Ground-state baryon masses in an equally mixed scalar-vector linear potential model

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Taking into account the pionic self-energy of the baryons, the color-electrostatic and magnetostatic energies due to one-gluon exchange, and the corrections due to the center-of-mass motion, the ground-state masses of the octet baryons are calculated in a chiral symmetric potential model of independent quarks. The effective potential representing phenomenologically the nonperturbative gluon interactions, including gluon self-couplings, is chosen with equally mixed scalar and vector parts in a linear form. The physical masses of the baryons so obtained with the strong coupling constant $\alpha_c = 0.576$ agree very well with the corresponding experimental values. [S0556-2821(96)02323-5]

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I. INTRODUCTION

Several works based on nonrelativistic quark model approaches [1] have appeared in the literature to study the mass spectrum of octet baryons. The phenomenological description of the baryon masses at the nonrelativistic level is quite reasonable; still, a relativistic approach is indispensable in view of the fact that the baryonic mass splittings are of the same order as the constituent quark masses. In this respect the MIT bag model [2] has proved to be quite successful. In its improved versions, the chiral bag models [3] (CBM's) have included the effect of pion self-energy due to baryon-pion coupling at the vertex to give a better understanding of the physical mass of the baryons. Nevertheless, such models still contain some dubious phenomenological ingredients which are objectionable. The sharp spherical bag boundary, the zero point energy, the exclusion of pions from within the bag, or *ad hoc* inclusion of pions within it are a few such points to be noted in this context. Furthermore, it is somewhat difficult to believe that the static spherical bag remains unperturbed even after the creation of a pion. On closer scrutiny one finds that the static spherical bag boundary, which is ironically responsible for the success and simplicity of the CBM, is, on the other hand, at the root of all the difficulties encountered by the CBM. However, the rigid spherical bag boundary of the CBM, which is nonetheless arbitrary and phenomenological, can always be replaced by a suitable phenomenological average potential for individual quarks, preserving at the same time its good features.

The chiral quark models [4] which are comparatively more straightforward, are no doubt attempts in this direction. In such models the confining potentials which represent basically the interaction of quarks with the gluon field are usually assumed phenomenologically as Lorentz scalars. The term in the quark Lagrangian density corresponding to such a scalar potential is chirally noninvariant through all space and hence requires the introduction of an additional pionic component everywhere in order to preserve chiral symmetry, thus removing the *ad hoc* nature of the CBM in including pion field in the interior region as against its requirement in the exterior region only. However, the pure scalar potential provides an attractive force for both the $q\overline{q}$ and qq states, whereas the pure vector potential produces only $q\overline{q}$ states [5]. Since there are no diquark states, the q-q interaction must be weaker, which can possibly be provided by the repulsive nature of the vector potential. Thus, for the confinement of quarks, a mixed scalar and vector potential is a more appropriate choice. In fact, the choice of such a potential has been immensely successful in predictions of hadronic properties. Therefore, for the study of baryons, one can think of taking an effective individual quark potential $V_q(r)$ in the form of an equally mixed scalar and vector part purely on phenomenological grounds without searching for any justification for its physical origin as well as mathematical structure. However, such a choice is certainly guided by the usual aesthetic compulsion of providing simplicity and tractability to the model in the same spirit as that which led Bogolioubov [6] to introduce the idea of the spherical bag at one stage. The implications of such potential forms in the Dirac framework of independent quarks have been discussed by Smith and Tassie [7]. Bell and Ruegg [8] have also shown that the spin-orbit interaction is absent in such a scheme due to exact cancellation of such terms coming from vector and scalar parts of the potential if taken in equal proportions. This is clearly a welcome aspect of the model in the case of baryons since the contribution of the spin-orbit interaction term to the baryon mass splittings is already known [9] to be negligible. Eichten and Feinberg [10] provide further support to the above Lorentz structure from a gauge-invariant formalism where the confinement mechanism is assumed to be purely color electric in character. Furthermore, such a Lorentz structure of the two-body confining potential has been observed [11] phenomenologically in the study of the hyperfine splittings of heavy meson spectra.

Moreover, in a Lagrangian formalism with such a potential, its scalar part together with the quark mass term appearing in the quark Lagrangian density, $\mathcal{L}_q^0$, makes it chirally odd through all space, which requires the introduction of an additional pionic component everywhere in order to preserve chiral symmetry which is essential in models like the CBM. On the other hand, the vector part of $V_q(r)$ together with the accompanying scalar part in equal proportion at every point renders solvability by converting the Dirac equation of the independent quarks derivable from $\mathcal{L}_q^0$ into an effective Schrödinger-like equation producing no spin-orbit splittings as required by the experimental baryon spectrum.

Nevertheless, there are some definite theoretical justifications underlying this phenomenologically successful confining potential form. It may be possible to arrive at such potential forms in color-dielectric models with an appropriately chosen color-dielectric function having radial dependence which, however, would require further investigation and one cannot claim anything at this stage.

