Asymptotics of heavy-meson form factors

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Using methods developed for hard exclusive QCD processes, we calculate the asymptotic behavior of heavy-meson form factors at large recoil. It is determined by the leading- and subleading-twist meson wave functions. For $1 \ll |v\cdot v'| \ll m_Q/\Lambda$, the form factors are dominated by the Isgur-Wise function, which is determined by the interference between the wave functions of leading and subleading twist. At $|v\cdot v'| \gg m_Q/\Lambda$, they are dominated by two functions arising at order $1/m_Q$ in the heavy-quark expansion, which are determined by the leading-twist wave function alone. The sum of these contributions describes the form factors in the whole region $|v\cdot v'| \gg 1$. As a consequence, there is an exact zero in the form factor for the scattering of longitudinally polarized B^* mesons at some value $v\cdot v' \sim m_b/\Lambda$, and an approximate zero in the form factor of B mesons in the timelike region ($v\cdot v' \sim -m_b/\Lambda$). We obtain the evolution equations and sum rules for the wave functions of leading and subleading twist as well as for their moments. We briefly discuss applications to heavy-meson pair production in e^+e^- collisions. [S0556-2821(97)03201-3]

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I. INTRODUCTION

In heavy-quark effective theory (HQET) [1–5] (see [6–11] for reviews), the heavy-quark spin does not interact with gluons to leading order in $1/m_Q$ (where m_Q is the heavy-quark mass). Therefore, this spin can be rotated (spin symmetry) or even switched off (superflavor symmetry [12,13]) without affecting the dynamics. In the heavy-quark limit, the properties of the doublet of the ground-state pseudoscalar and vector mesons $(\overline{q}Q)$ are therefore characterized by the spin-parity quantum numbers $j^P = \frac{1}{2}^+$ of the light degrees of freedom [1,14]. In this paper, we shall use the superflavor symmetry to describe the ground-state mesons by a Dirac wave function. However, we collect in Appendix A the most important formulas using a more conventional formalism.

Let Q_v^* be a scalar field describing a heavy antiquark moving at four-velocity v with its spin switched off. Then the decay constant f of a heavy meson (moving at the same velocity) is defined as

$$\langle 0|Q_v^* q|M(v)\rangle = f u(v), \tag{1.1}$$

where $u(v)$ is the Dirac wave function of the meson, which satisfies

$$\not{v} u(v) = u(v). \tag{1.2}$$

A nonrelativistic (i.e., mass-independent) normalization of $u(v)$ and $|M(v)\rangle$ is assumed. In the heavy-quark limit, the relation between f and the usual meson decay constants reads

$$f_M = f_{M^*} = \frac{2f}{\sqrt{m_Q}}. \tag{1.3}$$

To leading order in $1/m_Q$, current-induced transitions between two ground-state mesons are described by a single Isgur-Wise form factor [1,15]:

$$\langle M(v')|Q_v^* Q_{v'}|M(v)\rangle = \xi(v\cdot v')\overline{u(v')}u(v), \tag{1.4}$$

where v, v' are the meson velocities. At next-to-leading order, there appear $1/m_Q$ corrections to the currents and to the Lagrangian of the HQET [16]. The first type of corrections can be expressed via the matrix element of a dimension-four operator:[1]

$$\langle M(v')|(iD^{\mu\dagger}Q_v^*)Q_{v'}|M(v)\rangle = \overline{u(v')}\xi^\mu(v,v')u(v), \tag{1.5}$$

where

$$\xi^\mu(v,v') = \xi_+(v\cdot v')(v+v')^\mu + \xi_-(v\cdot v')(v-v')^\mu + \xi_3(v\cdot v')\gamma^\mu. \tag{1.6}$$

The equations of motion, $iv\cdot DQ_v = 0$, can be employed to relate $\xi_\pm$ to ξ_3 [16]. The result is

$$\xi_+ = \frac{(1/2)(v\cdot v'-1)\overline{\Lambda}\,\xi - \xi_3}{v\cdot v'+1}, \quad \xi_- = \tfrac{1}{2}\,\overline{\Lambda}\,\xi, \tag{1.7}$$

where $\overline{\Lambda}$ is the "binding energy," i.e., the difference between the meson mass and the heavy-quark mass. Hence, there is only one new independent form factor. The $1/m_Q$ corrections to the Lagrangian give rise to the matrix elements of two nonlocal operators:

[1] We use $D^\mu = \partial^\mu - iA^\mu$ and $D^{\mu\dagger} = \partial^\mu + iA^{\mu\dagger}$, where $A^\mu = g_s t_a A_a^\mu$ is the gluon field.

