CP violation in the decay $B\to X_d e^+e^-$

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The decay $b\to de^+e^-$ has an amplitude containing comparable contributions proportional to $V_{tb}V_{td}^*$, $V_{cb}V_{cd}^*$, and $V_{ub}V_{ud}^*$. These pieces involve different unitarity phases produced by $c\bar{c}$ and $u\bar{u}$ loops. The simultaneous presence of different CKM phases and different dynamical phases leads to a calculable asymmetry in the partial widths of $b\to de^+e^-$ and $\bar{b}\to\bar{d}e^+e^-$. Using the effective Hamiltonian of the standard model, we calculate this asymmetry as a function of the e^+e^- invariant mass. The effects of ρ, ω, and J/ψ resonances are taken into account in the vacuum polarization of the $u\bar{u}$ and $c\bar{c}$ currents. As a typical result, an asymmetry of -5% (-2%) is predicted in the nonresonant domain $1\ \text{GeV}<m_{e^+e^-}<m_{J/\psi}$, assuming $\eta=0.34$ and $\rho=0.3$ (-0.3). The branching ratio in this domain is 1.2×10^{-7} (3.3×10^{-7}). Results are also obtained in the region of the J/ψ resonance, where an asymmetry of 3×10^{-3} is expected, subject to certain theoretical uncertainties in the $b\to dJ/\psi$ amplitude. [S0556-2821(97)04505-0]

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I. INTRODUCTION

The decays $B\to X_{s,d}l^+l^-$ are important probes of the effective Hamiltonian governing the flavor-changing neutral current transition $b\to s(d)l^+l^-$ [1]. The matrix element contains a term describing the virtual effects of the top quark proportional to $V_{tb}V_{tq}^*$, $q=s,d$, and in addition terms induced by $c\bar{c}$ and $u\bar{u}$ loops, proportional to $V_{cb}V_{cq}^*$ and $V_{ub}V_{uq}^*$. In the case of the decay $b\to sl^+l^-$, the relevant Cabibbo-Kobayashi-Maskawa (CKM) factors have the order of magnitude $V_{tb}V_{ts}^*\sim\lambda^3$, $V_{cb}V_{cs}^*\sim\lambda^3$, $V_{ub}V_{us}^*\sim\lambda^5$, where $\lambda=\sin\theta_C\simeq0.221$. This has the consequence that the $u\bar{u}$ contribution is very small, and the unitarity relation for the CKM factors reduces approximately to $V_{tb}V_{ts}^*+V_{cb}V_{cs}^*\approx0$. Thus the effective Hamiltonian for $b\to sl^+l^-$ essentially involves only one independent CKM factor $V_{tb}V_{ts}^*$, so that *CP* violation in this channel is strongly suppressed, within the standard model [2,3].

The situation is quite different for the transition $b\to dl^+l^-$. The internal top-quark contribution is proportional to $V_{tb}V_{td}^*$, while the terms related to $c\bar{c}$ and $u\bar{u}$ loops are proportional to $V_{cb}V_{cd}^*$ and $V_{ub}V_{ud}^*$. All of these CKM factors are of order λ^4, and, *a priori*, can have quite different phases. In addition, the $c\bar{c}$ and $u\bar{u}$ loop contributions are accompanied by different unitarity phases corresponding to real intermediate states. We thus have a situation in which the amplitude contains pieces with different CKM phases as well as different dynamical (unitarity) phases. These are precisely the desiderata for observing *CP*-violating asymmetries in partial rates. The purpose of this paper is to derive quantitative predictions for the *CP*-violating partial width asymmetry between the channels $b\to de^+e^-$ and $\bar{b}\to\bar{d}e^+e^-$.

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II. THE EFFECTIVE HAMILTONIAN FOR $b\to dl^+l^-$

The effective Hamiltonian for the decay $b\to dl^+l^-$ in the standard model can be written as

$$H_{\text{eff}}=-\frac{4G_F}{\sqrt{2}}V_{tb}V_{td}^*\Bigg\{\sum_{i=1}^{10}c_i(\mu)\mathcal{O}_i(\mu)-\lambda_u\{c_1(\mu)[\mathcal{O}_1^u(\mu)-\mathcal{O}_1(\mu)]+c_2(\mu)[\mathcal{O}_2^u(\mu)-\mathcal{O}_2(\mu)]\}\Bigg\},\qquad(2.1)$$

where we have used the unitarity of the CKM matrix $V_{tb}V_{td}^*+V_{ub}V_{ud}^*=-V_{cb}V_{cd}^*$, and $\lambda_u\equiv V_{ub}V_{ud}^*/V_{tb}V_{td}^*$. For the purpose of this paper it is convenient to use the Wolfenstein representation [4] of the CKM matrix with four real parameters $\lambda=\sin\theta_C\simeq0.221$, A, ρ, and η, where η is a measure of *CP* violation. In terms of these parameters

$$\lambda_u=\frac{\rho(1-\rho)-\eta^2}{(1-\rho)^2+\eta^2}-i\,\frac{\eta}{(1-\rho)^2+\eta^2}+\cdots,\qquad(2.2)$$

where the ellipsis denotes higher-order terms in λ. Furthermore, we will make use of

$$\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2}=\lambda^2[(1-\rho)^2+\eta^2]+O(\lambda^4).\qquad(2.3)$$

The operator basis $\{\mathcal{O}_i\}$ for H_{eff} is given in Refs. [5,6] with the obvious replacement $s\to d$, and the additional operators $\mathcal{O}_{1,2}^u$ read

$$\mathcal{O}_1^u=(\bar{d}_\alpha\gamma_\mu P_L u_\beta)(\bar{u}_\beta\gamma^\mu P_L b_\alpha),$$
$$\mathcal{O}_2^u=(\bar{d}_\alpha\gamma_\mu P_L u_\alpha)(\bar{u}_\beta\gamma^\mu P_L b_\beta),\qquad(2.4)$$

with $P_{L,R}=(1\mp\gamma_5)/2$. The evolution of the Wilson coefficients $c_i(\mu)$ in Eq. (2.1) from the scale $\mu=m_W$ down to $\mu=m_b$ by means of the renormalization group equation has been discussed in several papers, and we refer the reader to the review article of Buchalla *et al.* [7]. The resulting QCD-corrected matrix element can be written as

$$\mathcal{M}=\frac{4G_F}{\sqrt{2}}V_{tb}V_{td}^{*}\frac{\alpha}{4\pi}\bigg\{c_9^{\rm eff}(\overline{d}\gamma_\mu P_L b)\overline{l}\gamma^\mu l+c_{10}(\overline{d}\gamma_\mu P_L b)\overline{l}\gamma^\mu\gamma^5 l-2c_7^{\rm eff}\overline{d}i\sigma_{\mu\nu}\times\frac{q^\nu}{q^2}(m_b P_R+m_d P_L)b\overline{l}\gamma^\mu l\bigg\}. \tag{2.5}$$

