# New physics in *CP* asymmetries and rare *B* decays

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We review and update the effects of physics beyond the standard model on *CP* asymmetries in *B* decays. These asymmetries can be significantly altered if there are important new-physics contributions to $B_q^0$-$\overline{B_q^0}$ mixing. This same new physics will, therefore, also contribute to rare, flavor-changing *B* decays. Through a study of such decays, we show that it is possible to partially distinguish the different models of new physics. [S0556-2821(97)03003-8]

## I. INTRODUCTION

Within the standard model (SM), *CP* violation is due to nonzero complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. In the Wolfenstein parametrization [1], only the elements $V_{ub}$ and $V_{td}$ have non-negligible phases:

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

(1)

where $\lambda = 0.22$ is the Cabibbo angle. The phase information of the CKM matrix can be displayed elegantly using the so-called unitarity triangle (Fig. 1), which follows from the orthogonality of the first and third columns. *CP* violation is indicated by a nonzero area of the unitarity triangle; to date, the only evidence for *CP* violation comes from $|\epsilon_K|$ in the kaon system.

At present, constraints on the unitarity triangle come from a variety of sources [2]. The sides of the triangle can be probed directly—$|V_{ub}/V_{cb}|$ in charmless *B* decays, and $|V_{td}/V_{cb}|$ through $B_d^0$-$\overline{B_d^0}$ mixing. The three angles, $\alpha$, $\beta$, and $\gamma$, are constrained by the above measurements, as well as those of $|\epsilon_K|$ and *B* decays to charmed mesons ($|V_{cb}|$). However, with the exception of $|V_{cb}|$, in all cases there are large theoretical hadronic uncertainties in the extraction of the CKM matrix parameters from such measurements. As such, our current knowledge of the unitarity triangle is rather poor.

In the coming years, we will be able to precisely determine the unitarity triangle, and hence test the SM explanation of *CP* violation. The key measurements involve *CP*-violating asymmetries in *B* decays. Through such measurements, the weak phases $\alpha$, $\beta$, and $\gamma$ can be extracted with no hadronic uncertainty [3], and then compared with SM predictions.

It is useful at this point to review how the *CP* angles are probed in *CP* asymmetries. Most asymmetries of interest measure mixing-induced indirect *CP* violation, which comes about through the interference of the two amplitudes $B \to f$ and $B \to \overline{B} \to f$. In order to cleanly extract the weak phases from these *CP* asymmetries, two conditions must be met. First, in the neutral *B* system, one must have $\Gamma_{12} \ll M_{12}$. This relation holds within the SM, where $\Gamma_{12}/M_{12} \sim 3\pi m_b^2/m_t^2 \lesssim 10^{-2}$. Second, the direct decay $B \to f$ must be dominated by a single weak amplitude. If this is not the case, then one may have direct *CP* violation, which involves unknown strong phases. In fact, in the SM most *B* decays which are useful for *CP* asymmetries have more than one weak amplitude—in addition to the tree-level contribution, one may also have penguin diagrams [4]. However, for the cases of interest, the penguin contamination is either unimportant or can be eliminated using isospin [5] and other considerations. We refer to [3] for a more complete discussion of these issues.

Assuming that the above two conditions are met, the *CP* asymmetry measures Im $\lambda$, where $\lambda$ is a pure phase:

$$\lambda = \left(\frac{X}{X^*}\right)\left(\frac{Y}{Y^*}\right)\left(\frac{Z}{Z^*}\right).$$

(2)

The three pieces are defined as follows [6]. $X$ is the weak phase of the direct $B \to f$ decay amplitude. For example,
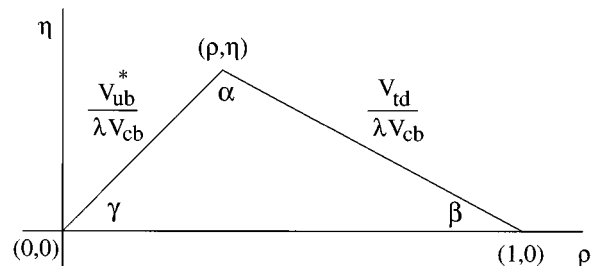


FIG. 1. The unitarity triangle. The angles $\alpha$, $\beta$, and $\gamma$ can be measured via *CP* violation in the *B* system.

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$X_{\bar{b}\to \bar{u}u\bar{d}} = V_{ub}V_{ud}^*$ in the SM. $Y$ is the phase of $B^0$-$\overline{B^0}$ mixing: e.g., for $B_d^0$-$\overline{B_d^0}$ mixing in the SM, $Y_d = V_{tb}^* V_{td}$. Finally, $Z$ is the phase of $K$-$\overline{K}$ mixing, which is important only if the final state $f$ contains a neutral kaon. In the SM, assuming that $K$-$\overline{K}$ mixing is dominated by box diagrams with virtual $c$ quarks, $Z = V_{cd}V_{cs}^*$, which is real to a good approximation in the Wolfenstein parametrization.

From the above equation, it is straightforward to establish which $CP$ angles are measured in different $CP$ asymmetries. For example, the $CP$ asymmetries in $\overset{(-)}{B_d} \to \pi^+\pi^-$ and $\overset{(-)}{B_d} \to \Psi K_s$ probe $\sin 2\alpha$ and $\sin 2\beta$, respectively. And the angle $\gamma$ can be extracted from the $CP$ asymmetry in $\overset{(-)}{B_s} \to D_s^\pm K^\mp$ [7] (the function in this case is $\sin^2\gamma$). Another way of measuring $\gamma$, which does not involve mixing-induced $CP$ violation, is via the asymmetry in $B^\pm \to D_{CP}K^\pm$ [8]. In all cases, the $CP$ phases can be obtained with no hadronic uncertainty. Of course, there are many other $CP$ asymmetries which can be used to obtain the angles $\alpha$, $\beta$, and $\gamma$. [For certain decays (e.g., $\overset{(-)}{B_s^0} \to \Psi\phi$) $CP$ asymmetries probe very small angles of other unitarity triangles, which are almost flat [75].]

Once these angles are measured, it will be possible to test the SM by comparing the measured values with the SM predictions, as well as with the angles expected from independent measurements of the sides of the unitarity triangle. (As we will see below, these two comparisons are not necessarily equivalent.) As mentioned above, the SM predictions are not very precise at present. Even so, the experimental data do somewhat constrain the $CP$ angles [2]. For example, $\sin 2\beta$ must be between 0.32 and 0.94 at 95% C.L. In addition, the predictions for $\alpha$, $\beta$, and $\gamma$ are correlated, since there is only a single complex phase in the CKM matrix and the three angles must add up to 180°. A special correlation was shown to exist between small values of $\sin 2\beta$ and large values of $\sin 2\alpha$ [9], and an almost linear correlation was found between $\alpha$ and $\gamma$ [10].

All this presupposes that the SM is the complete description of the weak interactions and $CP$ violation. However, it is widely accepted that there must be physics beyond the SM. There are a number of ways in which new physics can manifest itself through the measurement of $CP$ asymmetries:

(1) The relation $\alpha + \beta + \gamma = \pi$ is violated.

(2) Although $\alpha + \beta + \gamma = \pi$, one finds values for the $CP$ phases which are outside of the SM predictions.

(3) The $CP$ angles measured are consistent with the SM predictions, and add up to 180°, but are inconsistent with the measurements of the *sides* of the unitarity triangle.

In any of these cases, it is only natural to then ask what type of new physics could be responsible. [A special possibility is the so-called superweak-type model [11], in which the CKM phase vanishes (i.e., $\gamma=0$), and the $CP$ asymmetries in $\overset{(-)}{B_d} \to \pi^+\pi^-$ and $\overset{(-)}{B_d} \to \Psi K_s$ are equal in magnitude. It is possible, but not necessary, that these asymmetries vanish in such models, in which case the unitarity triangle becomes a straight line.]

A first step in answering this question was taken in [12], which examined the effects of a variety of models of new physics on the $CP$ asymmetries in $B$ decays. The authors of [12] concentrated on items (1) and (2) above. Their conclusion was that the predictions of the SM can be considerably altered in many of these models.

The reasoning goes as follows. First, it is very difficult to significantly change the relation $\Gamma_{12} \ll M_{12}$, even in the presence of new physics. Second, in most models of new physics, there are no new tree-level contributions to $B$ decays. Therefore $CP$ asymmetries continue to measure a well-defined $CP$ phase, as in Eq. (2). Of the three pieces in Eq. (2), only $Y$ is likely to be significantly altered by new physics. This is because (i) although $Z$ may be modified in the presence of new physics, only in extremely contrived models can $\arg(Z)$ be changed, and (ii) $X$ can be affected only if there are new amplitudes which can compete with the $W$-mediated tree-level decay, and there are very few models of new physics in which this occurs. (Small new-physics effects on $X$ are studied in Ref. [76].) Thus, the principal way that the SM predictions for $CP$ asymmetries can be significantly modified is if there are sizable new-physics contributions to $B^0$-$\overline{B^0}$ mixing with phases different than in the SM. It is therefore straightforward to establish, model by model, which types of new physics can do this. Examples of models of new physics which can significantly affect the $CP$ asymmetries include $Z$-mediated flavor-changing neutral currents, four generations, nonminimal supersymmetric models, etc. In some of these models, $|\epsilon_K|$ is also likely to obtain sizable new-physics contributions.

One point which was not emphasized in [12] is the third way of detecting new physics [item (3) above]. That is, even if the new-physics contributions to $B$ mixing have the same phase as in the SM their presence can still be detected. This is because, although the $CP$ phases $\alpha$, $\beta$, and $\gamma$ are unchanged from their SM values, the new physics affects one of the sides of the unitarity triangle, namely the extraction of $|V_{td}/V_{cb}|$ from $B_d^0$-$\overline{B_d^0}$ mixing. Thus, the measurements of the angles and those of the sides will be inconsistent with one another, indicating the presence of new physics. There are several models in which this may occur—two-Higgs-doublet models with natural flavor conservation, minimal supersymmetric models, etc.

The question of new physics and $CP$ asymmetries in $B$ decays has also been discussed in [13]. This paper focuses on how new physics can affect various relations among $CP$ asymmetries in the SM. One of the points made, which is of particular interest for our purposes, is the following. Suppose that $\alpha$ (which stands for $\pi - \beta - \gamma$) and $\beta$ are measured via $CP$ asymmetries involving $B_d^0$ decays, and that $\gamma$ is obtained through $B_s^0$ decays. In this case, if the phase in $B_s^0$-$\overline{B_s^0}$ mixing is identical to that of the SM, then the relation $\alpha + \beta + \gamma = \pi$ will hold, regardless of whether there is new physics in $B_d^0$-$\overline{B_d^0}$ or $B_s^0$-$\overline{B_s^0}$ mixing. In other words, any new-physics effects in $B_d^0$-$\overline{B_d^0}$ mixing cancel in the sum of $\alpha$ and $\beta$.

This has important experimental implications. The angles $\alpha$ and $\beta$ will probably be measured through $CP$ asymmetries in $\overset{(-)}{B_d} \to \pi^+\pi^-$ and $\overset{(-)}{B_d} \to \Psi K_S$, respectively. If the angle $\gamma$ is measured through the $CP$ asymmetry in $\overset{(-)}{B_s} \to D_s^\pm K^\mp$, then one might find that the three $CP$ angles do not add up to 180°, if there is new physics in $B_s^0$-$\overline{B_s^0}$ mixing. However, if

the angle $\gamma$ is obtained via $B^{\pm} \to D_{CP} K^{\pm}$, then, unless there are new contributions to $B$ decays, one *must* find $\alpha + \beta + \gamma = \pi$. This underlines the importance of measuring $\gamma$ (as well as $\alpha$ and $\beta$) in a variety of independent ways. This also demonstrates that it will be crucial to search for new-physics effects in all three ways—if only one of the methods (1)–(3) is used, one might miss the presence of physics beyond the SM.

This rather lengthy introduction summarizes previous work on new physics and *CP* asymmetries in the *B* system. However, these analyses only partially address the issue. Suppose that the *CP* angles $\alpha$, $\beta$, and $\gamma$ are measured, and it is found that, in fact, the presence of new physics is indicated. Reference [13] presents some tests to determine where the new physics might be found (e.g., $B_d^0$-$\overline{B_d^0}$ mixing, $B_s^0$-$\overline{B_s^0}$ mixing, etc.). Reference [12] identifies which models of new physics could be involved. However, neither of these references tells us how to distinguish among these various models. It is this question which we address in this paper.

