Bremsstrahlung correction for baryon β decays in the four-body region of the Dalitz plot. II. Neutral baryons

S. R. Juárez W.

Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9, Unidad Prof. Adolfo López Mateos, Col. Lindavista, C.P. 07738 México D.F., Mexico (Received 22 February 1996)

A very accurate analytical expression and the corresponding numerical values for the bremsstrahlung of semileptonic decays of unpolarized neutral baryons, in the region of the Dalitz plot that covers the four-body events, is obtained. The same approach as the one considered for the charged baryons case is used here. These results contain all the terms of the order α times the momentum transfer. They are suitable for a model-independent experimental analysis, when high statistics decays of ordinary baryons as well as medium statistics decays of heavy quark baryons are considered. [S0556-2821(97)02505-8]

PACS number(s): 13.40.Ks, 13.30.Ce

I. INTRODUCTION

Experiments for several hyperon semileptonic decays have now reached a level of precision where the radiative corrections (RCs) are expected to be significant compared with the experimental errors [1]. It is, therefore, important to take into account the most precise formulas available when comparing the experimental information with the predictions of the theory. The precise radiative corrections are necessary to reduce the theoretical and experimental uncertainties so that the theoretical parameters of these decays are determined rigorously.

The RCs to hyperon semileptonic decays have been discussed extensively, with different approaches (see Ref. [2] and references therein), because of their relevance to the problem of universality of weak interactions. Precise and reliable formulas for the RCs to various measurable quantities, in the three-body region (TBR) of the Dalitz plot (DP), and some in the four-body region (FBR) for the charged baryon decays [3], have been recently published.

The knowledge of the contributions of the RC in the FBR of the DP is relevant to obtain a complete information about the spectrum of the produced fermions and, therefore, to obtain the relative RC of the total decay rate.

In Ref. [3] the *charged* hyperon decay RCs in the FBR have been given. The results of this paper cannot be applied to the neutral hyperon decays because of the different photon couplings to the initial and final hyperon. This changes the kinematical variables in the matrix elements and the final precise result. The most important decay with initial neutral hyperon is $\Lambda \rightarrow p e \overline{\nu}$ and it is necessary to consider this case.

The subject of this paper is to obtain in a simplified analytical form the RC to the four-body leptonic decays of neutral hyperons and to calculate its numerical values. The same procedure is adopted as that used in Ref. [3] where the radiative corrections to the four-body leptonic decays of charged hyperons are given in terms of the energies of the produced fermions which are measurable quantities. The final result is obtained in an analytical way which is applicable for photon bremsstrahlung calculations in any neutral hyperon decay. The important feature of the analytic form is the possibility of explicit determination of the RC everywhere in the DP, even at the boundaries where the result is logarithmically divergent.

The plan of the paper is the following. In Sec. II the transition amplitude for the bremsstrahlung is presented. In Sec. III the differential decay rate with all the $\alpha q/\pi M_1$ terms included is discussed. A closed expression for the several bremsstrahlung contributions is obtained by analytical means. In Sec. IV all the partial results are collected into a final completely integrated analytical expression. The corresponding numerical results are obtained and compared to those published in Refs. [4,5].

II. TRANSITION AMPLITUDE

The emission of the real photon in the process

$$A^{0}(p_{1}) \to B^{+}(p_{2}) + e^{-}(\ell) + \overline{\nu}_{e}(p_{\nu}) + \gamma(k)$$
(1)

is described as a radiative correction to the semileptonic decay of the neutral hyperon, where A^0 and B^+ correspond to the neutral and charged baryons and e^- and $\overline{\nu}_e$ denote the lepton and its neutrino counterpart, respectively, and the γ corresponds to the photon.

