$\pi\pi$ final state interaction in $K\to\pi\pi$, $pp\to pp\pi\pi$, and related processes

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Final state interactions in the S-wave $\pi\pi$ system ($I=0,2$) are reexamined on the basis of the Omnès-Muskhelishvili equation and the coupled channel formalism. The contributions to the pion scalar form factor from ρ and $f_2(1270)$ exchange in the t channel and from the $f_0(980)$ s-channel resonance are separately evaluated and the role of the nontrivial polynomial in the Omnès function in a coupled channel situation is elucidated. Applications are made to $K\to\pi\pi$ and $pp\to pp\pi\pi$. It is found that the contribution from the f_0 resonance to the form factor is strongly reduced by a nearby zero. [S0556-2821(97)05505-7]

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I. INTRODUCTION

Final state interactions (FSI's) in the $\pi\pi$ system play an important role for many production reactions and meson decays. A case of long-standing interest is the $\Delta I=1/2$ rule in $K\to\pi\pi$ decays. The experimental ratio of the decay amplitudes A_I with isospin $I=0,2$ is [1]

$$\frac{A_0(K\to\pi\pi)}{A_2(K\to\pi\pi)}=22. \tag{1}$$

The calculated ratio is smaller [2] by at least a factor of 3 where this result includes perturbative QCD and soft-gluon corrections at the weak interaction vertex but no long-distance $\pi\pi$ FSI. In this paper we shall discuss the pionic FSI in the S wave aiming at a concrete application to the $\Delta I=1/2$ rule for the $K\to\pi\pi$ decay and the pion production reaction $pp\to\pi\pi pp$. Our analysis shows general features of FSI's which are relevant to other reactions involving pions or other hadrons.

The $\Delta I=1/2$ amplitude of the $K\to\pi\pi$ decay, as a function of $s=p_K^2$ where p_K is the kaon four-momentum, can be written in the form (see [2,3] and references therein)

$$\langle\pi\pi|A|K\rangle=iCf_\pi(s-m_\pi^2)F^0(s), \tag{2}$$

where C is a constant describing the short-range part of the process (by its order of magnitude C is close to the Fermi constant G_F), f_π is the pion decay constant, the factor $(s-m_\pi^2)$ explicitly takes the chiral symmetry into account, and the scalar form-factor $F^0(s)$ describes the final state interaction. Here the pions are assumed to be on the mass shell. Note that the definition of C depends on the normalization of $F^0(s)$. The calculations of the decay amplitude in the framework of the operator expansion [4] exploit the limit of zero pion momenta and therefore provide the combined strength of the product $CF^0(0)$. In order to get the physical amplitude, the FSI factor $F^0(m_K^2)/F^0(0)$ must be used. In the framework of the chiral perturbation theory, the expression $C(s-m_\pi^2)$ is interpreted as the first two terms of the power series expansion [3] and the form factor is normalized by the condition $F^0(m_\pi^2)=1$. The parameter C can then be determined from the data using the scalar form factor at $s=m_K^2$.

Several methods for the evaluation of FSI have been used in the literature. In one approach rescattering diagrams are evaluated directly. At low energies this has been done by applying chiral perturbation theory (CHPT) [5,6]. The relevant application in our context is the calculation of the scalar form factor of the pion in next-to-leading chiral order at low energies [7,8]. To extend the calculations to $s\underset{\sim}{<}1$ GeV^2, s-channel resonances and the coupling to the $K\bar{K}$ channel must be included. As a general tool the dispersive method based on the Omnès-Muskhelishvili (OM) equation [9,10] has turned out to be very efficient. It exploits analyticity and unitarity in order to connect the production or decay amplitude (or its form factor) with the amplitude of elastic $\pi\pi$ scattering. To solve the OM equation we shall take the scattering phases either from phase shift analysis or from a theoretical model. We shall choose a model which satisfies the requirements of unitarity and analyticity, and hence the OM equation automatically. The model with parameters fitted to the experimental constraints is described in Sec. III.

For the $K\to\pi\pi$ decay it was realized a long time ago that the nonperturbative long-distance effects must be included, and the combination of the attraction in the $I=0$ channel with the repulsion in the $I=2$ channel favors the $\Delta I=1/2$ rule. An enhancement of about a factor of 2 in the $I=0$ amplitude was estimated to result from the broad $\sigma(J^{PC}=0^{++})$ meson [4]. The FSI enhancement $F^0(m_K^2)/F^0(m_\pi^2)=1.4$ was obtained in [3] using the OM equation with a simple parametrization of the scattering phase consistent with the Weinberg low energy expansion. The analysis was done in CHPT to one loop in [8,11]. Rescattering in simple potential model was evaluated in [12,13] without regard to the energy dependence of the form factor. An extensive study of the FSI effects in the S-wave $\pi\pi$ system in production reactions and J/ψ and ψ' decays was conducted in [14–17]. Unitarity and analyticity of the production amplitudes was taken into account in a self-consistent way. It was noticed, in particular, that a narrow resonance (f_0 in the present notation) in the $\pi\pi$ scattering phase $\delta^{I=0}_{J=0}(s)$ corresponds to a shoulder in the $\pi\pi$ effective mass distribution in the reaction $pp\to pp\pi\pi$ [15,16]. The occurrence of a shoulder rather than a peak results from an interplay of the resonant pole and a nearby zero. We shall discuss this feature in