As argued above, the new physics can affect the *CP* asymmetries mainly through its contributions to $B_d^0$-$\overline{B_d^0}$ or $B_s^0$-$\overline{B_s^0}$ mixing, which are flavor-changing processes. This same new physics will therefore also affect rare flavor-changing decays, such as $b \to sX$ or $b \to dX$. (In this paper we generically refer to such processes as ''penguin'' decays.) This is the key point. As we will show, some models of new physics can be distinguished by their contributions to these rare processes. In fact, for certain models, the new-physics parameter space leading to large contributions to $B$-$\overline{B}$ mixing also predicts large deviations from the SM predictions for certain penguin decays. Conversely, if no deviation from the SM is found, this would so constrain the parameters of the new physics as to render its effects in $B$-$\overline{B}$ mixing, and hence the *CP* asymmetries, unimportant. It is an experimental question whether or not measurements of the rates for such penguin decays can be made before the *CP* asymmetries are measured. Regardless, it is clear that measurements of *CP* asymmetries and penguin decays will give complementary information. And in fact, unless the new particles are discovered in future colliders, it will be necessary to appeal to such measurements to infer their existence indirectly.

The paper is organized as follows. In Sec. II we review and update the contributions of various models of new physics to $B$-$\overline{B}$ mixing, and hence to *CP* asymmetries in $B$ decays. We summarize the current experimental constraints on the new-physics parameters which determine these contributions. For those models which can affect the SM predictions for the *CP* asymmetries, in Sec. III we examine their contributions to flavor-changing penguin decays. We conclude in Sec. IV.

## II. $B$-$\overline{B}$ MIXING AND NEW PHYSICS

There are a variety of models of new physics which can contribute to $B_q^0$-$\overline{B_q^0}$ mixing ($q = d, s$), and which therefore can affect *CP* asymmetries in $B$ decays. In this section we review and update the contributions of these models to this mixing. (Note that we include several models not discussed in [12].) We also examine how the new-physics parameters

are constrained by current experimental data. In all cases, we search for new contributions to $B_q^0$-$\overline{B_q^0}$ mixing at least comparable to that of the SM [14]:

$$M_{12}^{SM}(B_q) = \frac{G_F^2 M_{B_q} \eta_{B_q} M_W^2}{12\pi^2} f_{B_q}^2 B_{B_q} x_t f_2(x_t)(V_{tq}V_{tb}^*)^2,$$
(3)

where the mass difference $\Delta M$ is related to $M_{12}$ by $\Delta M_q = 2|M_{12}(B_q)|$, $x_t = m_t^2/M_W^2$, and

$$f_2(x) = \left[\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2} - \frac{3}{2}\frac{x^2\ln x}{(1-x)^3}\right].$$
(4)

New-physics contributions to $B_q^0$-$\overline{B_q^0}$ mixing are constrained by the measurements of the neutral *B*-meson mass differences, $\Delta M_d$ [15] and $\Delta M_s$ [16]:

$$\Delta M_d = (0.470 \pm 0.017) \ \text{ps}^{-1}, \quad \Delta M_s > 7.8 \ \text{ps}^{-1}. \quad (5)$$

These values are consistent with the SM prediction [Eq. (3)], and constrain the CKM elements $V_{td}$ and $V_{ts}$ as follows [2]:

$$0.15 < \left|\frac{V_{td}}{V_{cb}}\right| < 0.34, \quad \left|\frac{V_{ts}}{V_{cb}}\right| > 0.6. \quad (6)$$

These limits include the experimental errors on $m_t$ and $V_{cb}$, as well as the theoretical error on $f_{B_q}\sqrt{B_{B_q}}$. The bounds on these quantities due to the unitarity of the $3 \times 3$ CKM matrix alone are

$$0.11 < \left|\frac{V_{td}}{V_{cb}}\right| < 0.33, \quad \left|\frac{V_{ts}}{V_{cb}}\right| \simeq 1. \quad (7)$$

With the addition of new contributions to $B_q^0$-$\overline{B_q^0}$ mixing, the constraints of Eq. (6) on $V_{td}$ and $V_{ts}$ are relaxed, although the degree of relaxation is model dependent. In certain models, the CKM matrix remains unitary, which implies that the bounds of Eq. (7) still hold. In other models, the $3 \times 3$ CKM matrix is not unitary, so that $V_{td}$ or $V_{ts}$ can be much smaller and in principle even vanish, in which case $B_d^0$-$\overline{B_d^0}$ or $B_s^0$-$\overline{B_s^0}$ mixing comes entirely from new physics.

In order to see how large the new-physics contributions to $B_q^0$-$\overline{B_q^0}$ mixing can be in specific models, it is convenient to normalize these terms by the corresponding *W* box-diagram terms which appear in the SM, which are proportional to $V_{tq}^2$. However, one should note that in some cases the latter parameters can take values outside the SM constraints. To avoid confusion in this respect, we will denote the *W*-box contributions to $B_q^0$-$\overline{B_q^0}$ mixing by $M_{12}^W$ rather than by $M_{12}^{SM}$.

### A. Four generations [17]

This is a model with an additional generation of quarks and leptons, including a new charge 2/3 quark, $t'$. The CKM matrix is $4 \times 4$, which can be parametrized by six angles and three phases. The unitarity triangle thus now becomes a quadrangle. There are new loop-level contributions, involving internal $t'$ quarks, to both $B_q^0$-$\overline{B_q^0}$ mixing and penguin decays. The additional phases in the CKM matrix can play a role in the *CP* asymmetries.

There is a model-independent lower bound of 45 GeV on the mass of the $t'$ coming from LEP. There are stronger constraints on $m_{t'}$ of $O(100)$ GeV coming from hadron colliders, but these can be evaded since they depend on how strongly the $t'$ couples to the $b$ quark. There is an upper bound of 550 GeV on $m_{t'}$ coming from partial-wave unitarity [18]. A heavier $t'$ will lead to a breakdown of perturbation theory. The strongest constraints on the CKM matrix elements involving the $t'$ quark come from unitarity. There are additional constraints on the $t'$ mass and its charged-current couplings coming from the $K_L$-$K_S$ mass difference, from $|\epsilon_K|$, from $B_d^0$-$\overline{B}_d^0$ mixing, and from $b \to s\gamma$. Since the measurements of all these observables agree with the predictions of the SM, they provide upper limits on their respective $t'$ contributions, assuming no accidental cancellations. However, if one allows for such cancellations, the constraints become correspondingly weaker. Finally, we note that the fourth-generation neutrino must have a mass $m_\nu > M_Z/2$ due to constraints from the CERN $e^+e^-$ collider LEP. Since this is quite unlike the first three generations, many argue that the four-generation model is much less plausible. Still, it is a logical possibility.

In this model, the extra phases in the $4 \times 4$ CKM matrix enter through the new contributions to $B_q^0$-$\overline{B}_q^0$ mixing. Assuming that this mixing is dominated by box diagrams with $t$ and $t'$ quarks, we have

$$M_{12}^{\text{four gen}}(B_q) = \frac{G_F^2 M_{B_q} \eta_{B_q} M_W^2}{12\pi^2} f_{B_q}^2 B_{B_q}[E(x_t,x_t)(V_{tq}V_{tb}^*)^2$$

$$+ 2E(x_t,x_{t'})(V_{tq}V_{tb}^*)(V_{t'q}V_{t'b}^*)$$

$$+ E(x_{t'},x_{t'})(V_{t'q}V_{t'b}^*)^2], \quad (8)$$

where

$$E(x_i,x_j) = x_i x_j \left\{ \left[ \frac{1}{4} + \frac{3}{2} \frac{1}{(1-x_j)} - \frac{3}{4} \frac{1}{(1-x_j)^2} \right] \frac{\ln x_j}{x_j - x_i} \right.$$

$$\left. + (x_i \leftrightarrow x_j) - \frac{3}{4} \frac{1}{(1-x_i)(1-x_j)} \right\}. \quad (9)$$

Since the additional contributions to the mixing can be of a similar size to that of the SM, but with different phases, the $CP$ asymmetries can be considerably altered. For example, the experimental value of $B_d^0$-$\overline{B}_d^0$ mixing can be explained in the SM if $V_{td} = 0.01$ (for $m_t = 170$ GeV, $V_{tb} = 1$); the phase of the mixing is then $\arg(V_{td}V_{tb}^*)^2$. In a four-generation model, in which the $3 \times 3$ CKM matrix is no longer unitary, this mixing can be dominated by the fourth generation: e.g., $V_{td} \sim 0$, $V_{t'd} = 0.005$, $V_{tb} = V_{t'b} \simeq 1/\sqrt{2}$, and $m_{t'} = 480$ GeV. The phase of the mixing is then $\arg(V_{t'd}V_{t'b}^*)^2$, which may be quite different from the SM.

### B. Z-mediated flavor-changing neutral currents [19]

In these models, one introduces an additional vector-singlet charge $-1/3$ quark, and allows it to mix with the ordinary down-type quarks. Since the weak isospin of the exotic quark is different from that of the ordinary quarks, flavor-changing neutral currents (FCNC's) involving the $Z$

are induced. The $Zb\overline{d}$ and $Zb\overline{s}$ FCNC couplings, which affect $B$ decays, are parametrized by independent parameters $U_{db}$ and $U_{sb}$, respectively, which contain new phases:

$$\mathcal{L}_{\text{FCNC}}^Z = -\frac{g}{2\cos\theta_W} U_{qb} \overline{q}_L \gamma^\mu b_L Z_\mu. \quad (10)$$

There are, however, constraints on the FCNC couplings coming from the process $B \to \mu^+\mu^- X$. The current experimental bound on the branching ratio of this process is [20]

$$B(B \to \mu^+\mu^- X) < 5 \times 10^{-5}, \quad (11)$$

while the contributions of $Z$-mediated FCNC's to this process are

$$\frac{B(B \to \mu^+\mu^- X)}{B(B \to \mu\nu X)} = [(g_L^\mu)^2 + (g_R^\mu)^2] \frac{|U_{db}|^2 + |U_{sb}|^2}{|V_{ub}|^2 + F_{\text{ps}}|V_{cb}|^2}, \quad (12)$$

where $g_L^\mu = -1/2 + \sin^2\theta_W$, $g_R^\mu = \sin^2\theta_W$, and $F_{\text{ps}} \simeq 0.5$ is a phase-space factor. The FCNC couplings $U_{qb}$, $q = d,s$ are then constrained to be

$$\left| \frac{U_{qb}}{V_{cb}} \right| < 0.044, \quad (13)$$

or, taking $|V_{cb}| = 0.0388 \pm 0.0036$ [2],

$$|U_{qb}| < 0.0017 \pm 0.0002. \quad (14)$$

(Similar constraints can be obtained from the bound on $B \to \nu\overline{\nu}X$ [21].)

The $Z$-mediated flavor-changing couplings $U_{qb}$ can contribute to $B_q^0$-$\overline{B}_q^0$ mixing:

$$M_{12}^Z(B_q) = \frac{\sqrt{2}G_F M_{B_q} \eta_{B_q}}{12} f_{B_q}^2 B_{B_q}(U_{qb}^*)^2. \quad (15)$$

Recall that there can be new, independent phases in $U_{qb}$.

Comparing the contribution of this new physics to $B_q^0$-$\overline{B}_q^0$ mixing with that of the SM, we find

$$\frac{\Delta M_d^Z}{\Delta M_d^W} = \frac{\sqrt{2}\pi^2}{G_F M_W^2} \frac{1}{x_t f_2(x_t)} \frac{|U_{db}|^2}{|V_{td}V_{tb}|^2} = 80 \frac{|U_{db}|^2}{|V_{td}|^2}, \quad (16)$$

where we have taken $|V_{tb}| = 1$ and $m_t = 170$ GeV.