The uncorrected matrix element M_0 (without the emission of the real photon) for the decay in Eq. (1) is given by the product of the matrix elements of the baryonic weak current and of the leptonic current:

$$M_0 = \frac{G_v}{\sqrt{2}} \overline{u}_B W_\mu u_A \overline{u}_{\mathscr{O}} O_\mu v_\nu, \qquad (2)$$

where $G_v = G_\mu V_{ij}$, G_μ is the muon decay coupling constant, and V_{ij} is the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix element. We have

2889

© 1997 The American Physical Society

$$W_{\mu} = f_{1}(q^{2}) \gamma_{\mu} + \frac{f_{2}(q^{2})}{M_{1}} \sigma_{\mu\nu}q_{\nu} + \frac{f_{3}(q^{2})}{M_{1}}q_{\mu} + \left[g_{1}(q^{2}) \gamma_{\mu} + \frac{g_{2}(q^{2})}{M_{1}} \sigma_{\mu\nu}q_{\nu} + \frac{g_{3}(q^{2})}{M_{1}}q_{\mu}\right]\gamma_{5},$$
(3)

and

$$D_{\mu} = \gamma_{\mu}(1 + \gamma_{5}).$$

$$p_{1} = (E_{1}, \vec{p}_{1}), \quad p_{2} = (E_{2}, \vec{p}_{2}), \quad \ell = (E, \vec{\ell}),$$

$$p_{\nu} = (E_{\nu}, \vec{p}_{\nu}), \quad \text{and} \ k = (k_{0}, \vec{k})$$

are the baryonic, leptonic, and photonic four-momenta, respectively. The M_1 , M_2 , m, m_{ν} , and m_k denote their masses, and $q = p_1 - p_2$ is the four-momentum transfer. We shall assume throughout this paper that the neutrino and the photon are massless particles. Our metric and γ -matrix conventions are those of Ref. [6]. The order α bremsstrahlung amplitude is given by using the Low theorem, Refs. [7,8]. In order to obtain the bremsstrahlung correction to the DP we will follow the discussion and adapt the results of Secs. III and IV of Ref. [6]. We are interested in the process in Eq. (1), when it takes place in the FBR of the DP defined in Sec. II in Ref. [3].

We shall first obtain the amplitude of this process and afterwards we shall consider separately the amplitude which generates the infrared divergence in the TBR. Next, we shall obtain a complete expression for the differential bremsstrahlung decay rate that gives the DP with RC of process in Eq. (1).

What we want is the amplitude with all the $\alpha q/\pi M_1$ terms. It has been shown in Ref. [9] that these terms can be obtained in a model-independent fashion.

We only have to reproduce the model-independent amplitude, in terms of the Dirac form factors, given in Eq. (23) of Ref. [6].

The total transition amplitude can be written as

$$M_{BN} = M_{1N} + M_{2N} + M_{3N}, \qquad (4)$$

with

$$M_{1N} = e M_0 \left(\frac{\epsilon \cdot \ell}{\ell \cdot k} - \frac{\epsilon \cdot p_2}{p_2 \cdot k} \right), \tag{5}$$

$$M_{2N} = \frac{eG_v}{\sqrt{2}} \epsilon_{\mu} \overline{u_B} W_{\lambda} u_A \overline{u_\ell} \frac{\gamma_{\mu} k}{2\ell \cdot k} O_{\lambda} v_{\nu}, \qquad (6)$$

and

$$M_{3N} = \frac{-G_{v}}{\sqrt{2}} \overline{u}_{\ell} O_{\lambda} v_{v} \epsilon_{\mu} \overline{u}_{B} \left\{ \frac{e \gamma_{\mu} k W_{\lambda}}{2p_{2} \cdot k} + \kappa_{1} W_{\lambda} \frac{\not{p}_{1} + M_{1}}{2p_{1} \cdot k} \right.$$
$$\times \sigma_{\mu\rho} k_{\rho} - \kappa_{2} \sigma_{\mu\rho} k_{\rho} \frac{\not{p}_{2} + M_{2}}{2p_{2} \cdot k} W_{\lambda} - e \left(\frac{p_{2\mu} k_{\rho}}{p_{2} \cdot k} - g_{\mu\rho} \right)$$
$$\times \left[\left(\frac{f_{2} + g_{2} \gamma_{5}}{M_{1}} \right) \sigma_{\lambda\rho} + g_{\lambda\rho} \left(\frac{f_{3} + g_{3} \gamma_{5}}{M_{1}} \right) \right] \right] u_{A}. \tag{7}$$