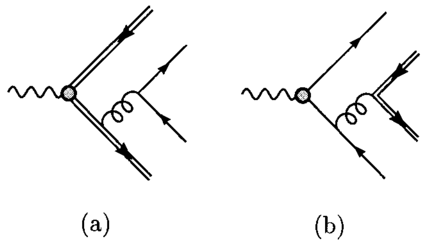

FIG. 1. Hard-gluon exchange contributions to heavy-meson form factors. The external current is presented by the wave line; the heavy antiquark is represented by a double line.

$$\langle M(v')|i\int dx T\{Q_v^* Q_{v'}(0), Q_v^*(iD)^2 Q_v(x)\}|M(v)\rangle$$
$$=2\chi_1(v\cdot v')\overline{u(v')}u(v),$$
$$\langle M(v')|i\int dx T\{Q_v^* Q_{v'}(0), Q_v^* iG^{\mu\nu} Q_v(x)\}|M(v)\rangle$$
$$=2\overline{u(v')}\chi^{\mu\nu}(v,v')u(v), \tag{1.8}$$

where $iG^{\mu\nu}=[iD^\mu,iD^\nu]=ig_s t_a G_a^{\mu\nu}$ is the gluon field strength, and ($\sigma^{\mu\nu}=(i/2)[\gamma^\mu,\gamma^\nu]$)

$$\chi^{\mu\nu}(v,v')=\chi_2(v\cdot v')(\gamma^\mu v'^\nu-\gamma^\nu v'^\mu)+2i\chi_3(v\cdot v')\sigma^{\mu\nu}. \tag{1.9}$$

In the second matrix element in Eq. (1.8), the indices μ and ν are restricted to the subspace orthogonal to v. Explicit expressions for the meson form factors in terms of the functions ξ, ξ_3 and χ_i can be found in [9,16]. At moderate values of $v\cdot v'$, these functions have been studied extensively in the framework of QCD sum rules, both at leading [17–21] and next-to-leading [22–24] order in the heavy-quark expansion. In the present paper, we shall consider the behavior of the form factors in the large-recoil region $|v\cdot v'|\gg 1$. This region is inaccessible in the weak semileptonic decays, but it can be explored (at least in principle) in the production reaction $e^+e^-\to M^{(*)}\overline{M}^{(*)}$. Using methods developed for hard exclusive processes, we calculate the asymptotic behavior of the form factors in a model-independent way.

Our results can be summarized as follows: For $|v\cdot v'|\gg 1$, there is a large momentum transfer to the light quark: $q^2_{\rm light}\sim-\Lambda^2 v\cdot v'$, where Λ is of the order of a typical hadronic mass scale. As shown in Fig. 1(a), this momentum is transferred by the exchange of a hard gluon, and the methods developed for hard exclusive processes in QCD [25–28] (see [29–31] for reviews) are applicable.[2] In the ''brick wall'' frame, where $\vec{v}\,'=-\vec{v}$, the projection of the total angular momentum on the z axis (directed along $\vec{v}$) is equal to the projection of the meson spin. (Recall that the heavy-quark spin has been switched off.) Since it is conserved, the meson helicity changes its sign [33]. The asymptotic behavior of the Isgur-Wise form factor is thus determined by the interference between the leading-twist (quark-antiquark) wave function and the subleading-twist (quark-antiquark and quark-antiquark-gluon) wave functions. It is given by

$$\xi(v\cdot v')\sim\frac{\alpha_s f^2}{\Lambda^3(v\cdot v')^2}. \tag{1.10}$$

Indeed, the situation is similar to the well-known case of the π-ρ form factor [29]. At order $1/m_Q$ in the heavy-quark expansion, there are however contributions involving the leading-twist wave function only. They conserve the meson helicity and behave as

$$\frac{\xi_3(v\cdot v')}{m_Q}\sim\frac{v\cdot v'\chi_2(v\cdot v')}{m_Q}\sim\frac{\alpha_s f^2}{m_Q\Lambda^2 v\cdot v'}. \tag{1.11}$$

The leading contribution in the heavy-quark expansion, which is given by the Isgur-Wise function, dominates as long as $|v\cdot v'|\ll m_Q/\Lambda$. For $|v\cdot v'|\gg m_Q/\Lambda$, however, the contributions of ξ_3 and χ_2 in Eq. (1.11) become the dominant ones. Note that they violate the heavy-quark spin symmetry; i.e., they contribute in a nonuniversal way to the various meson form factors. Higher-order terms in the heavy-quark expansion (of order $1/m_Q^2$ and higher) cannot fall off slower than $1/(v\cdot v')$ because this behavior corresponds to the leading twist, and hence they always remain small corrections.

It is instructive to consider the same situation from an opposite point of view. At asymptotically large values $|q^2|\simeq 2m_Q^2|v\cdot v'|\gg m_Q^2/x$ (where q is the momentum transferred to the mesons, and $x\sim\Lambda/m_Q$ is the momentum fraction carried by the light quark), the form factor of a heavy meson behaves like that of the pion [25–31]:

$$F(q^2)\sim\frac{\alpha_s f_M^2}{x^2q^2}, \tag{1.12}$$

which exactly corresponds to Eq. (1.11). However, there is a contribution to the subleading-twist ($1/q^4$) correction which is proportional to m_Q^2. It becomes important for moderate values of q^2:

$$F(q^2)\sim\frac{\alpha_s m_Q^2 f_M^2}{x^3q^4}. \tag{1.13}$$

This contribution exactly corresponds to Eq. (1.10). It dominates for $|q^2|\ll m_Q^2 x$. Higher-twist ($1/q^6$ and higher) corrections cannot be more enhanced than m_Q^2 because otherwise the form factor would diverge in the heavy-quark limit, and hence they always remain small corrections.

Until now, we considered form factors for transitions induced by a current containing heavy quarks only. In the case of, say, the electromagnetic current, which also contains light-quark fields, contributions of the type shown in Fig. 1(b) appear. However, they lead to the behavior

$$\frac{\alpha_s f^2}{m_Q^3 v\cdot v'}\sim\frac{\alpha_s f_M^2}{q^2} \tag{1.14}$$

and can thus be safely neglected, since $m_Q\gg\Lambda$ for a heavy quark.