In this model the CKM matrix is not unitary:

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = U_{db}, \quad (17)$$

so that the constraint of Eq. (7) on $V_{td}$ does not hold. Since $|U_{db}|$ is bounded by Eq. (14), we find

$$0.07 < \left| \frac{V_{td}}{V_{cb}} \right| < 0.37. \quad (18)$$

Consequently,

$$\frac{\Delta M_d^Z}{\Delta M_d^W} = (0.9 - 26) \left[ \frac{|U_{db}/V_{cb}|}{0.04} \right]^2, \quad (19)$$

where the numerical coefficients 0.9 and 26 correspond to the largest and smallest values of $|V_{td}/V_{cb}|$. The sum of the $W$ and $Z$ contributions to $B_d^0$-$\overline{B}_d^0$ mixing is consistent with measurement for the entire range of $V_{td}$.

$B_s^0$-$\overline{B}_s^0$ mixing can be analyzed similarly. However, in this case, the effect of $U_{sb}$ on the violation of $(sb)$ CKM unitarity is small, so that

$$\frac{\Delta M_s^Z}{\Delta M_s^W} = 0.15 \left[ \frac{|U_{sb}/V_{cb}|}{0.04} \right]^2. \qquad (20)$$

From this we see that $B_d^0$-$\overline{B}_d^0$ mixing can in fact be dominated by $Z$-mediated FCNC. And although $B_s^0$-$\overline{B}_s^0$ mixing is still mainly due to the $W$-box contribution, the new-physics contribution may be non-negligible, so that the new phases in $U_{sb}$ can be important. Thus, in both cases, measurements of $CP$ asymmetries can differ considerably from the predictions of the SM.

It is interesting to note that there exist specific models with seesaw-like predictions for the flavor-changing $Z$ couplings [22]:

$$U_{qb} = \sqrt{\frac{m_q m_b}{M^2}}, \quad M \sim 0.1\text{--}1 \text{ TeV}. \qquad (21)$$

Depending on the precise value of $M$, these couplings do not lie too far below the present limit [Eq. (14)], and may thus give sizable contributions to $B_d^0$-$\overline{B}_d^0$ and $B_s^0$-$\overline{B}_s^0$ mixing.

In one particular flavor-changing $Z$ model [23], $CP$ is violated spontaneously, the CKM matrix is essentially real, and the new contributions to $B_q^0$-$\overline{B}_q^0$ mixing lead to very small phases. The unitarity triangle becomes a straight line and all $CP$ asymmetries are expected to be tiny.

### C. Multi-Higgs-doublet models

Models with more than one Higgs doublet can be classified into two types: (i) models with natural flavor conservation [24], in which there are no flavor-changing neutral currents, and (ii) models in which flavor-changing interactions can be mediated by neutral scalars [25]. We discuss these in turn.

#### *1. Natural flavor conservation*

In models with natural flavor conservation [26], the new charged scalars may give significant contributions to $B_q^0$-$\overline{B}_q^0$ mixing if their masses lie in the range of 50 GeV to about 1 TeV [27]. The Yukawa couplings of the charged scalars to up- and down-type quarks are given by

$$\mathcal{L}_{H^\pm} = \frac{g}{\sqrt{2} M_W} \sum_i (X_i \overline{U}_L V_{\text{CKM}} M_D D_R$$

$$+ Y_i \overline{U}_R M_U V_{\text{CKM}} D_L) H_i^+ + \text{H.c.} \qquad (22)$$

Here $U(D)$ is a vector of up-type (down-type) quarks, and $M_U(M_D)$ is the diagonal charge 2/3 (charge $-1/3$) quark mass matrix. $X_i$ and $Y_i$ are complex coupling constants arising from the mixing in the scalar sector. In the case of two Higgs doublets, in which one doublet provides masses to all

quarks and the other decouples from the quark sector (model I), $Y = -X = \cot\beta$, where $\tan\beta$ is the ratio of the two vacuum expectation values. In a more popular version of the two-Higgs-doublet model (model II), found in supersymmetric models, for example (see discussion below), one scalar $(\phi_1)$ gives mass to the up-type quarks while the other scalar $(\phi_2)$ gives mass to the down-type quarks. In this case $X = \tan\beta$ and $Y = \cot\beta = v_2/v_1$.

For simplicity of presentation in the following we assume that only one charged Higgs is light; the others are heavy and decouple. This leaves two complex coupling constants, $X$ and $Y$. There are several useful observations regarding Eq. (22). First, the $Y$ term is dominated by the $t$ quark: $\mathcal{L}_{H^\pm}^Y \sim Y(m_t/M_W) H^+ \overline{t}_R V_{ti} d_{iL}$. Second, because of the smallness of the down-type quark masses, the $X$ term is important only if $|X| \gg |Y|$. In this region of parameter space, $CP$ violation can appear in charged-Higgs exchange if $X$ and $Y$ have a nonzero relative phase. However, the observed rate of $b \to s\gamma$ constrains $\text{Im}(XY^*) < 2$–4 [28], thus ruling out the possibility that $CP$-violating effects due to charged-Higgs-boson exchange can compete with those due to $W$ exchange. Since the inclusion of the $X$ term does not lead to new $CP$ violation, and since in general it is much smaller than the $Y$ term, from here on we will generally ignore the $X$ term altogether. We will refer to the possibility of very large $X$ only when its effect is particularly important.

There are two types of box diagrams involving charged Higgs bosons which contribute to $B_q^0$-$\overline{B}_q^0$ mixing: those with one $H$ and one $W$, and those with two $H$'s. The total charged-Higgs-boson contribution is given by [29]

$$M_{12}^{H^+}(B_q) = \frac{G_F^2 M_{B_q} \eta_{B_q} M_W^2}{48\pi^2} f_{B_q}^2 B_{B_q} (V_{tq} V_{tb}^*)^2 [I_{HH} + I_{HW}], \qquad (23)$$

where

$$I_{HH} = x_t y_t I_1(y_t) |Y|^4,$$

$$I_{HW} = 2x_t y_t [4I_2(x_t, y_t) + I_3(x_t, y_t)] |Y|^2, \qquad (24)$$

with

$$I_1(y) = \frac{1+y}{(1-y)^2} + \frac{2y \ln y}{(1-y)^3}, \qquad (25)$$

$$4I_2(x,y) + I_3(x,y) = \frac{(x-4y)\ln y}{(y-x)(1-y)^2} + \frac{3x \ln x}{(y-x)(1-x)^2}$$

$$+ \frac{x-4}{(1-x)(1-y)}. \qquad (26)$$

Here, $x_q \equiv m_q^2/M_W^2$ and $y_q \equiv m_q^2/M_{H^+}^2$. Note that $M_{12}^{H^+}(B_q)$ involves the same CKM factors and has the same phase as the $W$-box contributions. Therefore, the ratio of the two terms is independent of CKM factors and is positive, such that the two contributions add up constructively. For $M_{H^+}$ in the range 100–400 GeV, this ratio may be approximated to within about 20% by an inversely linear relation:

$$\frac{M_{12}^{H^+}(B_q)}{M_{12}^W(B_q)} \approx \left(\frac{100 \text{ GeV}}{M_{H^+}}\right)[0.24|Y|^4 + 1.05|Y|^2]. \quad (27)$$

For smaller and larger Higgs masses this simplified expression holds within about 30%.

The parameters $|Y|$ and $M_{H^+}$ are constrained by comparing the observed rate of $b \to s\gamma$ [30] with the SM prediction [31]. These constraints also depend on $X$ [32]. At $3\sigma$ the bounds are

$$-0.56 < |Y| \frac{1}{3} G_W(y_t) + XY^* G_H(y_t) < 0.27, \quad (28)$$

where

$$G_W(y) = \frac{y}{12(1-y)^4}[(7-5y-8y^2)(1-y)$$
$$+ 6y(2-3y)\ln(y)],$$

$$G_H(y) = \frac{y}{6(y-1)^3}[(3-5y)(1-y) + 2(2-3y)\ln(y)]. \quad (29)$$

The implications of these bounds on $Y$ and $M_{H^+}$ depend somewhat on the details of the model [33]. In a two-Higgs-doublet model of type II ($XY=1$), charged-Higgs-boson masses below about 300 GeV are already excluded, independent of the value of $Y$. In a two-Higgs-doublet model of type I ($X=-Y$), Higgs-boson masses in the entire range $M_{H^+} < 800$ GeV are excluded if one assumes $|Y| > 2.7$. However, in a general multi-Higgs-doublet model, in which $X$ and $Y$ are independent parameters, the constraints become weaker. This leaves a large region of $|Y| - M_{H^+}$ parameter space in which the charged-Higgs-boson contribution to $B_q^0$-$\overline{B_q^0}$ mixing [Eq. (27)] can be significant and even dominant. For example, in a general model, the values $M_{H^+} = 400$ GeV and $Y=3$ are allowed, which implies $M_{12}^{H^+}/M_{12}^W = 7$. In this case the sum of the $W$ and $H^+$ terms (dominated by $H^+$) is consistent with the measurement of $B_d^0$-$\overline{B_d^0}$ mixing for the smallest values of $V_{td}$ in the unitarity range [Eq. (7)].

There is also a bound on the parameters $|Y|$ and $M_{H^+}$ from the latest ALEPH measurement of $R_b \equiv \Gamma(Z \to b\overline{b})/\Gamma(Z \to \text{hadrons}) = 0.2158 \pm 0.0014$ [34]. Assuming the neutral-Higgs-boson contribution to $R_b$ is small (i.e., $X$ is not too large) [35], at $3\sigma$ this results in the constraint [36]

$$|Y|^2 F(y_t) < 1.7, \quad (30)$$

where $y_t = m_t^2/M_{H^+}^2$ and

$$F(y) \equiv \frac{y}{(y-1)^2}[y-1-\ln y]. \quad (31)$$

This constraint is somewhat weaker than that from $b \to s\gamma$.

It is important to note that the phase of $B_q^0$-$\overline{B_q^0}$ mixing is unaffected by these new contributions. Also, the unitarity of the $3 \times 3$ CKM matrix holds in these models. Consequently, although the extraction of $|V_{td}|$ through the measurement of $\Delta M_d$ has to take the $H^+$ contribution into account, the measurement of its phase in the asymmetry of $\overset{(-)}{B_d} \to \Psi K_s$ is unaffected by the presence of the new physics.

Particularly interesting are models of spontaneous $CP$ violation, in which the entire Lagrangian is $CP$ invariant while the vacuum is not. (In the Weinberg three-Higgs-doublet model [24], this possibility seems to have already been ruled out by the experimental upper limit on the neutron electric dipole moment [28]). In this case $CP$ is violated in (neutral and charged) Higgs-boson exchange, while natural flavor conservation leads to a real CKM matrix [37]. Thus, the unitarity triangle becomes a straight line, and the amplitude of $B_q^0$-$\overline{B_q^0}$ mixing is real.

### 2. Flavor-changing neutral scalars

In the second class of models, flavor-changing neutral scalar interactions between quarks $i$ and $j$ exist, but are suppressed by factors $F_{ij}$ due to an approximate global symmetry. Such models have recently received special attention [38–42]. In this case $B_q^0$-$\overline{B_q^0}$ mixing may also receive large contributions from tree-level neutral-Higgs-boson exchange amplitudes which carry new phases. Denoting the flavor-changing neutral-Higgs-boson couplings by $(m_i/v)F_{ij}$ ($m_i > m_j$, $v^{-2} = \sqrt{2}G_F$), their contributions to $B_q^0$-$\overline{B_q^0}$ mixing are given in the vacuum insertion approximation by

$$M_{12}^{H^0}(B_q) \approx \frac{5\sqrt{2}}{24} \frac{G_F f_{B_q}^2 m_{B_q}^3}{M_{H^0}^2} F_{qb}^2, \quad (32)$$

where $M_{H^0}$ includes possible complex mixing among several neutral Higgs fields.

Comparing with mixing in the SM, we find

$$\frac{M_{12}^{H^0}(B_q)}{M_{12}^W(B_q)} \approx 0.50 \left(\frac{F_{qb}}{V_{tq}V_{tb}^*}\right)^2 \left(\frac{100 \text{ GeV}}{M_{H^0}}\right)^2. \quad (33)$$

Thus the neutral Higgs contributions may substantially modify the SM prediction for $B_q^0$-$\overline{B_q^0}$ mixing. The unitarity triangle holds in this model. However neither the magnitude of $V_{td}$ nor its phase can be directly measured through $\Delta M_d$ and the asymmetry in $\overset{(-)}{B_d} \to \Psi K_s$, respectively.