In view of the success of this scheme, effective confining potentials with such Lorentz character in linear [12,13], harmonic [14], and non-Coulombic power-law form [15] have been used by several authors in the recent past for the study of baryons. In our earlier work we have also employed such a potential in linear form [13] to obtain a reasonable prediction of the core contributions to the static properties of the octet baryons. The same scheme has also been adopted by us to study the nucleon form factors, as well as the weak electric and magnetic form factors for semileptonic baryon decays [13]. Then, incorporating chiral symmetry in the SU(2)-flavor sector in the usual manner, we have obtained the electromagnetic properties of nucleons [16] as well as the magnetic moments of octet baryons [17] in reasonable agreement with experimental data. But, in this model, we have not yet taken into account the effects of the residual one-gluon-exchange interaction assuming its effects to be not so much significant for the electromagnetic properties of baryons. This, however, plays an important role in providing color-electrostatic and magnetostatic energies to the quark core. Therefore, in the present work, we are interested in employing such a chiral potential model to study the mass spectrum of the octet baryons by taking into account the corrections due to (i) the energy associated with center-of-mass motion, (ii) the pionic self-energy of the baryons arising out of the baryon-pion coupling at the vertex, and (iii) the color-electric and -magnetic energy arising out of the residual one-gluon-exchange interaction. We treat all these corrections, leading ultimately to the baryon physical masses, independently, as though they are of the same order of magnitude. This model with a linear form in particular for the scalar-vector mixed potential, turns out to be quite simple and tractable in these respects, yielding very satisfactory results for the physical masses of the low-lying baryons.

The plan of the present work is as follows. In Sec. II we outline the framework of the potential model used with the solutions for the relativistic bound states of the individually confined quarks in the ground state of baryons. Section III provides an account of various energy corrections such as those due to center-of-mass motion and quark-gluon and quark-pion interactions. Finally in Sec. IV, we present the results for the ground state baryon masses, which come out in very good agreement with the corresponding experimental values.

II. POTENTIAL MODEL

We start with the assumption that the constituent quarks in a baryon core are assumed to move independently in an average effective potential of the form

$$V_q(r)=\tfrac{1}{2}(1+\gamma^0)V(r), \tag{2.1}$$

where

$$V(r)=a^2r+V_0, \quad a>0. \tag{2.2}$$

$V_q(r)$ represents phenomenologically the confining interaction due to the dominant nonperturbative multigluon mechanism including the gluon self-couplings. Then, leaving aside for the moment any further residual interactions such as quark-pion and quark-gluon interactions to be treated perturbatively, one can write the zeroth order Lagrangian density for independent quarks in a baryon core as

$$\mathcal{L}_q^0(x)=\overline{\psi_q}(x)\left[\frac{i}{2}\,\gamma^\mu\overleftrightarrow{\partial}_\mu-m_q-V_q(r)\right]\psi_q(x). \tag{2.3}$$

Assuming all the quarks in a baryon core are in their ground states with $J^P=\frac{1}{2}^+$, the normalized quark wave functions $\psi_q(\vec{r})$ satisfying the Dirac equation, derivable from $\mathcal{L}_q^0(x)$ as

$$[\gamma^0E_q-\vec{\gamma}\cdot\vec{P}-m_q-V_q(r)]\psi_q(\vec{r})=0, \tag{2.4}$$

can be written in a two-component form as

$$\psi_q(\vec{r})=N_q\begin{bmatrix}\phi_q(r)\\ \dfrac{\vec{\sigma}\cdot\vec{P}}{\lambda_q}\,\phi_q(r)\end{bmatrix}\chi\uparrow, \tag{2.5}$$

where

$$\begin{aligned}\lambda_q&=E_q+m_q,\\ \phi_q(r)&=A_q\,\frac{U_q(r)}{r}\,Y_0^0(\theta,\phi)\end{aligned} \tag{2.6}$$

is the normalized radial angular part of $\psi_q(r)$ with normalization constant A_q. Taking $E_q'=E_q-V_0/2$, $m_q'=m_q+V_0/2$, and $\lambda_q=E_q'+m_q'$, the reduced radial part $U_q(r)$ can be found to satisfy a Schrödinger-type equation

$$U_q''(r)+\lambda_q[E_q'-m_q'-a^2r]U_q(r)=0, \tag{2.7}$$

which can be transformed into a convenient dimensionless form

$$U''(\rho)+(\epsilon_{ns}-\vartheta)U_q(\rho)=0, \tag{2.8}$$

where $\rho=r/r_{0q}$ is a dimensionless variable with $r_{0q}=(a^2\lambda_q)^{-1/3}$ and

$$\epsilon_{ns}=\left(\frac{\lambda_q}{a^4}\right)^{1/3}(E_q'-m_q'). \tag{2.9}$$

Now, with $z=\rho-\epsilon_{ns}$, Eq. (2.8) reduces to

$$U_q''(z)-zU_q(z)=0, \tag{2.10}$$

whose solution $U_q(z)$ is the Airy function Ai(z). Since, at $r=0$, $U_q(r)=0$, we have Ai$(z)=0$ at $z=-\epsilon_{ns}$. If z_n are the roots of the Airy function such that Ai$(z_n)=0$, then we have $z_n=-\epsilon_{ns}$. For the ground state of quarks, the ϵ_{ns} value is given by the first root z_1 of the Airy function so that

$$\epsilon_{ns}=\epsilon_{1s}=\epsilon_q=-z_1, \tag{2.11}$$

the value of the root being $z_1 = -2.338\,11$. Now the ground state individual quark binding energy $E_q = E'_q + V_0/2$ is obtainable from the energy eigenvalue condition (2.9) through the relation

$$E'_q = (m'_q + a x_q), \tag{2.12}$$

where x_q is the root of the equation

$$x_q^4 + b x_q^3 - \epsilon_q^3 = 0, \tag{2.13}$$

with $b = 2m'_q/a$.

