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Estimation of the muon energy spectrum emitted by high energy γ rays from the Crab nebula

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The vertical integral spectrum of TeV muons produced by the UHE γ rays incident on Earth from the direction of the Crab nebula is calculated. The analytical estimate of the muon spectrum is made using the standard cascade equation for muons and the latest photopion production cross section obtained from the fit to the DESY *ep* collider experiment. The derived integral muon energy spectrum generated by the photoproduced pions is compared with the estimated muon energy spectra from the direct production of muon pairs and photoproduced pions of charmed particle decay. The present analytical estimates have been compared with the earlier simulation estimate of Halzen and co-workers along with the analytical results of Drees and co-workers. The present results show a little bit flatter muon energy spectrum when compared to the earlier simulation result. The muon energy spectrum has also been estimated from photopions using the simulation formulation of Halzen and Stanev with latest parametrization and has been found fairly comparable with the present result. [S0556-2821(97)03403-6]

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I. INTRODUCTION

Investigation of γ-ray bursts (GRBs) is of great interest in particle astronomy for searching the emission of neutrinos [1–3] from the source of γ rays. Halzen and Jaczko [4] have recently considered ultrarelativistic fire balls [5] and cosmic strings [6] to interpret the experimental results. Gamma rays and neutrinos are produced in beam dumps during interactions of high-energy accelerated protons with stellar targets. In such reactions γ rays and neutrinos are generated simultaneously through the decay of neutral and charged pions, respectively. The discovery of the cosmic microwave background also reveals the fact that the Universe would not be transparent to γ rays above about 100 TeV due to photon-photon pair production. Underground muon detectors are designed in a fashion so that they can measure the arrival direction of downgoing muons originating from γ-ray-induced electromagnetic showers in the Earth's atmosphere. Usually, the γ-ray showers are muon poor, and so a sufficient statistics of muon events are required in a GeV or TeV muon telescope to confirm the arrival direction of the incident γ rays from the astronomical source. Barwick *et al.* [7] have pointed out that TeV muons arising from such sources compete with a background of downgoing cosmic-ray muons provided that they can be recorded by a telescope with a sufficient effective area having precise angular resolution. The GRBs may also originate in the halo population of neutron stars. The theoretical search on neutrino emission depends on the reduced luminosity since it is compensated by large redshift sources by a reduced distance of the source. The search of GRBs provides the concept of dark matter and requires a kilometer-size muon telescope for the identification of the sources of terrestrial γ rays and neutrinos. This also helps in the investigation of dark matter for the exploration of the active galactic nuclei. It is evident that the Deep Underground Muon and Neutrino Detector (DUMAND) [8] and NESTOR [9] detectors are suitable for the selection of short bursts of high-energy muons initiated by muon neutrinos. The shallower detectors of Antarctic Muon and Neutrino Detector Array (AMANDA) [10] and BAIKAL [11] can detect muons originating from extensive air showers (EASs) initiated by TeV γ rays.

The investigation on the point source of the cosmic rays is of astrophysical importance for muon content estimation in γ-ray-induced air showers. Usually, in hadron showers the generated pions are the source of muons through $\pi\rightarrow\mu\nu$ decays. The photon-induced electromagnetic showers are dominated by pair production and bremsstrahlung processes and generate a muon component via processes relative to the pair production cross section of the value around 500 mb in air. The photopion production processes followed by the decay of charged pions are responsible for muon production in photon-induced showers. Earlier, Halzen *et al.* [12] made a detailed Monte Carlo simulation study of the muon content of air showers in the range 1–10^5 TeV. Later, Drees *et al.* [13] investigated analytically the relative abundance of muons in photon and proton-induced air cascades. The conventional calculations [14,15] of TeV muons produced by γ-ray showers are governed by accelerator data on pion photoproduction cross sections. The slow logarithmic energy dependence of the photopion production was accounted for in the simulation calculations. The simulations do not explain the physics of the interactions as it is dominated by the QCD model. Drees and Halzen [16] pointed out that a higher γ-ray threshold can result in large photoproduction cross sections and the photon develops a significant gluon structure function.