In models with specific predictions for $F_{ij}$, this leads to large effects in an interesting range of Higgs-boson masses. For instance, neutral-Higgs-boson contributions to $B_q^0$-$\overline{B_q^0}$ mixing are sizable for Higgs-boson masses around 100 GeV when flavor-changing couplings have the form [39,40]

$$F_{qb} = V_{tq}V_{tb}^*, \quad (34)$$

and for Higgs-boson masses of a few hundred GeV up to about a TeV in models in which [38]

$$F_{qb} = \sqrt{m_q/m_b}. \quad (35)$$

In general, these new contributions carry unknown phases and have to be added to the charged Higgs contributions [Eq. (27)].

In the presence of flavor-changing scalar interactions, the special case of spontaneous $CP$ violation does not, in general, forbid a phase in the CKM matrix. For a particular

choice of the softly broken symmetry this phase may, however, be very small [42] or may even vanish [40]. This would imply that the unitarity triangle becomes a straight line, while the $B_q^0$-$\overline{B_q^0}$ mixing amplitude carries a complex phase.

### D. Left-right symmetric models

In left-right symmetric models [43], the gauge group is extended to $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L}$, along with a discrete $L \leftrightarrow R$ symmetry. The right-handed CKM matrix is then related to its left-handed counterpart: $V^R = V^L$ or $V^R = (V^L)^*$. The right-handed $W_R$ can participate in weak processes in the same way as the ordinary $W$ (although certain decays are forbidden if the $\nu_R$ is too heavy). In particular, the $W_R$ can contribute to $B_q^0$-$\overline{B_q^0}$ mixing through box diagrams, as in the SM. However, limits from the $K_L$-$K_S$ mass difference constrain the $W_R$ to be heavier than 1.4 TeV [44], which would render its effects in the $B$ system negligible.

If one abandons the discrete $L \leftrightarrow R$ symmetry, so that $V^R$ is unrelated to $V^L$, the constraints from the $K_L$-$K_S$ mass difference can be evaded. For judicious choices of the form of $V^R$, *CP*-conserving experimental data permit the $W_R$ to be considerably lighter, $M_R \gtrsim 300$ GeV [45]. However, unless the elements of $V^R$ are considerably fine-tuned, there will be large contributions to the *CP*-violating parameter $|\epsilon_K|$ [46]. Assuming no such fine-tuning, the $W_R$ is again constrained to be heavy, $M_R \gtrsim 5$ TeV.

The above analysis assumes that $V^L$ has the same form as in the SM. However, this need not be the case. For example, it was suggested [47] that $B$ decays might in fact be mediated by the $W_R$, instead of the ordinary $W$. The long $B$ lifetime would then be interpreted as being due to the heaviness of the $W_R$, rather than to the smallness of $V_{cb}$. A variety of different forms for $V^L$ and $V^R$ were proposed [47,48]. In all cases, the $V_{cd}^R$ element was considerably smaller than in the SM, leading to the prediction that $B(b \to c\bar{c}d)/B(b \to c\bar{c}s)$ is at most $\sim 10^{-4}$. However, the decay $B \to \Psi\pi$ has since been observed with a branching ratio in agreement with the SM [49], effectively ruling out all such models.

Our conclusion is therefore that there are no important new-physics effects in the $B$ system within left-right symmetric models. The one possible exception is if one considerably fine-tunes the right-handed CKM matrix [50], but we do not consider such possibilities here.

### E. Supersymmetry

In the supersymmetric standard model (SSM) [51], the gauge group is unchanged, but a plethora of new particles is added. These include the supersymmetric partners of the SM particles, as well as a second Higgs-boson doublet (in some versions, additional Higgs-boson representations are also present). In the SSM there are thus a variety of new contributions to $B_q^0$-$\overline{B_q^0}$ mixing. These come from box diagrams with internal (i) charged Higgs bosons and charge 2/3 quarks, (ii) charginos and charge 2/3 squarks, (iii) gluinos and charge $-1/3$ squarks, and (iv) neutralinos and charge $-1/3$ squarks. The relative sizes of these new contributions, as well as their phase information, depend on the version of the SSM.

We first consider the minimal supersymmetric standard model (MSSM), which is usually taken to mean the low-

energy limit of the minimal spontaneously broken $N=1$ supergravity model. Here one typically imagines that there is unification at some high scale ($M_x$), and that supersymmetry (SUSY) is broken at this scale by some unknown mechanism (e.g., a ''hidden sector'') which interacts only gravitationally with the known fields. SUSY breaking is parametrized by soft breaking terms in the supergravity Lagrangian [52]. The low-energy effects of SUSY breaking, as well as the masses of the superpartners, are calculated by running the renormalization-group equations down from $M_x$ to the weak scale. The net effect is that, in addition to the usual gauge and Yukawa couplings, the MSSM is described by only four new parameters. The masses and mixings of all superpartners at low energy can be described in terms of these four parameters. If one also requires that the spontaneous breaking of $\text{SU}(2)_L \times \text{U}(1)_Y$ be induced radiatively, there is a further reduction in the number of SUSY parameters from 4 to 3 [53].

Although there are several new SUSY contributions to $B_q^0$-$\overline{B_q^0}$ mixing, in the MSSM these all have the same phase as in the SM, to a good approximation. We consider them in turn [54].

(i) *Charged Higgs bosons and charge 2/3 quarks:* These contributions have already been described above in the discussion of the two-Higgs-doublet model (Sec. II C 1). Unless one goes to extremely large values of $\tan\beta$, the charged Higgs couples the down-type quarks only to the $t$ quark:

$$\mathcal{L}_{H^\pm} \simeq \frac{g}{\sqrt{2}} \cot\beta \frac{m_t}{M_W} H^+ \bar{t}_R V_{ti} d_{iL}. \tag{36}$$

Thus the contribution to $B_q^0$-$\overline{B_q^0}$ mixing from charged Higgs bosons is proportional to $(V_{tb}^* V_{tq})^2$, as in the SM.

(ii) *Charginos and charge 2/3 squarks:* The couplings of the $\widetilde{W}^\pm$ and $\widetilde{H}^\pm$ to up-type quarks are very similar to those of the $W^\pm$ and $H^\pm$. (Note that the physical charginos are in general linear combinations of $\widetilde{W}^\pm$ and $\widetilde{H}^\pm$). In particular, the $\widetilde{W}^\pm$ couples only to left-handed up-type squarks:

$$\mathcal{L}_{\widetilde{W}} = \frac{g}{\sqrt{2}} (\widetilde{u}, \widetilde{c}, \widetilde{t})_L V_{\text{CKM}} \overline{\widetilde{W}} \gamma_L \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{37}$$

while in the limit of negligible down-type quark masses, the $\widetilde{H}^\pm$ couples mainly to right-handed squarks (assuming non-extreme values of $\tan\beta$):

$$\mathcal{L}_{\widetilde{H}} \simeq \frac{g}{\sqrt{2}} \frac{1}{\sin\beta} \frac{m_t}{M_W} \widetilde{t}_R V_{ti} \overline{\widetilde{H}} \gamma_L d_i, \tag{38}$$

where $\gamma_L = (1 - \gamma_5)/2$. The contributions to $B_q^0$-$\overline{B_q^0}$ mixing of both the $\widetilde{W}^\pm$ and $\widetilde{H}^\pm$ are proportional to $(V_{tb}^* V_{tq})^2$, as in the SM. For the $\widetilde{H}^\pm$ this follows directly from the above equation, while in the case of the $\widetilde{W}^\pm$, one uses the unitarity of the CKM matrix, along with the fact that $m_{\widetilde{u}_L} = m_{\widetilde{c}_L} \neq m_{\widetilde{t}_L}$ in the MSSM, to arrive at this result.

(iii) *Gluinos and charge $-1/3$ squarks:* The important coupling of the gluino ($\widetilde{g}$) to down-type quarks and squarks is

$$\mathcal{L}_{\widetilde{g}} = \sqrt{2} g_3 (\widetilde{d}, \widetilde{s}, \widetilde{b})_L V_{\text{CKM}} \bar{\widetilde{g}} \gamma_L \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{39}$$

(There is also a generation-diagonal coupling involving right-handed down squarks, but this cannot contribute to $B_q^0$-$\overline{B_q^0}$ mixing.) Note that the coupling is proportional to $V_{\text{CKM}}$. This, along with the fact that $m_{\widetilde{d}_L} = m_{\widetilde{s}_L} \neq m_{\widetilde{b}_L}$ in the MSSM, leads to a contribution to $B_q^0$-$\overline{B_q^0}$ mixing proportional to $(V_{tb}^* V_{tq})^2$, as in the SM.

(iv) *Neutralinos and charge $-1/3$ squarks:* The physical neutralinos are linear combinations of the photino ($\widetilde{\gamma}$), the Zino ($\widetilde{Z}$), and the two neutral Higgsinos ($\widetilde{H}_{1,2}^0$). The couplings of the $\widetilde{\gamma}$ and $\widetilde{Z}$ to down-type quarks and squarks are similar to that of the $\widetilde{g}$ [Eq. (39)]:

$$\mathcal{L}_{\widetilde{\gamma}} = \left( -\frac{1}{3} e \right) (\widetilde{d}, \widetilde{s}, \widetilde{b})_L V_{\text{CKM}} \bar{\widetilde{\gamma}} \gamma_L \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

$$\mathcal{L}_{\widetilde{Z}} = \frac{g}{\cos\theta_w} \left[ -\frac{1}{2} + \frac{1}{3} \sin^2\theta_w \right] (\widetilde{d}, \widetilde{s}, \widetilde{b})_L V_{\text{CKM}} \bar{\widetilde{Z}} \gamma_L \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{40}$$

The dependence on $V_{\text{CKM}}$ of the couplings of the $\widetilde{\gamma}$ and $\widetilde{Z}$ is just like that of the gluino, leading to a contribution to $B_q^0$-$\overline{B_q^0}$ mixing which is proportional to $(V_{tb}^* V_{tq})^2$. As for the neutral Higgsinos, their coupling to down-type quarks and squarks is proportional to $M_D/M_W \cos\beta$, which is negligible (unless one goes to extremely large values of $\tan\beta$).

Thus, to a good approximation, in the MSSM all new SUSY contributions to $B_q^0$-$\overline{B_q^0}$ mixing have the same phase as in the SM. Therefore the $CP$ asymmetries in $B$ decays will not be modified. The full expressions for these contributions are quite complicated, so we do not reproduce them here (we refer the reader to [54]). The strongest constraints on the SUSY parameters come from direct searches. For example, there are lower bounds of 176 and 45 GeV on squark and chargino masses, respectively [55]. The parameter space is sufficiently complicated that it is very difficult to establish firm constraints from loop-level processes such as $b \rightarrow s\gamma$. We note, however, that the effect of supersymmetry on $B_q^0$-$\overline{B_q^0}$ can be quite significant. For certain values of the parameters, the total SUSY contribution to $B_d^0$-$\overline{B_d^0}$ mixing can be twice as large as that of the SM.

Recently [56] it was suggested that $CP$ violation could possibly come from SUSY-breaking alone, with a real CKM matrix. In this case, which is essentially a superweak-type model, the unitarity triangle becomes a straight line and all $CP$ asymmetries in $B$ decays are very small.

We now turn to nonminimal SUSY models. For generic squark masses, supersymmetric contributions enhance flavor-changing processes such as $K^0$-$\overline{K^0}$ well beyond their experimental values. Any nonminimal SUSY model should address this problem (in the MSSM this problem is resolved since, to a good approximation, the squarks are degenerate).

One class of nonminimal SUSY models solves the FCNC problem by imposing an Abelian horizontal symmetry on the

Lagrangian [57]. In this case the quark mass matrices are approximately aligned with the squark mass-squared matrices. This has the effect that the mixing matrix for quark-squark-gluino couplings is close to a unit matrix, so that FCNC are suppressed, even though the squarks may not be degenerate. In most such models, the SUSY contributions to $B_q^0$-$\overline{B_q^0}$ mixing are quite small. However, it is possible to construct models in which $M_{12}^{\text{SUSY}}(B_d^0)/M_{12}^{\text{SM}}(B_d^0)$ is as large as 0.15, with a negligible effect on $B_s^0$-$\overline{B_s^0}$ mixing. Since the phase of the SUSY contribution is unknown, this can lead to measurable deviations from the SM predictions in $CP$ asymmetries involving $B_d^0$ decays. In another class of nonminimal SUSY models, known as effective supersymmetry [77], the suppression of FCNC's applies only to the first two families of squarks. In this case new-physics effects on $B_q^0$-$\overline{B_q^0}$ mixing can be much larger [78].

