of the photon wave function has an important distinction compared with a hadron case: in addition to the perturbative tail of the soft part of the wave function of Eq. (9), the hard component of the photon wave function contains also a standard pointlike QED $q\bar{q}$ component such that

$$\Psi_H^\gamma=V^{\alpha_s}\otimes\Psi_S^\gamma+\Psi_{PT}^\gamma. \tag{11}$$

The corresponding expansion of the photon-meson transition form factor is described by the diagrams of Fig. 1 and has the form

$$F_{\gamma\pi}=F^{SS}+F^{SPT}+F^{SH(1)}+F^{SH(2)}. \tag{12}$$

At small Q^2, the F^{SS} part dominates the transition form factor. At large Q^2, the soft-point term F^{SPT} gives the leading $1/Q^2$ falloff whereas the contribution of the $O(\alpha_s)$ terms $F^{SH(1)}+F^{SH(2)}$ is suppressed by the additional factor α_s: the behavior of the photon-pion transition form factor differs from that of the elastic pion form factor in which case the soft-point term is absent and the soft-hard terms dominate in the asymptotic region.

At intermediate momentum transfers, Eq. (12) provides substantial corrections to the $1/Q^2$ falloff. These corrections are due to the transverse motion of quarks in the soft-point term as well as contributions related to a nontrivial $q\bar{q}$ structure of the soft photon (the terms involving Ψ_S^γ).

Allowing for the transverse motion is a standard procedure in moving from asymptotically large to intermediate momentum transfers within the modified hard scattering approach [12,13] and the hard scattering approach with transverse motion taken into account [14]. These considerations describe the amplitude of the process with a triangle graph with pointlike $q\bar{q}\gamma$ vertices. However, the quark between the photon vertices is an off-shell particle with the virtuality k^2 and thus one should take into account the off-shell quark form factors $f_c(m^2,k^2,Q^2)$ in both photon vertices. The off-shell quark form factor is a complicated quantity [in particular, $f_c(m^2,k^2,0)\neq 1$] and cannot be neglected. This situation is quite different from calculating the $\pi\to 2\gamma$ transition form factor through the axial anomaly in which case the anomaly comes from the lowest order triangle diagram with all pointlike vertices and is not renormalized by higher order corrections, see discussion in Ref. [15]. Using the light-cone technique, we do not face the problem of the off-shell form factor because the intermediate-state spectral density is calculated by placing intermediate particles on mass shell (see Ref. [1] for details). But in the light-cone technique we have to allow a nontrivial soft $q\bar{q}$ wave function of the photon which becomes a parameter of the consideration.

The proposed method allows a self-consistent description of the photon-pion transition form factor in a broad range of Q^2. We use the soft pion wave function which has been determined in Ref. [1] by fitting the elastic pion form factor. The description of the photon-pion transition form factor reveals a similarity of the low-s $q\bar{q}$ structure of soft photon and the pion.

Summing up, we obtain the following results.

(1) We determine the soft $q\bar{q}$ wave function of the photon by fitting the available data on the photon-pion transition form factor at $Q^2=0-8$ GeV^2. The pion wave function is taken from the description of elastic pion form factor [1]. The soft photon wave function turns out to be close to the ground-state meson wave function at low s, i.e., $\Psi_S^{\gamma}(s)\sim\Psi_S^{\pi}(s)$ at $s\leqslant 2$ GeV^2. The similarity of Ψ_S^{γ} and Ψ_S^{π} at $s\leqslant 2$ GeV^2 seems to be quite natural and corresponds to the vector meson dominance in the vertex $\gamma\to q\bar{q}$. At $s=s_0$ the photon wave function satisfies the boundary condition $\Psi_S^{\gamma}(s_0)=\Psi_{\rm PT}^{\gamma}(s_0)$, which provides a correct sewing of the $F^{\rm SS}$ and $F^{\rm SPT}$ terms. The soft wave functions determined, we calculate the photon-pion transition form factor in a broad range of Q^2. Calculations show that several kinematical regions with different dominant contributions may be isolated: (i) At small $Q^2=0-5$ GeV^2 the transition form factor is dominated by the soft-soft term which corresponds to a nontrivial hadronic structure of the soft photon; (ii) at large $Q^2\geqslant 50$ GeV^2 the QED pointlike component of the photon gives the main contribution reproducing the PQCD result; (iii) in the intermediate region the transition form factor is an interplay of the soft-soft, soft-point, and soft-hard contributions. Numerically, the form factor behavior is very close to the interpolation formula proposed by Brodsky and Lepage [4]:

$$F_{\gamma\pi}(Q^2)=\frac{\sqrt{2}f_\pi}{Q^2+4\pi^2 f_\pi^2}. \tag{13}$$

It should be pointed out, that the boundaries of the regions (i) and (ii) depend on the particular model of the wave function decomposition into the soft and the hard components. However, the very existence of such regions is a model-independent result.

(2) We calculate $\gamma\eta$ and $\gamma\eta'$ transition form factors assuming universality of the $\vec{k}$ dependence of wave functions of all the ground-state pseudoscalar mesons where $\vec{k}$ is relative momentum of constituent quarks such that $s=4m^2+4\vec{k}^2$. This assumption is in line with the conventional quark model. In accordance with vector meson dominance, the same ansatz is used for relating the nonstrange and strange components of the soft photon. Then, $\gamma\eta$ and $\gamma\eta'$ form factors are calculated with no free parameters. The results for the $\gamma\eta$ and $\gamma\eta'$ transition form factors are in excellent agreement with the experimental data both on the shape of the form factors and on the decay partial widths, $\Gamma(\eta\to\gamma\gamma)$ and $\Gamma(\eta'\to\gamma\gamma)$. It provides an argument for small admixture of a glueball (two-gluon) component into η and η': within the experimental accuracy we estimate corresponding probabilities as W_η(glueball)$<10\%$ and $W_{\eta'}$(glueball)$<20\%$.

The paper is organized as follows. In Sec. II the calculation of the $\gamma\pi^0$ transition form factor is performed and the soft transition vertex $\gamma\to q\bar{q}$ is reconstructed. Section III is devoted to the calculations of $\gamma\eta$ and $\gamma\eta'$ transition form factors. Conclusive remarks are given in Sec. IV. The Appendix presents calculation details.

II. $\gamma\pi^0$ TRANSITION FORM FACTOR

We consider the $\gamma\pi^0$ transition form factor using the method presented in detail in Ref. [1]. So, we omit here a discussion of the basic points of the technique, only outlining the calculation procedure.

The form factor $F_{\gamma\pi}$ is connected with the amplitude of the process $\gamma^*\gamma\to\pi^0$ as

$$T_{\mu\nu}^{\gamma^*\gamma\pi}(q^2)=e^2\varepsilon_{\mu\nu\alpha\beta}q^\alpha P^\beta F_{\gamma\pi}(-q^2), \tag{14}$$

where P is the pion momentum. The partial width of the decay $\pi^0\to\gamma\gamma$ reads

$$\Gamma(\pi^0\to\gamma\gamma)=\frac{\pi}{4}\alpha^2 m_\pi^3 F_{\gamma\pi}^2(0), \quad \alpha=1/137. \tag{15}$$

A. Soft-soft term $F_{\gamma\pi}^{\rm SS}(Q^2)$

The soft-soft contribution $F_{\gamma\pi}^{\rm SS}(Q^2)$ corresponds to the diagram of Fig. 1(a). The $q\bar{q}$ structure of the pion and the photon in the soft region is described by the vertices

$$\frac{\overline{qi}\gamma_5 q}{\sqrt{N_c}}G_\pi(s) \tag{16}$$

and

$$e_q\overline{q}\gamma_\nu q G_\gamma(s'), \tag{17}$$

respectively. The contribution of the diagram Fig. 1(a) can be written as the double spectral representation

$$F^{\rm SS}_{\gamma\pi}(Q^2)=Z_\pi 2 f_q(Q^2)\sqrt{N_c}\int \frac{ds G_\pi(s)\,\theta(4m^2<s<s_0)}{\pi(s-m_\pi^2)}\frac{ds' G_\gamma(s')\,\theta(4m^2<s'<s_0)}{\pi s'}\Delta_{\pi\gamma^*\gamma}(s,s',Q^2). \quad (18)$$

Here, $f_q(Q^2)$ is the quark form factor; Z_π is the charge factor, $Z_\pi=(e_u^2-e_d^2)/\sqrt{2}$, with $e_u=2/3$ and $e_d=-1/3$; and m is the nonstrange quark mass.