In summary, for $1\ll|v\cdot v'|\ll m_Q/\Lambda$ the dominant contribution to meson form factors comes from the universal

[2] There are also soft endpoint contributions [32], which are difficult to estimate.

Isgur-Wise function. It involves the exchange of a longitudinal hard gluon and a change of the meson helicity, corresponding to the interference between the leading- and subleading-twist wave functions. For $|v\cdot v'|\gg m_Q/\Lambda$, on the other hand, the situation is the same as in massless QCD; the asymptotic behavior of meson form factors is determined by leading-twist contributions, which are not universal. They are governed by the exchange of a hard transverse gluon, which conserves the meson helicity. The sum of these two contributions describes the asymptotic behavior of the form factors in the whole region $|v\cdot v'|\gg 1$. The simple picture described here is only slightly modified by the emission of gluon bremsstrahlung, which can be dealt with in renormalization-group improved perturbation theory. It leads to an additional, moderate power suppression of the form factors at large recoil.

The remainder of the paper is organized as follows: In Sec. II, we introduce the (quark-antiquark) meson wave functions of leading and subleading twist, $\varphi_\pm(\omega)$. Unlike in QCD, wave functions defined in the HQET depend on a dimensional argument ω. We investigate the moments of these wave functions and derive the symmetry relations between the various meson wave functions, which arise in the heavy-quark limit. In Sec. III, we derive the evolution equations for $\varphi_\pm(\omega)$, which are analogous to the Brodsky-Lepage equations [28]. This calculation extends the calculation of the HQET anomalous dimension of local heavy-light current operators [34–39]. A new kind of ultraviolet divergence appears in the relation between the local operators and the operators defining the wave function. Therefore, the Brodsky-Lepage kernels do not determine the renormalization properties of the local operators completely. A similar situation is encountered in the case of the (Altarelli-Parisi) equations describing the evolution of distribution functions in the HQET [40]. In Sec. IV, we investigate the properties of the wave functions $\varphi_\pm(\omega)$ using the QCD sum-rule approach. This extends the heavy-meson sum rules [18,41–43] to the case of nonlocal operators. After considering the sum rules for the lowest moments, we construct the sum rules for the wave functions themselves, taking into account the nonlocality of the quark condensate [44]. In Sec. V, we apply our results to derive the leading asymptotic behavior of meson form factors at large recoil. First, we calculate the contribution of the interference between the leading-twist and the (quark-antiquark) subleading-twist wave functions to the asymptotic behavior of the Isgur-Wise function.[2] Then we calculate the leading-twist contributions to the form factors appearing at order $1/m_Q$ in the heavy-quark expansion. Finally, in Sec. VI we discuss the implications of our results for the reactions $e^+e^-\to B^{(*)}\overline{B}^{(*)}$ and $e^+e^-\to D^{(*)}\overline{D}^{(*)}$. Technical details of our calculations are presented in four appendices.

Before we proceed, some comments on the existing literature on the application of perturbative QCD to the calculation of heavy-meson form factors are in order. A simple model of a meson made out of two heavy quarks with unequal masses was considered in [45]. There it was noted that some form factors must have zeros in the physical region. We confirm this interesting observation, although we disagree with some other results of this work (see Sec. V). A similar model was considered in [46–48]. There, a single spin structure of the heavy-meson wave function was used, which is determined from the condition that the light quark be at rest in the meson rest frame. Hence, all quark-antiquark wave functions were taken to have the same shape, and this shape was assumed to be $\delta(\omega-\mu)$, with μ being the constituent mass of the light quark. As we shall see later, for a realistic heavy meson the wave functions $\varphi_+(\omega)$ and $\varphi_-(\omega)$ do not coincide, and they are not well approximated by sharply peaked functions. Integrals for the form factors receive important contributions from the region of low ω values, which are missing in the peaking approximation. Therefore, the results obtained using such a static quark model can at best be taken as a crude estimate. Perturbative QCD and the constituent quark model were recently applied also to the semileptonic decays $\overline{B}\to D^{(*)}l\overline{\nu}$ [49], for which $1<v\cdot v'<1.6$. In our opinion, such small values of $v\cdot v'$ are far too low to treat the gluon with $k_g^2\sim-\Lambda^2 v\cdot v'$ perturbatively. Moreover, the calculations in [48] are done using a model wave function with an ad hoc $k_\perp$ dependence, whose longitudinal momentum dependence contradicts the expectations based on the HQET.

II. QUARK-ANTIQUARK WAVE FUNCTIONS

We shall define the quark-antiquark wave functions $\widetilde{\varphi}_\pm(t)$ of a heavy meson in terms of the matrix element of the bilocal operator

$$\widetilde{O}(t)=Q^*(0)E(0,z)q(z),\quad t=v\cdot z, \tag{2.1}$$

where z is a null vector on the light cone ($z^2=0$), and

$$E(x,y)=P\exp\left(-i\int_x^y dz^\mu A_\mu(z)\right) \tag{2.2}$$

is a string operator ensuring gauge invariance. In the light-cone gauge ($A_+=0$), one simply has $E(0,z)=1$. Since in this section we are considering operators containing a single heavy-antiquark field Q_v^*, we shall for simplicity omit the velocity label on the field. Similarly, we shall write M and u instead of $M(v)$ and $u(v)$. The meson matrix element of the operator $\widetilde{O}(t)$ has two independent Dirac structures, u and $\not{z}u$, and we define

$$\langle 0|\widetilde{O}(t)|M\rangle=f\left(\widetilde{\varphi}_+(t)+\frac{1}{2t}[\widetilde{\varphi}_-(t)-\widetilde{\varphi}_+(t)]\not{z}\right)u. \tag{2.3}$$