Neglecting terms of $O(m_q^2/m_W^2)$, $q=u,d,c$, the analytic expressions for all Wilson coefficients, except $c_9^{\rm eff}$, are the same as in the $b\to s$ analog, and can be found in Refs. [7–10]. Using the parameters given in Appendix A, we obtain, in the leading logarithmic approximation,

$$c_7^{\rm eff}=-0.315,\quad c_{10}=-4.642, \tag{2.6}$$

and, in the next-to-leading approximation,

$$\begin{aligned}c_9^{\rm eff}=&\,c_9+0.124\omega(\hat{s})+g(\hat{m}_c,\hat{s})(3c_1+c_2+3c_3\\&+c_4+3c_5+c_6)+\lambda_u[g(\hat{m}_c,\hat{s})-g(\hat{m}_u,\hat{s})](3c_1+c_2)\\&-\tfrac{1}{2}g(\hat{m}_d,\hat{s})(c_3+3c_4)-\tfrac{1}{2}g(\hat{m}_b,\hat{s})\\&\times(4c_3+4c_4+3c_5+c_6)+\tfrac{2}{9}(3c_3+c_4+3c_5+c_6),\end{aligned} \tag{2.7}$$

with

$$c_1=-0.249,\quad c_2=1.108,\quad c_3=1.112\times10^{-2},\quad c_4=-2.569\times10^{-2},\quad c_5=7.404\times10^{-3},\quad c_6=-3.144\times10^{-2},\quad c_9=4.227, \tag{2.8}$$

and the notation $\hat{s}=q^2/m_b^2$, $\hat{m}_q=m_q/m_b$. In the above formula $\omega(\hat{s})$ represents the one-gluon correction to the matrix element of the operator $\mathcal{O}_9$ (see Appendix B), while the function $g(\hat{m}_q,\hat{s})$ arises from the one-loop contributions of the four-quark operators $\mathcal{O}_1-\mathcal{O}_6$, i.e.,

$$\begin{aligned}g(\hat{m}_q,\hat{s})=&-\tfrac{8}{9}\ln(\hat{m}_q)+\tfrac{8}{27}+\tfrac{4}{9}y_q-\tfrac{2}{9}(2+y_q)\\&\times\sqrt{|1-y_q|}\bigg\{\Theta(1-y_q)\bigg(\ln\bigg(\frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}}\bigg)-i\pi\bigg)\\&+\Theta(y_q-1)2\arctan\frac{1}{\sqrt{y_q-1}}\bigg\},\end{aligned} \tag{2.9}$$

with $y_q\equiv4\hat{m}_q^2/\hat{s}$.

III. LONG-DISTANCE EFFECTS: ρ, ω, AND THE J/ψ FAMILY

A more complete analysis of the above decay has to take into account long-distance contributions, which have their origin in real $u\overline{u}$, $d\overline{d}$, and $c\overline{c}$ intermediate states, i.e., ρ,ω, and $J/\psi,\psi'$, etc., in addition to the short-distance interaction defined by Eqs. (2.5)–(2.8). In the case of the J/ψ family this is usually accomplished by introducing a Breit-Wigner distribution for the resonances through the replacement [11]

$$g(\hat{m}_c,\hat{s})\to g(\hat{m}_c,\hat{s})-\frac{3\pi}{\alpha^2}\sum_{V=J/\psi,\psi'\cdots}\frac{\hat{m}_V B(V\to l^+l^-)\hat{\Gamma}_{\rm total}^V}{\hat{s}-\hat{m}_V^2+i\hat{m}_V\hat{\Gamma}_{\rm total}^V}, \tag{3.1}$$

where the properties of the vector mesons are listed in Ref. [12].

We prefer to follow a different procedure, discussed in our previous paper [13], which uses the renormalized photon vacuum polarization $\Pi_{\rm had}^\gamma(\hat{s})$, related to the measurable quantity $R_{\rm had}(\hat{s})\equiv\sigma_{\rm tot}(e^+e^-\to{\rm hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$. This allows us to implement the long-distance contributions using experimental data. The absorptive part of the vacuum polarization is given by

$$\text{Im}\,\Pi_{\rm had}^\gamma(\hat{s})=\frac{\alpha}{3}R_{\rm had}(\hat{s}), \tag{3.2}$$

whereas the dispersive part may be obtained via a once-subtracted dispersion relation [14]

$$\text{Re}\,\Pi_{\rm had}^\gamma(\hat{s})=\frac{\alpha\hat{s}}{3\pi}P\int_{4\hat{m}_\pi^2}^\infty\frac{R_{\rm had}(\hat{s}')}{\hat{s}'(\hat{s}'-\hat{s})}d\hat{s}', \tag{3.3}$$

where P denotes the principal value.