 κ_1 and κ_2 are the anomalous magnetic moments of A and B given in Eqs. (21) and (22) in Ref. [10]. M_0 and W_{λ} are

given in Eqs. (2) and (3). ϵ_{μ} is the photon polarization fourvector and *e* is the charge of the produced charged lepton.

In order to calculate the bremsstrahlung contribution to the RC in the required order $\alpha q/\pi M_1$, we shall trace a close parallelism with the calculation of Ref. [6]. We shortly summarize our strategy in the following section.

III. DIFFERENTIAL DECAY RATE

The evaluation of the differential decay rate is performed by standard trace calculations leaving as the relevant independent variables, the energies E_2 and E of the emitted baryon, and the electron, respectively. The phase space integration is performed according to the kinematical limits given in Sec. II in Ref. [3].

The square of M_B summed over spins can be split, after trace calculations, into the sum of three contributions:

$$\sum_{\text{spins}} |M_{BN}|^2 = \sum_{\text{spins}} [|M_{1N}|^2 + |M_{2N}|^2 + 2Re(M_{1N}M_{2N}^{\dagger} + M_{1N}M_{3N}^{\dagger} + M_{2N}M_{3N}^{\dagger})].$$
(8)

The term $|M_{3N}|^2$ will contribute to order $\alpha q^2 / \pi M_1^2$ and higher and thus it is not included in Eq. (8).

In order to proceed as in Refs. [6,3], let us write the differential decay rate of Eq. (1) as

$$d\Gamma_{BN} = d\Gamma_{BN}^{0} + d\Gamma_{BN}^{I} + d\Gamma_{BN}^{II} + d\Gamma_{BN}^{III}, \qquad (9)$$

 $d\Gamma_{BN}^{0}$ and $d\Gamma_{BN}^{1}$ contain the otherwise TBR-infrared divergent and convergent terms of the $\Sigma_{\text{spins}}|M_{1N}|^{2}$ term, $d\Gamma_{B}^{\text{II}}$ and $d\Gamma_{B}^{\text{III}}$, contain the contributions from the second and third summands in Eq. (8), respectively. After integrating the first term of Eq. (9), one obtains

$$d\Gamma_{BN}^{0} = \frac{\alpha}{\pi} A_{1N}^{\prime} I_0^{NT}(E, E_2) d\Omega, \qquad (10)$$

where

$$A_{1N}' = Q_1 E E_{\nu}^0 - Q_2 E(\vec{p}_2^2 + |\vec{p}_2||\vec{\ell}|y_0) - Q_3(\vec{\ell}^2 + |\vec{p}_2||\vec{\ell}|y_0) + Q_4 E_{\nu}^0 |\vec{p}_2||\vec{\ell}|y_0,$$
(11)

with the coefficients Q_i , $i=1,\ldots,4$, given in Eqs. (16)–(19) in Ref. [11],

$$d\Omega = \frac{G_v^2}{2} \frac{dE_2 dE d\Omega_{/d} \phi_2}{(2\pi)^5} 2M_1,$$
 (12)

and $I_0^{NT}(E, E_2)$ is derived from Eq. (28) in Ref. [6] (see also Ref. [12]) by replacing the lower limit λ^2 by x_{\min} :