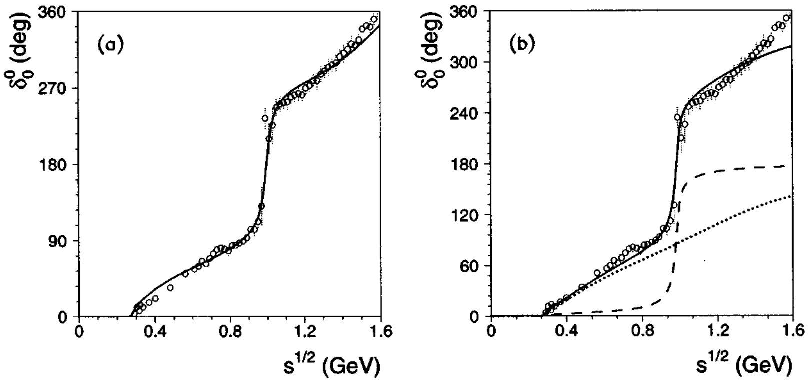

FIG. 1. The $\pi\pi$ S-wave scattering phase δ_0^0 vs $\sqrt{s}$: (a) the $K1$ fit from [16], (b) the meson exchange model described in the text (solid line: the total phase; dotted line: $\rho+f_2$ t exchange; dashed line: f_0 resonance). The experimental data are from [24–26].

detail in Sec. III. Resonance phenomena in the $\pi\pi$ S wave were emphasized in [18] where the f_0 resonance was discussed within a single-resonance model for the decay of a light Higgs boson. The prediction of a drastic enhancement because of the f_0 resonance is in striking contrast with the findings for the $pp\to pp\pi\pi$ reaction in [16]. An analysis of the $\pi\pi$ final state interaction in the framework of the coupled channel OM equation was performed in [7] for the decay of a light Higgs boson decay $H\to\pi\pi$. In this evaluation the f_0 resonance also produced significant effects far below the $K\bar{K}$ threshold.

The dynamics of the $I=0$ S-wave $\pi\pi$ interaction is characterized by several overlapping resonances [16,17,19,20], both narrow and broad. In the present paper we shall analyze the relative importance of the dynamical mechanisms in $\pi\pi$ scattering for the calculation of the form factors occurring in meson decays and in the pion pair production in pp scattering.

In Sec. II we prepare the ground with an evaluation of the OM equation for a restricted energy range (the cutoff used excludes the f_0 resonance). With respect to the pion dynamics we shall mainly use the picture of [21] which combines the ρ and f_2 exchanges in the t channel with the f_0 resonance in the s channel. The phases of the $I=0,2$ S-wave scattering are reproduced quite accurately in this model. To understand the role of the f_0 resonance for the calculation of the form factor in the $I=0$ channel, we shall introduce a coupled channel ansatz in Sec. III. The final state interaction effects in the $K\to\pi\pi$ decay are evaluated and the conclusions are presented in Sec. IV.

II. FORM FACTORS FROM THE OMNÈS-MUSKHELISHVILI EQUATION

The form factor $F(s)$ represents the effect of the FSI in the decay amplitude. The OM equation [9,10] relates $F(s)$ to the elastic final state scattering phase $\delta(s)$. For a single-channel problem the OM equation is

$$F(s)=1+\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{e^{-i\delta(s')}\sin\delta(s')F(s')}{s'(s'-s)}ds', \qquad (3)$$

where a once-subtracted form has been used. The general solution of Eq. (3) has the form

$$F(s)=P(s)\exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta(s')}{s'(s'-s)}ds'\right) \qquad (4)$$

as long as

$$\delta(s)\to\text{ const},\quad \frac{|F(s)|}{s}\to 0 \quad \text{for}\quad s\to\infty. \qquad (5)$$

The polynomial $P(s)$ is real for s real. In special cases, such as potential scattering without bound states, $P(s)$ is a constant, but in general additional information is required to determine it. In the following we normalize $F(0)$ to unity since we are not concerned with the overall normalization representing the short-range properties of the decay amplitude discussed in Sec. I. Note that the factor $(s-m_\pi^2)$ in the decay amplitude (2) is real for real s and could be combined with the polynomial $P(s)$; however, we prefer to use the form (2) explicitly displaying the chiral symmetry behavior.