About 15 different high-energy γ-ray experiments were summarized in [12] on the emission of very-high-energy γ rays from the same direction and with the similar characteristics time structure of binary system in Cygnus X3. The γ-ray spectrum has been found to follow the approximate form

$$N_\gamma(>E)=4\times10^{-11}[E/1\ \ \mathrm{TeV}]^{-2}\ \ (\mathrm{cm}^2\ \mathrm{sec})^{-1},$$

where E is the γ-ray energy in TeV and the spectrum has a cutoff value at the high-energy end for the emission of the

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*Electronic address: ytpdpb@iacs.ernet.in spectrum at around $E=10^5$ TeV. This spectrum of gamma rays has been considered by [12,13] for the photoproduced muon flux calculations. Recently, a search has been made on the upper limit of the γ-ray flux using scintillators at Canary Island La Palama by Merck *et al.* [17] for the detection of ultra high energy (UHE) γ rays from the direction of the Crab nebula. The latest precise investigations of the γ-ray flux from the Crab by THEMISTOCLE group [18] have pointed out that their earlier results on Crab γ-ray flux was noticeably decreased due to a shift in the energy which was calibrated on cosmic ray flux without a good accounting of the heavy nuclei and an insufficient modeling of their interaction. Their corrected γ-ray flux reduces appreciably, which reveals the fact that the integral spectrum has an energy spectral index of value $-(1.4\pm0.2)$ for energies up to 15 TeV. We have used the γ-ray spectrum based on the observed results [17–25] for the derivation of the integral spectrum of muons in the energy range 1–8 TeV initiated by γ-ray interactions in air by following the conventional analytic method the cascade equation after Drees *et al.* [13] through photopion production. The muon fluxes from direct $\mu^+\mu^-$ pairs and that from the decay of charmed mesons initiated in γN interactions in EAS have been estimated by adopting the analytical procedure of Berezinsky *et al.* [26]. The present results have been compared with the earlier Monte Carlo simulation results of Halzen *et al.* [12] where the input γ-ray spectrum of Cygnus X3 was considered along with the analytic results of Drees *et al.* [13].

We have also adopted the latest methodology of Halzen and Stanev [27], which is based on their Monte Carlo simulation results with a new parametrization to estimate the spectrum of muons initiated by photopions emitted from the Crab [17–25] and has been compared with the analytical muon flux estimate from photopions, direct muons, and charm meson decays.

II. NUCLEAR PHYSICS AND KINEMATICS

A. Analytical procedure

We outline the standard procedure for calculating the muon flux produced in the atmosphere from an incident γ-ray spectrum from a source following the work of Drees *et al.* [13]. The primary photon spectrum at the top of the atmosphere after Rossi [28] follows a power law fit of the form

$$\frac{dN\gamma}{dE}\,(E,t=0)=aE^{-\gamma}. \tag{1}$$

The photon spectrum inside the atmosphere is independent of depth and follows the form

$$\frac{dN\gamma}{dE}\,(E,t)=\gamma(E,t)=0.5aE^{-(\gamma+1)}=\Gamma(E_0). \tag{2}$$

The atmospheric cascade is generating pions by the photopion production process in the collision of pions with the atmosphere. The linear cascade equation for pion fluxes after Rossi [28] and that modified to a generalized form by Halzen *et al.* [12] obey the relation

$$\frac{d\pi}{dt}=\left(-\frac{1}{\lambda_\pi}-\frac{1}{d_\pi}\right)\pi^{\pm}+\int_0^1\frac{dx}{x}\,\frac{\pi(E/x,t)}{\lambda_\pi}\,\frac{1}{\sigma_{\pi N}}\,\frac{d\sigma}{dx}\;\pi\rightarrow\pi^{\pm}$$
$$+\int_0^1\frac{dx}{x}\,\frac{\gamma(E/x,t)}{\lambda_{\gamma N}}\,\frac{1}{\sigma_{\gamma N}}\,\frac{d\sigma}{dx}\;\gamma\rightarrow\pi^{\pm}. \tag{3}$$