To derive an expression that relates $g(\hat{m}_q,\hat{s})$ and $R_{\rm had}(\hat{s})$, let us start with the electromagnetic current involving u, d, and c quarks, which is relevant to the production of ρ, ω, and J/ψ resonances:

$$j_\mu^{\rm em}=\tfrac{2}{3}\,\overline{u}\gamma_\mu u-\tfrac{1}{3}\,\overline{d}\gamma_\mu d+\tfrac{2}{3}\,\overline{c}\gamma_\mu c+\cdots. \tag{3.4}$$

Using Eq. (3.4), the vacuum polarization may then be written as

$$\Pi_{\rm had}^\gamma=\tfrac{4}{9}\,\Pi^{c\overline{c}}+\tfrac{4}{9}\,\Pi^{u\overline{u}}+\tfrac{1}{9}\,\Pi^{d\overline{d}}+\cdots. \tag{3.5}$$

The vacuum polarization $\Pi^{q\overline{q}}$ associated with a $q\overline{q}$ loop is related to $g(\hat{m}_q,\hat{s})$ via

$$\Pi^{q\overline{q}}=\frac{9}{4}\frac{\alpha}{\pi}\bigg(g(\hat{m}_q,\hat{s})+\frac{4}{9}+\frac{8}{9}\ln\hat{m}_q\bigg). \tag{3.6}$$

Next we define currents corresponding to the quantum numbers of ρ, ω, and J/ψ:

$$j_\mu^\rho=\tfrac{1}{2}(j_\mu^u-j_\mu^d),\quad j_\mu^\omega=\tfrac{1}{6}(j_\mu^u+j_\mu^d),\quad j_\mu^{J/\psi}=\tfrac{2}{3}j_\mu^c, \tag{3.7}$$

with $j_\mu^q=\overline{q}\gamma_\mu q$, in terms of which the vacuum polarization, Eq. (3.5), can be rewritten as

$$\Pi_{\rm had}^\gamma=\Pi^{J/\psi}+\Pi^\omega+\Pi^\rho+\cdots. \tag{3.8}$$

With the assumption $m_u=m_d$ it follows immediately that $\Pi^{u\overline{u}}=\Pi^{d\overline{d}}$, and we arrive at

$$\Pi^{c\overline{c}}=\tfrac{9}{4}\,\Pi^{J/\psi}, \tag{3.9}$$

$$\Pi^{u\overline{u}}=\Pi^{d\overline{d}}=\tfrac{9}{5}(\Pi^\omega+\Pi^\rho), \tag{3.10}$$

so that

$$\mathrm{Im}\; g(\hat{m}_c,\hat{s})=\frac{\pi}{3}\,R^{J/\psi}_{\mathrm{had}}(\hat{s}), \tag{3.11}$$

$$\mathrm{Im}\; g(\hat{m}_u,\hat{s})=\mathrm{Im}\; g(\hat{m}_d,\hat{s})=\frac{4\pi}{15}\,[R^{\rho}_{\mathrm{had}}(\hat{s})+R^{\omega}_{\mathrm{had}}(\hat{s})]. \tag{3.12}$$

For the real part of the one-loop function $g(\hat{m}_q,\hat{s})$ one finds

$$\mathrm{Re}\; g(\hat{m}_c,\hat{s})=-\frac{8}{9}\ln\hat{m}_c-\frac{4}{9}+\frac{\hat{s}}{3}\,P\int_{4\hat{m}_D^2}^{\infty}\frac{R^{J/\psi}_{\mathrm{had}}(\hat{s}')}{\hat{s}'(\hat{s}'-\hat{s})}\,d\hat{s}', \tag{3.13}$$

and

$$\mathrm{Re}\; g(\hat{m}_q,\hat{s})=-\frac{8}{9}\ln\hat{m}_q-\frac{4}{9}+\frac{4\hat{s}}{15}\,P\int_{4\hat{m}_\pi^2}^{\infty}\frac{R^{\rho}_{\mathrm{had}}(\hat{s}')+R^{\omega}_{\mathrm{had}}(\hat{s}')}{\hat{s}'(\hat{s}'-\hat{s})}\,d\hat{s}',\quad q=u,d. \tag{3.14}$$

Note that in many cases the evaluation of the dispersion integral may be carried out analytically (see, e.g., Ref. [15]). The cross-section ratios appearing in Eqs. (3.11)–(3.14) may be written as

$$R^{J/\psi}_{\mathrm{had}}(\hat{s})=R^{c\bar{c}}_{\mathrm{cont}}(\hat{s})+R^{J/\psi}_{\mathrm{res}}(\hat{s}), \tag{3.15}$$

$$R^{\rho}_{\mathrm{had}}(\hat{s})+R^{\omega}_{\mathrm{had}}(\hat{s})=R^{u\bar{u}+d\bar{d}}_{\mathrm{cont}}(\hat{s})+R^{\rho}_{\mathrm{res}}(\hat{s})+R^{\omega}_{\mathrm{res}}(\hat{s}), \tag{3.16}$$

where the subscripts ''cont'' and ''res'' refer to the contributions from the continuum and the resonances respectively. The J/ψ resonances and ω are well described through a relativistic Breit-Wigner form; i.e.,

$$R^{J/\psi}_{\mathrm{res}}(\hat{s})=\sum_{V=J/\psi,\psi',\cdots}\frac{9\hat{s}}{\alpha^2}\,\frac{B(V\to l^+l^-)\hat{\Gamma}^V_{\mathrm{total}}\hat{\Gamma}^V_{\mathrm{had}}}{(\hat{s}-\hat{m}_V^2)^2+\hat{m}_V^2\hat{\Gamma}^{V^2}_{\mathrm{total}}}, \tag{3.17}$$

and

$$R^{\omega}_{\mathrm{res}}(\hat{s})=\frac{9\hat{s}}{\alpha^2}\,\frac{B(\omega\to l^+l^-)\hat{\Gamma}^{\omega^2}_{\mathrm{total}}}{(\hat{s}-\hat{m}_\omega^2)^2+\hat{m}_\omega^2\hat{\Gamma}^{\omega^2}_{\mathrm{total}}}, \tag{3.18}$$

with a $\hat{s}$-independent total width, which is quite adequate for our purposes. The ρ resonance may be introduced through

$$R^{\rho}_{\mathrm{res}}(\hat{s})=\frac{1}{4}\left(1-\frac{4\hat{m}_\pi^2}{\hat{s}}\right)^{3/2}|F_\pi(\hat{s})|^2, \tag{3.19}$$

$F_\pi(\hat{s})$ being the pion form factor, which is represented by a modified Gounaris-Sakurai formula [16]. The continuum contributions can be parametrized using the experimental data from Ref. [17], and are given in Appendix A.