For $K\to\pi\pi$ in the simplest evaluations, single-channel $\pi\pi$ scattering data are used below the $K\bar{K}$ threshold (the coupling to the 4π channel is known to be small). To exploit Eq. (4) we need a smooth parametrization of $\delta(s)$. To demonstrate the sensitivity of the result to the input phases, we use two parametrizations of the $\pi\pi$ $J=I=0$ scattering phase $\delta_0^0(s)$. Figure 1(a) shows $\delta_0^0(s)$ from the phase shift analysis [16]. In Fig. 1(b) we show the same phase from the meson exchange model mentioned earlier and developed in [21–23]. We briefly recapitulate the ingredients for the benefit of the later discussion. The phases in Fig. 1(b) correspond to unitarized ρ and f_2 exchange with the f_0 resonance added in the s channel. The individual contributions are shown in the figure as explained in the caption. The Born term for the ρ exchange is

$$T(s,t)_{BA}^{I=0}=2G\left(\frac{s-u}{m_\rho^2-t}+\frac{s-t}{m_\rho^2-u}\right), \qquad (6)$$

$$T(s,t)_{BA}^{I=2}=-\frac{1}{2}T(s,t)_{BA}^{I=0}, \qquad (7)$$

where m_ρ is the mass of the ρ meson, $G=g_{\rho\pi\pi}^2/32\pi$, and $g_{\rho\pi\pi}$ is the $\rho\pi\pi$ coupling constant. The $I=2$ amplitude will be needed later. The S-wave projection is

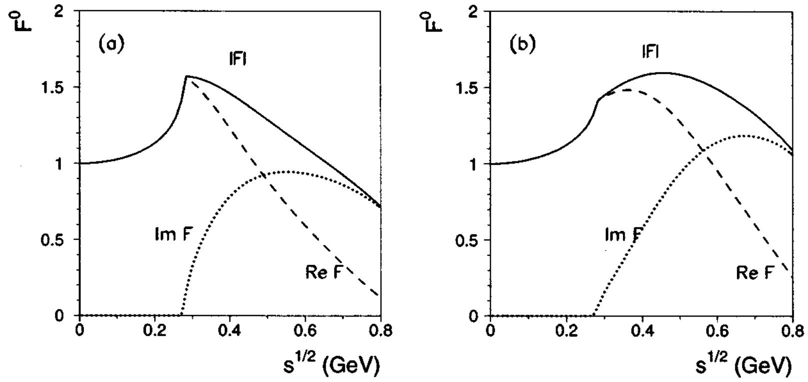

FIG. 2. The pion scalar form-factor $F^{I=0}(s)$ vs $\sqrt{s}$ evaluated using the OM equation with the cutoff $\Lambda=0.975$ GeV and the $\pi\pi$ scattering phases as shown in Figs. 1(a) and 1(b), correspondingly.

$$T^{I=0}_{BA-S}(s)=4G\left[\frac{2s+m_\rho^2-4m_\pi^2}{s-4m_\pi^2}\ln\left(1+\frac{s-4m_\pi^2}{m_\rho^2}\right)-1\right], \tag{8}$$

K-matrix unitarization is introduced by

$$T^I_S(s)=\frac{K^I_S(s)}{1-i\rho(s)K^I_S(s)}, \tag{9}$$

where

$$K^I_S(s)=T^I_{BA-S}(s), \tag{10}$$

and $\rho(s)=(1-4m_\pi^2/s)^{1/2}$. The coupling constant $g_{\rho\pi\pi}$ is determined from the ρ-meson decay width in the crossed $I=1$ channel after K-matrix unitarization [21]. The corresponding value is $g_{\rho\pi\pi}=6.04$ which is close to the result obtained from the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [27], $g_{\rho\pi\pi}=m_\rho/\sqrt{2}f_\pi$.

The corresponding expression for f_2 exchange in Born approximation [21] is

$$K_{f_2}(s)=2G_{f_2}\left\{\frac{-11}{3}s-\frac{2}{3}m_{f_2}^2+4m_\pi^2+\frac{(2s+m_{f_2}^2-4m_\pi^2)^2-(m_{f_2}^2-4m_\pi^2)^2/3}{s-4m_\pi^2}\times\ln\left(1+\frac{s-4m_\pi^2}{m_{f_2}^2}\right)\right\}, \tag{11}$$

where $G_{f_2}\simeq 0.19\ \mathrm{GeV}^{-2}$.

The s-channel f_0 resonance is included using the Dalitz-Tuan representation [28], i.e., the S matrix is considered to be the product of the S matrices corresponding to the individual mechanisms. The corresponding Breit-Wigner parametrization is taken from [21]

$$S(s)=\frac{s-M_r^2-ig_1\rho_1(s)+ig_2\rho_2(s)}{s-M_r^2+ig_1\rho_1(s)+ig_2\rho_2(s)}, \tag{12}$$

where

$$\rho_1(s)=\sqrt{1-4m_\pi^2/s},\quad \rho_2(s)=\sqrt{1-4m_K^2/s}, \tag{13}$$

and the resonance parameters are $M_r=0.9535$ GeV, $g_1=0.1108\ \mathrm{GeV}^2$, $g_2=0.4229\ \mathrm{GeV}^2$, respectively.