The overall normalization constant N_q of $\psi_q(\vec{r})$ appearing in Eq. (2.5) can be obtained in a simplified form as

$$N_q^2 = \frac{3(E'_q + m'_q)}{2(2E'_q + m'_q)}. \tag{2.14}$$

III. ENERGY CORRECTIONS TO BARYON MASSES

The binding energies of the individual constituent quarks contribute additionally to the mass of the baryon core. Equations (2.9)–(2.12) provides the quark binding energy E_q, which immediately leads to the mass of the baryonic core in zeroth order as

$$M_B^0 = E_B = \sum_q E_q. \tag{3.1}$$

Such a contribution needs corrections due to center-of-mass motion, quark pion interaction, and quark-gluon interaction, which need to be calculated separately so as to obtain the physical masses of the baryon.

A. Center-of-mass correction

In this model there would be a sizable spurious contribution to the energy E_q from the motion of the c.m. of the three-quark system. Unless this aspect is duly accounted for, the concept of the independent motion of quarks inside the baryon core will not lead to a physical baryon state of definite momentum. Although there is still some controversy on this subject, we follow the technique adopted by Bartelski *et al.* [18] and Eich *et al.* [19], which is just one way of accounting for the c.m. motion. Following their prescription, a ready estimate of the center-of-mass momentum $\vec{P}_B$ can be obtained as

$$\langle \vec{P}_B^2 \rangle = \sum_q \langle \vec{P}^2 \rangle_q, \tag{3.2}$$

where $\langle \vec{P}^2 \rangle_q$ is the average value of the square of the individual quark momentum taken over the $1S_{1/2}$ single quark states and is given in the present model as

$$\langle \vec{P}^2 \rangle_q = \frac{(E_q'^2 - m_q'^2)(4E'_q + m'_q)}{5(2E'_q + m'_q)}. \tag{3.3}$$

In the same manner one can get the expression for the physical mass M_B of the bare baryon core as

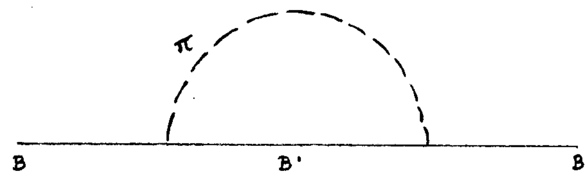

FIG. 1. Baryon self-energy due to coupling with pion.

$$\langle M_B^2/E_B^2 \rangle = 1 - \sum_q \langle \vec{P}^2 \rangle_q / E_B^2, \tag{3.4}$$

which provides the energy correction to the baryon mass in Eq. (3.1) as

$$(\delta E_B)_{\mathrm{c.m.}} = \left[\left(E_B^2 - \sum_q \langle P^2 \rangle_q \right)^{1/2} - E_B \right]. \tag{3.5}$$

B. Pionic self-energy corrections

Looking at the zeroth order Lagrangian density $\mathcal{L}_q^0$ described in Sec. II, one can note that under a global infinitesimal chiral transformation at least in the (u,d)-flavor sector the axial vector current of quarks is not conserved due to the fact that the scalar term in $\mathcal{L}_q^0$, which is proportional to $S(r) = [m_q + V(r)/2]$, is chirally odd. Of course, the vector part of the potential poses no problem in this respect.

In order to restore the chiral SU(2)×SU(2) symmetry within the PCAC (partial conservation of axial vector current) limit, one can introduce in the usual manner an elementary pion field $\vec{\phi}(x)$ of small but finite mass $m_\pi = 0.14$ GeV with linearized interaction Lagrangian density

$$\mathcal{L}_I^\pi(x) = \frac{1}{f_\pi} S(r) \overline{\psi_q}(x) \gamma^5 (\vec{\tau} \cdot \vec{\phi}) \psi_q(x), \tag{3.6}$$

where $f_\pi = 93$ MeV is the phenomenological pion decay constant. Then the four-divergence of the total axial vector current becomes

$$\partial_\mu A^\mu(x) = -f_\pi m_\pi^2 \phi(x), \tag{3.7}$$

yielding the usual PCAC relation. Consequently, the pion coupling of the nonstrange quarks would give rise to pionic self-energy of the baryons which would ultimately contribute to the physical masses of the baryon.

Though this consideration can be generalized to include the strange flavor sector for a chiral SU(3)×SU(3) symmetry, we would ignore it because of the large mass of the kaon involved in the process.

Then following the Hamiltonian techniques [20] as has been used in the CBM, we can describe the effect of pion coupling in low order perturbation theory as given below.

The pionic self-energy of the baryons can be evaluated with the help of the single-loop self-energy diagram (Fig. 1) as

$$\sum_B (E_B) = \sum_K \sum_{B'} \frac{V_j^{\dagger BB'}(\vec{K}) V_j^{BB'}(\vec{K})}{E_B - w_k - M_{B'}^0}, \tag{3.8}$$

TABLE I. Baryon-pion coupling constant and spin-isospin reduced matrix elements for various baryon states.