It is convenient to introduce two light-cone vectors $n_\pm^\mu=(1,0,0,\mp1)$ such that $n_\pm^2=0$ and $n_+\cdot n_-=2$. Any vector a^μ can be decomposed as $a^\mu=\frac{1}{2}(a_+n_-^\mu+a_-n_+^\mu)+a_\perp^\mu$, where $a_\pm=a\cdot n_\pm$. This implies $a\cdot b=\frac{1}{2}(a_+b_-+a_-b_+)-\vec{a}_\perp\cdot\vec{b}_\perp$. We shall also use the light-cone components of the Dirac matrices, defined as $\gamma_\pm=n_\pm^\mu\gamma_\mu=\not{n}_\pm$. If the meson is at rest, then $v^\mu=(1/2)(n_+^\mu+n_-^\mu)$, i.e., $v_+=v_-=1$. Using $\not{v}u=u$, we can then rewrite Eq. (2.3) as

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[2]The properties of quark-antiquark-gluon subleading-twist wave functions and their contribution to $\xi(v\cdot v')$ will be discussed elsewhere.

$$\langle 0|\tilde{O}(t)|M\rangle=\tfrac{1}{2}f[\tilde{\varphi}_+(t)\gamma_-+\tilde{\varphi}_-(t)\gamma_+]u. \tag{2.4}$$

For a meson with an arbitrary velocity in the n_+–n_- plane, this formula becomes

$$\langle 0|\tilde{O}(t)|M\rangle=\tfrac{1}{2}f[\tilde{\varphi}_+(t)v_+\gamma_-+\tilde{\varphi}_-(t)v_-\gamma_+]u. \tag{2.5}$$

If we introduce the rapidity ϑ by writing

$$v^\mu=(\cosh\vartheta,0,0,\sinh\vartheta), \tag{2.6}$$

then $v_+=e^\vartheta$ and $v_-=e^{-\vartheta}$. This shows that for a fast-moving meson ($\vartheta\gg 0$), $\tilde{\varphi}_+$ is the leading-twist wave function, whereas $\tilde{\varphi}_-$ has subleading twist. It is convenient to project onto these wave functions by writing

$$\langle 0|\tilde{O}_\pm(t)|M\rangle=f\tilde{\varphi}_\pm(t)\gamma_\pm u,\quad \tilde{O}_\pm(t)=\gamma_\pm\tilde{O}(t). \tag{2.7}$$

This result is valid in an arbitrary reference frame, as can be seen by using the relations

$$\gamma_\pm^2=0,\quad \gamma_\pm\gamma_\mp=\frac{2}{v_\pm}\gamma_\pm\not{v}. \tag{2.8}$$

The wave functions $\tilde{\varphi}_\pm(t)$ depend on the separation t on the light cone. We define the corresponding wave functions in momentum space by

$$\varphi_\pm(\omega)=\frac{1}{2\pi}\int dt\tilde{\varphi}_\pm(t)e^{i\omega t},$$

$$\tilde{\varphi}_\pm(t)=\int d\omega\varphi_\pm(\omega)e^{-i\omega t}. \tag{2.9}$$

The variable ω has the meaning of the light-cone projection p_+ of the light-quark momentum in the heavy-meson rest frame. The positions of the singularities in the complex t plane are such that $\varphi_\pm(\omega)$ vanish for $\omega<0$. The wave functions are normalized such that

$$\tilde{\varphi}_\pm(0)=\int_0^\infty d\omega\varphi_\pm(\omega)=1. \tag{2.10}$$

We can formally introduce operators $O_\pm(\omega)$ such that

$$\langle 0|O_\pm(\omega)|M\rangle=f\varphi_\pm(\omega)\gamma_\pm u. \tag{2.11}$$

This implies

$$O_\pm(\omega)=\frac{1}{2\pi}\int dt\tilde{O}_\pm(t)e^{i\omega t}=Q^*(0)\gamma_\pm\delta(iD_+-\omega)q(0),$$

$$\tilde{O}_\pm(t)=\int d\omega O_\pm(\omega)e^{-i\omega t}. \tag{2.12}$$

Expanding in powers of t in the definitions (2.9) and (2.12), we obtain

$$\tilde{O}_\pm(t)=\sum_{n=0}^\infty O_\pm^{(n)}\frac{(-it)^n}{n!},$$

$$\tilde{\varphi}_\pm(t)=\sum_{n=0}^\infty \langle\omega^n\rangle_\pm\frac{(-it)^n}{n!}, \tag{2.13}$$

where

$$O_\pm^{(n)}=\int d\omega O_\pm(\omega)\omega^n=Q^*\gamma_\pm(iD_+)^n q,$$

$$\langle\omega^n\rangle_\pm=\int d\omega\varphi_\pm(\omega)\omega^n. \tag{2.14}$$

Equation (2.11) then implies a relation between the moments of the momentum-space wave functions and the local, higher-dimensional operators $O_\pm^{(n)}$:

$$\langle 0|O_\pm^{(n)}|M\rangle=f\langle\omega^n\rangle_\pm\gamma_\pm u. \tag{2.15}$$