The scattering phase in the meson exchange model gives a good description of the data for $s<1.4\ \mathrm{GeV}^2$ [see Fig. 1(b)] which justifies using the simple approach based on Eqs. (9) and (10) as long as we just need a smooth parametrization of the scattering phase for the OM equation. A more elaborate meson exchange model [29] where the scattering amplitude is calculated from the Lippmann-Schwinger equation would lead to similar results. The $\pi\pi$ $I=0$ scattering length in the meson exchange model $a_0^0=0.24m_\pi^{-1}$ is in good agreement with the Weinberg low energy theorems $a_0^0=0.20m_\pi^{-1}$ [30] and the $\pi\pi$ scattering data $a_0^0=(0.28\pm0.05)m_\pi^{-1}$ [26].

In order to show the sensitivity of the form factors to the variations in the phase, we shall evaluate the OM equation using the two sets shown in Figs. 1(a) and 1(b). For small s the integral of Eq. (4) is dominated by low energies, see [8]. As a first step we evaluate in this section the OM equation with a cutoff Λ. Choosing $\Lambda=0.975$ GeV we exclude the f_0 and the $K\bar{K}$ threshold region as in some early applications. We therefore write

$$F(s)=F_\Lambda(s)P(s), \tag{14}$$

where

$$F_\Lambda(s)=\exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\Lambda^2}\frac{\delta(s')}{s'(s'-s)}ds'\right). \tag{15}$$

The polynomial $P(s)$ represents the contribution from high energies and any other dynamics not included so far. We observe that any additive contributions in the phase lead to a multiplicative factor in F, see Eq. (4). We shall use

$$P(s)=1+bs, \tag{16}$$

where the parameter b is related to the scalar radius of the pion by

$$F(s)=1+\frac{1}{6}\langle r_s^2\rangle\ s\ . \tag{17}$$

When plotting Fig. 2, we have adjusted the polynomial (16) in order to have $\langle r_s^2\rangle=0.6\ \mathrm{fm}^2$ [8] in both cases. This leads to $b=0.32\ \mathrm{GeV}^{-2}$ for Fig. 2(a) and $b=0.83\ \mathrm{GeV}^{-2}$ for Fig. 2(b).

Figure 2 demonstrates that the form factor is rather sensitive to the $\pi\pi$ scattering phase shift at low energies, one of the important parameters being the scattering length a_0^0. For the meson exchange model $a_0^0=0.24m_\pi^{-1}$ in good agreement with the data, and the absolute value of the form factor continues to rise between the $\pi\pi$ threshold and $\sqrt{s}\approx 0.5$ GeV. On the other hand, the form factor corresponding to the phase in Fig. 2(a) displays a more prominent cusp and decreases above the $\pi\pi$ threshold. This difference is not surprising because the phase of Fig. 2(a) was obtained from the fit of the data at $\sqrt{s}>0.6$ GeV and does not provide a good description of the low energy region (the corresponding value of $a_0^0=0.51m_\pi^{-1}$ is too large). For $\sqrt{s}<0.5$ GeV the result in Fig. 2(b) is close to the solution of the coupled channel OM equation in [8].

III. PROTECTIVE ZERO AND THE F_0 RESONANCE

For energies around $s=1$ GeV^2 the truncation in the calculation of the form factor must be abandoned and the role of the f_0 and the $K\bar{K}$ threshold discussed. The resonant part of the phase will be defined by

$$\delta_{\rm res}(s)=\arctan\frac{gk(s)}{(M_r^2-s)},\tag{18}$$

corresponding to the resonance amplitude

$$T_{\rm res}(s)=\frac{gk(s)}{s-M_r^2+igk(s)},\tag{19}$$

where $k(s)=\sqrt{s-4m_\pi^2}/2$, leading to

$$F_{\rm res}(s)=\exp\left(\frac{s}{\pi}\int_{s_0}^{\infty}\frac{\delta_{\rm res}(s')}{s'(s'-s)}ds'\right)\tag{20}$$

$$=\frac{M_r^2+gm_\pi}{M_r^2-s-igk(s)}.\tag{21}$$

Inserting this phase naively into Eq. (15) has the undesirable feature that

$$|F_{\rm res}(s)|\xrightarrow{s\rightarrow\pm\infty}0\tag{22}$$

rather than unity which would be expected at high energies where the resonance contribution should vanish. The wrong asymptotic form is actually imposed on the whole solution by means of phase additivity. In Fig. 3 the dotted line corresponds to the naive evaluation of $F(s)$, with $P(s)$ being set to unity. Apart from the wrong asymptotics it is also seen that the f_0 resonance dominates the form factor far outside the resonance region $M_{f_0}\pm\Gamma_{f_0}$. Recall that the experimental width is $\Gamma_{f_0}\approx 60$ MeV. It is clear that this defect should be compensated by a nontrivial polynomial $P(s)$ in the solution of the OM equation.