IV. BRANCHING RATIO AND CP-VIOLATING ASYMMETRY

The differential branching ratio for $b\to dl^+l^-$ in the variable $\sqrt{\hat{s}}$ including next-to-leading order QCD corrections is given by

$$\frac{dB}{d\sqrt{\hat{s}}}=\frac{\alpha^2}{2\pi^2}\,\frac{|V_{tb}V^*_{td}|^2}{|V_{cb}|^2}\,\frac{B(B\to X_c e\overline{\nu_e})}{f(\hat{m}_c)\kappa(\hat{m}_c)}\times\lambda^{1/2}(1,\hat{s},\hat{m}_d^2)\sqrt{\hat{s}-4\hat{m}_l^2}\;\Sigma, \tag{4.1}$$

where we have neglected nonperturbative corrections of $O(1/m_b^2)$ [18]. The various factors appearing in Eq. (4.1) are defined by

$$\lambda(a,b,c)=a^2+b^2+c^2-2(ab+bc+ac), \tag{4.2}$$

$$\Sigma=\left\{\left(12\,\mathrm{Re}(c_7^{\mathrm{eff}}c_9^{\mathrm{eff}})F_1(\hat{s},\hat{m}_d^2)+\frac{4}{\hat{s}}\,|c_7^{\mathrm{eff}}|^2F_2(\hat{s},\hat{m}_d^2)\right)\times\left(1+\frac{2\hat{m}_l^2}{\hat{s}}\right)+(|c_9^{\mathrm{eff}}|^2+|c_{10}|^2)F_3(\hat{s},\hat{m}_d^2,\hat{m}_l^2)+6\hat{m}_l^2(|c_9^{\mathrm{eff}}|^2-|c_{10}|^2)F_4(\hat{s},\hat{m}_d^2)\right\}, \tag{4.3}$$

with

$$F_1(\hat{s},\hat{m}_d^2)=(1-\hat{m}_d^2)^2-\hat{s}(1+\hat{m}_d^2),$$

$$F_2(\hat{s},\hat{m}_d^2)=2(1+\hat{m}_d^2)(1-\hat{m}_d^2)^2-\hat{s}(1+14\hat{m}_d^2+\hat{m}_d^4)-\hat{s}^2(1+\hat{m}_d^2),$$

$$F_3(\hat{s},\hat{m}_d^2,\hat{m}_l^2)=(1-\hat{m}_d^2)^2+\hat{s}(1+\hat{m}_d^2)-2\hat{s}^2+\lambda(1,\hat{s},\hat{m}_d^2)\,\frac{2\hat{m}_l^2}{\hat{s}},$$

$$F_4(\hat{s},\hat{m}_d^2)=1-\hat{s}+\hat{m}_d^2, \tag{4.4}$$

while the ratio of CKM matrix elements in terms of the Wolfenstein parameters ρ and η has already been given in Eq. (2.3). In order to remove the uncertainties in Eq. (4.1) due to an overall factor of m_b^5, we have introduced the inclusive semileptonic branching ratio via the relation

$$\Gamma(B\to X_c e\overline{\nu_e})=\frac{G_F^2m_b^5}{192\pi^3}\,|V_{cb}|^2f(\hat{m}_c)\kappa(\hat{m}_c), \tag{4.5}$$

where $f(\hat{m}_c)$ and $\kappa(\hat{m}_c)$ represent the phase space and the one-loop QCD corrections [19] to the semileptonic decay respectively, and are given in Appendix B. Integrating the distribution in Eq. (4.1) for $l=e,\ \mu$, and τ over $\sqrt{\hat{s}}$, we obtain the branching ratio $B(B\to X_d l^+l^-)$, depending on the specific choice of ρ and η. The results are shown in Table I, for typical values of (ρ,η) in the experimentally allowed domain

TABLE I. Branching ratio $B(B\to X_d l^+ l^-)$, where $l=e$, μ or τ, for different values of (ρ,η) excluding the region (± 20 MeV) around the J/ψ and ψ' resonances.

(ρ,η)	$B(B\to X_d e^+ e^-)$	$B(B\to X_d \mu^+\mu^-)$	$B(B\to X_d \tau^+\tau^-)$
(0.3,0.34)	2.7×10^{-7}	1.8×10^{-7}	0.7×10^{-8}
(−0.07,0.34)	5.5×10^{-7}	3.8×10^{-7}	1.6×10^{-8}
(−0.3,0.34)	7.9×10^{-7}	5.4×10^{-7}	2.3×10^{-8}

[1].[1] Note that the branching ratio is quite sensitive to the Wolfenstein parameter ρ. For instance, the branching ratio for $B\to X_d e^+ e^-$ varies from 2.7 to 7.9×10^{-7}, when ρ is varied from $+0.3$ to -0.3.

Let us now turn to the CP-violating rate asymmetry, which is defined as

$$A_{CP}(\sqrt{\hat{s}})=\frac{d\Gamma/d\sqrt{\hat{s}}-d\overline{\Gamma}/d\sqrt{\hat{s}}}{d\Gamma/d\sqrt{\hat{s}}+d\overline{\Gamma}/d\sqrt{\hat{s}}}, \tag{4.6}$$

where

$$\frac{d\Gamma}{d\sqrt{\hat{s}}}\equiv\frac{d\Gamma(b\to dl^+l^-)}{d\sqrt{\hat{s}}},\quad \frac{d\overline{\Gamma}}{d\sqrt{\hat{s}}}\equiv\frac{d\Gamma(\overline{b}\to\overline{d}l^+l^-)}{d\sqrt{\hat{s}}}. \tag{4.7}$$

The physical origin of a CP-violating asymmetry in the reaction can be understood by considering the term proportional to c_9^{eff} in the matrix element, which can be written symbolically as

$$\mathcal{M}\sim A+\lambda_u B. \tag{4.8}$$

The corresponding matrix element for $\overline{b}\to\overline{d}l^+l^-$ is

$$\overline{\mathcal{M}}\sim A+\lambda_u^* B, \tag{4.9}$$

giving an asymmetry

$$A_{CP}=\frac{-2\,\text{Im}\,\lambda_u\text{Im}(A^*B)}{|A|^2+|\lambda_u B|^2+2\,\text{Re}\,\lambda_u\,\text{Re}(A^*B)}, \tag{4.10}$$

which provides a measure for CP violation. The asymmetry results from the presence of CP violation in the CKM matrix ($\text{Im}\,\lambda_u\neq0$) and unequal unitarity phases in the amplitudes A and B [$\text{Im}(A^*B)\neq0$].