Another approach which is often taken is to ignore the FCNC problem altogether. One simply assumes that all SUSY parameters take the maximum allowed values permitted by experiment. In such ''models'' one can have non-negligible contributions to $B_q^0$-$\overline{B_q^0}$ mixing which have different phases than in the SM. These contributions typically involve right-handed squarks. For example [58], the general coupling of a $\widetilde{H}^\pm$ to down-type quarks and up-type squarks can be written

$$\mathcal{L}_{\widetilde{H}} \simeq \frac{g}{\sqrt{2}} \frac{1}{M_W \sin\beta} (\widetilde{u}, \widetilde{c}, \widetilde{t})_R \widetilde{U}_R^{u\dagger} U_R^u M_U V_{\text{CKM}} \bar{\widetilde{H}} \gamma_L \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{41}$$

where $\widetilde{U}_R^u$ ($U_R^u$) is the transformation matrix of the right-handed up-type squarks (quarks) needed to diagonalize the squark (quark) mass matrix. In the MSSM, $\widetilde{U}_R^u = U_R^u$, leading to a contribution to $B_q^0$-$\overline{B_q^0}$ mixing proportional to $(V_{tb}^* V_{tq})^2$ [Eq. (38)]. However, in general this relation need not hold, in which case there can be new phases in this contribution to $B_q^0$-$\overline{B_q^0}$ mixing.

As another example, consider again the contribution of gluinos and charge $-1/3$ squarks to $B_q^0$-$\overline{B_q^0}$ mixing. In the MSSM there is no intergenerational mixing among right-handed down-type squarks. As a consequence, the contribution to $B_q^0$-$\overline{B_q^0}$ mixing of gluinos and charge $-1/3$ squarks involves only left-handed squarks [Eq. (39)]. However, in nonminimal SUSY models, this need not be the case [59], there can be intergenerational mixing among right-handed down-type squarks. In general, this mixing matrix is unrelated to $V_{\text{CKM}}$, so that there can be new phases in this contribution to $B_q^0$-$\overline{B_q^0}$ mixing.

The main problem with this approach is that there is little predictivity. There are, in general, a very large number of parameters: the masses of the superpartners, their mixings, etc. Thus, although one can describe how new phases can enter $B_q^0$-$\overline{B_q^0}$ mixing, it is virtually impossible to analyse such effects in a systematic way.

### III. PENGUIN DECAYS AND NEW PHYSICS

As discussed in the Introduction, any new physics which contributes to $B_q^0$-$\overline{B_q^0}$ mixing will also contribute to flavor-

changing $B$ decays. Before examining the new-physics contributions to such penguin decays, we first review the SM predictions. Two aspects of these predictions are of particular interest to us: (i) the actual size of the branching ratios for various penguin decays, and (ii) the uncertainties, both experimental and theoretical, of the predictions. New-physics effects will be considered important in a particular penguin decay only if they change the branching ratio by quite a bit more than the uncertainty in the SM prediction; in other words, we are looking for ''smoking gun'' signals of new physics in such decays.

### A. The standard model

$b \to q \gamma$, $q = d$, $s$: The lowest-order amplitude for the decay $b \to q \gamma$ is [31]

$$A(b \to q \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi} \sum_i V_{ib}^* V_{iq} F_2(x_i) q^\mu \epsilon^\nu \overline{s} \sigma_{\mu\nu}$$
$$\times (m_b \gamma_R + m_s \gamma_L) b, \quad (42)$$

where the sum is over the up-type quarks, and $q^\mu$ and $\epsilon^\mu$ are the photon's four-momentum and polarization, respectively. The function $F_2$ is given by

$$F_2(x) = \frac{x}{24(x-1)^2} [6x(3x-2)\ln x$$
$$- (x-1)(8x^2 + 5x - 7)]. \quad (43)$$

Due to the smallness of the $u$- and $c$-quark masses, $F_2(x_u), F_2(x_c) \ll F_2(x_t)$, so that the $b \to q \gamma$ amplitude is dominated by $t$-quark exchange. A full quantitative treatment of these decays requires the calculation of the important QCD corrections. Including these, the SM branching ratios are [31]

$$B(B \to X_s \gamma) = (3.2 \pm 0.58) \times 10^{-4},$$
$$B(B \to X_d \gamma) = (1.0 \pm 0.8) \times 10^{-5}. \quad (44)$$

The uncertainties include both experimental errors ($m_t$, $B$ semileptonic branching ratio) and theoretical errors [$\mu$ (the renormalization scale), $\Lambda_{\rm QCD}$, and the ambiguity in the interpretation of $m_t$ (pole or running mass)], combined in quadrature. For $b \to d\gamma$, $|V_{td}/V_{ts}| = 0.24 \pm 0.11$ has been used and combined in quadrature with the other errors. The decay $b \to s\gamma$ has actually been measured by the CLEO Collaboration [30]:

$$B(B \to X_s \gamma) = (2.32 \pm 0.67) \times 10^{-4}. \quad (45)$$

This measurement can be used to constrain models of new physics, as already demonstrated in Eq. (28).

$b \to q l^+ l^-$, $q = d, s$: This class of decays is rather complicated theoretically [31]. First, one must calculate, at next-to-leading order, the Wilson coefficients of ten local operators which mix under renormalization. Second, one needs the matrix elements relevant for $B \to X_{s,d} l^+ l^-$, which can be calculated using the spectator model, along with $O(1/m_b^2)$ corrections. Finally, long-distance effects due to $J/\Psi$ and $\Psi'$ resonances must also be taken into account. The short-

distance contribution for $b \to s l^+ l^-$, which can be measured far away from the resonances, gives the following branching ratios, taken from [31]:

$$B(B \to X_s e^+ e^-) = (8.4 \pm 2.2) \times 10^{-6},$$
$$B(B \to X_s \mu^+ \mu^-) = (5.7 \pm 1.3) \times 10^{-6},$$
$$B(\to X_s \tau^+ \tau^-) = (2.6 \pm 0.5) \times 10^{-7}. \quad (46)$$

For $b \to d l^+ l^-$, the branching ratios are

$$B(B \to X_d e^+ e^-) = (4.9 \pm 4.3) \times 10^{-7},$$
$$B(B \to X_d \mu^+ \mu^-) = (3.3 \pm 2.8) \times 10^{-7},$$
$$B(B \to X_d \tau^+ \tau^-) = (1.5 \pm 1.3) \times 10^{-8}. \quad (47)$$

For all decays, the errors come from the same sources as in $b \to q\gamma$: $m_t$, $B$ semileptonic branching ratio, the renormalization scale $\mu$, $\Lambda_{\rm QCD}$, and $|V_{td}/V_{ts}| = 0.24 \pm 0.11$.

$B_q^0 \to l^+ l^-$, $q = d, s$: This decay can be calculated quite precisely in the SM. By including the QCD corrections [60], the renormalization-scale uncertainty is reduced to $O(1\%)$. There is still some hadronic uncertainty, parametrized by the $B$-meson decay constant $f_B$. The branching ratios are [31]

$$B(B_s^0 \to \tau^+ \tau^-) = (7.4 \pm 2.1) \times 10^{-7} \quad (f_{B_s}/232 \text{ MeV})^2,$$
$$B(B_s^0 \to \mu^+ \mu^-) = (3.5 \pm 1.0) \times 10^{-9} \quad (f_{B_s}/232 \text{ MeV})^2,$$
$$B(B_d^0 \to \tau^+ \tau^-) = (3.1 \pm 2.9) \times 10^{-8} \quad (f_{B_d}/200 \text{ MeV})^2,$$
$$B(B_d^0 \to \mu^+ \mu^-) = (1.5 \pm 1.4) \times 10^{-10} \quad (f_{B_d}/200 \text{ MeV})^2. \quad (48)$$

(The branching ratios to $e^+ e^-$ are some 5 orders of magnitude smaller than those for $\mu^+ \mu^-$.) The error in the $B_s^0$ branching ratios is due to the uncertainty, both experimental and theoretical, in the top-quark mass. The $B_d^0$ branching ratios have a larger error due to the $V_{td}$ CKM matrix element: $|V_{td}/V_{ts}| = 0.24 \pm 0.11$. At present, the best upper limits are $B(B_s^0 \to \mu^+ \mu^-) < 8.4 \times 10^{-6}$ and $B(B_d^0 \to \mu^+ \mu^-) < 1.6 \times 10^{-6}$ [61], with no significant limits on the $\tau^+ \tau^-$ final state.

*Gluon-mediated exclusive hadronic decays:* These arise from the quark-level process $b \to q q' \overline{q}'$. Throughout this paper we will refer to such loop-level decays as ''hadronic penguins.'' There are two ingredients needed to calculate the rates for hadronic penguin decays in the SM. First, the rates for the quark-level decays $b \to s q \overline{q}$ and $b \to d q \overline{q}$ are computed. This is done similarly to the decays $b \to s l^+ l^-$ and $b \to d l^+ l^-$: the Wilson coefficients of a variety of operators are calculated as one renormalizes down from the weak scale to the $b$ mass [62]. This can be done with reasonable precision. Second, one calculates the hadronic matrix elements for the hadronization of the final-state quarks into particular final states [63]. It is this step which introduces enormous uncertainty. These hadronic matrix elements are typically evaluated using the factorization approximation. Unfortunately, it is difficult to estimate the error incurred by applying this approximation to penguin decays. The predicted rates for

exclusive hadronic penguin decays can easily be in error by a factor of 2–3. (Much of this uncertainty cancels in the ratio of rates of corresponding $b \to d$ and $b \to s$ processes, which is given in the SM by $|V_{td}/V_{ts}|^2$ [64].)

Since the SM predictions for hadronic penguins have considerable uncertainties, if one wants an unmistakable signal of physics beyond the SM in such decays, the new-physics effects must be *enormous*: they must change the SM rates by an order of magnitude or more. It is therefore sufficient for our purposes to obtain approximate, order-of-magnitude estimates for both the SM and new-physics effects. To this end, we will use the following approximate form for the amplitude of the SM gluonic penguin contribution to the decay $b \to q q' \bar{q}'$:

$$A_{\text{penguin}} \sim \frac{\alpha_s(m_b)}{12\pi} \ln\left(\frac{m_t^2}{m_b^2}\right) V_{tq} V_{tb}^* \simeq 0.04 V_{tq} V_{tb}^*. \quad (49)$$

(Note that the coefficient 0.04 is about the same size as the largest of the Wilson coefficients of penguin operators [62].)

This expression can be used to estimate the order-of-magnitude rates for $b \to s$ and $b \to d$ penguins in the SM. For example, the branching ratio for $B_d^0 \to \pi^+ \pi^-$, which is dominated by the tree-level $b \to u \bar{u} d$ amplitude, is $\sim 10^{-5}$. Comparing this decay with $b \to s$ penguins, which dominate $B_d^0 \to \pi^- K^+$, we find

$$\left| \frac{A_{\text{penguin}}(b \to s)}{A_{\text{tree}}(b \to u \bar{u} d)} \right| \sim \left| \frac{0.04 V_{tb}^* V_{ts}}{V_{ub}^* V_{ud}} \right| \sim 0.5, \quad (50)$$

where we have used $|V_{ts}| \simeq |V_{cb}|$ and $|V_{ub}/V_{cb}| = 0.08$. This is consistent with the observation of a combined sample of $B_d^0 \to \pi^+ \pi^-$ and $B_d^0 \to \pi^- K^+$ decays [65], in which about equal mixtures of both modes are most likely. Thus, assuming that the hadronic matrix elements of tree and penguin operators have similar magnitudes, we expect pure $b \to s$ penguin hadronic decays (e.g., $B^+ \to \pi^+ K_s$, $B_d^0 \to \phi K_s$) to have branching ratios of $O(10^{-5})$. $b \to d$ penguins can be analyzed similarly:

$$\left| \frac{A_{\text{penguin}}(b \to d)}{A_{\text{tree}}(b \to u \bar{u} d)} \right| \sim \left| \frac{0.04 V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right|. \quad (51)$$

Since $1.4 \lesssim |V_{td}/V_{ub}| \lesssim 4.6$ [2], the penguin amplitude is about 1/10 the size of the tree amplitude, and may be larger if the hadronic penguin matrix elements are enhanced relative to those of tree amplitudes. Thus, pure $b \to d$ penguin hadronic decays (e.g., $B^+ \to K^+ K_s$) should have branching ratios $\sim 10^{-7}$ or somewhat larger.