Using the equations of motion, the first moments of the wave functions can be calculated in terms of the parameter $\bar{\Lambda}$ encountered in Eq. (1.7) [50]. In general, we may write

$$\langle 0|Q^*iD^\mu q|M\rangle=f(av^\mu+b\gamma^\mu)u. \tag{2.16}$$

The equations of motion for the light quark, $i\not{D}q=0$, imply that $(a+4b)=0$. The equations of motion for the heavy quark, $iv\cdot DQ=0$, can be used to write

$$\langle 0|Q^*iv\cdot Dq|M\rangle=iv\cdot\partial\langle 0|Q^*q|M\rangle=\bar{\Lambda}\langle 0|Q^*q|M\rangle, \tag{2.17}$$

where $\bar{\Lambda}=m_M-m_Q$ is the effective mass of the meson M in the HQET [51]. This relation implies that $(a+b)=\bar{\Lambda}$, and, therefore,

$$\langle 0|Q^*iD^\mu q|M\rangle=\tfrac{1}{3}f\bar{\Lambda}(4v^\mu-\gamma^\mu)u. \tag{2.18}$$

Using this result, we find that the first moments of the wave functions are given by

$$\langle\omega\rangle_+=\tfrac{4}{3}\bar{\Lambda},\quad \langle\omega\rangle_-=\tfrac{2}{3}\bar{\Lambda}. \tag{2.19}$$

A similar analysis can be performed for the second moments. Consider the matrix element

$$\langle 0|Q^*iD^\mu iD^\nu q|M\rangle=f\Theta^{\mu\nu}u, \tag{2.20}$$

where the most general form of $\Theta^{\mu\nu}$ is

$$\Theta^{\mu\nu}=c_1v^\mu v^\nu+c_2g^{\mu\nu}+c_3(\gamma^\mu v^\nu+\gamma^\nu v^\mu)+c_4(\gamma^\mu v^\nu-\gamma^\nu v^\mu)+ic_5\sigma^{\mu\nu}. \tag{2.21}$$

The equations of motion impose three independent relations among the five parameters c_i, which imply that the matrix element in Eq. (2.20) is completely determined by its antisymmetric part [52], i.e., by the matrix element of the gluon field $iG^{\mu\nu}=[iD^\mu,iD^\nu]$. For reasons to become clear below, we find it convenient to introduce two hadronic parameters λ_E^2 and λ_H^2 by

$$c_4=\tfrac{1}{6}(\lambda_H^2-\lambda_E^2),\quad c_5=\tfrac{1}{6}\lambda_H^2. \tag{2.22}$$

In terms of these quantities, we obtain

$$\langle 0|Q^* i G^{\mu\nu} q|M\rangle=\tfrac{1}{3}f[(\lambda_H^2-\lambda_E^2)(\gamma^\mu v^\nu-\gamma^\nu v^\mu)+i\lambda_H^2\sigma^{\mu\nu}]u,$$

$$\langle 0|Q^*\tfrac{1}{2}\{iD^\mu,iD^\nu\}q|M\rangle=\tfrac{1}{3}f[(6\bar\Lambda^2+2\lambda_E^2+\lambda_H^2)v^\mu v^\nu-(\bar\Lambda^2+\lambda_E^2+\lambda_H^2)g^{\mu\nu}-(\bar\Lambda^2+\tfrac{1}{2}\lambda_E^2)(\gamma^\mu v^\nu+\gamma^\nu v^\mu)]u. \tag{2.23}$$

From the second relation, it follows that the second moments of the wave functions are given by

$$\langle\omega^2\rangle_+=2\bar\Lambda^2+\tfrac{2}{3}\lambda_E^2+\tfrac{1}{3}\lambda_H^2,$$

$$\langle\omega^2\rangle_-=\tfrac{2}{3}\bar\Lambda^2+\tfrac{1}{3}\lambda_H^2. \tag{2.24}$$

According to the first equation in Eq. (2.23), the moments $\langle\omega^2\rangle_\pm$ are thus related to normalization integrals of quark-antiquark-gluon wave functions.

Our definition in Eq. (2.22) is such that, in the rest frame of the heavy meson, the quantities λ_E^2 and λ_H^2 parametrize the matrix elements of the chromoelectric and chromomagnetic fields, respectively. Defining[3] $E_i=G_{0i}$, $H_i=-\frac{1}{2}\epsilon_{ijk}G_{jk}$, and $\alpha_i=\gamma^0\gamma^i$, we find

$$\langle 0|Q^* i\vec\alpha\cdot\vec E q|M\rangle=f\lambda_E^2 u,$$

$$-\langle 0|Q^*\vec\sigma\cdot\vec H q|M\rangle=f\lambda_H^2 u. \tag{2.25}$$

To finish this section, let us switch the heavy-quark spin on and relate the numerous quark-antiquark wave functions of the ground-state pseudoscalar and vector mesons, M and M^*, to the HQET wave functions $\varphi_\pm$. These relations are very conveniently obtained using the covariant tensor formalism described in Appendix A. For a pseudoscalar meson M, the matrix elements of the pseudoscalar, axial, and tensor currents are nonzero, and we define a set of four wave functions in the following way:[4]