The first term of the above expression considers the loss of π by interactions and decay. The second term is taking into account the regeneration of π by π with $E'=E/x$ energy. The third term indicates the production of $\pi^{\pm}$ by photoproduction in $\gamma N\rightarrow\pi^{\pm}X$ reactions with cross section $\sigma_{\gamma N}$. One can neglect pion decay term since, for TeV pions with $E\gg\varepsilon_\pi\lambda_\pi/t\cos\theta$, the pion decay is negligible as a result of its small contribution and Feynman scaling can be assumed for inclusive cross sections $\sigma_{\gamma\rightarrow\pi^{\pm}}$ and $\sigma_{\pi\rightarrow\pi^{\pm}}$. One can safely consider

$$\pi(E,t)=\Gamma_0(E)\pi_2(t), \tag{4}$$

and the simplified form follows after Drees *et al.* [13]:

$$\frac{d\pi_2}{dt}=-\frac{1}{\Lambda_\pi}\,\pi_2+\frac{Z_{\gamma\pi}}{\lambda_{\gamma N}}, \tag{5}$$

where

$$\Lambda_\pi=\frac{\lambda_\pi}{1-Z_{\pi\pi}},$$

$$Z_{\pi\pi}=\frac{1}{\sigma_{\pi N}}\int_0^1 dx\;x^{\gamma-1}\,\frac{d\sigma}{dx}\;\pi\rightarrow\pi^{\pm},$$

and

$$Z_{\gamma\pi}=\frac{1}{\sigma_{\gamma N}}\int_0^1 dx\;x^{\gamma-1}\,\frac{d\sigma}{dx}\;\gamma\rightarrow\pi^{\pm},$$

where $\lambda_{\gamma N}$ is the photointeraction length associated with π photoproduction and Λ_π ($>\lambda_\pi$) is the absorption length of pions in air. The solution comes out to be of the form

$$\pi_2=\frac{Z_{\gamma\pi}}{\lambda_{\gamma N}}\,\Lambda_\pi[1-\exp(-t/\Lambda_\pi)]. \tag{6}$$

For TeV muons the limiting solution for the muon flux estimation after Drees *et al.* [13] follows the form

$$\frac{dN\mu}{dE}=\Gamma_0(E)\,\frac{\Lambda_\pi}{\lambda_{\gamma N}}\,Z_{\gamma\pi}\,\frac{L_\gamma}{1+(L_\gamma/H_\gamma)(E\ \cos\theta/\varepsilon_\pi)}, \tag{7}$$

where

$$L_\gamma=\frac{(1-r_\pi^2)t_{\max}}{2(1-r_\pi)\Lambda_\pi},\qquad H_\gamma=\frac{1-r_\pi^3}{3(1-r_\pi)}\,[1+\ln(t_{\max}/\Lambda_\pi)],$$

$r_\pi=m_\mu^2/m_\pi^2$ is the square of the ratio of muon to pion masses, $\varepsilon_\pi=m_\pi c^2H/(c\tau_\pi)$ represents the critical energy of the pion decay for the scale height of the isothermal atmosphere $H=6.7$ km, the pion lifetime in air $\tau_\pi=2.603\times10^{-9}$ sec, $m_\pi=0.139\,568$ GeV, and $m_\mu=0.105\,68$ GeV.

Following the analytic procedure based on QED calculations developed by Berezinsky *et al.* [26], the generation of muons by γ rays through the direct production of muon pairs in the Coulomb field of nuclei of charge Z in reactions $\gamma+Z\rightarrow Z+\mu^{+}+\mu^{-}$ has been calculated. This process dominates at high energy since nonprompt mesons have a small probability of decay on the interaction length in the atmosphere. We used the conventional relation from [26] to calculate the muon flux from direct pair production emitted from γ-ray flux J_γ $(>E)$, which follows:

$$F_\mu^{\mu^{+}\mu^{-}}(>E)=j_\gamma(>E)N_A[x_{\mathrm{rad}}/A]\sigma_{\gamma N}(E)[0.572/(\gamma-1)], \tag{8}$$

where $\sigma_{\gamma N}(E)$ is photopion production cross section, N_A is Avogadro's number, x_{rad} is the radiation length in air, and γ is the γ-ray spectral index.