As an aside, we note that Eq. (51) demonstrates why penguin contamination is a concern in the extraction of $\sin 2\alpha$ using the $CP$ asymmetry in $\overset{(-)}{B_d} \to \pi^+ \pi^-$. According to this estimate, the penguin amplitude can be as much as $\sim 15\%$ of the tree amplitude in magnitude (or even larger, if penguin matrix elements are enhanced), and with a different phase. This can lead to considerable uncertainty in the extraction of $\sin 2\alpha$, and shows why isospin techniques[5] are necessary to remove the penguin contamination.

*Electroweak-penguin-dominated exclusive hadronic decays:* Here, the diagrams contributing to the quark-level process $b \to q q' \bar{q}'$ consist of $\gamma$ and $Z$ penguins and box dia-grams. Of these, the diagram involving $Z$ exchange is the most important since it is enhanced by a factor of $m_t^2/M_W^2$. Throughout this paper we will refer to such processes as ''electroweak penguins.'' As with hadronic penguins, the calculation of SM rates for exclusive electroweak penguin decays suffers from large uncertainties in the hadronic matrix elements. We will therefore again use an approximate form for the amplitude of the SM electroweak penguin contribution to the decay $b \to q q' \bar{q}'$. This can be obtained from Eq. (49) through the replacements $\alpha_s(m_b) \to \alpha_2(m_b)$ and $\ln(m_t^2/m_b^2) \to m_t^2/M_W^2$. There is an additional factor of 2 due to a larger Wilson coefficient for the electroweak penguin operator. Therefore,

$$A_{\text{EWP}} \sim \frac{\alpha_2(m_b)}{6\pi} \left( \frac{m_t^2}{M_W^2} \right) V_{tq} V_{tb}^* \simeq 0.008 V_{tq} V_{tb}^*. \quad (52)$$

Note that, if one combines the final-state $q$ and $\bar{q}'$ quarks to form a meson, there is an additional color-suppression factor, $a_2$ [66]. Compared to color-allowed decays (i.e., forming a meson from $q'$ and $\bar{q}'$), which are parametrized by $a_1$, this suppression is $a_2/a_1 = 0.2$ [67].

Comparing Eqs. (49) and (52), we note that electroweak penguin amplitudes are suppressed relative to their hadronic penguin counterparts by a factor 0.2. Therefore we expect $b \to s$ and $b \to d$ electroweak penguin decays to have branching ratios in the range $10^{-7}$–$10^{-6}$ and $10^{-9}$–$10^{-8}$, respectively [68]. Some examples of decays which are dominated by electroweak penguins are $B_s^0 \to \phi \pi^0 (b \to s)$ and $B^+ \to \phi \pi^+ (b \to d)$.

At this point several observations are in order. From the above summary, we see that the SM $b \to d$ penguins have branching ratios which are about 1–2 orders of magnitude smaller than their $b \to s$ counterparts. Therefore, unless the new physics has a large influence on the $b \to d$ FCNC, its effects on $b \to s$ penguins will be detected first. On the other hand, $CP$ asymmetries involving $B_d^0$ decays are likely to be measured—and hence will reveal the presence of new physics—well before those involving $B_s^0$ mesons. So, from a practical point of view, this poses a bit of a problem. That is, even if new physics is detected in $CP$ violation in $B_d^0$ decays, it will be possible to test its nature by looking at $b \to d$ penguin decays only if the small SM branching ratios for these processes are significantly enhanced. Conversely, if one finds new-physics effects in $b \to s$ penguins, it will be difficult to determine its origin by looking at $CP$ asymmetries in $B_s^0$ decays.

This having been said, however, the situation is not quite so bleak. Although it is possible to construct models of physics beyond the SM in which only one of the $b \to d$ or $b \to s$ FCNC's is changed, in practice both FCNC's are affected in most new-physics models. For example, in models with $Z$-mediated FCNC's the $b \to d$ or $b \to s$ FCNC's are indeed described by different parameters. However, both of these FCNC's arise due to the mixing of the charge $-1/3$ quarks with an exotic vector singlet quark. It would be difficult to imagine that only one of the two FCNC's is induced, although this is a logical possibility. As another example, consider models with four quark generations. The enlarging of the CKM matrix to $4 \times 4$ will, in general, affect both FCNC's. Thus, in looking for new physics, the measure-

ments of *CP* asymmetries and *B* penguins are complementary, and it is quite likely that, should new physics be discovered, its nature will be revealed only by studying both *B* asymmetries and decays.

We now turn to the new-physics contributions to the above penguin decays.

### B. Four generations

Models with four generations have a number of new parameters which can enter in *B* decays: $m_{t'}$ and the CKM matrix elements involving the $t'$ quark. Furthermore, the strongest constraints on $V_{ts}$ and $V_{td}$ in the SM come from the unitarity of the $3 \times 3$ CKM matrix. When this matrix is enlarged to be $4 \times 4$, these constraints are weakened, so that, in effect, $V_{td}$ and $V_{ts}$ are also unknown parameters.

The parameter space of four-generation models is therefore quite large. Rather than exploring the entire space, we will simply present an ''existence proof.'' That is, we will show that it is possible to choose values of $m_{t'}$ and the CKM matrix elements, consistent with experimental data, which significantly affect both *CP* asymmetries and *B* penguin decays.

As discussed in Sec. II A, the experimental value of $B_d^0$-$\overline{B}_d^0$ mixing can be reproduced if $V_{td} \sim 0$, $V_{t'd} = 0.005$, $V_{tb} = V_{t'b} \simeq 1/\sqrt{2}$, and $m_{t'} = 480$ GeV. The phase of this mixing may, however, be quite different than in the SM. In this scenario, all penguin decays involving the *b-d* FCNC will also be dominated by the fourth generation. Below we consider the effects of this choice of parameters on $b \to d$ penguins.

We first consider the decays $b \to q \gamma$. The experimental measurement of $b \to s \gamma$ can be easily accommodated by adjusting the *t* and $t'$ contributions. However, the decay $b \to d \gamma$ will involve only $t'$ exchange. Using Eq. (42), we find that $|V_{t'd} V_{t'b}^* F_2(x_{t'})| = 0.001$, as compared to the SM value of $|V_{td} V_{tb}^* F_2(x_t)| = 0.002$. Therefore, this choice of parameters will result in a branching ratio for $b \to d \gamma$ which is roughly four times smaller than in the SM. Given the large uncertainties in the SM prediction, this cannot be considered an unmistakable signal of new physics.

Now consider hadronic penguins. From Eq. (49), we have

$$\left| \frac{A_{\text{penguin}}^{\text{four gen}}}{A_{\text{penguin}}^{\text{SM}}} \right| = \left| \frac{\ln(m_{t'}^2/m_b^2) V_{t'd} V_{t'b}^*}{\ln(m_t^2/m_b^2) V_{td} V_{tb}^*} \right| = 0.46. \quad (53)$$

Thus, in this model, the branching ratios for exclusive $b \to d$ hadronic penguins will be a factor of $\sim 5$ smaller than in the SM. Given the uncertainties in the SM predictions this is also a marginal ''smoking gun'' signal of new physics.

The situation is better for electroweak penguins. From Eq. (52),

$$\left| \frac{A_{\text{EWP}}^{\text{four gen}}}{A_{\text{EWP}}^{\text{SM}}} \right| = \left| \frac{(m_{t'}^2/M_W^2) V_{t'd} V_{t'b}^*}{(m_t^2/M_W^2) V_{td} V_{tb}^*} \right| = 2.8. \quad (54)$$

The branching ratios for exclusive $b \to d$ electroweak penguins are thus a factor of $\sim 8$ larger than in the SM. This same enhancement applies to the decays $B_d^0 \to l^+ l^-$. In both cases, this would be a quite convincing signal of physics beyond the SM.

Thus, in models with four generations, we have shown that there are regions of parameter space in which both *B* *CP* asymmetries and *B* penguin decays are significantly affected. If a discrepancy with the SM is found in the measurement of the *CP* asymmetries, the study of the decays can help pin down the new-physics parameters. Admittedly, in the particular example we have chosen, the branching ratios for the affected processes are all small, of $O(10^{-7})$ or smaller. But the key point here is that the various penguin processes depend differently on the masses of the *t* and $t'$ quarks. It is thus straightforward to find a set of parameters, consistent with current experimental data, in which the $b \to s$ FCNC decays are affected. In this case it would be *CP* asymmetries involving $B_s^0$ decays which would be altered. Of course, since the four-generation CKM matrix is $4 \times 4$, in the general case both the $b \to d$ and $b \to s$ FCNC's will be changed from the SM, affecting *all CP* asymmetries and penguin decays.

### C. Z-mediated FCNC's

For the process $b \to q f \bar{f}$, the amplitude due to Z-mediated FCNC's is

$$\mathcal{M}_{Z \text{ FCNC}} = \frac{4 G_F}{\sqrt{2}} U_{qb} \overline{q} \gamma^\mu \gamma_L b \overline{f} [g_L^f \gamma_\mu \gamma_L + g_R^f \gamma_\mu \gamma_R] f. \quad (55)$$

The rate for $b \to q f \bar{f}$ is simply

$$\Gamma(b \to q f \bar{f}) = \frac{G_F^2 m_b^5}{192 \pi^3} |U_{qb}|^2 [(g_L^f)^2 + (g_R^f)^2], \quad (56)$$

while the rate for $B_q^0 \to l^+ l^-$ is

$$\Gamma(B_q^0 \to l^+ l^-) = \frac{G_F^2}{16\pi} \tau_{B_q} f_{B_q}^2 M_{B_q} m_L^2 |U_{qb}|^2. \quad (57)$$

With these expressions in hand, we can now calculate or estimate the contributions to penguin decays due to Z-mediated FCNC's. In all cases, when presenting numbers, we use the upper limit $U_{qb} < 0.0017$ [Eq. (14)].

The upper limit of $U_{qb} < 0.0017$ is, in fact, derived from the experimental limit on the branching ratio of $B \to \mu^+ \mu^- X$ (see Sec. II B). Thus, with this value of $U_{qb}$, Z-mediated FCNC models ''predict'' that $B(B \to \mu^+ \mu^- X) = 5 \times 10^{-5}$. For $b \to s$ FCNC's this is roughly an order of magnitude larger than the SM prediction, while for $b \to d$ transitions it is about 2 orders of magnitude larger. If the branching ratios for the decays $B \to X_s l^+ l^-$ and $B \to X_d l^+ l^-$ are observed to be consistent with the SM predictions, this will imply that $|U_{sb}| \lesssim 6 \times 10^{-4}$ and $|U_{db}| \lesssim 1 \times 10^{-4}$. In both cases, this implies that the new-physics effects in $B_q^0$-$\overline{B}_q^0$ mixing are negligible. Conversely, if $B_q^0$-$\overline{B}_q^0$ mixing is significantly affected by Z-mediated FCNC's then one expects to see a substantial enhancement of the branching ratios for $B \to X_s l^+ l^-$ and/or $B \to X_d l^+ l^-$.

We now turn to $B_q^0 \to l^+ l^-$, $q = d,s$. From Eq. (57), the rates for these processes, due only to Z-mediated FCNC's are

$$B(B_s^0 \to \tau^+ \tau^-)|_{Z\ \mathrm{FCNC}} < 1.6 \times 10^{-5} \quad (f_{B_s}/232\ \mathrm{MeV})^2,$$
$$(58)$$

$$B(B_s^0 \to \mu^+ \mu^-)|_{Z\ \mathrm{FCNC}} < 5.8 \times 10^{-8} \quad (f_{B_s}/232\ \mathrm{MeV})^2,$$

$$B(B_d^0 \to \tau^+ \tau^-)|_{Z\ \mathrm{FCNC}} < 1.2 \times 10^{-5} \quad (f_{B_d}/200\ \mathrm{MeV})^2,$$

$$B(B_d^0 \to \mu^+ \mu^-)|_{Z\ \mathrm{FCNC}} < 4.2 \times 10^{-8} \quad (f_{B_d}/200\ \mathrm{MeV})^2.$$

Thus, if the FCNC parameters $U_{qb}$ have values near the present upper limits, the predicted rates for $B_s^0 \to l^+ l^-$ and $B_d^0 \to l^+ l^-$ are, respectively, about 20 and 300–400 times larger than those expected in the SM.