$$I_{0}^{NT}(E,E_{2}) = -2\ln\left|\frac{x_{\max}}{x_{\min}}\right| + \frac{1}{2}(\ln^{2}v_{\max}^{+} - \ln^{2}v_{\min}^{+}) - \frac{1}{2}(\ln^{2}v_{\min}^{-} - \ln^{2}v_{\max}^{-}) + \frac{1}{\beta_{N}}\left[\ln v^{+}\ln\left|\frac{v^{+} - a(1+\beta_{N})}{v^{+} - a(1-\beta_{N})}\right|\right|_{v^{+}=v_{\min}^{+}} + \ln v^{-}\ln\left|\frac{v^{-} - a(1+\beta_{N})}{v^{-} - a(1-\beta_{N})}\right|_{v^{-}=v_{\min}^{-}}\right] - \frac{1}{\beta_{N}}[I_{1}^{N} - I_{2}^{N} + I_{3}^{N} - I_{4}^{N}], \qquad (13)$$

where

$$\begin{aligned} x_{\max} &= 2|\vec{p}_2||\vec{\ell}|(y_0+1), \quad x_{\min} = 2|\vec{p}_2||\vec{\ell}|(y_0-1), \\ v_{\max}^{\pm} &= 2(E_2 \pm |\vec{p}_2|)(E \pm |\vec{\ell}|), \\ v_{\min}^{\pm} &= 2\{EE_2 - |\vec{p}_2||\vec{\ell}| \pm |E_2|\vec{\ell}| - |\vec{p}_2|E|\}, \\ &= M_1^2 - H^2 - q^2, \end{aligned}$$

and

$$\beta_{N} = \left(1 - \frac{4m^{2}M_{2}^{2}}{a^{2}}\right)^{1/2}, \quad H^{2} = (p_{1} - l)^{2}, \quad q^{2} = (p_{1} - p_{2})^{2}.$$
(14)

To formulate the I_i^N i = 1, ..., 4 in Eq. (13) we introduce the Heaviside function

$$\theta(x) = \begin{cases} 1 & \text{if } x \ge 0, \\ 0 & \text{if } x < 0. \end{cases}$$

Then,

$$I_i^N = I_{iA}\theta(r_{Ai}) + I_{iB}\theta(r_{Bi})\theta(r'_{Bi}) + I_{iC}\theta(r_{Ci}), \quad (15)$$

where

$$\begin{split} I_{1A,2A} = L \bigg(\frac{a(1 \pm \beta_N)}{v_{\min}^+} \bigg) - L \bigg(\frac{a(1 \pm \beta_N)}{v_{\max}^+} \bigg) \\ &+ \frac{1}{2} (\ln^2 v_{\max}^+ - \ln^2 v_{\min}^+), \\ I_{1B,2B} = \frac{-\pi^2}{3} - L \bigg(\frac{v_{\min}^+}{a(1 \pm \beta_N)} \bigg) - L \bigg(\frac{a(1 \pm \beta_N)}{v_{\max}^+} \bigg) \\ &+ \frac{1}{2} \ln^2 \frac{a(1 \pm \beta_N)}{v_{\min}^+} + \frac{1}{2} (\ln^2 v_{\max}^+ - \ln^2 v_{\min}^+), \\ I_{1C,2C} = L \bigg(\frac{v_{\max}^+}{a(1 \pm \beta_N)} \bigg) - L \bigg(\frac{v_{\min}^+}{a(1 \pm \beta_N)} \bigg) \\ &+ \ln |a(1 \pm \beta_N)| \ln \bigg| \frac{v_{\max}^+}{v_{\min}^+} \bigg|, \end{split}$$

$$I_{3A,4A} = L\left(\frac{a(1\pm\beta_N)}{v_{\min}^{-}}\right) - L\left(\frac{a(1\pm\beta_N)}{v_{\max}^{-}}\right) - \frac{1}{2}(\ln^2 v_{\min}^{-} - \ln^2 v_{\max}^{-}),$$
(16)