The quantity $\Delta_{\pi\gamma^*\gamma}(s,s',Q^2)$ is the double spectral density of the Feynman diagram of Fig. 1(a) with pointlike vertices and the off-shell pion and photon momenta $\tilde{q}=\tilde{P}'-\tilde{P}$, $\tilde{P}^2=s$, $\tilde{P}'^2=s'$, $\tilde{q}^2=q^2$:

$$\varepsilon_{\mu\nu\alpha\beta}\tilde{q}^\alpha\tilde{P}^\beta\Delta_{\pi\gamma^*\gamma}(s,s',Q^2)=-\mathrm{disc}_s\mathrm{disc}_{s'}\int\frac{d^4k_2}{i(2\pi)^4}\frac{Sp(i\gamma_5(m-\hat{k}_2)\gamma_\nu(m+\hat{k}_1')\gamma_\mu(m+\hat{k}_1))}{(m^2-k_1^2)(m^2-k_2^2)(m^2-k_1'^2)},\quad k_1+k_2=\tilde{P},\quad k_1'+k_2=\tilde{P}'. \quad (19)$$

The trace reads

$$Sp(i\gamma_5(m-\hat{k}_2)\gamma_\nu(m+\hat{k}_1')\gamma_\mu(m+\hat{k}_1))=-4m\varepsilon_{\mu\nu\alpha\beta}\tilde{q}^\alpha\tilde{P}^\beta. \quad (20)$$

Then, one finds

$$\Delta_{\pi\gamma^*\gamma}(s,s',Q^2)=\frac{m}{4}\frac{\theta(s's Q^2-m^2\lambda(s,s',Q^2))}{\lambda^{1/2}(s,s',Q^2)}, \quad (21)$$

with $\lambda(s,s',Q^2)=(s'-s)^2+2Q^2(s'+s)+Q^4$.

Introducing the light-cone variables, $x=k_{2+}/P_+$, $k_\perp=k_{2\perp}$, one finds, in the reference frame $q_+=0$, $P_\perp=0$ ($q_\perp^2=Q^2$),

$$\Delta_{\pi\gamma^*\gamma}(s,s',Q^2)=\frac{m}{4\pi}\int\frac{dx d^2k_\perp}{x(1-x)^2}\delta\left(s-\frac{m^2+k_\perp^2}{x(1-x)}\right)\times\delta\left(s'-\frac{m^2+(k_\perp-xq_\perp)^2}{x(1-x)}\right). \quad (22)$$

In terms of these variables the soft-soft form factor takes the form

$$F^{\rm SS}_{\gamma\pi}(Q^2)=2Z_\pi f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int\frac{dx d^2k_\perp}{x(1-x)^2}\Psi_\pi(s)\times\theta(s_0-s)\frac{G_\gamma(s')}{s'}\theta(s_0-s'), \quad (23)$$

with

$$s=\frac{m^2+k_\perp^2}{x(1-x)},\quad s'=\frac{m^2+(k_\perp-xq_\perp)^2}{x(1-x)}, \quad (24)$$

and

$$\Psi_\pi(s)\equiv\frac{G_\pi(s)}{s-m_\pi^2}. \quad (25)$$

The function $\Psi_\pi(s)$ satisfies the normalization condition [1]

$$\frac{1}{8\pi^2}\int ds\Psi_\pi^2(s)s\sqrt{1-\frac{4m^2}{s}}=1. \quad (26)$$

The pion leptonic constant is expressed through Ψ_π as

$$\frac{m\kappa\sqrt{N_c}}{4\pi^2}\int ds\Psi_\pi(s)\sqrt{1-\frac{4m^2}{s}}=f_\pi, \quad (27)$$

where κ is the weak decay constant of the constituent quark [1].

At $Q^2=0$ one finds

$$F^{\rm SS}_{\gamma\pi}(0)=Z_\pi\frac{m}{2\pi}\int_{4m^2}^{s_0}\frac{ds}{\pi}\Psi_\pi(s)\frac{G_\gamma(s)}{s}\ln\frac{1+\sqrt{1-4m^2/s}}{1-\sqrt{1-4m^2/s}}. \quad (28)$$

On the other hand, $F^{\rm SS}_{\gamma\pi}(0)$ is connected with the $\pi^0\to\gamma\gamma$ decay width through Eq. (15). We use this equation for fixing an overall magnitude of $G_\gamma(s)$.

$F^{\rm SS}_{\gamma\pi}$ involves the constituent quark form factor for which we use the same prescription as in Ref. [1]:

$$f_q(Q^2)=\begin{cases}1, & Q<Q_0,\\ S(Q^2), & Q>Q_0,\end{cases} \quad (29)$$

where $Q_0=1$ GeV and $S(Q^2)$ is the Sudakov factor taken in the form [16]

$$S(Q^2)=\exp\left[-\frac{\alpha_s}{2\pi}C_F\ln^2\left(\frac{Q^2}{Q_0^2}\right)\right],\quad C_F=\frac{N_c^2-1}{2N_c}. \quad (30)$$

The choice of the infrared regulator $Q_0\simeq 1$ GeV in the Sudakov form factor is motivated in Ref. [10].

The coupling constant $\alpha_s(Q^2)$ is assumed to be frozen below 1 GeV [17]:

$$\alpha_s(Q^2)=\begin{cases}\text{const}, & Q<1\text{ GeV},\\ \frac{4\pi}{9}\ln^{-1}\left(\frac{Q^2}{\Lambda^2}\right), & Q>1\text{ GeV},\end{cases} \quad (31)$$

where $\Lambda=220$ MeV. This corresponds to freezing the coupling constant at the scale $\alpha_s/\pi\simeq 0.15$ which is a reasonable value for constituent quark models.

B. Soft-point term $F^{\rm SPT}_{\gamma\pi}(Q^2)$

The contribution of the diagram of Fig. 1(b) with $s'\geqslant s_0$ is denoted as $F^{\rm SPT}_{\gamma\pi}(Q^2)$ and reads

$$F^{\rm SPT}_{\gamma\pi}(Q^2)=2Z_\pi f_q(Q^2)\sqrt{N_c}\frac{m}{4}\int\frac{ds}{\pi}\frac{ds'}{\pi s'}\times\Psi_\pi(s)\frac{\theta(s'sQ^2-m^2\lambda(s,s',Q^2))}{\lambda^{1/2}(s,s',Q^2)}\times\theta(4m^2\leqslant s\leqslant s_0)\,\theta(s_0\leqslant s'). \quad (32)$$

The pointlike $q\bar{q}$ component of the photon at large s corresponds to $G_\gamma\equiv 1$ for $s\geqslant s_0$. Hence, the continuity of G_γ requires

$$G_\gamma(s_0)=1. \quad (33)$$

Notice that $F^{\rm SPT}_{\gamma\pi}(0)=0$ as follows from the constraints $s\leqslant s_0$ and $s'\geqslant s_0$.

In terms of the light-cone variables, $F^{\rm SPT}_{\gamma\pi}(Q^2)$ reads

$$F^{\rm SPT}_{\gamma\pi}(Q^2)=2Z_\pi f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int\frac{dx\,d^2k_\perp}{x(1-x)^2}\Psi_\pi(s)\frac{1}{s'}\times\theta(s_0-s)\,\theta(s'-s_0), \quad (34)$$

with s and s' from Eq. (24).