Turning to hadronic and electroweak penguins, we note that there are three types of comparisons which can be made. First consider decays such as $B_s^0 \to K_s K_s (b \to s)$ or $B_d^0 \to K_s K_s (b \to d)$, which receive contributions from ordinary (gluonic) penguins and color-suppressed $Z$-mediated FCNC's. For these decays we have

$$\left| \frac{A_{Z\ \mathrm{FCNC}}}{A_{\mathrm{SM}}} \right| \sim \left| \frac{a_2}{a_1} \frac{U_{qb}\sqrt{(g_L^d)^2 + (g_R^d)^2}}{0.04 V_{tb}^* V_{tq}} \right| = 2.2 \left| \frac{U_{qb}}{V_{tq}} \right|$$
$$< \begin{cases} 0.1, & q = s, \\ 0.4, & q = d, \end{cases} \quad (59)$$

where we have taken $|U_{qb}/V_{cb}| < 0.044$ and $|V_{td}/V_{cb}| \simeq 0.24$ in our estimates of the ratio. The branching ratios for this type of $B$ decays will therefore not be significantly affected by $Z$-mediated FCNC's.

Next we have decays such as $B_d^0 \to \phi K_s (b \to s)$ or $B_s^0 \to \phi K_s (b \to d)$. Here there are contributions from ordinary gluonic penguins and color-allowed $Z$-mediated FCNC's. Then

$$\left| \frac{A_{Z\ \mathrm{FCNC}}}{A_{\mathrm{SM}}} \right| \sim \left| \frac{U_{qb} g_V^s}{0.04 V_{tb}^* V_{tq}} \right| = 8.8 \left| \frac{U_{qb}}{V_{tq}} \right| < \begin{cases} 0.4, & q = s, \\ 1.6, & q = d. \end{cases}$$
$$(60)$$

In this case, $Z$-mediated FCNC's will not much affect the $b \to s$ penguin decays, but the branching ratios for $b \to d$ penguins can be increased by a factor of 3–4. Given the uncertainties in the SM predictions for such decays, this cannot be considered significant. However, if the value of $V_{td}$ is in fact smaller than 0.24 (in this model it can be as small as 0.07), then the branching ratios for $b \to d$ penguins will be correspondingly increased. We consider this to be a marginal prediction, since it depends sensitively on the true value of $V_{td}$.

Finally, decays such as $B_s^0 \to \phi \pi^0 (b \to s)$ or $B^+ \to \phi \pi^+ (b \to d)$ have contributions from ordinary electroweak penguins (but not gluonic penguins) and $Z$-mediated FCNC's. Here

$$\left| \frac{A_{Z\ \mathrm{FCNC}}}{A_{\mathrm{SM}}} (b \to s) \right| \sim \left| \frac{U_{sb}}{0.008 V_{tb}^* V_{ts}} \right| < 5.5,$$

$$\left| \frac{A_{Z\ \mathrm{FCNC}}}{A_{\mathrm{SM}}} (b \to d) \right| \sim \left| \frac{U_{db}}{0.008 V_{tb}^* V_{td}} \right| < 22.9. \quad (61)$$

The effects of $Z$-mediated FCNC's on such decays are clearly enormous. The branching ratios for pure electroweak penguin decays can be increased by as much as a factor of $\sim 25$ ($b \to s$) or $\sim 500$ ($b \to d$). Clearly this is a "smoking gun" signal of new physics.

On the topic of hadronic penguin decays, there is another possibility which should be mentioned. In the SM, the decay $B^+ \to \pi^+ \pi^0$ occurs principally via a tree-level amplitude—there is no gluonic penguin and the electroweak penguin is much suppressed. One therefore does not expect to find $CP$ violation in this mode. However, $Z$-mediated FCNC's also contribute to this decay. Comparing this new-physics contribution with the SM tree-level amplitude, we find

$$\left| \frac{A_{Z\ \mathrm{FCNC}}}{A_{\mathrm{SM}}} \right| \sim \left| \frac{U_{db}(g_A^u - g_A^d)}{V_{ub}^* V_{ud}} \right| \lesssim 0.5. \quad (62)$$

The $Z$-mediated FCNC contribution to this decay could therefore be substantial. If there is a significant strong phase difference between the two amplitudes, following for instance from different rescattering in the $I = 2$ channel, this could lead to direct $CP$ violation in this decay mode. If found, this would be another clear signal of this particular type of new physics.

$Z$-mediated FCNC's can also contribute to the decays $b \to q \gamma$ at the one-loop level. However, the calculations are highly model dependent, since it is necessary to include the new vector-singlet quark(s) with which the SM charge $-1/3$ quarks mix. The authors of Ref. [69] considered the case of a single vector-singlet quark, and included the contribution of the Higgs boson as well. For $|U_{qb}| < 0.0017$ they found that the contribution to $b \to s \gamma$ was unimportant, but that the branching ratio for $b \to d \gamma$ could be changed significantly, depending on the values of the masses of the exotic quark and the Higgs boson. Since there is quite a bit of model dependence, we do not consider this to be a clean signal of new physics.

In summary, if $Z$-mediated FCNC's contribute significantly to $B_q^0$-$\overline{B_q^0}$ mixing, they will also lead to large effects in a variety of penguin decays. The present experimental upper limit on the FCNC parameters is $|U_{qb}| < 0.0017$ ($q = d, s$). This value for the parameters leads to unmistakable effects in $b \to q l^+ l^-$, $B_q^0 \to l^+ l^-$, and electroweak penguins. In addition, $Z$-mediated FCNC's may lead to direct $CP$ violation in decay modes such as $B^+ \to \pi^+ \pi^0$.

### D. Multi-Higgs-doublet models

#### 1. Natural flavor conservation

In these models the new contributions to rare $B$ decays come about through amplitudes in which charged-Higgs boson exchange replaces the SM $W$ exchange. As noted in Eq. (28) and the surrounding discussion, the measurement of $b \to s \gamma$ already excludes a region of parameter space with low $M_{H^+}$ and high $|Y|$.

In spite of this, charged-Higgs-boson exchange may have significant effects on other rare $B$ decays. The processes $B_q^0 \to l^+ l^-$, $q = d, s$ were studied in multi-Higgs-doublet models in [70]. Neglecting small contributions from neutral Higgs pseudoscalar (which are proportional to $m_b^2/M_P^2$, where $M_P$ is the pseudoscalar mass) [71], one finds the following expression for the ratio of charged-Higgs-boson and SM amplitudes:

$$\frac{A_{H^+}(B_q^0 \to l^+ l^-)}{A_{\mathrm{SM}}(B_q^0 \to l^+ l^-)} = \frac{(1/2)x_t B(y_t)|Y|^2}{B(x_t) - C(x_t)} \approx -2.25 B(y_t)|Y|^2,$$

$$(63)$$

where

$$B(x) = \frac{1}{4}\left[\frac{x}{1-x} + \frac{x}{(1-x)^2}\ln x\right],$$

$$C(x) = \frac{x}{4}\left[\frac{3 - (1/2)x}{1-x} + \frac{(3/2)x + 1}{(1-x)^2}\ln x\right]. \quad (64)$$

Since $B(y_t)$ is negative, the two amplitudes add up constructively.

To illustrate the effect, let us consider the case $M_{H^+} = 400$ GeV, $Y = 3$ discussed in Sec. II C 1. For these parameters, the ratio of Eq. (63) becomes 2.5. Thus, the $B_q^0 \to l^+ l^{-1}$ rates are expected to be an order of magnitude larger than in the SM while charged-Higgs exchange dominates $B_q^0$-$\overline{B_q^0}$ mixing.

The decays $b \to q l^+ l^-$ in multi-Higgs-doublet models are more complicated than $B_q^0 \to l^+ l^-$, since there are more operators which can contribute to this process. We refer to [72] for the details of the computation, but the conclusions are as follows. For those values of $|Y|$ and $M_{H^+}$ for which $B_q^0$-$\overline{B_q^0}$ mixing is dominated by charged-Higgs-boson exchange (i.e., large $|Y|$), the decays $b \to q l^+ l^-$ can be enhanced by a factor of about 2. For $b \to d l^+ l^-$ this is within the error of the SM prediction, but it is a significant effect for $b \to s l^+ l^-$.

Finally, we turn to hadronic penguin decays. Model-dependent studies [73] show that, once the constraint from $b \to s\gamma$ is taken into account, the effect of charged-Higgs-boson exchange can change the SM predictions by no more than a few tens of percent. Since these predictions suffer from large hadronic uncertainties, the rates of these processes cannot signal charged-Higgs-boson effects.

### 2. Flavor-changing neutral scalars

In models without NFC, one can have flavor-changing processes mediated by neutral scalars (FCNS's). The flavor-changing couplings between quarks of flavor $i$ and $j$ are parametrized as $(m_i/v)F_{ij}(\gamma_5/2)$ $(m_i > m_j)$, while the flavor-conserving couplings are $m_f/v$. Thus there are also contributions to penguin decays due to neutral scalar exchange. Consider the decay $b \to q f \overline{f}$, where $f$ can be a quark or a lepton. The rate for this decay, due to FCNS alone, is

$$\Gamma(b \to q f \overline{f})|_{\mathrm{FCNS}} = \frac{G_F^2 m_b^5}{3072\pi^3}\left(\frac{m_b m_f}{M_{H^0}^2}\right)^2 |F_{qb}|^2. \quad (65)$$

This rate is clearly maximized when the fermion $f$ is as massive as possible. Consider then the decay $b \to q\tau^+\tau^-$. For $M_{H^0} = 100$ GeV, $F_{db} = 0.02$ is essentially the largest value possible due to constraints from $B_d^0$-$\overline{B_d^0}$ mixing. On the other hand, $F_{sb}$ has no similar constraint, so we take $F_{sb} = \sqrt{m_s/m_b} = 0.17$. Then

$$B(b \to q\tau^+\tau^-)|_{\mathrm{FCNS}} = \begin{cases} 2.4 \times 10^{-7}, & q = s, \\ 4.3 \times 10^{-9}, & q = d. \end{cases} \quad (66)$$

The branching ratio for $b \to d\tau^+\tau^-$ is only about 1/3 of the SM expectation, well within the errors of the prediction. This decay can therefore not be used to find effects of neutral scalars. On the other hand, the decay $b \to s\tau^+\tau^-$ could be significantly affected by flavor-changing neutral scalars, since its new-physics branching ratio is of the same order as the SM prediction. However, note that we have selected almost maximal values for the new-physics parameters. If $F_{sb}$ is smaller, or $M_{H^0}$ larger, than the values we have chosen, the new-physics contribution to $b \to s\tau^+\tau^-$ would then diminish considerably, although there could still be important effects in $B_s^0$-$\overline{B_s^0}$ mixing. Note also that, due to the mass suppression, the FCNS contribution to decays involving lighter fermions is completely negligible.

The one decay in which the mass suppression is not obviously a disadvantage is $B_q^0 \to l^+ l^-$, $q = d, s$, since such a suppression is present even in the SM. We find

$$\Gamma(B_q^0 \to l^+ l^-)|_{\mathrm{FCNS}} = \frac{5}{48\pi} G_F^2 M_{B_q} f_{B_q}^2 m_l^2 |F_{qb}|^2 \left(\frac{M_{B_q}}{M_{H^0}}\right)^4. \quad (67)$$

Again taking $M_{H^0} = 100$ GeV, $F_{sb} = 0.17$, and $F_{db} = 0.02$, this gives

$$B(B_s^0 \to \tau^+\tau^-)|_{\mathrm{FCNS}} = 2.3 \times 10^{-6} \quad (f_{B_s}/232 \text{ MeV})^2,$$

$$B(B_s^0 \to \mu^+\mu^-)|_{\mathrm{FCNS}} = 8.1 \times 10^{-9} \quad (f_{B_s}/232 \text{ MeV})^2,$$

$$B(B_d^0 \to \tau^+\tau^-)|_{\mathrm{FCNS}} = 2.1 \times 10^{-8} \quad (f_{B_d}/200 \text{ MeV})^2,$$

$$B(B_d^0 \to \mu^+\mu^-)|_{\mathrm{FCNS}} = 7.5 \times 10^{-11} \quad (f_{B_d}/200 \text{ MeV})^2. \quad (68)$$

For $B_d^0$ decays, the new-physics effects are within the errors of the SM prediction. For $B_s^0$ decays, the branching ratios due to flavor-changing neutral scalars are a factor of 2–3 times larger than in the SM. Since there are uncertainties in the SM prediction, and since we have taken optimal values for the new-physics parameters, this can only be considered a marginal signal of new physics.