The muon spectrum from γ-ray-emitted charmed meson decay in reactions such as $\gamma+A\rightarrow D+X$ has been estimated using the kinematical expression with their symbolic significance from [26] and the usual relation follows:

$$\begin{aligned}F_\mu^{\mathrm{ch}}(>E)=&j_\gamma(>E)1.144B_\mu N_A\sigma_{\gamma N}(E)[x_{\mathrm{rad}}/A]\\&\times[\gamma/(\gamma-1)]\int_0^1 dy\; y^{\gamma-1}f'(y)\\&\times\int_0^1 dx_D x_D^{\gamma}\frac{d\sigma_A}{dx_D},\end{aligned} \tag{9}$$

where $y=E_\mu/E_D$, $x_D=E_D/E_\gamma$, $E_\gamma=E_\mu/x_D y$, and $f/(y)=1-2y+2y^3-y^4$.

B. Simulation procedure

The majority of GeV or TeV gamma rays from different sources follow a power law $\sim E^{-(\gamma+1)}$ energy spectrum with $\gamma\cong 1$ [12]. Recent γ-ray investigations [17–25] exhibit that the CRAB emitted a photon spectrum available for a maximum energy of 1000 TeV. The number of photons from the point source may be estimated from the fluence F_γ by the relation

$$\frac{dN}{dE}\,\gamma=F_\gamma E^{-(\gamma+1)}\times 10^{-12}\ \mathrm{cm}^{-2}\,\mathrm{sec}^{-1}, \tag{10}$$

where γ is the integral γ-ray spectral index and E is the photon energy in TeV. The atmospheric cascades initiated by γ rays in due course, creating electrons, photons, and photoproduced pions from shower photons, which in turn decay to muons. Halzen and Stanev [27] have formulated the integral spectrum of muons, which is parametrized in a manner that reproduces explicitly their Monte Carlo simulation results of muon flux initiated by γ rays:

$$\begin{aligned}N_\mu(>E)=&\int_{E_{\gamma\,\min}}^{E_{\gamma\,\max}}F_\gamma\,\frac{10^{-12}}{E^{\gamma+1}}\left[\frac{2.14\times 10^{-5}}{E}\right]\left[\frac{E\,\cos\theta}{E}\right]dE\\=&2\times 10^{-17}\,\frac{F_\gamma}{\cos\theta}\,[E/\cos\theta]^{-(\gamma+1)}\\&\times\ln[E_{\gamma\,\max}/E_{\gamma\,\min}]f,\end{aligned} \tag{11}$$

where E is the vertical muon threshold energy of the detector. The photon ranges with the minimum energy $E_{\gamma\min}\cong[10E/\cos\theta]$ at θ zenith angle of the observed muon with respect to the source direction to the maximum energy of the source $E_{\gamma\max}$ generating the observed muons. The term f represents a correction factor $f=[E\cos\theta/0.04]^{0.53}$. The highest energies of photons at the source are responsible for the generation of TeV muon fluxes.

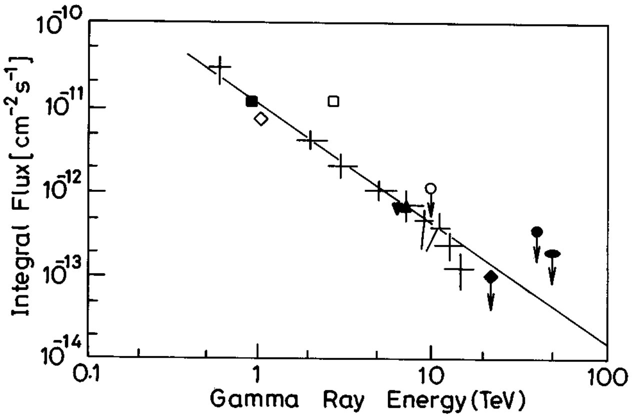

FIG. 1. Observed and extrapolated spectrum of γ rays from the Crab Nebula and other γ-ray flux limits surveyed by Themistocle Group [18]: Experimental data. (□) HEGRA and (●) HEGRA SCINT [17], (+) THEMISTOCLE [18], ○↓ TIBET [19], ◆↓ AEROBIC [20], ▲ CANGARO [21], (■) CRIMEA [22], (+) ASGAT [23], (◇) WHIPPLE [24], and ●↓ CYGNUS [25]. The solid line is the power law after relation (12).