C. Soft-hard term $F^{\rm SH(1)}_{\gamma\pi}(Q^2)$

The soft-hard term $F^{\rm SH(1)}_{\gamma\pi}(Q^2)$ is given by the diagram of Fig. 1(c) which can be written as the dispersion representation

$$T^{\rm SH(1)}_{\mu\nu}=2e^2Z_\pi\sqrt{N_c}\int\frac{ds\,ds'\,ds''}{\pi\pi\pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''}\times D^{(1)}_{\mu\nu}(\tilde{P},\tilde{P}',\tilde{P}'')\,\theta(s_0-s)\,\theta(s_0-s')\,\theta(s''-s_0), \quad (35)$$

where $\tilde{P}^2=s$, $\tilde{P}'^2=s'$, $\tilde{P}''^2=s''$, and $\tilde{q}=\tilde{P}''-\tilde{P}$, $\tilde{\delta}=\tilde{P}''-\tilde{P}'$ such that $\tilde{q}^2=-Q^2$, $\tilde{\delta}^2=0$, $\tilde{\delta}\tilde{q}=0$. In this expression $D^{(1)}_{\mu\nu}$ is the spectral density of the diagram of Fig. 1(c):

$$D^{(1)}_{\mu\nu}(\tilde{P},\tilde{P}',\tilde{P}'')=\frac{C_F}{64\pi^3}\int d^4k_2\,d^4k_2'\frac{4\pi\alpha_s(t)}{m_G^2-t}Sp^{(1)}_{\mu\nu}\times\delta(m^2-k_1^2)\,\delta(m^2-k_2^2)\,\delta(m^2-k_1''^2)\times\delta(m^2-k_1'^2)\,\delta(m^2-k_2'^2),$$

$$k_1=\tilde{P}-k_2,\quad k_1''=\tilde{P}''-k_2,$$

$$k_1'=\tilde{P}'-k_2',\quad t=(k_2'-k_2)^2. \quad (36)$$

Here,

$$Sp^{(1)}_{\mu\nu}\equiv Sp[i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}_2')\gamma_\nu(m+\hat{k}_1')\times\gamma_\alpha(m+\hat{k}_1'')\gamma_\mu(m+\hat{k}_1)]=\varepsilon_{\mu\nu\alpha\beta}\tilde{q}^\alpha\tilde{P}^\beta\cdot S(1), \quad (37)$$

with

$$S(1)=4m[s'-12b'+4b''+(s''-s+Q^2)(a_1'-a_2')]. \quad (38)$$

Details of the calculation and the analytic expressions for a_1',a_2',b',b'' are given in the Appendix. It should be pointed out that although $D^{(1)}_{\mu\nu}$ from Eq. (36) is an infrared-safe quantity, we introduce an effective gluon mass m_G which is not a regulator but the quantity relevant for the description of the gluon of a small virtuality [18]. However, numerically, the introduction of the soft gluon mass provides minor changes in the results.

Thus, one obtains

$$D^{(1)}_{\mu\nu}(\tilde{P},\tilde{P}',\tilde{P}'')=\varepsilon_{\mu\nu\alpha\beta}\tilde{q}^\alpha\tilde{P}^\beta\cdot\Delta^{(1)}(s,s',s'',Q^2),$$

where

$$\Delta^{(1)}(s,s',s'',Q^2)=\frac{C_F}{64\pi^3}\int d^4k_2\,d^4k_2'\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1)\times\delta(m^2-k_1^2)\,\delta(m^2-k_2^2)\,\delta(m^2-k_1''^2)\times\delta(m^2-k_1'^2)\,\delta(m^2-k_2'^2). \quad (39)$$

The dispersion representation for the soft-hard form factor $F^{\rm SH(1)}_{\gamma\pi}(Q^2)$ takes the form

$$F^{\rm SH(1)}_{\gamma\pi}(Q^2)=2Z_\pi\sqrt{N_c}\int\frac{ds\,ds'\,ds''}{\pi\pi\pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''}\times\Delta^{(1)}(s,s',s'',Q^2)\,\theta(s_0-s)\,\theta(s_0-s')\,\theta(s''-s_0). \quad (40)$$

Introducing the light-cone variables, $x=k_{2+}/P_+$, $k_\perp=k_{2\perp}$, $x'=k'_{2+}/P_+$, $k'_\perp=\vec{k}'_{2\perp}$ (for details see the Appendix), one finds

$$\Delta^{(1)}(s,s',s'',Q^2)=\frac{C_F}{256\pi^3}\int\frac{dx\,d^2k_\perp}{x(1-x)^2}\times\frac{dx'\,d^2k'_\perp}{x'(1-x')}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1)\times\delta\left(s'-\frac{m^2+(k'_\perp-x'k_\perp)^2}{x'(1-x')}\right)\times\delta\left(s-\frac{m^2+k_\perp^2}{x(1-x)}\right)\times\delta\left(s''-\frac{m^2+(k_\perp-xq_\perp)^2}{x(1-x)}\right). \quad (41)$$

Substituting Eq. (41) into Eq. (40) yields the expression of $F^{\rm SH(1)}_{\gamma\pi}(Q^2)$ in terms of the light-cone variables:

$$F^{\rm SH(1)}_{\gamma\pi}(Q^2)=\frac{2Z_\pi C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dx\,d^2k_\perp}{x(1-x)^2}\frac{dx'\,d^2k'_\perp}{x'(1-x')}\times\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1)\times\theta(s_0-s)\,\theta(s_0-s')\,\theta(s''-s_0), \quad (42)$$

where

$$s=\frac{m^2+k_\perp^2}{x(1-x)},\quad s'=\frac{m^2+(k'_\perp-x'q_\perp)^2}{x'(1-x')},$$

$$s''=\frac{m^2+(k_\perp-xq_\perp)^2}{x(1-x)},\tag{43}$$

$$t=-\frac{m^2(x'-x)^2+(xk'_\perp-x'k_\perp)^2}{x'x}.\tag{44}$$

With the light-cone expressions for a'_1,a'_2,b',b'' [see Eq. (A12)] the function $S(1)$ takes the form

$$S(1)=4m\left[s'-12\left(\frac{(k_\perp q_\perp)(k'_\perp q_\perp)}{Q^2}-(k_\perp k'_\perp)\right)+4\left(\frac{(k'_\perp q_\perp)^2}{Q^2}-k'^2_\perp\right)-(s''-s+Q^2)\frac{(k'_\perp q_\perp)}{Q^2}\right].\tag{45}$$

D. Soft-hard term $F_{\gamma\pi}^{\mathrm{SH}(2)}(Q^2)$

The calculation of the soft-hard term $F_{\gamma\pi}^{\mathrm{SH}(2)}(Q^2)$ looks very much like the calculation of $F_{\gamma\pi}^{\mathrm{SH}(1)}(Q^2)$, namely, $F_{\gamma\pi}^{\mathrm{SH}(2)}(Q^2)$ is given by the diagram Fig. 1(d) which can be written as the spectral representation

$$T_{\mu\nu}^{\mathrm{SH}(2)}=2e^2Z_\pi\sqrt{N_c}\int\frac{ds\,ds'\,ds''}{\pi\pi\pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''-m_\pi^2}\times D_{\mu\nu}^{(2)}(\tilde{P},\tilde{P}',\tilde{P}'')\,\theta(s_0-s)\,\theta(s_0-s')\,\theta(s''-s_0),\tag{46}$$

with $\tilde{P}^2=s$, $\tilde{P}'^2=s'$, $\tilde{P}''^2=s''$, and $\tilde{q}=\tilde{P}'-\tilde{P}''$, $\tilde{\delta}=\tilde{P}-\tilde{P}''$ such that $\tilde{q}^2=-Q^2$, $\tilde{\delta}^2=0$, $\tilde{\delta}\tilde{q}=0$. The quantity $D_{\mu\nu}^{(2)}$ is the spectral density of the diagram of Fig. 1(d)

$$D_{\mu\nu}^{(2)}(\tilde{P},\tilde{P}',\tilde{P}'')=\frac{C_F}{64\pi^3}\int d^4k_2d^4k'_2\frac{4\pi\alpha_s(t)}{m_G^2-t}Sp_{\mu\nu}^{(2)}\times\delta(m^2-k_1^2)\,\delta(m^2-k_2^2)\,\delta(m^2-k_1''^2)\times\delta(m^2-k_1'^2)\,\delta(m^2-k_2'^2),$$

$$k_1=\tilde{P}-k_2,\quad k''_1=\tilde{P}''-k'_2,\quad k'_1=\tilde{P}'-k'_2,\tag{47}$$

with

$$Sp_{\mu\nu}^{(2)}\equiv Sp[i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\times\gamma_\mu(m+\hat{k}''_1)\gamma_\alpha(m+\hat{k}_1)]=\varepsilon_{\mu\nu\alpha\beta}\tilde{q}^\alpha\tilde{P}^\beta\cdot S(2),\tag{48}$$

$$S(2)=8m[s-(a'_1-a'_2)(k''_1P)-a'_2(k'_1k'_2)-2b''-m^2(1+a'_2)].\tag{49}$$

Details of the calculation and the expressions for a'_1,a'_2,b',b'' are given in the Appendix. So,

$$D_{\mu\nu}^{(2)}(\tilde{P},\tilde{P}',\tilde{P}'')=\varepsilon_{\mu\nu\alpha\beta}\tilde{q}^\alpha\tilde{P}^\beta\cdot\Delta^{(2)}(s,s',s'',Q^2),$$

$$\Delta^{(2)}(s,s',s'',Q^2)=\frac{C_F}{64\pi^3}\int d^4k_2d^4k'_2\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2)\times\delta(m^2-k_1^2)\,\delta(m^2-k_2^2)\,\delta(m^2-k_1''^2)\times\delta(m^2-k_1'^2)\,\delta(m^2-k_2'^2).\tag{50}$$