Baryon B	Baryon intermediate baryon states BB'	$\frac{f_{BB'\pi}}{f_{NN\pi}}$	$(\sigma^{BB'}\cdot\sigma^{BB'})$	$(\tau^{BB'}\cdot\tau^{BB'})$	$C_{BB'}$
N	NN	1	3	3	9
	$N\Delta$	$\frac{6\sqrt{2}}{5}$	2	2	4
Δ	$\Delta\Delta$	1/5	15	15	225
	ΔN	$\frac{6\sqrt{2}}{5}$	1	1	1
Λ	$\Lambda\Lambda$	0	3	0	0
	$\Lambda\Sigma$	$\frac{-2\sqrt{3}}{5}$	3	3	9
	$\Lambda\Sigma^*$	$-6/5$	2	3	6
Σ	$\Sigma\Sigma$	4/5	3	2	6
	$\Sigma\Lambda$	$\frac{-2\sqrt{3}}{5}$	3	1	3
	$\Sigma\Sigma^*$	$\frac{-2\sqrt{3}}{5}$	2	2	4
Σ^*	$\Sigma^*\Sigma^*$	2/5	15	2	30
	$\Sigma^*\Lambda$	$-6/5$	1	1	1
	$\Sigma^*\Sigma$	$\frac{-2\sqrt{3}}{5}$	1	2	2
Ξ	$\Xi\Xi$	$-1/5$	3	3	9
	$\Xi\Xi^*$	$\frac{-2\sqrt{3}}{5}$	2	3	6
Ξ^*	$\Xi^*\Xi^*$	1/5	15	3	45
	$\Xi^*\Xi$	$\frac{-2\sqrt{3}}{5}$	1	3	3

where

$$\sum_K = \sum_j \int d^3\vec{k}/(2\pi)^3.$$

Here j corresponds to the pion-isospin index and B' is the intermediate baryon state. $V_j^{BB'}(\vec{k})$ is the general baryon pion absorption vertex function obtained [17] in the present model as

$$V_j^{BB'}(\vec{k}) = i\sqrt{4\pi}\,\frac{f_{BB'\pi}}{m_\pi}\,\frac{ku(k)}{(2w_k)^{1/2}}\,(\vec{\sigma}^{BB'}\cdot\hat{k})\,\tau_j^{BB'}, \quad (3.9)$$

where $\sigma_j^{BB'}$ and $\tau_j^{BB'}$ are spin and isospin matrices and $w_k^2 = \vec{k}^2 + m_\pi^2$. The form factor $u(k)$ in this model can be expressed as

$$u(k) = \frac{5N_q^2}{3\lambda_u g_A}\,[2m_q'\langle\langle j_0(kr)\rangle\rangle + a^2\langle\langle r j_0(kr)\rangle\rangle + a^2\langle\langle j_1(kr)/k\rangle\rangle], \quad (3.10)$$

where $j_0(kr)$ and $j_l(kr)$ represent the zeroth and first order spherical Bessel functions, respectively. The double angular brackets stand for the expectation values with respect to $\phi_q(r)$. The baryon pion coupling constants $f_{BB'\pi}$ can be expressed in terms of the nucleon-pion coupling constant $f_{NN\pi}$ as given in Table I.

Now with the vertex function $V_j^{BB'}(K)$ on hand, it is possible to calculate the pionic self-energy for various baryons with appropriate baryon intermediate states contributing to the process. For degenerate intermediate states on mass shell with $M_B^0 = M_{B'}^0$, the self-energy correction becomes

$$(\delta E_B)_P = \sum_B (E_B = M_B^0 = M_{B'}^0) = -\sum_{K,B'} \frac{V_j^{\dagger BB'} V_j^{BB'}}{w_k}. \quad (3.11)$$

Now, using Eq. (3.9), we get

$$(\delta E_B)_P = -\tfrac{1}{3} I_\pi \sum_{B'} C_{BB'} f_{BB'\pi}^2, \quad (3.12)$$

where

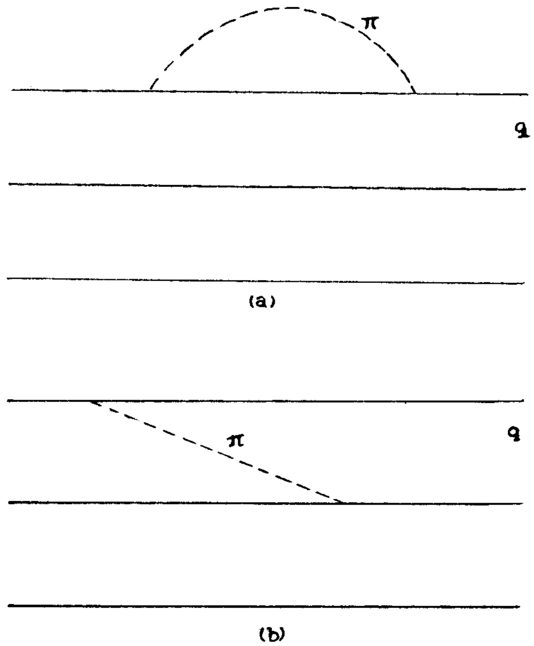

FIG. 2. One-pion-exchange contribution to the energy.