The complete result contains an additional term due to the interference of c_7^{eff} with c_9^{eff}, and the asymmetry takes the final form

$$A_{CP}(\sqrt{\hat{s}})=\frac{-2\,\text{Im}\,\lambda_u\Delta}{\Sigma+2\,\text{Im}\,\lambda_u\Delta}\approx-2\,\text{Im}\,\lambda_u\frac{\Delta}{\Sigma}=\left(\frac{2\eta}{(1-\rho)^2+\eta^2}\right)\frac{\Delta}{\Sigma}, \tag{4.11}$$

with Σ defined in Eq. (4.3), and

$$\Delta=\text{Im}(\xi_1^*\xi_2)f_+(\hat{s})+\text{Im}(c_7^{\text{eff}}\xi_2)f_1(\hat{s}),$$
$$c_9^{\text{eff}}\equiv\xi_1+\lambda_u\xi_2,$$
$$f_1(\hat{s})=6F_1(\hat{s},\hat{m}_d^2)\left(1+\frac{2\hat{m}_l^2}{\hat{s}}\right),$$
$$f_+(\hat{s})=F_3(\hat{s},\hat{m}_d^2,\hat{m}_l^2)+6\hat{m}_l^2F_4(\hat{s},\hat{m}_d^2), \tag{4.12}$$

where the phase-space functions F_1 and $F_{3,4}$ are given in Eq. (4.4). Notice that A_{CP} vanishes as $m_u\to m_c$, since in that limit $\xi_2\to0$ [see Eq. (2.7)].

Our numerical results for the asymmetry together with the differential branching ratio, Eq. (4.1), are shown in Figs. 1–3 for different values of ρ and η.[2] It is interesting to note that the ρ resonance is barely visible in the invariant mass spectrum, but has a strong influence on the asymmetry in the region up to 1 GeV. We have evaluated the branching ratio and average asymmetry $\langle A_{CP}\rangle$ for different regions of $\sqrt{s}$ using Eq. (4.6), and our results are displayed in Tables II–IV.[3]

V. CONCLUSIONS

The principal results of our analysis are as follows.

(1) In the region excluding the J/ψ resonances, we find a sizeable CP-violating asymmetry between the decays $b\to de^+e^-$ and $\overline{b}\to\overline{d}e^+e^-$. This asymmetry amounts to -5.3% (-1.9%) for the invariant mass region 1 GeV $<\sqrt{s}_{e^+e^-}<m_{J/\psi}-20$ MeV, assuming $\eta=0.34$ and $\rho=0.3$ (-0.3). The corresponding branching ratio is 1.2×10^{-7} (3.3×10^{-7}). The asymmetry scales approximately as $\eta[(1-\rho)^2+\eta^2]^{-1}$, while the branching ratio scales as $(1-\rho)^2+\eta^2$. For a nominal asymmetry of 5% and a branching ratio of 10^{-7}, a measurement at 3σ level requires 4×10^{10} B mesons. In view of the clear dilepton signal, such a measurement might be feasible at future hadron colliders. It should be noted, however, that identification of the reaction $b\to de^+e^-$ in the presence of the much stronger reaction $b\to se^+e^-$ would require a study of the decay vertex, in order to select final states such as π^+, $\pi^+\pi^-\pi^+$, etc. (accompanied by any numbers of neutrals). In the inclusive analysis of e^+e^- pairs, only those with invariant mass in the range $(M_B-M_K)<\sqrt{s}<(M_B-M_\pi)$ can be unambiguously ascribed to $b\to de^+e^-$.

(2) In the neighborhood of the J/ψ resonance ($m_{J/\psi}-20$ MeV$<\sqrt{s}<m_{J/\psi}+20$ MeV), the branching ratio is substantial ($B=3.7\times10^{-6}$), but the asymmetry is very small ($\langle A_{CP}^{J/\psi}\rangle=0.6\times10^{-3}$). This smallness in asymmetry is the inevitable result of a very large $c\overline{c}$ amplitude near the J/ψ,

[1]The branching ratio for different regions of $\sqrt{\hat{s}}$ will be discussed below.

[2]We have also calculated the asymmetry in the $b\to s$ transition, which is roughly one order of magnitude smaller than in $b\to d$. Our results for the asymmetry differ somewhat from those given in Ref. [3], which uses an incorrect sign for the absorptive part of the one-loop function $g(\hat{m}_q,\hat{s})$. The correct sign is given in Refs. [8] and [10].

[3]A variation of m_t in the interval 176 ± 10 GeV changes these numbers by $\lesssim10\%$.