Finally, the soft-hard form factor $F_{\gamma\pi}^{\mathrm{SH}(2)}(Q^2)$ takes the form

$$F_{\gamma\pi}^{\mathrm{SH}(2)}(Q^2)=2Z_\pi\sqrt{N_c}\int\frac{ds\,ds'\,ds''}{\pi\pi\pi}\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''-m_\pi^2}\times\Delta^{(2)}(s,s',s'',Q^2)\,\theta(s_0-s)\,\theta(s_0-s')\,\theta(s''-s_0).\tag{51}$$

Introducing the light-cone variables, one obtains

$$\Delta^{(2)}(s,s',s'',Q^2)=\frac{C_F}{256\pi^3}\int\frac{dx\,d^2k_\perp}{x(1-x)}\frac{dx'\,d^2k'_\perp}{x'(1-x')^2}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2)\times\delta\left(s-\frac{m^2+k_\perp^2}{x(1-x)}\right)\delta\left(s''-\frac{m^2+k'^2_\perp}{x'(1-x')}\right)\times\delta\left(s'-\frac{m^2+(k'_\perp-x'q_\perp)^2}{x'(1-x')}\right).\tag{52}$$

Substituting Eq. (52) into Eq. (51) yields

$$F_{\gamma\pi}^{\mathrm{SH}(2)}(Q^2)=\frac{2Z_\pi C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dx\,d^2k_\perp}{x(1-x)}\frac{dx'\,d^2k'_\perp}{x'(1-x')^2}\times\Psi_\pi(s)\frac{G_\gamma(s')}{s'}\frac{1}{s''-m_\pi^2}\frac{4\pi\alpha_s(t)}{m_G^2-t}\times S(2)\,\theta(s_0-s)\,\theta(s_0-s')\,\theta(s''-s_0),\tag{53}$$

where

$$s=\frac{m^2+k_\perp^2}{x(1-x)},\quad s'=\frac{m^2+(k'_\perp-x'q_\perp)^2}{x'(1-x')},\quad s''=\frac{m^2+k'^2_\perp}{x'(1-x')}.\tag{54}$$

Using the light-cone expressions for a'_1,a'_2,b',b'' from Eq. (A12), one finds

$$S(2)=8m\left[s+\frac{(k'_\perp q_\perp)}{Q^2}(k''_1P)-\left(x'+\frac{(k'_\perp q_\perp)}{Q^2}\right)(k'_1k'_2)-2\left(\frac{(k'_\perp q_\perp)^2}{Q^2}-k'^2_\perp\right)-m^2\left(1+x'+\frac{(k'_\perp q_\perp)}{Q^2}\right)\right].\tag{55}$$

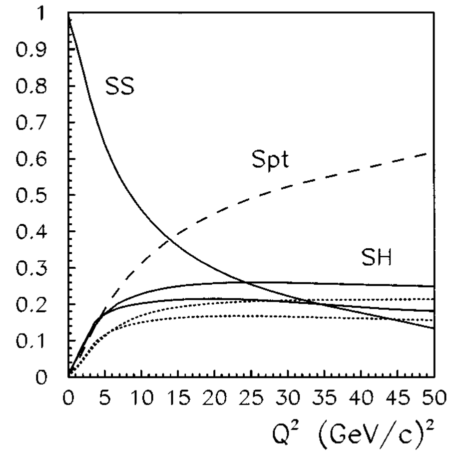

FIG. 2. Relative contributions of different terms to the form factor: the ratios of different terms contributing to the whole form factor are shown. Four variants for the SH term correspond to: (1) $m_G=0$, $\alpha_s(t)$, the upper solid curve; (2) $m_G=0$, $\alpha_s(Q^2/4)$, the lower solid curve; (3) $m_G=m_G(t)$, $\alpha_s(t)$, the upper dotted curve; (4) $m_G=m_G(t)$, $\alpha_s(Q^2/4)$, the lower dotted curve.

E. Calculation results

For calculating the form factors at low and intermediate Q^2 one needs masses which are relevant for the soft physics: constituent quark mass and effective gluon mass. For the constituent quark mass we use the value $m=0.35$ GeV. For the effective gluon mass $m_G(t)$ we consider, just as in Ref. [1], the following two variants (masses are given in GeV):

$$\text{(i)}\quad m_G=0,$$

$$\text{(ii)}\quad m_G=\begin{cases} 0.7\left(1+\dfrac{t}{0.5}\right), & |t|<0.5\quad \text{GeV}^2,\\ 0, & |t|>0.5\quad \text{GeV}^2.\end{cases} \tag{56}$$

In the second variant the effective gluon mass depends on its virtuality, namely, $M_G(t=0)=0.7\simeq M_{\rm glueball}/2$ [18], above $t\simeq 0.5$ GeV^2 the gluon is massless, and in the region $0\leqslant t\leqslant 0.5$ GeV^2 a simple linear interpolation is used.

For $\alpha_s(t)$ we use the two options

$$\text{(i)}\quad \alpha_s(t)=\begin{cases} \text{const}, & |t|<1\,\text{GeV}^2,\\ \dfrac{4\pi}{9}\ln^{-1}\left(\dfrac{|t|}{\Lambda^2}\right), & |t|>1\,\text{GeV}^2,\end{cases}$$

$$\text{(ii)}\quad \alpha_s(t)=\alpha_s\left(\frac{Q^2}{4}\right). \tag{57}$$

The first choice corresponds to freezing $\alpha_s(t)$ at low t just as in Eqs. (29). The second choice takes $\alpha_s(t)$ in the middle point, $|t|=Q^2/4$. So, Eqs. (56) and (57) provide four possibilities for the soft-hard form factors. The corresponding results for the soft-hard term are shown in Fig. 2. One can see that a particular choice of m_G and α_s in the soft region does not influence the result significantly.

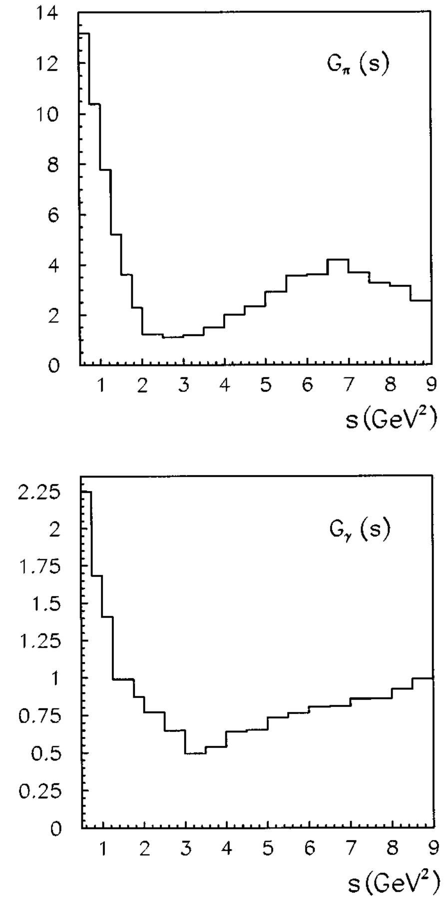

FIG. 3. $q\bar{q}$-distribution function for pion (a) and for photon (b).

Much more important is the shape of the soft photon wave function. There are two constraints on the soft photon wave function: the condition (28) which guarantees the correct decay width $\pi^0\to\gamma\gamma$, and the continuity condition (33). In addition, we impose the third constraint in the spirit of vector dominance model and SU(6) symmetry: the slope of $G_\gamma(s)$ in the region $4m^2<s<1.5$ GeV^2 is the same as that of $G_\pi(s)$. Using these conditions we reconstruct the photon $q\bar{q}$-distribution function $G_\gamma(s)$ which is shown in Fig. 3, together with $G_\pi(s)$ for comparison. The reconstructed distribution function has a dip in the region $s\sim 2-4$ GeV^2 similar to that of $G_\pi(s)$. In Ref. [1] we termed the $q\bar{q}$ distribution of such type a quasizone structure, as this distribution looks like a smeared zone structure in solid state: there is a forbidden zone (the dip in the region of $s\sim 2-4$ GeV^2) where the probability of finding the $q\bar{q}$ pair is low and an allowed zone above the forbidden one (the second bump in G). We could not propose an explanation of this fact but only observed it.