$$C_{BB'}=(\vec{\sigma}^{BB'}\cdot\vec{\sigma}^{BB'})(\vec{\tau}^{BB'}\cdot\vec{\tau}^{BB'})$$

and

$$I_\pi=\frac{1}{\pi m_\pi^2}\int_0^\infty \frac{dk\ k^4u^2(k)}{w_k^2}. \tag{3.13}$$

For the intermediate baryon states B', we consider only the octet and decuplet ground states. Using the values of $f_{BB'}\pi$ and $C_{BB'}$ summarized in Table I according to Ref. [1], the pionic self-energy for different baryons can be computed as

$$(\delta E_N)_P=-\tfrac{171}{25}f_{NN\pi}^2 I_\pi,$$

$$(\delta E_\Delta)_P=-\tfrac{99}{25}f_{NN\pi}^2 I_\pi,$$

$$(\delta E_\Lambda)_P=-\tfrac{108}{25}f_{NN\pi}^2 I_\pi, \tag{3.14}$$

$$(\delta E_\Sigma)_P=(\Delta E_{\Sigma*})_\pi=-\tfrac{12}{5}f_{NN\pi}^2 I_\pi,$$

$$(\delta E_\Xi)_P=(\Delta E_{\Xi*})_\pi=-\tfrac{27}{25}f_{NN\pi}^2 I_\pi,$$

and, of course, $(\delta E_{\Omega^-})_\pi=0$ since the strange quarks in Ω^- have no interaction with the pion. The self-energy δE_B calculated here contains both the quark self-energy [Fig. 2(a)] and the one-pion-exchange contribution [Fig. 2(b)].

The bare pseudovector nucleon-pion coupling constant $f_{NN\pi}$ can be computed from the usual relation [21]

$$\sqrt{4\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)=\left(\frac{g_{NN\pi}}{2M_P}\right), \tag{3.15}$$

where $g_{NN\pi}$ is the pseudoscalar nucleon-pion coupling constant defined as $g_{NN\pi}=G_{NN\pi}(q^2=m_\pi^2)$, $G_{NN\pi}$ being the nucleon-pion form factor given by

$$G_{NN\pi}(q^2)=\left(\frac{M_P}{f_\pi}\right)g_A u(q^2). \tag{3.16}$$

Here M_P is the mass of the proton and g_A is the axial vector coupling constant for the β decay of the neutron. In the present model the expression for g_A is corrected for center-of-mass motion and is given by

$$g_A=\frac{5}{3}\frac{(4N_u^2-1)}{(1+2\delta_N)}, \tag{3.17}$$

where

$$\delta_N=\frac{M_N}{E_N}, \tag{3.18}$$

with M_N and E_N given in Eqs. (3.4) and (3.1).

C. One-gluon-exchange correction

The weak one-gluon-exchange interaction arising inside the quark core is provided by the Lagrangian density

$$\mathcal{L}_I^g=\sum_{a=1}^{8} J_i^{\mu a}(x)A_\mu^a(x), \tag{3.19}$$

where $A_\mu^a(x)$ are eight vector gluon fields and $J_i^{\mu a}$ is the ith quark color current. Since at small distances the quarks should be almost free, it is reasonable to calculate the energy shift in the mass spectrum arising out of the quark interaction energy due to their coupling to the colored gluons, using a first order perturbation theory.

If we keep only terms of the order α_c, the quark-gluon coupling constant, then the problem reduces to evaluating the diagrams shown in Figs. 3(a) and 3(b), where Fig. 3(a) corresponds to the one-gluon-exchange part between different quarks, while Fig. 3(b) implies the quark self-energy that normally contributes to the renormalization of the quark masses.

If $\vec{E}_i^a$ and $\vec{B}_i^a$ are the color-electric and -magnetic parts, respectively, generated by the ith quark color-current

$$J_i^{\mu a}(x)=g_c\overline{\psi_i}(x)\gamma^\mu\lambda_i^a\psi_i(x), \tag{3.20}$$

with λ_i^a being the usual Gell-Mann SU(3) matrices and $\alpha_c=(g_c^2/4\pi)$, then the contribution to the mass due to the relevant diagrams can be written as a sum of color-electric and magnetic parts as

$$(\delta E_B)_g=(\delta E_B)_g^e+(\delta E_B)_g^m, \tag{3.21}$$

where

$$(\delta E_B)_g^e=\frac{1}{8\pi}\sum_{i,j}\sum_{a=1}^{8}\int\int\frac{d^3\vec{r}_i d^3\vec{r}_j}{|\vec{r}_i-\vec{r}_j|}\langle B|J_i^{0a}(\vec{r}_j)J_j^{0a}(\vec{r}_j)|B\rangle, \tag{3.22}$$

$$(\delta E_B)_g^m=-\frac{1}{4\pi}\sum_{i<j}\sum_{a=1}^{8}\int\int\frac{d^3\vec{r}_i d^3\vec{r}_j}{|\vec{r}_i-\vec{r}_j|}\times\langle B|\vec{J}_i^a(\vec{r}_i)\cdot\vec{J}_j^a(\vec{r}_j)|B\rangle. \tag{3.23}$$