$$\begin{split} I_{3B, 4B} &= \frac{\pi^2}{3} + L \left(\frac{v_{\max}^-}{a(1 \pm \beta_N)} \right) \\ &+ L \left(\frac{a(1 \pm \beta_N)}{v_{\min}^-} \right) - \frac{1}{2} \ln^2 \frac{a(1 \pm \beta_N)}{v_{\max}^-} + \frac{1}{2} (\ln^2 v_{\max}^-) \\ &- \ln^2 v_{\min}^-), \end{split}$$

$$I_{3C, 4C} = L\left(\frac{v_{\text{max}}}{a(1 \pm \beta_N)}\right) - L\left(\frac{v_{\text{min}}}{a(1 \pm \beta_N)}\right) + \ln|a(1 \pm \beta_N)|\ln\left|\frac{v_{\text{max}}}{v_{\text{min}}}\right|,$$

and the arguments of the Heaviside function are given by the expressions

$$r_{A1,A2} = v_{\min}^{+} - a(1 \pm \beta_N), \quad r_{B1,B2} = a(1 \pm \beta_N) - v_{\min}^{+},$$

$$r'_{B1,B2} = v_{\max}^{+} - a(1 \pm \beta_N), \quad r_{C1,C2} = a(1 \pm \beta_N) - v_{\max}^{+},$$

$$(17)$$

$$r_{A3,A4} = v_{\max}^{-} - a(1 \pm \beta_N), \quad r_{B3,B4} = a(1 \pm \beta_N) - v_{\max}^{-},$$

$$r'_{B3,B4} = v_{\min}^{-} - a(1 \pm \beta_N), \quad r_{C3,C4} = a(1 \pm \beta_N) - v_{\min}^{-}.$$

The left (right) subindex on the left-hand side (LHS) of Eqs. (16) and (17) corresponds to the upper (lower) sign in the right-hand side (RHS) in the same equations. L(x) is the Spence function. Within our approximation, the terms of order $\alpha q^2/M_1^2$ that are contained in Eq. (11) can be neglected.

The remaining part,

$$d\Gamma^{R}_{BN} = d\Gamma^{I}_{BN} + d\Gamma^{II}_{BN} + d\Gamma^{III}_{BN}, \qquad (18)$$

can be integrated following the same procedure as in Ref. [6] considering the FBR integration limits. The analytical integration leads us to the result

$$d\Gamma_{BN}^{R} = \frac{\alpha}{\pi} d\Omega \left[(H_{0}' + N_{0}') \theta_{0}^{T} + \sum_{i=2}^{16} (H_{i}' + N_{i}') \theta_{i}^{T} + N_{17}' \theta_{17}^{T} \right].$$
(19)

The H'_i 's with $i=0,\ldots,16$ and the θ_i^T 's with $i=0,2,\ldots,16$ are given in Eqs. (37) and in Eqs. (33), respectively, in Ref. [3]. The $\theta_{17}^T=2I_1$, and the I_1 is given in Eq. (35) in the same reference. The N'_i are given as in Ref. [6] by

TABLE I. Numerical values of the form factors and of the anomalous and the total magnetic moments used in our calculations. κ_1 , κ_2 , μ_1 , and μ_2 are given in nuclear magnetons.

Process	\mathbf{f}_1	\mathbf{f}_2	\mathbf{g}_1	κ_1	κ2	μ_1	μ_2
$\Lambda \rightarrow - pev$	1.0	0.974	0.699	-0.6130	1.7928	-0.613	2.7928