$$\langle 0|\overline{Q}(0)\gamma_5 q(z)|M\rangle=-if_M m_M\tilde\varphi_P,$$

$$\langle 0|\overline{Q}(0)\gamma^\mu\gamma_5 q(z)|M\rangle=f_M[i\tilde\varphi_{A1}p^\mu-m_M\tilde\varphi_{A2}z^\mu],$$

$$\langle 0|\overline{Q}(0)\sigma^{\mu\nu}\gamma_5 q(z)|M\rangle=if_M\tilde\varphi_T(p^\mu z^\nu-p^\nu z^\mu), \tag{2.26}$$

where $\tilde\varphi_i=\tilde\varphi_i(p\cdot z)=\tilde\varphi_i(m_M t)$. For simplicity, we have omitted the string operator $E(0,z)$, i.e., we have adopted the light-cone gauge $A_+=0$. In the heavy-quark limit, we obtain, using the results of Appendix A:

$$\tilde\varphi_P=\frac{\tilde\varphi_+(t)+\tilde\varphi_-(t)}{2},\quad \tilde\varphi_{A1}=\tilde\varphi_+(t),$$

$$\tilde\varphi_{A2}=\tilde\varphi_T=\frac{i}{2}\,\frac{\tilde\varphi_+(t)-\tilde\varphi_-(t)}{t}. \tag{2.27}$$

For $t=0$, we obtain in this limit the normalization conditions:

$$\tilde\varphi_P(0)=\tilde\varphi_{A1}(0)=1,\quad \tilde\varphi_{A2}(0)=\tilde\varphi_T(0)=\frac{\bar\Lambda}{3}, \tag{2.28}$$

where the second relation is a consequence of Eqs. (2.13) and (2.19).

For a vector meson M^* with polarization vector e, the matrix elements of the scalar, vector, axial vector, and tensor currents are nonzero, and we introduce a set of six wave functions as follows:

$$\langle 0|\overline{Q}(0)q(z)|M^*\rangle=if_{M^*}m_{M^*}\tilde\varphi_S z\cdot e,$$

$$\langle 0|\overline{Q}(0)\gamma^\mu q(z)|M^*\rangle=f_{M^*}[m_{M^*}\tilde\varphi_{V1}e^\mu+i\tilde\varphi_{V2}(p\cdot z e^\mu-e\cdot z p^\mu)],$$

$$\langle 0|\overline{Q}(0)\gamma^\mu\gamma_5 q(z)|M^*\rangle=f_{M^*}\tilde\varphi_A\epsilon^{\mu\alpha\beta\gamma}z_\alpha p_\beta e_\gamma,$$

$$\langle 0|\overline{Q}(0)\sigma^{\mu\nu}q(z)|M^*\rangle=f_{M^*}[i\tilde\varphi_{T1}(e^\mu p^\nu-e^\nu p^\mu)-m_{M^*}\tilde\varphi_{T2}(e^\mu z^\nu-e^\nu z^\mu)]. \tag{2.29}$$

In the heavy-quark limit, we find the relations

$$\tilde\varphi_{V1}=\tilde\varphi_{T1}=\tilde\varphi_+(t),$$

$$\tilde\varphi_S=\tilde\varphi_{V2}=\tilde\varphi_A=\tilde\varphi_{T2}=\frac{i}{2}\,\frac{\tilde\varphi_+(t)-\tilde\varphi_-(t)}{t}, \tag{2.30}$$

and the corresponding normalization conditions

$$\tilde\varphi_{V1}(0)=\tilde\varphi_{T1}(0)=1,$$

$$\tilde\varphi_S(0)=\tilde\varphi_{V2}(0)=\tilde\varphi_A(0)=\tilde\varphi_{T2}(0)=\frac{\bar\Lambda}{3}. \tag{2.31}$$

The QCD wave functions in momentum space are defined as

$$\varphi_i(x)=\frac{1}{2\pi}\int d(p\cdot z)\tilde\varphi_i(p\cdot z)e^{-ixp\cdot z}, \tag{2.32}$$

so that $x=\omega/m_M$. On the basis of the behavior of the eigenfunctions of the evolution equations as well as sum-rule inspired arguments, it is usually assumed that for pseudoscalar mesons $\varphi_{A1}(x)\sim x$ and $\varphi_P(x)\sim 1$ as $x\to 0$ [29]. For the HQET wave functions, this implies the behavior

$$\varphi_+(\omega)\sim\omega,\quad \varphi_-(\omega)\sim 1 \tag{2.33}$$

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[3]If we define $D^\mu=(D^0,-\vec D)$, then $\vec E=i[D^0,\vec D]$ and $\vec H=-i\vec D\times\vec D$.

[4]Contrary to the notation used in the rest of this paper, here and in Eq. (2.29) we use the standard relativistic normalization of states, which adds a factor $\sqrt{m_M}$ on the right-hand side of the equations.

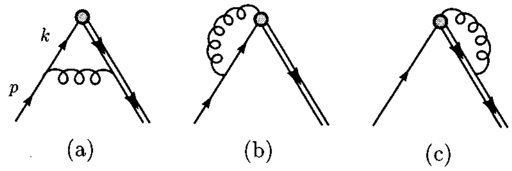

FIG. 2. One-loop diagrams contributing to the matrix elements $\langle 0|O_\pm^{\rm bare}(\omega)|\omega'\rangle$. The bare current operators are represented by a circle.

as $\omega\to 0$. In Sec. IV, we will indeed find these scaling laws from an explicit calculation of the wave functions using QCD sum rules.