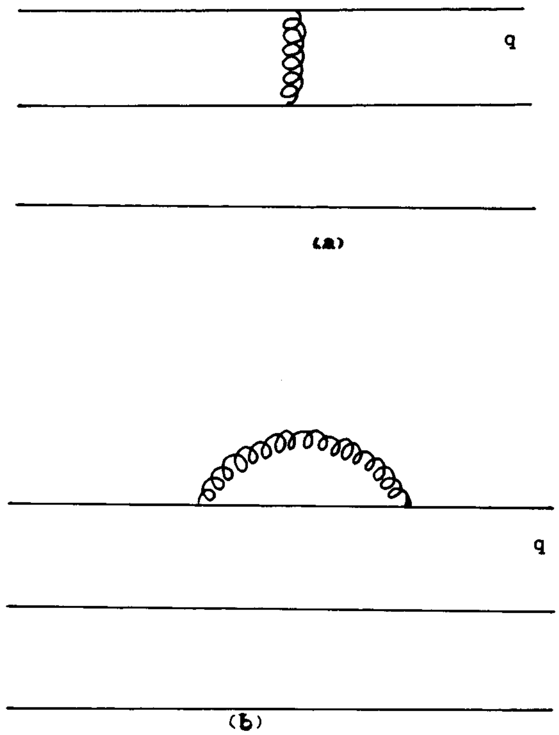

FIG. 3. One-gluon-exchange contribution to the energy.

Here the self-energy diagram contributing to the renormalization of quark masses has not been included in the calculation of the magnetic part of the interaction as it can possibly be accounted for in the phenomenological quark masses. The exclusion of this diagram, however, requires that each B_i^a should satisfy the boundary condition $\hat{r}\times\vec{B}_i^a=0$, separately at the edge of the confining region, which is a possible case. On the other hand, as the electric field E_i^a is necessarily in the radial direction, it is only possible to satisfy the boundary condition $\hat{r}\cdot(\Sigma_i E_i^a)=0$ for a color-singlet state $|B\rangle$ for which $(\Sigma_i \lambda_i^a)=0$. Therefore, in order to preserve the boundary conditions, we are forced to take into account the self-energy diagrams in Fig. 3(b) in the calculation of electric part only.

Now, using Eq. (2.5) in Eq. (3.20), we find

$$J_i^{0a}(\vec{r}_i)=g_c\lambda_i^a N_i^2\left[\phi^2(r_i)+\frac{\phi'^2(r_i)}{\lambda_i^2}\right],$$
$$\vec{J}_i^a(\vec{r}_i)=-2g_c\lambda_i^a N_i^2(\vec{\sigma}_i\times\hat{r}_i)\phi(r_i)\phi'(r_i)/\lambda_i. \tag{3.24}$$

Again, using Eq. (3.24) together with the identity

$$\frac{1}{|\vec{r}_i-\vec{r}_j|}=\frac{1}{2\pi^2}\int\frac{d^3k}{k^2}\exp[i\vec{k}\cdot(\vec{r}_i-\vec{r}_j)]$$

in Eqs. (3.22) and (3.23), we obtain

$$(\delta E_B)_g^e=\frac{\alpha_c}{4\pi^2}\sum_{i,j}\left\langle\sum_a\lambda_i^a\lambda_j^a\right\rangle N_i^2N_j^2\int\frac{d^3\vec{k}}{k^2}F_i^e(k)F_j^e(k), \tag{3.25}$$

TABLE II. Coefficients appearing in the calculation of the color-electric and -magnetic energy corrections due to one-gluon exchange.

Baryons	a_{uu}	a_{us}	a_{ss}	b_{uu}	b_{us}	b_{ss}
N	0	0	0	−3	0	0
Δ	0	0	0	+3	0	0
Λ	1	−2	1	−3	0	0
Σ	1	−2	1	1	−4	0
Ξ	1	−2	1	0	−4	1
Σ^*	1	−2	1	1	2	0
Ξ^*	1	−2	1	0	2	1
Ω^-	0	0	0	0	0	+3

$$(\delta E_B)_g^m=\frac{2\alpha_c}{\pi^2}\sum_{i<j}\left\langle\sum_a\lambda_i^a\lambda_j^a\right\rangle\frac{N_i^2N_j^2}{\lambda_i\lambda_j}\times\int\frac{d^3\vec{k}}{k^2}\vec{F}_i^m(k)\cdot\vec{F}_j^m(k), \tag{3.26}$$

where

$$F_i^e(k)=\frac{1}{2\lambda_i^2}[(4E_i'\lambda_i-k^2)\langle\langle j_0(kr_i)\rangle\rangle-2\lambda_i a^2\langle\langle r_i j_0(kr_i)\rangle\rangle], \tag{3.27}$$

$$\vec{F}_i^m(k)=\frac{i}{2}\langle\langle j_0(kr_i)\rangle\rangle(\vec{\sigma}_i\times\vec{k}). \tag{3.28}$$