The reconstructed $G_\gamma(s)$ behavior provides an argument that the quasizone structure in the $q\bar{q}$ distributions is a general property. The quantity $(\pi/4)\alpha^2 m_\pi^3 F_{\gamma\pi}^2(Q^2)$ with the reconstructed $G_\gamma(s)$ is presented in Fig. 4. We tried other shapes of G_γ but did not succeed in describing the data. To illustrate the form factor sensitivity to the photon $q\bar{q}$-distribution function we present $F_{\gamma\pi}(Q^2)$ calculated with the following alternative $G_\gamma(s)$:

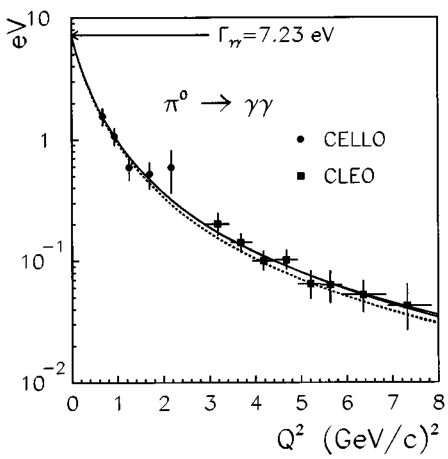

FIG. 4. $\gamma\pi^0$ transition form factor: the quantity $(\pi/4)\alpha^2 m_\pi^3 F_{\gamma\pi}^2(Q^2)$ is shown. Different curves correspond to different sets of parameters in the SH term; the curve notation is the same as in Fig. 2. Experimental data are taken from Ref. [2].

$$(1)\quad G_\gamma^{(1)}(s)=\begin{cases}0.98G_\gamma(s), & s\leqslant 1.5\ \text{GeV}^2,\\ 1, & s>1.5\ \text{GeV}^2,\end{cases}$$

$$(2)\quad G_\gamma^{(2)}(s)=1. \tag{58}$$

The results are shown in Fig. 5. The variant (1) corresponds to the low-s $q\bar{q}$ distribution similar to $G_\pi(s)$ but without a dip at $s\sim 3$ GeV^2 (the factor 0.98 is due to renormalization of the $q\bar{q}$ distribution to have $\Gamma_{\gamma\gamma}=7.23$ eV). The absence of a dip in the photon $q\bar{q}$ distribution results in raising the calculated curve at large Q^2. The variant (2), which corresponds to the pointlike vertex $q\bar{q}\to\gamma$ at low s, reproduces neither $\Gamma_{\gamma\gamma}$ nor low-Q^2 form factor correctly. It should be noticed that the value of $\Gamma_{\pi\to\gamma\gamma}$ strongly depends on the constituent quark mass, namely, using the value $m=0.22$ GeV one can easily find the pion wave function to reproduce the correct decay width with pointlike vertex $G_\gamma(s)\equiv 1$ at all s [19]. At the same time such a description provides the correct large Q^2 asymptotic behavior of $F_{\pi\gamma}$. Taking into account that the latter is a smooth function, the description at intermediate Q^2 will be also good. However, real problem arises when one is trying to describe both F_π and $F_{\pi\gamma}$ simultaneously with the same parameters. Our numerical analysis of both the elastic pion and the transition pion-photon form factors suggests an extended soft region with $s_0\simeq 8-9$ GeV^2 and the photon $q\bar{q}$ distribution with a dip in the region $s\simeq 2-4$ GeV^2 just as in the pion case.

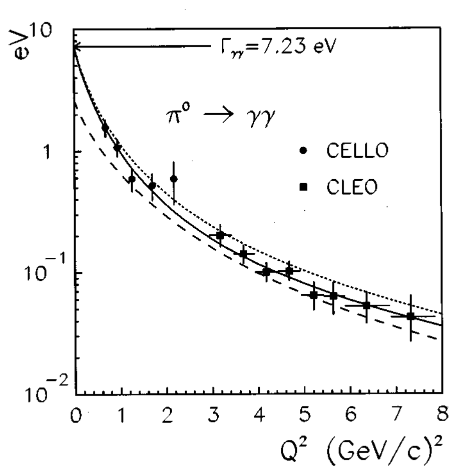

FIG. 5. The quantity $(\pi/4)\alpha^2 m_\pi^3 F_{\gamma\pi}^2(Q^2)$ for the variant (1) of Eq. (58), dotted line, and for the variant (2), dashed line. The solid line is the same as in Fig. 4.

III. $\gamma\eta$ AND $\gamma\eta'$ TRANSITION FORM FACTORS

For the $\gamma\eta$ and $\gamma\eta'$ transition form factors we assume, in the spirit of the quark model, the universality of soft wave functions of the 0^- nonet. This universality is usually formulated in the r representation, or in terms of the relative quark momenta. So, let us rewrite the pion wave function $\Psi_\pi(s)$ of Eq. (25) as a function of $\vec{k}$ which is connected with the energy squared by $\vec{k}^2=(s-4m^2)/4$.

In terms of the relative momenta, the pion wave function has the form

$$\Psi_\pi(s)=\psi_\pi(\vec{k}^2)=\frac{g(\vec{k}^2)}{\vec{k}^2+\kappa_0^2}, \tag{59}$$

where according to Ref. [1] $\kappa_0^2=0.1176$ GeV^2.

The pseudoscalar mesons η and η' are mixtures of the nonstrange and strange components, $n\bar{n}=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $s\bar{s}$: $\eta=n\bar{n}\cos\theta-s\bar{s}\sin\theta$, $\eta'=n\bar{n}\sin\theta+s\bar{s}\cos\theta$. Respectively, the wave functions of the η and η' mesons are described by the two components

$$\Psi_\eta=\cos\theta\psi_{n\bar{n}}(\vec{k}^2)-\sin\theta\psi_{s\bar{s}}(\vec{k}^2),$$

$$\Psi_{\eta'}=\sin\theta\psi_{n\bar{n}}(\vec{k}^2)+\cos\theta\psi_{s\bar{s}}(\vec{k}^2). \tag{60}$$

The universality of the pseudoscalar meson wave function suggests that the function

$$\psi_{n\bar{n}}(\vec{k}^2)=\frac{g(\vec{k}^2)}{\vec{k}^2+\kappa_0^2}, \tag{61}$$

normalized as

$$\frac{2}{\pi^2}\int_0^{\vec{k}_0^2} d\vec{k}^2|\vec{k}|\sqrt{\vec{k}^2+m^2}\,\psi_{n\bar{n}}^2(\vec{k}^2)=1,\quad \vec{k}_0^2=\frac{s_0}{4}-m^2, \tag{62}$$

is just the same function as for the π meson.

For the $s\bar{s}$ component we should take into account the SU(3)-breaking effects which are described mainly by the strange or nonstrange quark mass difference; hence,

$$\psi_{s\bar{s}}(\vec{k}^2)=N\frac{g(\vec{k}^2)}{\vec{k}^2+\kappa_0^2+\Delta^2}, \tag{63}$$

where $\Delta^2\equiv m_s^2-m^2$, $m_s=0.5$ GeV and $\vec{k}^2=(s-4m_s^2)/4$. The factor N corresponds to the renormalization of $\psi_{s\bar{s}}(\vec{k}^2)$ after introducing Δ, i.e., replacing $m\to m_s$. The strange component is normalized as

$$\frac{2}{\pi^2}\int_0^{\vec{k}_0^2} d\vec{k}^2 |\vec{k}| \sqrt{\vec{k}^2+m_s^2}\,\psi_{s\bar{s}}^2(\vec{k}^2)=1. \tag{64}$$

Similarly, for the momentum representation of the soft photon wave function we use the expressions

$$\Psi_\gamma(s)\equiv\psi_{\gamma\to n\bar{n}}(\vec{k}^2)=\frac{g_\gamma(\vec{k}^2)}{\vec{k}^2+m^2} \tag{65}$$

and

$$\psi_{\gamma\to s\bar{s}}(\vec{k}^2)=\frac{g_\gamma(\vec{k}^2)}{\vec{k}^2+m^2+\Delta^2}. \tag{66}$$

The $n\bar{n}$ and $s\bar{s}$ components of the meson and soft photon wave function fixed, one can proceed with calculations of the transition $\gamma\eta$ and $\gamma\eta'$ form factors, which read

$$F_{\gamma\eta}(Q^2)=\cos\theta F_{n\bar{n}}(Q^2)-\sin\theta F_{s\bar{s}}(Q^2),$$

$$F_{\gamma\eta'}(Q^2)=\sin\theta F_{n\bar{n}}(Q^2)+\cos\theta F_{s\bar{s}}(Q^2). \tag{67}$$

The following two subsections present a summary of formulas for $F_{n\bar{n}}(Q^2)$ and $F_{s\bar{s}}(Q^2)$.