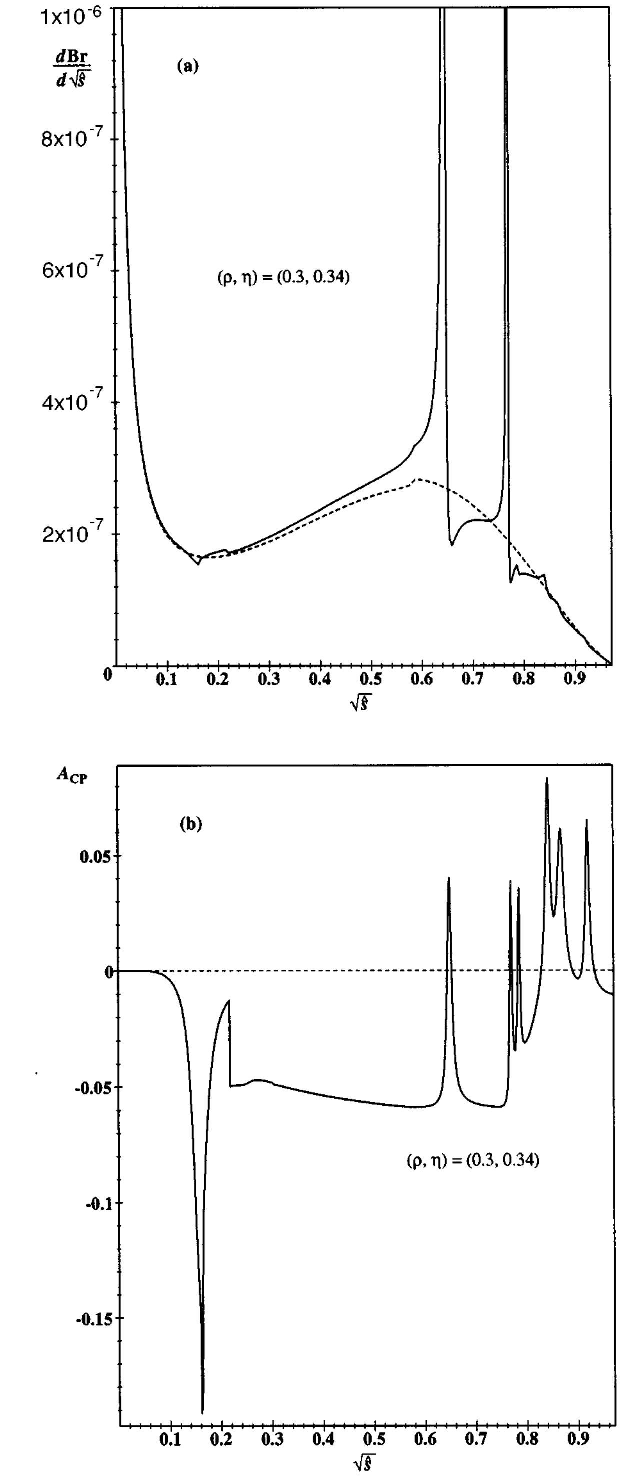

FIG. 1. Branching ratio $B(B\rightarrow X_d e^+ e^-)$ (a) and CP-violating asymmetry A_{CP} (b) including next-to-leading order QCD corrections as well as long-distance contributions (solid line), i.e., ρ, ω, and the J/ψ family, as a function of $\sqrt{\hat{s}}$, $\hat{s}\equiv q^2/m_b^2$. The dashed line in (a) corresponds to the nonresonant invariant mass spectrum. The Wolfenstein parameters are chosen to be $(\rho,\eta)=(0.3,\ 0.34)$.

interfering with a small nonresonant background.

(3) It is pertinent to ask if some refinement of the effective Hamiltonian underlying our calculation might lead to a higher asymmetry in the J/ψ region. In this connection, the following comments are in order.

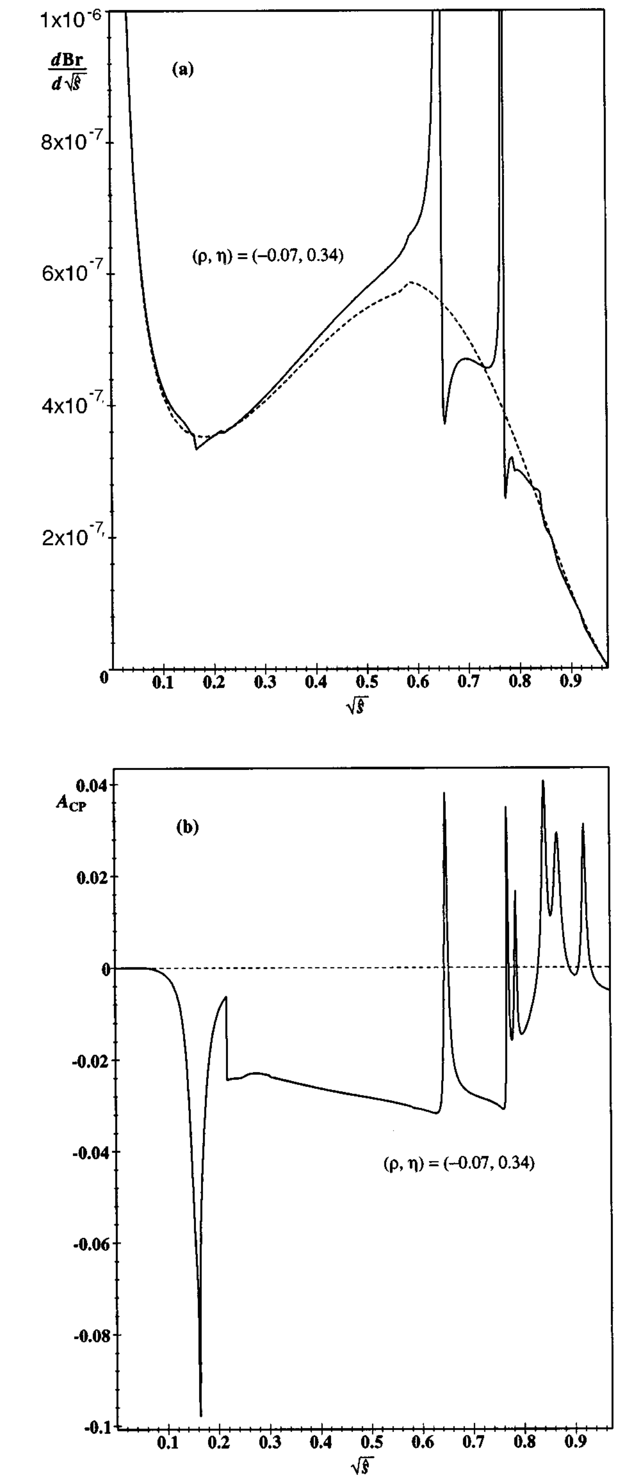

FIG. 2. Branching ratio $B(B\rightarrow X_d e^+ e^-)$ (a) and CP-violating asymmetry A_{CP} (b) for $(\rho,\eta)=(-0.07,\ 0.34)$. The dashed line in (a) represents the nonresonant spectrum.