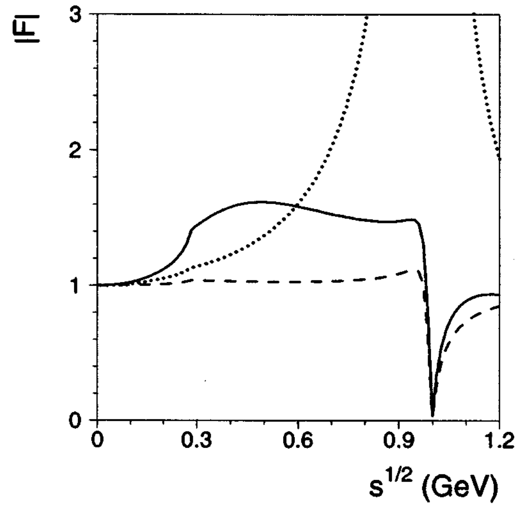

FIG. 3. The pion scalar form-factor $F^{I=0}(s)$ vs $\sqrt{s}$. The solid line corresponds to ρ and f_2 exchange and an f_0 resonance including the polynomial (protective zero). The f_0 resonance alone leads to the dotted line (OM equation without a polynomial) and to the dashed line with polynomial.

A. The OM equation for a resonance in the Weisskopf-Wigner model

We shall study the modification required for a sensible inclusion of a direct channel resonance into the OM equation by means of a very simple coupled channel model. The following nonrelativistic ansatz, which is a variant of the Weisskopf-Wigner (WW) model with two channels, already has all the necessary ingredients. Channel 1 is the scattering channel of interest which has no diagonal potential. It will be denoted by its relative momentum $|k\rangle$. Channel 2 has a bound state $|b\rangle$, and the rest of the dynamics in this channel is ignored. The only interaction in the model results from the coupling of the first channel (the $\pi\pi$ channel) to the bound state $|b\rangle$ (the bare f_0 resonance considered as a bound state of either a $K\bar{K}$ or a quark-antiquark system). We assume a channel coupling of the form

$$\langle k|V|b\rangle=\gamma\xi(k)=\frac{\gamma}{k^2+\mu^2},\tag{23}$$

where γ is the coupling constant (dimension $[\gamma]=[k]^{3/2}$) and μ characterizes the range of interaction. The T matrix satisfies the Lippmann-Schwinger equation

$$T(E)=V\frac{|b\rangle\langle b|}{(E-E_r)}V[1+G_0(E)T(E)],\tag{24}$$

where $G_0(E)$ is the free Green function and $E=k^2/2m$ (m is the reduced mass). The solution for the scattering amplitude has the form

$$f(k)=-2m\langle k|T(E)|k\rangle\tag{25}$$

$$=\frac{-2m\gamma^2\xi^2(k)}{\dfrac{k^2}{2m}-E_b-\gamma^2 D(k)},\tag{26}$$

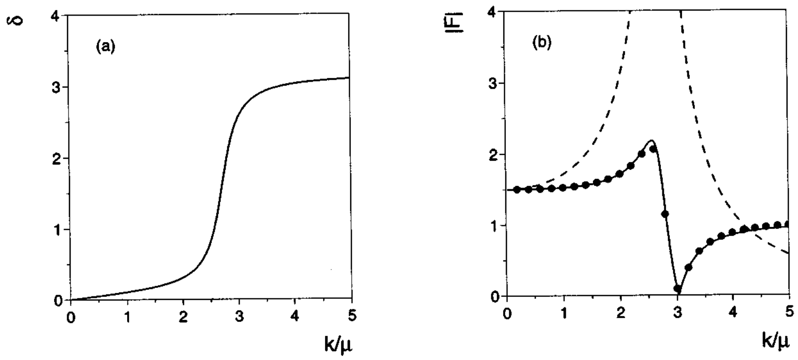

FIG. 4. The momentum dependence of the scattering phase (a) and the form factor (b) for a resonance in the WW model. The solid line is the exact solution (29), the dashed line is the solution of the OM equation without polynomial factor. The dots show the approximate solution of the OM equation with the factor $(E-E_z)$.

with

$$D(k)=\langle b|VG_0(E)V|b\rangle=\frac{m}{\mu(k+i\mu)^2} \quad . \tag{27}$$

In our model the form factor $F(k)$ describing the final state interaction is equal to the scattering wave function at zero distance according to standard results from scattering theory [31]:

$$F(k)=\langle r=0|k^{(+)}\rangle=\langle r=0|k\rangle+\langle r=0|G_0(E)T(E)|k\rangle \tag{28}$$

$$=1+\frac{\gamma^2 Z(k)\xi(k)}{\frac{k^2}{2m}-E_r-\gamma^2 D(k)}, \tag{29}$$

with

$$Z(k)=\frac{-2im}{k+i\mu} \quad . \tag{30}$$

In Fig. 4 we show the scattering phase and the form factor for the WW model for $\mu=5m$, $\gamma=10\mu^{3/2}$, $E_r=4m$. The pole produces a resonance peak in the energy dependence of the form factor which is damped by a nearby zero restoring the right limit $F\rightarrow 1$ for $E\rightarrow\infty$. The reduction imposed by this zero is enormous.