Then Eqs. (3.25) and (3.26) can be written as

$$(\delta E_B)_g^e=\frac{\alpha_c}{\pi}\sum_{i,j}\left\langle\sum_a\lambda_i^a\lambda_j^a\right\rangle N_i^2N_j^2I_{ij}^e, \tag{3.29}$$

$$(\delta E_B)_g^m=-\frac{4\alpha_c}{3\pi}\sum_{i<j}\left\langle\sum_a\lambda_i^a\lambda_j^a(\vec{\sigma}_i\cdot\vec{\sigma}_j)\right\rangle\frac{N_i^2N_j^2}{\lambda_i\lambda_j}I_{ij}^m, \tag{3.30}$$

where

$$I_{ij}^e=\int_0^\infty dk\, F_i^e(k)F_j^e(k), \tag{3.31}$$

$$I_{ij}^m=\int_0^\infty dk\, k^2\langle\langle j_0(kr_i)\rangle\rangle\langle\langle j_0(kr_j)\rangle\rangle. \tag{3.32}$$

Finally, taking into account the specific quark flavor and spin configurations in various ground state baryons and using the relations $\langle\Sigma_a(\lambda_i^a)^2\rangle=\frac{16}{3}$ and $\langle\Sigma_a\lambda_i^a\lambda_j^a\rangle_{i\neq j}=-\frac{8}{3}$ for baryons, one can write in general the energy correction due to one-gluon exchange as

$$(\delta E_B)_g^e=\alpha_c(a_{uu}T_{uu}^e+a_{us}T_{us}^e+a_{ss}T_{ss}^e),$$
$$(\delta E_B)_g^m=\alpha_c(b_{uu}T_{uu}^m+b_{us}T_{us}^m+b_{ss}T_{ss}^m), \tag{3.33}$$

where a_{ij} and b_{ij} are the numerical coefficients depending on each baryon, listed in Table II, and the terms $T_{i,j}^{e,m}$ are

TABLE III. Energy corrections and physical masses of ground state baryons (in MeV).

Baryon	E_B	$(\Delta E_B)_{\text{c.m.}}$	$(\Delta E_B)_g^M$	$(\Delta E_B)_g^E$	δM_B	M_B Present calculation	M_B Experiment
N			−110.612		−168.092	940	940
	1459.863	−241.159		0			
Δ			+110.612		−97.3165	1232	1232
Λ			−110.612		−106.164	1129.628	1111.6
Σ	1575.373	−236.239	−98.164	7.27	−58.980	1189.26	1193
Σ^*			+104.387		−58.980	1391.811	1385
Ξ			−103.551		−26.540	1335.946	1321
	1690.882	−232.115		7.27			
Ξ^*			+98.999		−26.540	1538.496	1533
Ω^-	1806.392	−228.605	94.448	0	0	1672.235	1672

$$T^e_{i,j}=\frac{12(E_i'+m_i')(E_j'+m_j')}{\pi(2E_i'+m_i')(2E_j'+m_j')}\,I^e_{ij}\,, \qquad (3.34)$$

$$T^m_{i,j}=\frac{8}{\pi(2E_i'+m_i')(2E_j'+m_j')}\,I^m_{ij}\,. \qquad (3.35)$$

One can note from Table III that the color-electric contribution for the baryon masses vanishes when all the constituent-quark masses in a baryon are equal, whereas it is nonzero otherwise. However, even in the case of strange baryons, it would be seen, subsequently, that the color-electric contribution is quite small. Therefore the degeneracy among the baryons is essentially removed through the spin-spin interaction energy in the color-magnetic part.

IV. FIXATION OF PARAMETERS, RESULTS, AND CONCLUSIONS

In the foregoing sections we have shown that the zeroth order mass $M_B^0=E_B$ of a ground-state baryon arising out of the binding energies of the constituent quarks confined independently by a phenomenological average potential $V_q(r)$, which presumably represents the dominant nonperturbative gluon interactions, must be subjected to certain corrections due to the residual quark-pion and quark-gluon interactions together with that due to the spurious center-of-mass motion. All these corrections can be treated independently as though they are of the same order of magnitude so that we can obtain the physical mass of a low-lying ground-state baryon as

$$M_B=E_B+(\delta E_B)_{\text{c.m.}}+(\delta E_B)_P+(\delta E_B)_g^m+(\delta E)_g^e\,, \qquad (4.1)$$

where $(\delta E_B)_{\text{c.m.}}$ is the energy associated with the spurious c.m. motion [Eq. (3.5)], $(\delta E_B)_P$ is the pionic self-energy of the baryon [Eq. (3.14)], and $[(\delta E_B)_g^e+(\delta E_B)_g^m]$ is the color-electric and -magnetic interaction energies arising out of the residual one-gluon-exchange processes [Eq. (3.32)].

To calculate the terms on the right-hand side (RHS) of Eq. (4.1), we first all assume the potential parameters a and V_0 to be flavor independent and take the quark masses as $m_u=m_d\neq m_s$. However, for convenience the parameter V_0 is absorbed appropriately in m_q and E_q of Eq. (2.4) so as to obtain solutions leading to individual quark binding energy in terms of $m_q'=(m_q+V_0/2)$ and $E_q'=(E_q-V_0/2)$ through Eqs (2.9)–(2.13). Consequently, the computation of the energy correction terms in Eq. (4.1), and hence the physical mass M_B of the ground-state baryon, is found to depend on the choice of the effective Lagrangian mass parameters $m_q'(m_u'=m_d',m_s')$ and the potential parameter a alone.