(i) Our prescription for incorporating resonances into the effective Hamiltonian via the vacuum polarization function $\Pi^{\gamma}_{\rm had}(\hat{s})$ implicitly assumes that the transition $b\rightarrow dJ/\psi$ is adequately described by the leading term $(3c_1+c_2+3c_3+c_4+3c_5+c_6)$ appearing in the Wilson coefficient $c_9^{\rm eff}$, Eq. (2.7). With the values of c_i $(i=1,\ldots,6)$ given in Eq. (2.8), the theoretical branching ratio for the

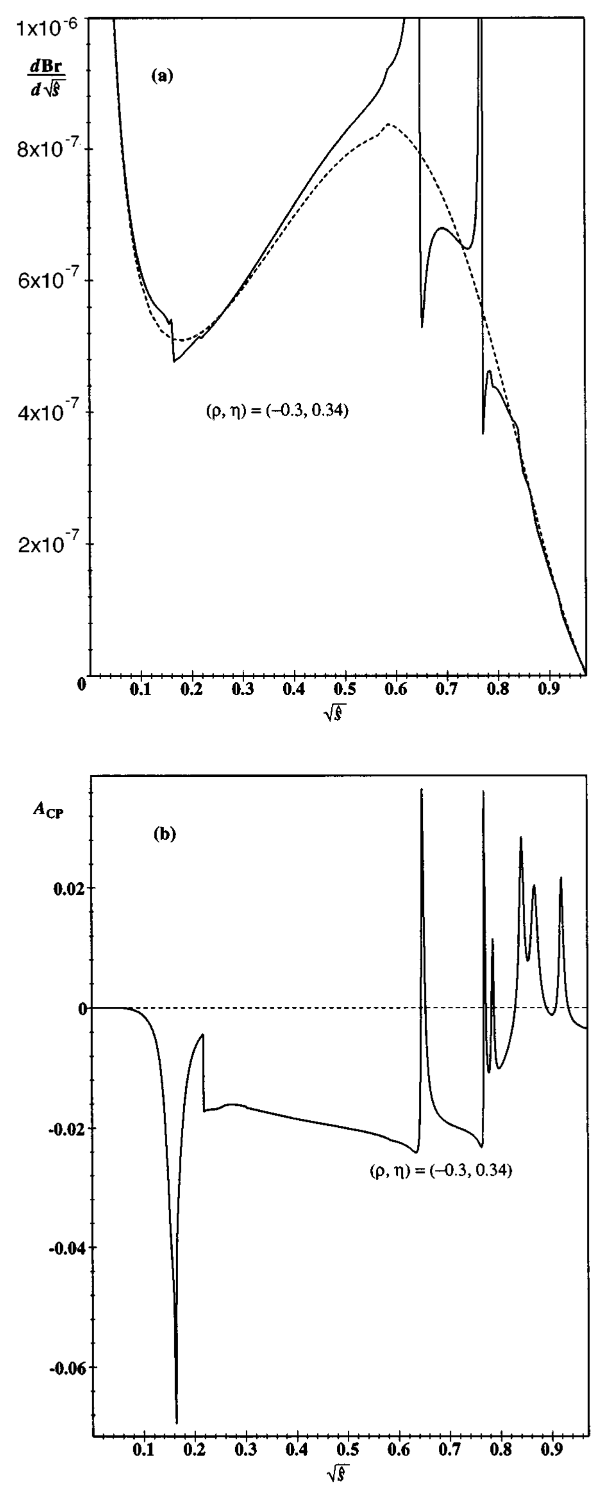

FIG. 3. Branching ratio $B(B\to X_d e^+e^-)$ (a) and CP-violating asymmetry A_{CP} (b) for $(\rho,\eta)=(-0.3,\ 0.34)$. The dashed line in (a) represents the nonresonant spectrum.

related reaction $b\to sJ/\psi$ is known to be $\sim$5 times smaller than measured [1,13,20]. It could be argued that for the purposes of calculating the $b\to dJ/\psi$ amplitude the coefficient $(3c_1+c_2+\cdots)$ should accordingly be corrected to $\kappa_V(3c_1+c_2+\cdots)$, with $\kappa_V\sim\sqrt{5}$. While such a procedure *enhances* the branching ratio of $b\to dJ/\psi$ by a factor κ_V^2, it *reduces* the asymmetry by a factor κ_V. Outside the J/ψ and ψ' regions, the branching ratio is essentially independent of κ_V. The asymmetry for $\sqrt{s}<m_{J/\psi}$ is likewise unaffected, while that between J/ψ and ψ' is reduced by $\sim\kappa_V$. In the region $\sqrt{s}>m_{\psi'}$ the asymmetry is quite sensitive to κ_V and can even be enhanced by an order of magnitude. This corner of phase space accounts, however, for only about 6% of the decay rate.

TABLE II. Branching ratio $B(B\to X_d e^+e^-)$ and average asymmetry $\langle A_{CP}\rangle$ for different regions of $\sqrt{s}$, below the J/ψ resonance (ε=20 MeV).

	(ρ,η)	$2m_e<\sqrt{s}<1$ GeV	1 GeV$<\sqrt{s}<(m_{J/\psi}-\varepsilon)$
B	(0.3,0.34)	1.1×10^{-7}	1.2×10^{-7}
	(−0.07,0.34)	2.4×10^{-7}	2.3×10^{-7}
	(−0.3,0.34)	3.4×10^{-7}	3.3×10^{-7}
$\langle A_{CP}\rangle$	(0.3,0.34)	-8.4×10^{-3}	-5.3×10^{-2}
	(−0.07,0.34)	-4.0×10^{-5}	-2.7×10^{-2}
	(−0.3,0.34)	-2.9×10^{-3}	-1.9×10^{-2}

(ii) The asymmetry may be slightly enhanced if one takes into account mixing of the $c\bar{c}$ current with the $u\bar{u}$ and $d\bar{d}$ currents. Such a mixing can give rise to an Okubo-Zweig-Iizuka- (OZI-) rule-violating transition $u\bar{u}\to J/\psi$, mediated by a one-photon (or three-gluon) intermediate state [21,22]. The QED effect can be incorporated into our calculation of the asymmetry near the J/ψ resonance by the replacement

$$\lambda_u g(\hat{m}_c,\hat{s})\to\lambda_u(1+i\ \tfrac{4}{9}\ \alpha)g(\hat{m}_c,\hat{s}) \tag{5.1}$$

in the coefficient c_9^{eff}. The resulting asymmetry increases from 0.6×10^{-3} to 2.9×10^{-3} (see Table IV).