A. The $n\bar{n}$ contributions to the form factor

The $n\bar{n}$ contributions can be obtained from the corresponding terms in $F_{\gamma\pi}(Q^2)$ by replacing the charge factor and the wave functions. The final results are listed below.

The soft-soft term $F_{n\bar{n}}^{\rm SS}(Q^2)$:

$$F_{n\bar{n}}^{\rm SS}(Q^2)=2Z_{n\bar{n}}f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int\frac{dx d^2k_\perp}{x(1-x)^2}\psi_{n\bar{n}}(\vec{k}^2)$$
$$\times\theta(s_0-s)\psi_{\gamma\to n\bar{n}}(\vec{k}'^2)\theta(s_0-s'), \tag{68}$$

$$Z_{n\bar{n}}=\frac{e_u^2+e_d^2}{\sqrt{2}},\quad \vec{k}^2=\frac{s}{4}-m^2,\quad \vec{k}'^2=\frac{s'}{4}-m^2, \tag{69}$$

$$F_{n\bar{n}}^{\rm SS}(0)=Z_{n\bar{n}}\frac{m}{2\pi}\int_{4m^2}^{s_0}\frac{ds}{\pi}\psi_{n\bar{n}}(\vec{k}^2)\psi_{\gamma\to n\bar{n}}(\vec{k}^2)$$
$$\times\ln\frac{1+\sqrt{1-4m^2/s}}{1-\sqrt{1-4m^2/s}}, \tag{70}$$

s and s' are given by Eq. (24).

The soft-point term $F_{n\bar{n}}^{\rm SPT}(Q^2)$:

$$F_{n\bar{n}}^{\rm SPT}(Q^2)=2Z_{n\bar{n}}f_q(Q^2)\sqrt{N_c}\frac{m}{4\pi^5}\int\frac{dx d^2k_\perp}{x(1-x)^2}$$
$$\times\psi_{n\bar{n}}(\vec{k}^2)\frac{1}{s'}\theta(s_0-s)\theta(s'-s_0). \tag{71}$$

The soft-hard term $F_{n\bar{n}}^{\rm SH(1)}(Q^2)$:

$$F_{n\bar{n}}^{\rm SH(1)}(Q^2)=\frac{2Z_{n\bar{n}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dx d^2k_\perp}{x(1-x)^2}\frac{dx' d^2k'_\perp}{x'(1-x')}$$
$$\times\psi_{n\bar{n}}(\vec{k}^2)\psi_{\gamma\to n\bar{n}}(\vec{k}'^2)\frac{1}{s''}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1)$$
$$\times\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \tag{72}$$

and s, s', and s'' are given by Eq. (43).

The soft-hard term $F_{n\bar{n}}^{\rm SH(2)}(Q^2)$:

$$F_{n\bar{n}}^{\rm SH(2)}(Q^2)=\frac{2Z_{n\bar{n}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dx d^2k_\perp}{x(1-x)}\frac{dx' d^2k'_\perp}{x'(1-x')^2}$$
$$\times\psi_{n\bar{n}}(\vec{k}^2)\psi_{\gamma\to n\bar{n}}(\vec{k}'^2)\frac{1}{s''-m_{\eta,\eta'}^2}$$
$$\times\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2)\theta(s_0-s)$$
$$\times\theta(s_0-s')\theta(s''-s_0), \tag{73}$$

and s, s', and s'' are given by Eq. (54).

B. The $s\bar{s}$ contributions to the form factor

The $s\bar{s}$ expressions are obtained from the corresponding terms in $F_{\gamma\pi}(Q^2)$ by replacing the charge factor, the wave functions, and the quark masses. The results have the following form.

The soft-soft term $F_{s\bar{s}}^{\rm SS}(Q^2)$:

$$F_{s\bar{s}}^{\rm SS}(Q^2)=2Z_{s\bar{s}}f_q(Q^2)\sqrt{N_c}\frac{m_s}{4\pi^5}\int\frac{dx d^2k_\perp}{x(1-x)^2}\psi_{s\bar{s}}(\vec{k}^2)$$
$$\times\theta(s_0-s)\psi_{\gamma\to s\bar{s}}(\vec{k}'^2)\theta(s_0-s'), \tag{74}$$

$$Z_{s\bar{s}}=e_s^2,\quad \vec{k}^2=\frac{s}{4}-m_s^2,\quad \vec{k}'^2=\frac{s'}{4}-m_s^2, \tag{75}$$

$$s=\frac{m_s^2+k_\perp^2}{x(1-x)},\quad s'=\frac{m_s^2+(k_\perp-xq_\perp)^2}{x(1-x)}; \tag{76}$$

$$F_{s\bar{s}}^{\rm SS}(0)=Z_{s\bar{s}}\frac{m_s}{2\pi}\int_{4m_s^2}^{s_0+4\Delta^2}\frac{ds}{\pi}$$
$$\times\psi_{s\bar{s}}(\vec{k}^2)\psi_{\gamma\to s\bar{s}}(\vec{k}^2)\ln\frac{1+\sqrt{1-4m_s^2/s}}{1-\sqrt{1-4m_s^2/s}}. \tag{77}$$

The soft-point term $F_{s\bar{s}}^{\rm SPT}(Q^2)$:

$$F_{s\bar{s}}^{\rm SPT}(Q^2)=2Z_{s\bar{s}}f_q(Q^2)\sqrt{N_c}\frac{m_s}{4\pi^5}\int\frac{dx d^2k_\perp}{x(1-x)^2}$$
$$\times\psi_{s\bar{s}}(\vec{k}^2)\frac{1}{s'}\theta(s_0-s)\theta(s'-s_0). \tag{78}$$

The soft-hard term $F_{s\bar{s}}^{\rm SH(1)}(Q^2)$:

$$F_{s\bar{s}}^{\mathrm{SH}(1)}(Q^2)=\frac{2Z_{s\bar{s}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dxd^2k_\perp}{x(1-x)^2}\frac{dx'd^2k'_\perp}{x'(1-x')}\times\psi_{s\bar{s}}(\vec{k}^2)\psi_{\gamma\to s\bar{s}}(\vec{k}'^2)\frac{1}{s''}\frac{4\pi\alpha_s(t)}{m_G^2-t}S(1)\times\theta(s_0-s)\theta(s_0-s')\theta(s''-s_0), \tag{79}$$

$$s=\frac{m_s^2+k_\perp^2}{x(1-x)},\quad s'=\frac{m_s^2+(k'_\perp-x'q_\perp)^2}{x'(1-x')},$$

$$s''=\frac{m_s^2+(k_\perp-xq_\perp)^2}{x(1-x)}, \tag{80}$$

$$t=-\frac{m_s^2(x'-x)^2+(xk'_\perp-x'k_\perp)^2}{x'x}. \tag{81}$$

The soft-hard term $F_{s\bar{s}}^{\mathrm{SH}(2)}(Q^2)$:

$$F_{s\bar{s}}^{\mathrm{SH}(2)}(Q^2)=\frac{2Z_{s\bar{s}}C_F\sqrt{N_c}}{(16\pi^3)^2}\int\frac{dxd^2k_\perp}{x(1-x)}\frac{dx'd^2k'_\perp}{x'(1-x')^2}\times\psi_{s\bar{s}}(\vec{k}^2)\psi_{\gamma\to s\bar{s}}(\vec{k}'^2)\frac{1}{s''-m^2_{\eta,\eta'}}\times\frac{4\pi\alpha_s(t)}{m_G^2-t}S(2)\theta(s_0-s)\times\theta(s_0-s')\theta(s''-s_0), \tag{82}$$

$$s=\frac{m_s^2+k_\perp^2}{x(1-x)},\quad s'=\frac{m_s^2+(k'_\perp-x'q_\perp)^2}{x'(1-x')},\quad s''=\frac{m_s^2+k'^2_\perp}{x'(1-x')}. \tag{83}$$

C. Calculation results for the $\gamma\eta$ and $\gamma\eta'$ transition form factors

The calculation of the $\gamma\eta$ and $\gamma\eta'$ transition form factors does not require any additional parameter, as all unknown quantities are fixed by the $\gamma\pi$ case. We use only the universality of wave functions of the ground-state pseudoscalar meson nonet. For the mixing angle we take the value $\sin\theta=0.61$ [20]. The results show an excellent agreement with data both in the Q^2 dependence of the form factors (see Fig. 6) and in absolute values of $F_{\gamma\eta}(0)$ and $F_{\gamma\eta'}(0)$: experimental data for the partial decay widths are $\Gamma_{\eta\to\gamma\gamma}=0.514\pm0.052$ keV [20] and $\Gamma_{\eta'\to\gamma\gamma}=4.57\pm0.69$ keV [20] and our calculation gives $\Gamma_{\eta\to\gamma\gamma}=0.512$ keV and $\Gamma_{\eta'\to\gamma\gamma}=4.81$ keV.