Formula (29) can be rewritten explicitly showing the interplay of the pole and the zero:

$$F(k)=\frac{A(E)}{B(E)}, \tag{31}$$

$$A(E)=E-E_r-\frac{\gamma^2 m}{\mu}\frac{1}{(k^2+\mu^2)}, \tag{32}$$

$$B(E)=E-E_r-\frac{\gamma^2 m}{\mu}\frac{(k^2-\mu^2-2i\mu k)}{(k^2+\mu^2)^2} \quad . \tag{33}$$

In the limit of weak coupling the resonance in the scattering channel is directly connected to the bound state in the continuum which has an energy shift ΔE_r and a width Γ_r:

$$\Delta E_r=\frac{\gamma^2 m(k^2-\mu^2)}{\mu(k^2+\mu^2)^2}, \tag{34}$$

$$\Gamma_r=\frac{4\gamma^2 mk}{(k_r^2+\mu^2)^2}, \tag{35}$$

where $E_r=k_r^2/2m$. The form factor in the vicinity of the resonance has the form

$$F(k)=\frac{E-E_z}{E-(E_r+\Delta E_r-i\Gamma_r/2)}, \tag{36}$$

and the zero E_z is located near the resonance energy $E_r+\Delta E_r$ at

$$E_z=E_r+\Delta E_r+\frac{\mu}{k_r}\Gamma_r . \tag{37}$$

If $|E_z-E_r-\Delta E_r|>\Gamma_r$, the resonance produces a pronounced peak followed by a dip in the energy dependence of the form factor. In case $|E_z-E_r-\Delta E_r|<\Gamma_r$, the energy dependence coming from the pole is damped completely by the zero in the nominator, and only a dip is visible in the form factor. Notice that the zero is of dynamical nature and disappears for vanishing channel coupling: $F\rightarrow 1$ as $\gamma\rightarrow 0$.

Since $A(E)$ is a real symmetric function of momentum k, it does not contribute to the elastic scattering amplitude. The solution of the OM equation without a polynomial factor reflects only the resonance pole in formula (36) as shown in Fig. 4, dashed line. By including the factor $(E-E_z)$, one gets

$$F(E)=F(0)\frac{(E_z-E)}{E_z}\exp\left(\frac{E}{\pi}\int_0^\infty\frac{\delta(E')}{E'(E'-E)}dE'\right), \tag{38}$$

which is very close to the exact solution[1] of the WW model.

These results characterize a *coupled channel* resonance. The scattering phase beyond the resonance does not decrease as occurs for a direct channel *potential* resonance[2] where no

[1]A careful analysis of the OM equation for the model considered shows that there is an extra factor $(k^2+\nu^2)/(k^2+\mu^2)$ resulting from the singularities in the upper half-plane of complex momentum k: a pole at $k=i\mu$ and a nearby zero at $k=i\nu$. For our example this factor is close to 1 in the region of the resonance.

[2]In the literature the first category is often called *normal resonance* and the second one *molecular* or *bootstrap resonance,* see, e.g., [16] and references therein.

extra polynomial factor appears in the solution of the OM equation. For a potential resonance the decrease of the phase for $s\rightarrow\infty$ guarantees that the asymptotic limit of the form factor is one.

It must be emphasized that in the WW model considered, the resonance-dip structure occurs only in processes where the particles in the scattering channel are produced at small distance due to some extraneous interaction which can be treated perturbatively, so that the momentum dependence of the production amplitude is entirely determined by the form-factor $F(k)$ given by Eq. (29) (this is relevant for the $K\rightarrow\pi\pi$ decay). This situation must be distinguished from a situation where the original bound state $|b\rangle$ is produced as a resonance with amplitude C and it then decays into the scattering channel. The corresponding amplitude with rescattering included is

$$T_b(k)=C\frac{\gamma\xi(k)}{\frac{k^2}{2m}-E_r-\gamma^2 D(k)}, \tag{39}$$

which has a purely resonant behavior, there is no nearby zero. Studying the energy dependence of the data in the vicinity of the resonance, one can determine whether this situation is realized for the process in question.