III. RESULTS AND DISCUSSION

The Crab nebula and the AGN MRK 421 have been observed to emit high-energy gamma rays up to 15 TeV energy and can be detected from the northern hemisphere. Recent collaborators [17–25] have investigated γ-ray emission from point sources with the help of an array of scintillators in various locations. Figure 1 shows the measured fluxes from the Crab nebula for energies up to 60 TeV together with upper limits obtained from different experiments [17–25], and the observed and extrapolated γ-ray spectrum from the Crab has been found to follow a power law fit of the form, in the spectral range 0.2–60 TeV,

$$N_\gamma(>E)=1.07\times 10^{-11}E^{-1.4}\ \text{particles}\ (\mathrm{cm}^2\,\mathrm{sec})^{-1}. \tag{12}$$

The differential γ-ray spectrum in the atmosphere has been estimated using Eq. (12) and follows the form:

$$\frac{dN\gamma}{dE}\,(E,t=0)=7.49\times 10^{-12}E^{-2.4}\ \text{per}\ (\mathrm{cm}^2\,\mathrm{sec\ TeV}). \tag{13}$$

Using the FNAL data for meson production in πp collisions after Brenner *et al.* [29] and by adopting $\gamma=1.4$, the Z factors for $\pi p\rightarrow\pi^{\pm}X$ reactions have been calculated and duly corrected for p-air collisions and EMC effects by using the procedure of Minorikawa and Mitsui [30]. The results are found to be 0.131 27 and 0.065 44 for $Z'^{A}_{\pi^+\pi^+}$ and $Z'^{A}_{\pi^+\pi^-}$,

TABLE I. Calculated values of $t_{\max}$, L_γ, H_γ, and the estimated differential intensity of photopion decay muons in the range 1–8 TeV.

Muon energy E_μ (TeV)	$t_{\max}$	L_γ	H_γ	Differential muon flux per (cm^2 sec TeV)
1	209.82	1.0950	0.8557	2.70×10^{-14}
2	183.48	0.9702	0.7700	2.37×10^{-15}
3	168.07	0.8887	0.7140	5.62×10^{-16}
4	157.14	0.8309	0.6711	2.00×10^{-16}
5	148.66	0.7861	0.6356	8.91×10^{-17}
6	141.73	0.7495	0.6051	4.58×10^{-17}
7	135.87	0.7185	0.5782	2.59×10^{-17}
8	130.80	0.6917	0.5539	1.58×10^{-17}

respectively. From the survey of Borione *et al.* [31] on the ultrahigh-energy γ-ray energy spectrum from the Crab Nebula, one can safely assume the maximum cutoff energy of the gamma rays at a value $E_{\max}=10^3$ TeV. The pion attenuation length Λ_π has been calculated and found to be 149.4 g cm^{-2}. By adopting the procedure of Drees *et al.* [13], $t_{\max}$ has been calculated by assuming $E_{\max}=10^3$ TeV, $\langle x\rangle_{\gamma\to\pi^\pm}=0.25$ and using the form

$$t_{\max}=\lambda_R \ln(E_{\max}\langle x\rangle_{\gamma\to\pi^\pm}/E)\cong 38\ \ln[250/E\ \ \mathrm{TeV}]\ \ \mathrm{g\ cm}^{-2} \tag{14}$$

the corresponding simplified forms of the L_γ and H_γ follow the forms

$$L_\gamma=5.28\times10^{-3}[t_{\max}\ \ \mathrm{cm}^2\ \mathrm{g}^{-1}] \tag{15}$$

and

$$H_\gamma=0.6388[1+\ln\{t_{\max}/149.4\ \ \mathrm{g\ cm}^{-2}\}]. \tag{16}$$

Taking $r_\pi=0.58$, the term $t_{\max}$ is estimated at TeV energies along with the parameters L_γ and H_γ and the results are displayed in Table I. We have taken the value of $Z_{\gamma\pi}$ as 0.6666 and $\sigma_{\gamma N}\cong0.332$ mb at 10^3 TeV obtained from the extrapolated results of the photopion production cross sections available from the DESY *ep* collider HERA [32]. The photopion production cross section $\sigma_{\gamma p}$ follows the form

$$\sigma_{\gamma p}\cong(0.147-0.017\ \ln E_\gamma+0.0022\ \ln^2 E_\gamma)\ \ \mathrm{mb}. \tag{17}$$