Assuming universality of the pseudoscalar meson wave functions, we can estimate the value of gluonic (or glueball) components in η and η'. With $\eta=C_1q\bar{q}+C_2gg$ and $\eta'=C'_1q\bar{q}+C'_2gg$, we obtain the following constraints for probabilities of the gluonic components in η and η': $C_2^2\leqslant0.1$ and $C'^2_2\leqslant0.2$.

IV. CONCLUSION

We have analyzed the $\gamma\pi^0$, $\gamma\eta$, and $\gamma\eta'$ transition form factors at low and moderately high Q^2, making use of the approach developed in Ref. [1] which allows covering a wide range of momentum transfers from the soft to PQCD physics. Our analysis shows that a hadronic $q\bar{q}$ structure of the soft photon similar to the pion structure at low s, that is quite natural in the framework of the vector dominance model, yields a good description of the photon-pion transition form factor.

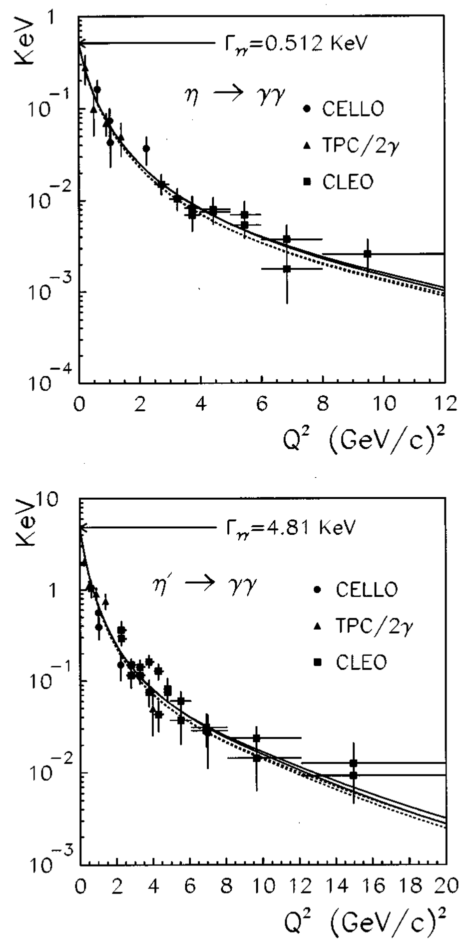

FIG. 6. Q^2 dependence for the quantities $(\pi/4)\alpha^2m^3_{\eta'}F^2_{\gamma\eta}(Q^2)$ and $(\pi/4)\alpha^2m'^3_{\eta}F^2_{\gamma\eta'}(Q^2)$. Different curves correspond to different sets of the parameters in the SH term (the curve notation is the same as in Fig. 2). Experimental data are taken from Ref. [2].

It should be underlined that we use the pion wave function which has been previously determined within the same approach from the data on the elastic pion form factor at $Q^2=0-10$ GeV^2. Thus, we have a simultaneous description of F_π and $F_{\pi\gamma}$ at low and intermediate Q^2 and guarantee the asymptotical behavior of these form factors in accordance with PQCD.

In the case when both of the photon virtualities are nonzero, our approach recovers at large Q^2 the $1/Q^2$ behavior as PQCD does, and not $1/Q_1^2Q_2^2$ as might be expected from the naive application of the vector meson dominance. The vector meson dominance reveals itself rather in a hadronic structure of the soft photon than in a naive $1/Q_1^2$ behavior of the form factors.

For the $\gamma\eta$ and $\gamma\eta'$ transition the universality of the soft wave functions of the ground-state pseudoscalar mesons, the members of the lightest nonet, gives unambiguous results found to be in perfect agreement with the experimental data.

The transition photon-meson form factors were calculated at moderately large Q^2 within the hard scattering approach [12–14] and at low Q^2 in the quark model [19,21]. A common feature of these considerations is describing the process by the triangle graph with pointlike $q\bar{q}\gamma$ vertices. This is equivalent to writing single dispersion representations in the pion channel and hence leaving the exchange quark between the photon vertices an off-shell particle with the virtuality k^2. In this case one should include into consideration the quark form factors $f_c(k^2,m^2,Q_i^2)$ in both photon vertices (see also discussion in Ref. [15]). The off-shell quark form factor is a complicated object, in particular $f_c(k^2,m^2,0) \neq 1$. This important point is usually neglected. We overcome this problem by considering a double dispersion representation whose spectral density is obtained by placing all quarks on mass shell and thus having in the vertices $f_c(m^2,m^2,q_i^2)$ such that $f_c(m^2,m^2,0)=1$. But in this case we have to allow a nontrivial $q\bar{q}$ structure of the soft photon. The quantity G_γ describing this structure has been a variational parameter of our consideration. As a matter of fact, the numerical analysis suggests a hadronic structure of the soft photon similar to the structure of the meson just in the spirit of the vector meson dominance. Although some particular details of the consideration are model dependent (the partition of wave function into the soft and the hard components and the corresponding description of the elastic and transition form factors in terms of the soft and the soft-hard parts at intermediate momentum transfers), the main conclusion on the importance of the hadronic structure of the soft photon and the similarity of the soft photon and meson structure remains valid and crucial for a simultaneous description of the pion elastic form factor and pion-photon transition form factor in a broad interval of momentum transfers.

Finally, we would like to comment on a possibility to distinguish between the pion distribution amplitudes of the Chernyak-Zhitnitsky- (CZ-) type and the asymptotic type by studying the photon-meson transition form factors at several GeV^2, whereas in an earlier application of the hard scattering picture [12,13], the CZ distribution is discarded as failing to describe the data at $Q^2=2-8\ \mathrm{GeV}^2$, a later analysis [14] shows that with the transverse corrections taken into account more properly, both of the distribution amplitudes agree with the data. These conclusions are based on ignoring the soft contribution in a few GeV region. We have found that once the soft part of the elastic pion form factor and a nontrivial hadronic structure of the soft photon in the transition pion-photon form factor are included, the reconstructed distribution amplitude is numerically very close to the asymptotic form in agreement with recent QCD sum rule results [22].

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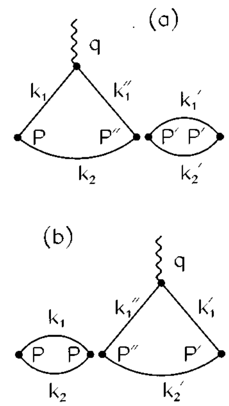

FIG. 7. Momentum notations in soft-hard terms $F^{\mathrm{SH}(1)}$ (a) and $F^{\mathrm{SH}(2)}$ (b).

APPENDIX: CALCULATION OF THE SOFT-HARD CONTRIBUTIONS

The soft-hard contributions are described by the two-loop diagrams of Figs. 1(c) and 1(d), the momentum notations in which are shown in Figs. 7(a) and 7(b), respectively. As the calculations of these graphs have many common steps, we present formulas in parallel denoting the specific formulas as case A and case B. For obtaining the spectral densities of the dispersion representations for the soft-hard contributions with only two-particle $q\bar{q}$ singularities taken into account we must set all intermediate quarks on the mass shell and allow off-shell $q\bar{q}$ momenta, namely,

$$\text{Case A:}\quad \tilde{P}=k_1+k_2,\quad \tilde{P}'=k_1'+k_2',\quad \tilde{P}''=k_1''+k_2,$$

$$\tilde{q}=\tilde{P}''-\tilde{P},\quad \tilde{\delta}=\tilde{P}''-\tilde{P}'; \tag{A1}$$

$$\text{Case B:}\quad \tilde{P}=k_1+k_2,\quad \tilde{P}'=k_1'+k_2',\quad \tilde{P}''=k_1''+k_2',$$

$$\tilde{q}=\tilde{P}'-\tilde{P}'',\quad \tilde{\delta}=\tilde{P}-\tilde{P}'', \tag{A2}$$

where $\tilde{P}^2=s$, $\tilde{P}'^2=s'$, $\tilde{P}''^2=s''$, $\tilde{q}^2=q^2$, $\tilde{\delta}^2=0$, $\tilde{\delta}\tilde{q}=0$, and $k_i^2=m^2$.