Therefore, in models with flavor-changing processes mediated by neutral scalars, there are no "smoking gun" signals in penguin decays. For maximal values of the new-physics parameters, there may be enhancements in the branching ratios of $b \to s\tau^+\tau^-$ and $B_s^0 \to l^+ l^-$. However, for other values of these parameters there will be no significant effects in these and other penguin decays, even though there may still be important contributions to $B_q^0$-$\overline{B_q^0}$ mixing.

### E. Supersymmetry

The parameter space of supersymmetric models is quite complex, so that definite predictions of effects in penguin decays are hard to obtain. For example, consider the decay $b \to s\gamma$ in the MSSM [33]. In addition to the charged-Higgs-boson effects, which always increase the rate (see Sec. II C 1), there are additional SUSY contributions from charginos + up-type squarks or gluinos + down-type squarks in the loop. In certain regions of parameter space the net effect is to cancel the contribution of the $H^\pm$, resulting in a branching

ratio at or below the SM value, while in other regions the branching ratio is always greater than in the SM. Thus, the experimental measurement of $b \to s\gamma$ does not constrain the SUSY parameter space in a simple way.

The decay $b \to sl^+l^-$ has recently been analyzed in supersymmetric models, taking into account the constraint from $b \to s\gamma$ [74]. In the MSSM, it is found that the branching ratios for $b \to se^+e^-$ and $b \to s\mu^+\mu^-$ can be changed by, at most, 23 and 12 %, respectively. These deviations are within the errors of the SM predictions, so these decay modes cannot be used to detect supersymmetry. However, the authors of [74] also study the $C$-odd lepton-antilepton energy asymmetry:

$$\mathcal{A} = \frac{N(E_{l^-} > E_{l^+}) - N(E_{l^+} > E_{l^-})}{N(E_{l^-} > E_{l^+}) + N(E_{l^+} > E_{l^-})}, \quad (69)$$

in which $N(E_{l^-} > E_{l^+})$ denotes the number of decays where the $l^-$ is more energetic in the $B$ meson rest frame than the $l^+$. They find that this asymmetry can be affected by up to 70% for $b \to se^+e^-$ and 48% for $b \to s\mu^+\mu^-$. Furthermore, these sizable deviations occur in a large region of SUSY parameter space. Thus, this asymmetry is an excellent place to look for effects of supersymmetry. Unfortunately, it is not clear how that region of parameter space which leads to large deviations in this asymmetry is correlated with that region of parameter space in which there are significant contributions to $B_q^0$-$\overline{B_q^0}$ mixing.

Reference [74] also examines $b \to sl^+l^-$ in a certain class of nonminimal SUSY models. In this case, the effects can be huge: the branching ratios for $b \to se^+e^-$ and $b \to s\mu^+\mu^-$ can be doubled, and the asymmetries enhanced by a factor of 3.

For the decays $B_q^0 \to l^+l^-$ and $b \to qq'\overline{q}'$, similar studies have not yet been carried out in the context of supersymmetric models. However, since the branching ratio for $b \to sl^+l^-$ is not substantially affected in the MSSM, this suggests that the SUSY contributions to $B_q^0 \to l^+l^-$ and $b \to qq'\overline{q}'$, which are similar decays, will also not change the branching ratios significantly. On the other hand, in nonminimal SUSY models, the branching ratios for these decays may receive important corrections.

## IV. SUMMARY

The phase information of the CKM matrix, which is the SM explanation of $CP$ violation, is represented by the unitarity triangle. At present, our knowledge of this triangle is rather poor: only the sides have been measured directly ($|V_{ub}/V_{cb}|$ is probed in charmless $B$ decays, and within the SM $|V_{td}/V_{cb}|$ can be extracted from $B_d^0$-$\overline{B_d^0}$ mixing), but these measurements suffer from large theoretical uncertainties. In the near future, the angles of the unitarity triangle will be extracted from $CP$ asymmetries in $B$-meson decays. Through the measurements of the $CP$ angles $\alpha$, $\beta$, and $\gamma$, it will be possible to test the consistency of this description. There are three distinct ways in which the presence of new physics might be revealed.

(1) The relation $\alpha + \beta + \gamma = \pi$ is violated. (Note that this relation can be tested only if $\gamma$ is measured in $CP$ asymmetries involving $B_s^0$ decays. If $\gamma$ is measured via $B^\pm \to D_{CP}K^\pm$

this relation will hold even in the presence of most types of new physics.)

(2) Although $\alpha + \beta + \gamma = \pi$, one finds values for the $CP$ phases which are outside of the SM predictions.

(3) The $CP$ angles measured are consistent with the SM predictions, and add up to 180°, but are inconsistent with the measurements of the *sides* of the unitarity triangle.

If any of these discrepancies is found, we will want to know what kind of new physics is involved. The principal way in which new physics can affect the $CP$ asymmetries is through new contributions to $B_q^0$-$\overline{B_q^0}$ mixing. By performing a model-by-model analysis of physics beyond the SM, it is possible to ascertain which types of new physics might be responsible for the discrepancy. This analysis allows us to separate new-physics models into two types: (i) those in which the phase of $B_q^0$-$\overline{B_q^0}$ mixing is changed, in which case each of items (1)–(3) may occur, and (ii) those in which the phase is unchanged, in which case only (3) is possible. However, this analysis does not tell us how to distinguish among models of a given type. It is this question which we have attempted to address in this paper.

Our main observation is quite simple. Any new physics which contributes to $B_d^0$-$\overline{B_d^0}$ or $B_s^0$-$\overline{B_s^0}$ mixing will necessarily contribute to the rare flavor-changing penguin decays $b \to dX$ and $b \to sX$. In some cases, the values of the new-physics parameters which yield significant effects in $B_q^0$-$\overline{B_q^0}$ mixing will also lead to large deviations from the SM predictions for certain penguin decays. It is therefore possible to partially distinguish among different models of new physics by examining their predictions for these penguin decays.

(i) *Four generations:* The phase of $B_q^0$-$\overline{B_q^0}$ mixing can be changed due to the new box-diagram contributions with internal $t'$ quarks. Although the new-physics parameter space is too large to make absolute predictions for branching ratios of penguin decays, we have shown that there are regions of parameter space in which both $B$ $CP$ asymmetries and $B$ penguin decays are significantly affected. The particular example we chose found important contributions to $B_d^0$-$\overline{B_d^0}$ mixing, and roughly an order-of-magnitude enhancement of the branching ratios for both exclusive $b \to d$ electroweak penguins and $B_d^0 \to l^+l^-$.

(ii) *Z-mediated flavor-changing neutral currents:* The phase of $B_q^0$-$\overline{B_q^0}$ mixing can be altered due to the tree-level exchange of a $Z$ with flavor-changing couplings. In fact, $B_d^0$-$\overline{B_d^0}$ mixing can be dominated by this new physics. If these new contributions are important, there will also be unmistakable effects in $b \to ql^+l^-$, $B_q^0 \to l^+l^-$, and electroweak penguins. In particular, the rates for $b \to s$ ($b \to d$) penguin processes can be enhanced by as much as 1 (2) orders of magnitude. In addition, $Z$-mediated FCNC's may lead to direct $CP$ violation in decay modes such as $B^+ \to \pi^+\pi^0$. On the other hand, if the branching ratios for $b \to ql^+l^-$ are found to be consistent with the SM, this will indicate that $Z$-mediated FCNC effects in $B_q^0$-$\overline{B_q^0}$ mixing are negligible.

(iii) *Multi-Higgs-doublet models with natural flavor conservation:* There are new box-diagram contributions to $B_q^0$-$\overline{B_q^0}$ mixing with internal charged Higgs bosons, but the phase of this mixing is unchanged. (If one also has sponta-

neous *CP* violation in such models, this phase is zero, since the CKM matrix is real, and the unitarity triangle becomes a straight line.) For that region of parameter space in which $B_q^0$-$\overline{B_q^0}$ mixing is dominated by the charged-Higgs-boson contribution, the branching ratios for $B_q^0 \to l^+ l^-$ and $B \to X_q l^+ l^-$ are enhanced by up to an order of magnitude or a factor of 2, respectively. However, when the box diagrams with internal $W^\pm$ and $H^\pm$ bosons are about equal in magnitude, charged-Higgs-boson effects in penguin decays may not be sufficiently large to be detected, due to theoretical uncertainties in the SM rate calculations.

(iv) *Multi-Higgs-doublet models with flavor-changing neutral scalars:* The phase of $B_q^0$-$\overline{B_q^0}$ mixing can be changed due to the tree-level exchange of a neutral scalar with flavor-changing couplings. However, there are no significant effects in *B* penguin decays. This is due to the fact that the flavor-conserving coupling of the neutral scalar to a fermion is proportional to the fermion mass, and penguin decays all involve light fermions.

(v) *Left-right symmetric models:* There are no significant effects in the *B* system in these models. The only exception is the case where the right-handed CKM matrix is considerably fine-tuned, but we do not consider this possibility.

(vi) *Minimal supersymmetric models:* There are many new contributions to $B_q^0$-$\overline{B_q^0}$ mixing, but all have the same phase as in the SM. A search of the parameter space reveals that SUSY contributions to $b \to s l^+ l^-$ do not change its branching ratio significantly. This suggests that the branching ratios for the decays $B_q^0 \to l^+ l^-$ and $b \to q q' \overline{q'}$ will also be relatively unaffected. However, SUSY can be detected by examining the lepton-antilepton energy asymmetry. This asymmetry can be affected by up to 70% for $b \to s e^+ e^-$ and 48% for $b \to s \mu^+ \mu^-$.

(vii) *Nonminimal supersymmetric models:* In nonminimal SUSY models with quark-squark alignment, the SUSY contributions to $B_q^0$-$\overline{B_q^0}$ mixing (and hence to *B* penguin decays) are generally very small [though models do exist in which $M_{12}^{\text{SUSY}}(B_d^0)/M_{12}^{\text{SM}}(B_d^0) \approx 0.15$]. In alternative nonminimal SUSY ''models'' it is simply assumed that all SUSY parameters take the maximum allowed values allowed by experiment. These models are not terribly predictive, due to the very large numbers of parameters. It is possible to find new contributions to the mixing with different phases than in the SM, and to arrange the many parameters such that the branching ratios of penguin decays are enhanced or suppressed. However, it is virtually impossible to analyze the effects of nonminimal SUSY on $B_q^0$-$\overline{B_q^0}$ mixing and *B* penguins in any systematic way.

If some indication of new physics is found in the measurements of *CP* asymmetries, the above analysis may be used to distinguish different candidate models of physics beyond the SM. (In fact, in some cases it is likely that the new physics will be found first through measurements of rare *B* decays.) For example, suppose that new physics is found through a discrepancy of type (1) or (2). This would indicate that the new physics is probably either a fourth-generation, *Z*-mediated FCNC's or flavor-changing neutral scalars. Since each of these three models affects *B* penguin decays differently, they can be at least partially differentiated by a study of such decays. And if the new physics is found through a discrepancy of type (3), the new physics is likely to be either a multi-Higgs-doublet model with NFC, or the MSSM. In this case, it may be difficult to distinguish the two models of new physics since their effects on *B* penguin decays are similar. This is not surprising, since the MSSM contains two Higgs doublets. Still, there are signals, such as the lepton-antilepton energy asymmetry in $b \to s l^+ l^-$, which can differentiate these two models.

To sum up, most physics beyond the SM which can affect *CP* asymmetries in *B* decays will also contribute to rare, flavor-changing *B* decays. We have examined the effects of a number of models of new physics on both *CP* asymmetries and penguin decays. Although not all models of new physics have ''smoking gun'' signatures in these decays, we have shown that the measurements of *CP* asymmetries and rare penguin decays give complementary information, and both will be necessary if we hope to identify the new physics.

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