$$\begin{split} N_{0}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{E\beta}{2M_{1}} \left[2(E-E_{\nu}^{0})R^{+} + (E+2E_{\nu}^{0})R^{-} \\ &+ (1-y_{0})\frac{\left|\vec{p_{2}}\right|}{2\beta}R^{-} \right], \\ N_{2}^{\prime} &= N_{6}^{\prime} = N_{9}^{\prime} = N_{11}^{\prime} = N_{15}^{\prime} = 0, \\ N_{3}^{\prime} &= \left|\vec{p_{2}}\right| \frac{E\beta m^{2}}{M_{1}}R^{+}, \\ N_{4}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{E\beta}{2M_{1}} \left[2E^{2}R^{+} + E\beta(E\beta + 4\left|\vec{p_{2}}\right|y_{0})R^{-} \right], \\ N_{5}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{(E\beta)^{2}}{M_{1}} \left[ER^{+} + 2E_{\nu}^{0}R^{-} \right], \\ N_{5}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{\beta}{2M_{1}} \left[2m^{2} + EE_{\nu}^{0}(1-\beta x_{0}) \right]R^{+}, \\ N_{8}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{E\beta}{2M_{1}} \left(2E + E_{\nu}^{0} \right)R^{+}, \\ N_{10}^{\prime} &= -\frac{3(E\beta)^{3}}{2M_{1}} \left|\vec{p_{2}}\right|R^{-}, \quad N_{12}^{\prime} &= \left|\vec{p_{2}}\right|^{2} \frac{E\beta^{2}}{M_{1}} \left(2E - E_{\nu}^{0} \right)R^{+}, \\ N_{13}^{\prime} &= -\left|\vec{p_{2}}\right|^{2} \frac{(E\beta)^{2}}{2M_{1}} \left(2R^{+} - R^{-} \right), \quad N_{14}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{(E\beta)^{2}}{M_{1}}R^{+}, \\ N_{16}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{\beta}{4M_{1}}R^{+}, \\ N_{16}^{\prime} &= -\left|\vec{p_{2}}\right| \frac{E\beta}{4M_{2}} \left[2E_{\nu}^{0} + \left(1-y_{0}\right)\left|\vec{p_{2}}\right|\beta\right]R^{-}, \end{split}$$

and

$$E\beta = |\vec{\ell}|, \quad R^{\pm} = |f_1|^2 \pm |g_1|^2. \tag{21}$$

Equation (13) is one of the main results of this section. In this equation the first term contains the $\ln|x_{\min}|$ which is divergent at the upper boundary $E_2 = E_2^{\min}$ of the FBR, where $y_0 = 1$. This divergence is canceled by the virtual contribution at the lower boundary of the TBR in the tree body decay.

As a guide to perform the lengthy integrations and in order to reproduce our results, the definitions used in the different stages of the analytical integration are given in Appendix of Ref. [3].

IV. FINAL RESULT AND CONCLUSIONS

Now, our complete result up to order $\alpha q/\pi M_1$ is compactly given by

$$d\Gamma_{BN}(A^{0} \rightarrow B^{+}e^{-}\overline{\nu}\gamma) = \frac{\alpha}{\pi}d\Omega \bigg[(H_{0}'+N_{0}')\theta_{0}^{T}+A_{1N}'\theta_{1N}^{T} + \sum_{i=2}^{16} (H_{i}'+N_{i}')\theta_{i}^{T}+N_{17}'\theta_{17}^{T} \bigg],$$
(22)

where A'_{1N} is given in Eq. (11),

$$\theta_{1N}^T = I_0^{NT}(E, E_2) \quad , \tag{23}$$

and $I_0^{NT}(E, E_2)$ is given in Eq. (13).

With Eq. (22) we have the analytic result for the bremsstrahlung part of the DP of the semileptonic decay of neutral hyperons, at the FBR. With Eq. (61), given in Ref. [6], at the TBR, we have a full analytic result for the bremsstrahlung part of the complete DP of semileptonic decay of unpolarized neutral hyperons.

TABLE II. Comparison of the results of Ref. [4] with our results for the relative correction to the (E, E_2) distribution in the four-body region.

$\frac{1}{Process} \Lambda \rightarrow p e \overline{\nu}$													
$y = \frac{E_2}{M_1}$	x = 0.05		x = 0.15		x = 0.25		x = 0.35						
0.851	2.0	2.019											
0.849	1.7	1.706											
0.848	1.5	1.446											
0.847	1.3	1.221	2.7	2.605									
0.846	1.1	0.975	2.0	2.002									
0.844	0.9	0.656	1.6	1.515	3.3	3.245							
0.843	0.6	0.281	1.1	1.071	2.0	1.935							
0.842	0.4	-0.140	0.6	0.603	1.0	1.008	2.0	2.049					

Λ

Ν

2893

This result for unpolarized decays is of high precision, model independent, and useful for processes where the momentum transfer is not small and, therefore, cannot be neglected.