In the Lagrangian formulation adopted here, we chose to fix m_q' in the current quark limit as

$$(m_u'=m_d',\ m_s')=(10,\ 205)\ \text{MeV}. \qquad (4.2)$$

Then we find that with a suitable choice of the potential parameter

$$a=386.05\ \text{MeV}, \qquad (4.3)$$

the energy eigenvalue condition (2.9) yields through relation (2.11) the individual quark effective binding energies

$$(E_u'=E_d',\ E_s')=(735,\ 850)\ \text{MeV}. \qquad (4.4)$$

Now, using a standard numerical method, we evaluate the integral expression I_π in Eq. (3.13) as I_π=296.283 MeV, which enables us to obtain the pionic self-energies of different baryons through Eq. (3.14). The values of $(\delta E_B)_P$ so obtained with $f^2_{NN\pi}$=0.083 for various baryons are predicted in Table III.

Then we evaluate the integral expressions for $I^{e,m}_{i,j}$ in Eqs. (3.31) and (3.32) with the help of standard numerical methods and calculate the terms $T^{e,m}_{ij}$ from Eqs. (3.34) and (3.35) which are necessary for computing $(\delta E_B)_g^{e,m}$. We find

$$(T^e_{uu},\ T^e_{us},\ T^e_{ss})=(553.57,\ 581.373,\ 622.01)\ \text{MeV}, \qquad (4.5)$$

$$(T^m_{uu},\ T^m_{us},\ T^m_{ss})=(63.95,\ 58.55,\ 54.60)\ \text{MeV}. \qquad (4.6)$$

Referring to the physical masses of N and Δ, which are

$$M_\Delta=[E_N^2-E_u\langle\vec{P}^2\rangle_u]^{1/2}+(\delta E_\Delta)_P+3\alpha_c T^m_{uu}\,, \qquad (4.7)$$

$$M_N=[E_N-E_u\langle\vec{P}^2\rangle_u]^{1/2}+(\delta E_N)_P-3\alpha_c T^m_{uu}\,, \qquad (4.8)$$

we find the QCD splitting among the N and Δ masses as

$$6\alpha_c T^M_{uu} = (M_\Delta - M_N) - [(\delta E_\Delta)_P - (\delta E_N)_P]. \quad (4.9)$$

Since $(M_\Delta - M_N) \simeq 292$ MeV and $[(\delta E_\Delta)_P - (\delta E_N)_P] \simeq 71$ MeV, as seen from Table II, we find that $6\alpha_c T^M_{uu} = 221.011$ MeV. This gives $\alpha_c = 0.576$, which is comparable with 0.55 found by DeGrand *et al.* [21] and is not too much different from the value 0.3–0.4 obtained in the CBM [22]. It must be pointed out here that we do not need anywhere near as large a value of α_c as in the original MIT work, where without including pionic corrections the QCD splitting was equated with $(M_\Delta - M_N) \simeq 292$ MeV. Finally, using the combination

$$[M_\Delta - (\delta E_\Delta)_P] + [M_N - (\delta E_N)_P] = 2\left[E_N^2 - \sum_u \langle \vec{P}^2 \rangle_u\right]^{1/2}, \quad (4.10)$$

we find $E_N = 3(E'_u + V_0/2)$, which enables us to fix the potential parameter V_0 independent of α_c at a value $V_0 = -496.75$ MeV. It must be noted here that the value of the $NN\pi$-coupling constant $f_{NN\pi}$, which has been used in the evaluation of pionic corrections, is obtained from Eqs. (3.15) and (3.16) by using c.m.-corrected expressions (3.17) for g_A. With the value of E_N calculated from Eq. (4.10), we find $g_A = 1.1223$, which yields $f_{NN\pi} = 0.288$ as against the experimental value 0.283. Now, using all the results thus obtained, one can calculate all the individual terms leading to the physical masses of various ground-state baryons. The calculated values of the energy correction terms for various baryons considered here are presented in Table II. Consequently, the physical masses of baryons such as N, Δ, Λ, Σ, Ξ, and Ξ^* are found to be in very good agreement with the corresponding experimental values. The quark-gluon coupling constant $\alpha_s = 0.576$ taken in our calculation is quite consistent with the idea of treating one-gluon-exchange effects in lowest order perturbation theory.

Thus we draw the following conclusions in the present work.

(i) The SU(3)-breaking effect due to the quark masses $m_u = m_d \neq m_s$ lifts the degeneracy in baryon masses through the energy term $[E_B + (\delta E_B)_{\text{c.m.}}]$ among the groups (N,Δ), $(\Lambda,\Sigma,\Sigma^*)$, (Ξ,Ξ^*), and Ω^-.

(ii) The constraint of chiral symmetry imposed on the baryon core removes the degeneracy partially through the spin-isospin interaction energy δM_B between N and Δ, Λ, and Σ, whereas Σ^* still remains degenerate with Σ and Ξ^* with Ξ.

(iii) The color-electric and -magnetic interaction energy arising out of the one-gluon exchange with the dominant color-magnetic part giving a spin-spin contribution removes the mass degeneracy completely among these baryons.

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