(iii) Finally, it is possible to contemplate gluonic corrections to the effective Hamiltonian, that allow the transition $b\to dJ/\psi$ to take place not only through a color-singlet $(c\bar{c})_1$ intermediate state [i.e., $b\to d(c\bar{c})_1\to dJ/\psi$] but also through a color-octet intermediate configuration $[b\to d(c\bar{c})_8\to dJ/\psi]$. An illustrative calculation by Soares [22] yields an asymmetry of about 1% from such a mechanism.

Our general conclusion is that a measurement of the branching ratio and partial width asymmetry in the channel $b\to de^+e^-$ in the nonresonant continuum, would provide a theoretically clean and fundamental test of the idea that CP violation originates in the CKM matrix. The predicted asymmetry in the region 1 GeV$<m_{e^+e^-}<m_{J/\psi}$ is approximately

TABLE III. Branching ratio $B(B\to X_d e^+e^-)$ and average asymmetry $\langle A_{CP}\rangle$ for the large $\sqrt{s}$ region, excluding the J/ψ and ψ' resonances (ϵ=20 MeV).

	(ρ,η)	$(m_{J/\psi}+\varepsilon)<\sqrt{s}$ $<(m_{\psi'}-\varepsilon)$	$(m_{\psi'}+\varepsilon)$ $<\sqrt{s}<\sqrt{s}_{max}$
B	(0.3,0.34)	0.3×10^{-7}	1.6×10^{-8}
	(−0.07,0.34)	0.5×10^{-7}	3.4×10^{-8}
	(−0.3,0.34)	0.8×10^{-7}	4.9×10^{-8}
$\langle A_{CP}\rangle$	(0.3,0.34)	-5.1×10^{-2}	5.2×10^{-3}
	(−0.07,0.34)	-2.5×10^{-2}	2.1×10^{-3}
	(−0.3,0.34)	-1.8×10^{-2}	1.5×10^{-3}

TABLE IV. Branching ratio $B(B\to X_d e^+e^-)$ and average asymmetry $\langle A_{CP}\rangle$ near the J/ψ and ψ' resonances (ϵ=20 MeV).

	$(m_{J/\psi}-\varepsilon)<\sqrt{s}<(m_{J/\psi}+\varepsilon)$	$(m_{\psi'}-\varepsilon)<\sqrt{s}<(m_{\psi'}+\varepsilon)$
B	3.7×10^{-6}	1.8×10^{-7}
$\langle A_{CP}\rangle$	0.6×10^{-3}	4.4×10^{-3}
$\langle A_{CP}\rangle^{a}$	2.9×10^{-3}	6.7×10^{-3}

[a]Including OZI correction, induced by one-photon exchange as specified in Eq. (5.1).

$$-5.3\%\left(\frac{\eta}{0.34}\right)\left(\frac{1.2\times10^{-7}}{B}\right), \tag{5.2}$$

where B denotes the branching ratio in the above interval. Measurements near the J/ψ resonance are predicted to show a very small asymmetry ($\sim3\times10^{-3}$) that depends somewhat on the manner in which QCD modulates the effective interaction for $b\to dJ/\psi$.

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APPENDIX A: INPUT PARAMETERS

$$m_b=4.8\ \text{GeV},\quad m_c=1.4\ \text{GeV},$$

$$m_u=m_d=m_\pi=0.139\ \text{GeV},\quad m_t=176\ \text{GeV},$$

$$m_e=0.511\ \text{MeV},$$

$$m_\mu=0.106\ \text{GeV},\quad m_\tau=1.777\ \text{GeV},$$

$$\mu=m_b,\quad B(B\to X_c e\overline{\nu_e})=10.4\%,\quad \lambda=0.2205,$$

$$\Lambda_{\text{QCD}}=225\ \text{MeV},\quad \alpha=1/129,\quad \sin^2\theta_W=0.23,$$

$$M_W=80.2\ \text{GeV}. \tag{A1}$$

$$R_{\text{cont}}^{u\bar{u}+d\bar{d}}(\hat{s})=\begin{cases}0 & \text{for}\quad 0\leqslant\hat{s}\leqslant4.8\times10^{-2},\\ 1.67 & \text{for}\quad 4.8\times10^{-2}\leqslant\hat{s}\leqslant1.\end{cases} \tag{A2}$$

$$R_{\text{cont}}^{c\bar{c}}(\hat{s})=\begin{cases}0 & \text{for}\quad 0\leqslant\hat{s}\leqslant0.60,\\ -6.80+11.33\hat{s} & \text{for}\quad 0.60\leqslant\hat{s}\leqslant0.69,\\ 1.02 & \text{for}\quad 0.69\leqslant\hat{s}\leqslant1.\end{cases} \tag{A3}$$

APPENDIX B: USEFUL FUNCTIONS

As noted by Misiak [10], the function $\omega(\hat{s})$ can be inferred from [23] and is defined by

$$\begin{aligned}\omega(\hat{s})=&-\frac{2}{9}\pi^2-\frac{4}{3}\text{Li}_2(\hat{s})-\frac{2}{3}\ln\hat{s}\ln(1-\hat{s})\\&-\frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s})-\frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln\hat{s}\\&+\frac{5+9\hat{s}-6\hat{s}^2}{6(1-\hat{s})(1+2\hat{s})}.\end{aligned} \tag{B1}$$

$$f(\hat{m}_c)=1-8\hat{m}_c^2+8\hat{m}_c^6-\hat{m}_c^8-24\hat{m}_c^4\ln\hat{m}_c. \tag{B2}$$

$$\kappa(\hat{m}_c)=1-\frac{2\alpha_s(m_b)}{3\pi}\left[\left(\pi^2-\frac{31}{4}\right)(1-\hat{m}_c)^2+\frac{3}{2}\right]. \tag{B3}$$

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