III. EVOLUTION EQUATIONS

The definitions of the previous section are somewhat formal, because the operators involved require renormalization. In this section we discuss how the ultraviolet divergences in operator matrix elements can be removed in a consistent way. After doing this, however, we shall ignore renormalization effects in the further course of this paper. The reader not interested in the conceptual problem of renormalization can thus proceed directly with Sec. IV.

We use the modified minimal subtraction ($\overline{\rm MS}$) scheme in ($d=4-2\varepsilon$)-dimensional space-time. The bare and renormalized operators are related by

$$O_\pm^{\rm bare}(\omega)=\int d\omega' Z_\pm(\omega,\omega')O_\pm(\omega'), \tag{3.1}$$

where

$$Z_\pm(\omega,\omega')=\delta(\omega-\omega')-\frac{1}{2\varepsilon}z_\pm(\omega,\omega')+\cdots, \tag{3.2}$$

and the ellipses represent poles of higher order in $1/\varepsilon$. The operators $O_\pm(\omega)$ and hence their matrix elements $f\varphi_\pm(\omega)$ obey the renormalization-group equations

$$\frac{df\varphi_\pm(\omega)}{d\ln\mu}+\int d\omega'\Gamma_\pm(\omega,\omega')f\varphi_\pm(\omega')=0, \tag{3.3}$$

where the anomalous dimensions $\Gamma_\pm(\omega,\omega')$ are given by

$$\Gamma_\pm(\omega,\omega')=\alpha_s\frac{\partial}{\partial\alpha_s}z_\pm(\omega,\omega'). \tag{3.4}$$

Equation (3.3) is analogous to the Brodsky-Lepage evolution equation in QCD [28].

In order to obtain the anomalous dimensions $\Gamma_\pm(\omega,\omega')$ at the one-loop order, we consider the matrix elements $\langle 0|O_\pm^{\rm bare}(\omega)|\omega'\rangle$, where $|\omega'\rangle$ represents a state consisting of a scalar heavy antiquark at rest and a light quark with momentum $p_+=\omega'$. According to Eq. (3.1), these matrix elements equal $Z_\pm(\omega,\omega')\gamma_\pm u$. The relevant one-particle irreducible loop diagrams are shown in Fig. 2. Although the operators $O_\pm(\omega)$ in Eq. (2.12) take a particularly simple form in the light-cone gauge $A_+=0$, we refrain from adopting such a singular gauge and work instead in the Feynman gauge.[5] To obtain the Feynman rules for vertices involving the operators $O_\pm(\omega)$, we start from their definition as the Fourier transforms of the nonlocal operators $\widetilde{O}_\pm(t)$ and obtain, to first order in the gauge field,

$$O_\pm(\omega)=\frac{1}{2\pi}\int dt e^{i\omega t}\Big\{Q^*(0)\gamma_\pm e^{t\partial_+}q(0) -i\int d\tau Q^*(0)\gamma_\pm e^{\tau\partial_+}A_+(0)e^{(t-\tau)\partial_+}q(0) +\cdots\Big\}. \tag{3.5}$$

It is then straightforward to derive the Feynman rules shown in Fig. 3.

Let us then sketch the calculation of the diagrams in Fig. 2. The contribution of the first diagram is

$$-iC_F\frac{\alpha_s}{2\pi}\mu^{2\varepsilon}\int dk_+dk_-\frac{{\rm d}^{2-2\varepsilon}k_\perp}{(2\pi)^{2-2\varepsilon}} \times\frac{\delta(k_+-\omega)\gamma_\pm \not k u}{(k^0-i0)(k^2+i0)[(k-p)^2+i0]}, \tag{3.6}$$

where $C_F=(N_c^2-1)/(2N_c)$. The k_+ integral is trivial to perform, and the k_- integral is readily calculated by the method of residues. The poles of the integrand are located at $k_-=-\omega+i0$, $k_-=(\vec{k}_\perp^2-i0)/\omega$, and $k_-=(\vec{k}_\perp^2-i0)/(\omega-\omega')$. If $\omega<0$, all poles lie in the upper half plane, and the integral vanishes. For $\omega>0$, it is necessary to distinguish the cases $\omega>\omega'$ and $\omega<\omega'$. For $\omega>\omega'$, we close the contour in the upper half plane and set $k_-=-\omega$. For $\omega<\omega'$, we close the contour in the lower half plane and set $k_-=\vec{k}_\perp^2/\omega$. Only in the second case and for the minus projection there is an ultraviolet divergence, which arises from the $k_\perp$ integration. Keeping only the singular term, we obtain for the contribution to $Z_\pm(\omega,\omega')$:

$$C_F\frac{\alpha_s}{4\pi\varepsilon}(1\mp 1)\frac{1}{\omega'}\theta(\omega'-\omega). \tag{3.7}$$

The other two diagrams in Fig. 2 are evaluated in a similar way. For the second one, we find an ultraviolet divergence for the plus projection if $k_+<\omega'$. Its contribution to $Z_\pm(\omega,\omega')$ is

$$-C_F\frac{\alpha_s}{4\pi\varepsilon}(1\pm 1)\int dk_+\frac{k_+}{\omega'}\frac{\delta(k_+-\omega)-\delta(\omega'-\omega)}{k_+-\omega'} =-C_F\frac{\alpha_s}{4\pi\varepsilon}(1\pm 1)\Big\{\Big[\frac{1}{\omega'}+\frac{1}{(\omega-\omega')_+}\Big]\theta(\omega'-\omega) -\delta(\omega'-\omega)\Big\}. \tag{3.8}$$

[5]We have repeated the calculation of $\Gamma_+(\omega,\omega')$ in the light-cone gauge and obtained the same result as in the Feynman gauge; however, we could not find an easy way to recover the correct result for $\Gamma_-(\omega,\omega')$.