In our approach we have neglected the terms of order $\alpha q^2/\pi M_1^2$ and higher. Nevertheless, some of them are included via the Q_i factors. If the accompanying factors cause one order of magnitude increase, then the upper bound to the theoretical uncertainty is of the order 0.6% for charm decay and even smaller for Λ .

For the FBR contributions there are no previously published *analytical* results based in the theoretical framework which is outlined in the previous sections for the Dalitz (E,E_2) distribution for radiative decays of the type $A \rightarrow Be \nu \gamma$. We can compare earlier numerical predictions given in Table I(d) of Ref. [5] with those obtained from expressions in Eq. (22). The results in Ref. [5] do not include the anomalous magnetic moments of baryons.

Our calculations use the same values of the form factors employed in Ref. [4], which are displayed in Table I, along with the anomalous and total magnetic moments of the baryons involved.

In Table II, we repeat for each value of $x=E/E_m$ in the first column the numerical result from Ref. [5]. The numbers shown in the second column are obtained from Eq. (22),

taking into account the anomalous magnetic moments and following the definitions given in Eq. (4.2') in Ref. [5]. We have not considered in this case a third column in the table comparing with the exact results for the lowest order of the three-body process with effective form factors for consistency in the approximation scheme. Nevertheless, the q^2/M_1^2 dependence of the form factors has a significant effect in the zeroth order result [13] and, therefore, in the final relative corrections for the three-body process as one can see in Tables VII and Tables VIII in Ref. [14].

It is worth mentioning that there are some efforts to develop a Monte Carlo method for photon bremsstrahlung calculations in semileptonic decays [2]. The fully analytical, precise result we accomplished for the four particles in the final state and the Monte Carlo results might be complementary to each other.

ACKNOWLEDGMENTS

I am grateful to Professor P. Kielanowski for reading the manuscript and providing valuable suggestions. I thank Professor E. Virueña S. and F. Guzmán for their assistance while running the computational programs. I also acknowledge partial support by Comisión de Operación y Fomento de Actividades Académicas (Instituto Politécnico Nacional).

- Particle Data Group, L. Montanet *et al.*, Phys. Rev. D 50, 1173 (1994).
- [2] F. Glück and I. Joó, Report No. CERN-TH.73111-94 (unpublished); Phys. Lett. B 340, 240 (1994).
- [3] S. R. Juárez W., Phys. Rev. D 53, 3746 (1996).
- [4] K. Tóth et al., Phys. Rev. D 33, 3306 (1986).
- [5] F. Glück and K. Tóth, Phys. Rev. D 41, 2160 (1990).
- [6] S. R. Juárez W., Phys. Rev. D 48, 5233 (1993); 52, 556 (1995).
- [7] F. E. Low, Phys. Rev. 110, 974 (1958).
- [8] H. Chew, Phys. Rev. 123, 377 (1961).

- [9] S. R. Juárez W. and A. Martínez, Rev. Mex. Fis. 35, 13 (1989).
- [10] D. M. Tun, S. R. Juárez W., and A. García, Phys. Rev. D 44, 3589 (1991).
- [11] D. M. Tun, S. R. Juárez W., and A. García, Phys. Rev. D 40, 2967 (1989).
- [12] E. S. Ginsberg, Phys. Rev. D 1, 229 (1970); Phys. Rev. 171, 1675 (1968).
- [13] S. R. Juárez W., Phys. Rev. D 51, 6267 (1995).
- [14] A. Martínez, S. R. Juárez W., and A. García, Phys. Rev. D 46, 322 (1992).