From the relation (14) one can relate $\lambda_{\gamma N}=\sigma_R\lambda_R/(A\sigma_{\gamma N})$, which yields a value of 3789 g cm^{-2} for the adopted values $\sigma_R=480$ mb, $\lambda_R=38$ g cm^{-2}, and $A=14.5$. The simplified form of the differential muon spectrum obtained from UHE γ-ray cascades in the atmosphere has been found to follow the form

$$\frac{dN_\mu}{dE}=\frac{2.95\times10^{-13}E^{-2.4}L_\gamma}{[1+(L_\gamma E)/(H_\gamma\varepsilon_\pi)]}. \tag{18}$$

Taking L_γ and H_γ from Table I and $\varepsilon_\pi=0.115$ TeV the differential energy spectrum of muons from pions photoproduced by photon-induced showers has been calculated and found to follow the form

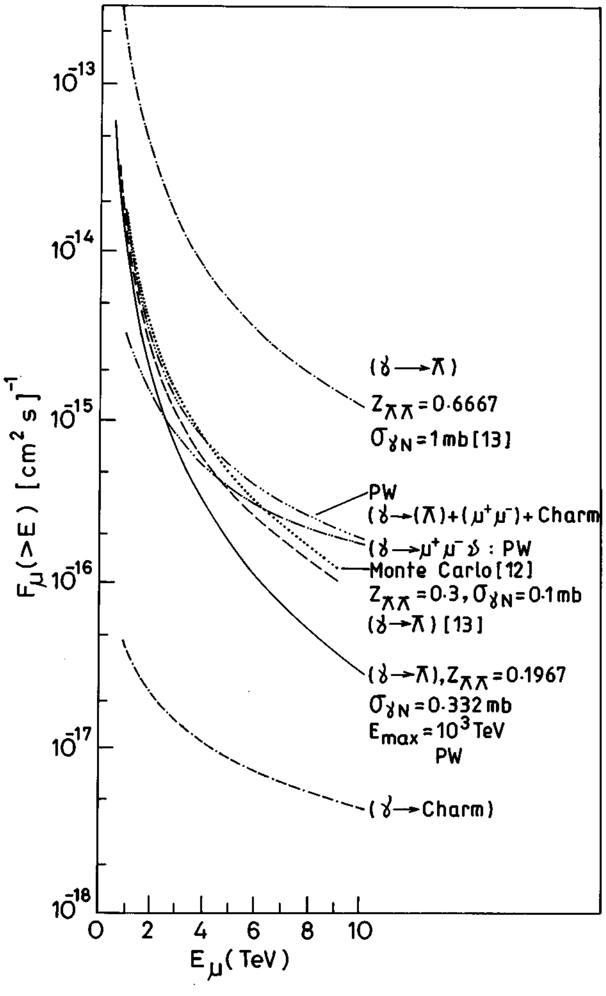

FIG. 2. The analytically derived integral spectra of muons generated by γ rays emitted from the Crab direction in the atmosphere and other results: Dotted curve shows the Monte Carlo results of Halzen *et al.* [12] and dashed and dot-dashed curves show the analytical result of Drees *et al.* [13] for $Z_{\pi\pi}=0.3$, $\sigma_{\gamma N}=100$ μb and $Z_{\pi\pi}=0.6667$, $\sigma_{\gamma N}=1$ mb, respectively. The solid line is the present work for muons from photopions ($\gamma\to\pi$) for $Z_{\pi\pi}=0.1967$ and $\sigma_{\gamma N}=0.332$ mb. The double-dot-dashed and dot-double-dashed curves represent the calculated muon spectra from the γ-ray produced direct muons and charmed mesons [26]. The triple-dot-dashed curve shows the derived total muon spectrum.