B. Application to the f_0 resonance and constraint from $pp\rightarrow pp\pi\pi$

To evaluate the role of the f_0 resonance for $K\rightarrow\pi\pi$ decay, we use the S matrix in Breit-Wigner form fitted to data [21], see Eq. (12). As we demonstrated in Sec. III A, the polynomial in the solution of the OM equation is expected to have a zero at $s=s_z$ close to the resonance:

$$P(s)=1-\frac{s}{s_z}. \tag{40}$$

Note that $s_z\rightarrow M_r^2$ as $g_1\rightarrow 0$. In order to fix the position of the zero, we use information from a related process and study the effective mass distribution ($M=\sqrt{s}$) of pion pairs produced in the reaction $pp\rightarrow pp\pi\pi$ [32], which can be expressed by [15]

$$\frac{d\sigma}{dM}\sim\frac{(M^2-4m_\pi^2)^{1/2}}{M^3}|F(M^2)|^2. \tag{41}$$

Including the polynomial (40) into the calculation of the form-factor $F(s)$ (f_0 plus ρ and f_2 exchange), we obtain $s_z=1.0\ \ \mathrm{GeV}^2$ for the position of the zero, see Fig. 5. The fit shown for the mass distribution $d\sigma/dM$ also contains a factor $(1+0.25s)$ in the polynomial and an overall normalization constant. The position of the zero, however, is determined very precisely from nearby data alone. The corresponding scalar form factor will be discussed in Sec. IV.

Note that the factor containing the zero can be incorporated into a formal solution of the OM equation, if a physically equivalent discontinuous scattering phase is introduced:

$$\overline{\delta(s)}=\delta(s)-\pi\theta(s-s_z)\ \ . \tag{42}$$

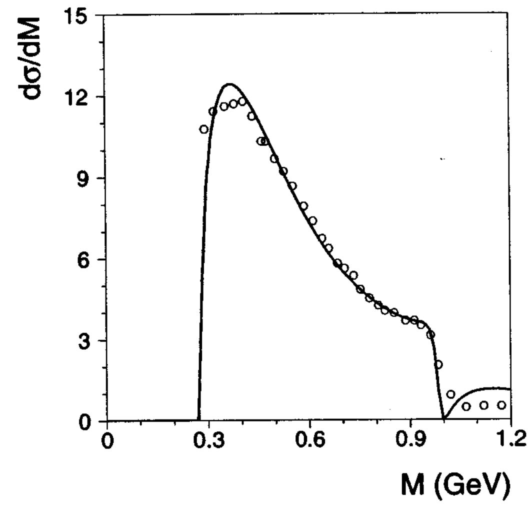

FIG. 5. The effective mass distribution of pion pairs in $pp\rightarrow pp\pi\pi$ vs $M=\sqrt{s}$. The data are from [32].

It is interesting to note that the description of the pion pair distribution $d\sigma/dM$ in [15,16] was seemingly achieved using the trivial polynomial $P(s)=1$. However, the elastic phase was calculated from the expression $\Phi=\arctan(\mathrm{Im}\ T_{11}/\mathrm{Re}\ \ T_{11})$. In the presence of inelasticities the phase of T_{11} is bounded to the interval $[0,\pi]$ by the requirement of continuity. When Re T_{11} changes from negative to positive due to the sharp resonance rise of $\delta(s)$, the phase Φ drops sharply by nearly π. With this choice the Omnès function develops a zero close to the point where $\delta=\pi$. While this provides a good description of the data, the introduction of the zero in this way appears to be accidental. For instance, in the model considered in Sec. III A there is no connection between the position of the zero and the condition $\overline{\delta}=\pi$. Also, if the scattering phase δ reached π before the KK threshold, the zero factor would not be obtained from using the phase prescription for Φ quoted above.

IV. RESULTS FOR THE $K\rightarrow\pi\pi$ DECAY

We have prepared the ground for the $I=0$ S-wave final state interaction in the preceding sections. The solid line in Fig. 3 shows the net result for the model combining the ρ and f_2 exchange with the f_0 resonance. The resonance in the form factor is protected by the zero at $s_z=1\ \ \mathrm{GeV}^2$ as determined from the pion pair production data. At the kaon mass the $I=0$ enhancement factor is $F(m_K^2)=1.62$, a result which is similar to the values obtained in the literature quoted above. From ρ exchange alone we obtain $F(m_K^2)=1.38$, ρ and f_2 give $F(m_K^2)=1.57$ while the enhancement from the f_0 resonance alone is $F(m_K^2)=1.03$. For the complete form factor the reduction of the f_0 contribution induced by the protective zero is, of course, crucial. The effects of the zero and the resonance largely cancel and only a very small contribution to the form factor far away from the pole (zero) survives. For example, at $s=0$ the pion scalar radius is: $\langle r_s^2\rangle=0.52\ \ \mathrm{fm}^2$ when only considering ρ and f_2 exchanges. When including the resonance protected by the zero, we have $\langle r_s^2\rangle=0.58\ \ \mathrm{fm}^2$. We see that the inclusion of the reso-