For calculating the traces in the double spectral densities of Eqs. (36) and (47) it is convenient to perform the Fierz rearrangements in the traces to isolate factors related to different loops as follows

$$\text{Case A:}\quad Sp^{(1)}_{\mu\nu}\equiv Sp[i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}_2')\gamma_\nu(m+\hat{k}_1') \times\gamma_\alpha(m+\hat{k}_1'')\gamma_\mu(m+\hat{k}_1)]$$

$$=\sum_{i=S,V,T,A,P} C_i Sp[(m+\hat{k}_1'')\gamma_\mu(m+\hat{k}_1) \times i\gamma_5(m-\hat{k}_2)O_i] \times Sp[O_i(m-\hat{k}_2')\gamma_\nu(m+\hat{k}_1')]; \tag{A3}$$

Case B:
$$Sp^{(2)}_{\mu\nu}\equiv Sp[i\gamma_5(m-\hat{k}_2)\gamma_\alpha(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\times\gamma_\mu(m+\hat{k}''_1)\gamma_\alpha(m+\hat{k}_1)]$$
$$=\sum_{i=S,V,T,A,P} C_i Sp[(m+\hat{k}_1)\times i\gamma_5(m-\hat{k}_2)O_i]\times Sp[O_i(m-\hat{k}'_2)\gamma_\nu(m+\hat{k}'_1)\gamma_\mu\times(m+\hat{k}''_1)], \quad (A4)$$

with $C_S=1, C_V=-\frac{1}{2}, C_T=0, C_A=\frac{1}{2}, C_P=-1$. Each term in Eqs. (A3) and (A4) is a product of two factors, related to two different loops (see Fig. 7). The nonzero terms in Eqs. (A3) and (A4) are the S,V,A and P,A terms, respectively.

Next, we have to calculate integrals of the form

$$\int dk_1 dk_2 dk''_1 dk'_1 dk'_2 \frac{\{k_\mu, k'_\mu, k_\mu k'_\nu, k'_\mu k'_\nu\}}{\mu_G^2-(k_2-k'_2)^2}\delta(k''^2_1-m^2)$$
$$\times\delta f(k_1^2-m^2)\delta(k_2^2-m^2)\delta(k'^2_1-m^2)\delta(k'^2_2-m^2)$$
$$\times\delta(\widetilde{P}-k_1-k_2)\delta(\widetilde{P}''-k''_1-k_2)\delta(\widetilde{P}'-k'_1-k'_2). \quad (A5)$$

Under the integral sign we can substitute

$$k_\mu\to a_1\widetilde{P}_\mu+a_2\widetilde{q}_\mu+a_3\widetilde{\delta}_\mu,$$
$$k'_\mu\to a'_1\widetilde{P}_\mu+a'_2\widetilde{q}_\mu+a'_3\widetilde{\delta}_\mu,$$
$$k_\mu k'_\nu\to\sum_{\widetilde{A},\widetilde{B}=\widetilde{P},\widetilde{q},\widetilde{\delta}} a'_{AB}\widetilde{A}_\mu\widetilde{B}_\nu+b'g_{\mu\nu},$$
$$k'_\mu k'_\nu\to\sum_{\widetilde{A},\widetilde{B}=\widetilde{P},\widetilde{q},\widetilde{\delta}} a''_{AB}\widetilde{A}_\mu\widetilde{B}_\nu+b''g_{\mu\nu}. \quad (A6)$$

For further calculations it is convenient to use the light-cone variables and work in the reference frame $q_+=0, P_\perp=0$ such that

$$P=\left(P_+,\frac{m_\pi^2}{2P_+},0\right),\quad P'=\left(P_+,\frac{Q^2}{2P_+},q_\perp\right),$$
$$q=\left(0,\frac{-m_\pi^2+Q^2}{2P_+},q_\perp\right),\quad q_\perp^2=Q^2. \quad (A7)$$

In this reference frame the vectors $\widetilde{P}$ and $\widetilde{P}'$ can be chosen such that

$$\widetilde{P}=\left(P_+,\frac{s}{2P_+},0\right),\quad \widetilde{P}'=\left(P_+,\frac{s'+Q^2}{2P_+},q_\perp\right). \quad (A8)$$

The vectors $\widetilde{P}''$, $\widetilde{\delta}$, and $\widetilde{q}$ are different in the A and B cases:

Case A:
$$\widetilde{P}''=\left(P_+,\frac{s''+Q^2}{2P_+},q_\perp\right),$$
$$q=\left(0,\frac{s''-s+Q^2}{2P_+},q_\perp\right),$$
$$\widetilde{\delta}=\left(0,\frac{s''-s'}{2P_+},0\right), \quad (A9)$$

Case B:
$$\widetilde{P}''=\left(P_+,\frac{s''}{2P_+},0\right),\quad \widetilde{q}=\left(0,\frac{s'-s''+Q^2}{2P_+},q_\perp\right),$$
$$\widetilde{\delta}=\left(0,\frac{s-s''}{2P_+},0\right). \quad (A10)$$

The light-cone variables $x, x', k_\perp$, and $k'_\perp$ are introduced as

$$k_2=\left(xP_+,\frac{m^2+k_\perp^2}{2xP_+},k_\perp\right),\quad k'_2=\left(x'P_+,\frac{m^2+k'^2_\perp}{2x'P_+},k'_\perp\right),$$
$$\widetilde{P}=\left(P_+,\frac{s}{2P_+},0\right),\quad \widetilde{P}'=\left(P_+,\frac{s'+Q^2}{2P_+},q_\perp\right). \quad (A11)$$

In terms of these variables the coefficients in Eq. (A6), which are needed for the trace calculation, take the form

$$a_1=x,\quad a'_1=x',\quad a_2=\frac{k_\perp q_\perp}{Q^2},\quad a'_2=x'+\frac{k'_\perp q_\perp}{Q^2},$$
$$b'=\frac{(k_\perp q_\perp)(k'_\perp q_\perp)}{Q^2}-(k_\perp k'_\perp),\quad b''=\frac{(k'_\perp q_\perp)^2}{Q^2}-k'^2_\perp. \quad (A12)$$

The final expressions for the traces read

$$Sp^{(1)}_{\mu\nu}=4m\varepsilon_{\mu\nu\alpha\beta}\widetilde{q}^\alpha\widetilde{P}^\beta[s'+4b''-12b'+(s''-s-q^2)(a'_1-a'_2)], \quad (A13)$$

$$Sp^{(2)}_{\mu\nu}=8m\varepsilon_{\mu\nu\alpha\beta}\widetilde{q}^\alpha\widetilde{P}^\beta[s-(a'_1-a'_2)(k''_1\widetilde{P})-2b''-a'_2(k'_1k'_2)-m^2(1+a'_2)], \quad (A14)$$

where

$$(k''_1\widetilde{P})=\frac{1}{2}[x's''+(1-x')s],\quad (k'_1k'_2)=\frac{1}{2}s'-m^2. \quad (A15)$$

The expressions for s and s' are

$$s=\frac{m^2+k_\perp^2}{x(1-x)},\quad s'=\frac{m^2+(k'_\perp-x'q_\perp)^2}{x'(1-x')},\qquad \text{(A16)}$$

and s'' is different in the A and B cases:

$$\text{Case A:}\quad s''=\frac{m^2+(k_\perp-xq_\perp)^2}{x(1-x)},$$

$$\text{Case B:}\quad s''=\frac{m^2+k'^2_\perp}{x'(1-x')}.\qquad \text{(A17)}$$

In the Lorentz structures of the final expressions for the traces one must set

$$\tilde{P}\to P,\quad \tilde{P}'\to P',\quad \tilde{q}\to q,\quad \tilde{\delta}\to 0,$$

$$\text{Case A:}\quad \tilde{P}''\to P',$$

$$\text{Case B:}\quad \tilde{P}''\to P.\qquad \text{(A18)}$$

These Lorentz structures determine the Lorentz structure of the amplitude, and the corresponding scalar factors enter into the spectral densities of the form factors. Let us point out that shifting from the mass shell is performed only in the $(-)$ components of the vectors, while the $(+)$ and $(\perp)$ components of $\tilde{P}$, $\tilde{P}'$, and $\tilde{P}''$ are equal to the corresponding components of the physical vectors P and P', in agreement with Ref. [4].

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