TABLE II. Muon fluxes with different interaction parameters for $E_{\gamma\max}=10^3$ TeV [31] along with the different values of $E_{\gamma\min}$, and correction factors f and F_γ for vertical incidence.

Muon energy (TeV)	$E_{\gamma\min}$ (TeV)	$\ln[E_{\gamma\max}/E_{\gamma\min}]$	$f=[E/0.04]^{0.53}$	$N_\mu(>E)$ $[\mathrm{cm}^2\ \mathrm{sec}]^{-1}$
1	10	4.6050	5.5069	7.60×10^{-15}
2	20	3.9120	7.9516	1.77×10^{-15}
3	30	3.5066	9.8579	7.43×10^{-16}
4	40	3.2189	11.4815	3.98×10^{-16}
5	50	2.9957	12.9230	2.44×10^{-16}
6	60	2.8134	14.2300	1.63×10^{-16}
7	70	2.6593	15.4458	1.15×10^{-16}
8	80	2.5257	16.5785	8.54×10^{-17}
9	90	2.4080	17.6464	6.53×10^{-17}
10	100	2.3026	18.6600	5.13×10^{-17}

$$\frac{dN_\mu}{dE}=2.79\times10^{-14}[E/\mathrm{TeV}]^{-3.58}\ \ (\mathrm{cm}^2\ \mathrm{sec\ TeV})^{-1}. \tag{19}$$

The corresponding integral γ-ray-induced muon spectrum follows the form

$$F_\mu(>E)=1.0822\times10^{-14}[E/\mathrm{TeV}]^{-2.58}\ \ (\mathrm{cm}^2\ \mathrm{sec})^{-1}. \tag{20}$$

Taking $A=14.5$, $x_{\mathrm{rad}}=38\ \mathrm{g\ cm}^{-2}$, and $N_A=6.02\times10^{23}$ and using the analytic relation (8) based on QED calculations of Berezinsky *et al.* [26], the muon generation by γ rays through direct production of muon pairs in the Coulomb field of the nuclei charge Z in reactions $\gamma+Z\rightarrow Z+\mu^++\mu^-$ has been calculated. The muon spectrum from γ-ray charmed meson decay in reactions $\gamma+A\rightarrow D+X$ has been estimated using the kinematical expression (9) after [26]. The derived γ-ray-induced muon spectra expected from the photopion production, direct muon production, and charm meson decay in the spectral range 1–8 TeV are displayed in Fig. 2 along with the Monte Carlo result of Halzen *et al.* [12] and also with the analytical estimate of Drees *et al.* [13]. The plot shows that muon spectrum appearing from direct production in $\gamma+Z\rightarrow Z+\mu^++\mu^-$ dominates at muon energies beyond 5 TeV. As a matter of fact, nonprompt mesons have a small decay probability on the interaction length in the atmosphere.

Auriema *et al.* [33] and Stanev *et al.* [34] have pointed out that the charm contribution dominates the muon spectrum for energies beyond 100 TeV. The present result displayed in Fig. 2 shows the simulation result of Halzen *et al.* [12], which arises from the flux of the primary γ rays emitted from the Cygnus X3 source and is also due to the different adoption of interaction parameters.

The photon spectral relation (12) yields the amplitude when multiplied by 10^{12} of the value $F_\gamma=15$ and $\gamma=1.4$. We have considered the maximum energy available from the Crab-emitted γ-ray spectrum from the measurements surveyed by the CASA MIA group [31] viz., $E_{\gamma\max}=10^3$ TeV and other parameters $E_{\gamma\min}$ and f are calculated and have been presented in Table II. We have adopted relation (11) and the other parameter $F_\gamma=15$ along with the values of