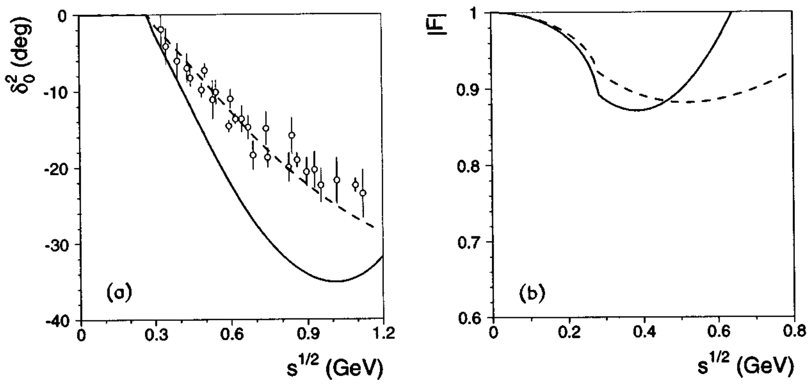

FIG. 6. The $\pi\pi$ $I=2$ S-wave scattering phase $\delta_0^{I=2}$ vs $\sqrt{s}$ (a) and the form-factor $F^{I=2}(s)$ (b). Solid line: $\rho+f_2$ exchange, dashed line: ρ exchange with vertex form factor. The experimental data are from [33].

nance does improve the result on the scalar radius but avoids too large an effect. Our full result is very close to the value obtained in [8] where $\langle r_s^2\rangle$ is determined from chiral perturbation theory. Without the zero we would have obtained a rather large value $\langle r_s^2\rangle=0.81$ fm^2.

In order to complete the evaluation of the overall $\Delta I=1/2$ enhancement factor, the contribution of the $I=2$ channel must be evaluated as well. Because of the signature of the crossing matrix, the contribution from ρ exchange is repulsive in the $I=2$ channel, see Eq. (6). On the other hand, f_2 exchange does not change sign relative to the $I=0$ channel leading to destructive interference between ρ and f_2 for the isotensor. The solid line in Fig. 6(a) shows the unitarized sum of ρ and f_2 exchange. Also shown is ρ exchange modified by a vertex form factor with monopole range $\Lambda_\rho=1.5$ GeV (dashed line) which is a good effective parametrization of the data. With this modification the $\pi\pi$ $I=2$ scattering length $a_0^2=-0.052m_\pi^{-1}$ is in fair agreement with the result of the soft-pion theory[3] $a_0^2=-0.06m_\pi^{-1}$ [30]. The phases at higher energies are not known, but fortunately the form factor at $\sqrt{s}=m_K$ is not sensitive to this region. The corresponding form-factor $F^{I=2}(s)$ is shown in Fig. 6(b). At the kaon mass we obtain a reduction factor $F^{I=2}(m_K^2)=0.9$ leading to a combined $\Delta I=1/2$ enhancement of $F^{I=0}(m_K^2)/F^{I=2}(m_K^2)=1.81$ which is satisfactory, but slightly less than the value required by the data.

[3]Note that the Born approximation for ρ exchange alone, Eqs. (6), (7), gives the ratio $a_0^0/a_0^2=-2$ while the low energy theorems predict $a_0^0/a_0^2=-7/2$ [30]. The construction of a chiral-invariant Lagrangian with ρ mesons was first considered in [34]. For more discussion of the scattering lengths in the ρ-exchange model see [23].

V. CONCLUSION

We conclude that the ρ- and f_2-exchange interactions remain the dominant mechanisms for the FSI enhancement factor in the $\Delta I=1/2$ rule in $K\to\pi\pi$. Since ρ exchange generates a broad pole in the $I=0$ S-wave amplitude [21,23], one can associate this enhancement with a σ meson. The f_0 resonance plays a minor role. This is because of the occurrence of a protective zero at $s=1$ GeV^2, modifying the polynomial in the OM equation. The nature and position of this zero has been verified by analyzing pion pair production in $pp\to pp\pi\pi$ where the f_0 resonance only leads to a small shoulder in the mass distribution.

A simple coupled channel model describes the resonance-dip structure in the form factor very adequately and illuminates the appearance of a nontrivial polynomial in the solution of the OM equation when the scattering involves a coupled channel resonance. We expect that this damping mechanism will be applicable to many other decay and production reactions in the vicinity of a coupled channel resonance.

As a consequence, all other higher resonances in the $\pi\pi$ channel (above 1 GeV), which have coupled channel origin, play a minor role in the FSI effects at $s=m_K^2$ and below. This makes the OM calculations of these effects only weakly model dependent if only the model reproduces the scattering data below the f_0 resonance. In particular, the previous OM calculations of the scalar form factor in the framework of the chiral perturbation theory (see [8] and references therein) will get only a few percent correction at $s=m_K^2$ if the higher resonances are taken into account.

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