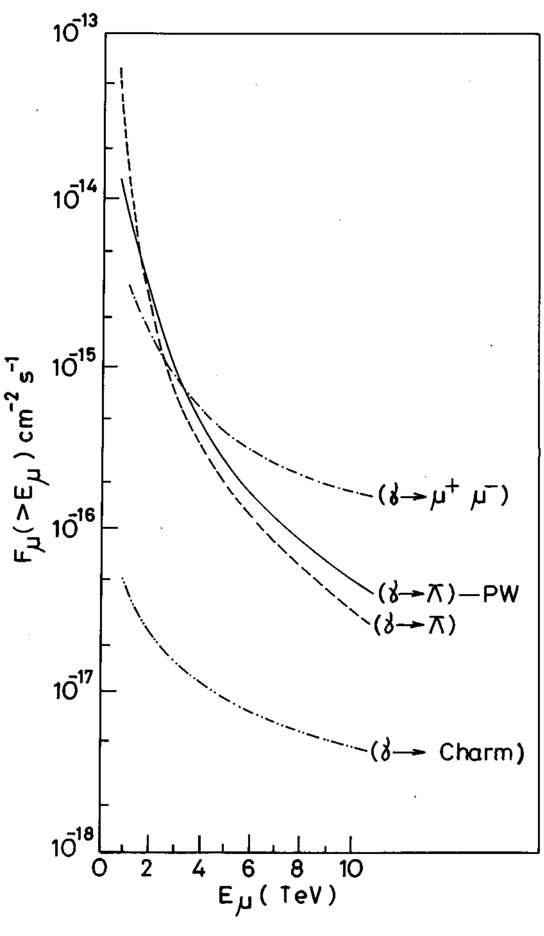

FIG. 3. Muon spectra derived from the Crab-emitted γ rays [17–25] using a HS [27] simulation formulation for photopions has been compared with those analytically estimated for photopions, direct muons, and charm meson decays: The solid curve shows the muons from the decay of photopions ($\gamma\rightarrow\pi$) is the spectrum expected from the HS [27] simulation formulation; the dashed curve is the similar result found for photopions from the analytical model [13] for $Z_{\pi\pi}=0.1967$, $\sigma_{\gamma N}=0.332$ mb, dot-dashed and double-dot-dashed curves show the muon fluxes expected from direct muons ($\gamma\rightarrow\mu^+\mu^-$) and charm particle decays ($\gamma\rightarrow$charm) from the standard model [26].

minimum γ-ray energy $E_{\gamma \min}$ and correction factor f from Table II. The simplified form of the relation comes out to be of the form

$$N_{\mu}(>E)=3.10^{-16}f\ \ln(100/E_{\mathrm{TeV}})E_{\mathrm{TeV}}^{-2.4}\ \mathrm{cm}^{-2}\ \mathrm{sec}^{-1}. \tag{21}$$

Using relation (21) along with the parametric values presented in Table II, the integral muon fluxes from the latest Crab-emitted γ-ray spectrum (11) have been derived and shown in Table II.

In Fig. 3 muon energy spectrum expected from an HS [27] simulation-based formulation has been compared with the analytical result expected from photopions ($\gamma \rightarrow \pi$) using the model of Drees *et al.* [13]. It is evident from Fig. 3 that the HS [27] formulation yields an excess of muon flux beyond 4 TeV energy when compared to our expected results obtained from the analytical calculations. In the same plot the analytical estimates of muon fluxes are shown from direct muons ($\gamma \rightarrow \mu^{+}\mu^{-}$) and charmed mesons ($\gamma \rightarrow$charm) obtained from the QCD-based procedure of Berezinsky *et al.* [26]. The spectrum of muons from photopions is much higher than that expected from charmed mesons. It may be pointed out that there are no significant differences between the muon spectra from the decay of photopions when obtained from simulation and analytical estimates. For muon energy above 3 TeV, the majority of the muon contribution occur from γ-air interactions through $\gamma+Z \rightarrow Z+\mu^{+}+\mu^{-}$ reactions. The muon production through the pion-generated in photonuclear γN reaction is adequate for muon energies below 3 TeV. So one can safely ignore the muons from direct contributions as well as from charmed meson decays below such energy.

CONCLUSION

The muon energy spectrum below 3 TeV analytically derived from the Crab-emitted γ-ray photopion decay is in fair agreement with the expected results from the Monte Carlo formulation of Halzen and Stanev when parametrized with the latest available results. The muon fluxes obtained from photopion decay through photonuclear γN reactions is adequate for muon energies below 3 TeV. But above that energy, the majority of the muon contribution occur from γ-air interactions through $\gamma+Z \rightarrow Z+\mu^{+}+\mu^{-}$ reactions.

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