The distribution $1/(\omega-\omega')_+$ is defined such that

$$\int d\omega \frac{f(\omega)}{(\omega-\omega')_+} = \int d\omega \frac{f(\omega)-f(\omega')}{\omega-\omega'} \tag{3.9}$$

for any smooth function $f(\omega)$. The third diagram has an ultraviolet divergence if $k_+ > \omega'$. Its contribution is

$$C_F \frac{\alpha_s}{2\pi\varepsilon} \frac{1}{(\omega-\omega')_+} \theta(\omega-\omega'). \tag{3.10}$$

Finally, we have to add the contributions from the wave-function renormalization of the external lines, which gives [36]

$$Z_Q^{1/2} Z_q^{1/2} \delta(\omega-\omega') = \left(1 + C_F \frac{\alpha_s}{8\pi\varepsilon}\right) \delta(\omega-\omega'). \tag{3.11}$$

Collecting all terms, we obtain, for the quantities $z_\pm(\omega,\omega')$ defined in Eq. (3.2),

$$\begin{aligned} z_\pm(\omega,\omega') = C_F \frac{\alpha_s}{\pi} \Bigg\{ & \pm \frac{\theta(\omega'-\omega)}{\omega'} - \frac{\theta(\omega-\omega')}{(\omega-\omega')_+} \\ & - \tfrac{3}{4}\,\delta(\omega-\omega') + \tfrac{1}{2}(1\pm 1) \\ & \times \left[\frac{\theta(\omega'-\omega)}{(\omega-\omega')_+} - \delta(\omega-\omega') \right] \Bigg\}. \end{aligned} \tag{3.12}$$

To one-loop order, the anomalous dimensions are given by the same expression. We thus obtain

$$\begin{aligned} \Gamma_+(\omega,\omega') = C_F \frac{\alpha_s}{\pi} \Bigg[& -\frac{1}{|\omega-\omega'|_+} + \frac{\theta(\omega'-\omega)}{\omega'} \\ & - \tfrac{5}{4}\,\delta(\omega-\omega') \Bigg], \end{aligned}$$

$$\begin{aligned} \Gamma_-(\omega,\omega') = C_F \frac{\alpha_s}{\pi} \Bigg[& -\frac{\theta(\omega-\omega')}{(\omega-\omega')_+} - \frac{\theta(\omega'-\omega)}{\omega'} \\ & - \tfrac{1}{4}\,\delta(\omega-\omega') \Bigg]. \end{aligned} \tag{3.13}$$

In the familiar case of QCD wave functions for light-quark systems, the renormalization of the nonlocal operators analogous to $O_\pm(\omega)$ would suffice to renormalize the tower

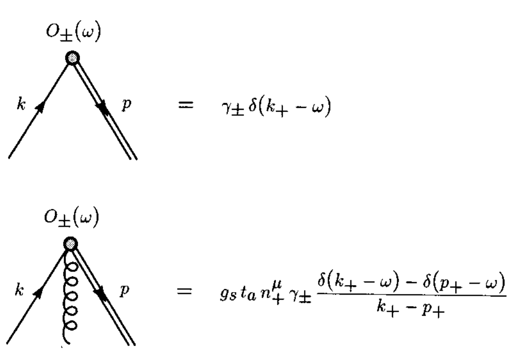

FIG. 3. Feynman rules for vertices involving $O_\pm(\omega)$.

of local operators defined analogous to $O_\pm^{(n)}$ in Eq. (2.14). Accordingly, the renormalization of the wave functions $\varphi_i(x)$ renders their moments $\langle x^n\rangle_i$ finite. Unfortunately, this situation does not hold in the case of wave functions for heavy mesons defined in the HQET. The reason for this unexpected fact is that in the HQET the wave functions depend on the dimensional variable ω, which takes values between 0 and ∞. As a consequence, Eq. (2.14), which defines the operators $O_\pm^{(n)}$ in terms of weighted integrals of $O_\pm(\omega)$, does not hold for the renormalized operators; the integral over ω has an additional ultraviolet divergence not yet removed by the renormalization of $O_\pm(\omega)$. This divergence must be removed separately.

Consider, as an example, the simplest case $n=0$. Then the bare operators $O_\pm^{(0),\text{bare}} = Q^* \gamma_\pm q$ are local heavy-light current operators with dimension three, which are renormalized in the following way:

$$O_\pm^{(0),\text{bare}} = Z_0 O_\pm^{(0)} = \left(1 - \frac{1}{2\varepsilon}\Gamma_0 + \cdots\right) O_\pm^{(0)}, \tag{3.14}$$

where

$$\Gamma_0 = -\frac{3}{4} C_F \frac{\alpha_s}{\pi} \tag{3.15}$$

is the well-known one-loop hybrid anomalous dimension [34–36]. On the other hand, the bare operators can be expressed in terms of integrals over the renormalized operators $O_\pm(\omega)$ using Eq. (3.1). This gives

$$\begin{aligned} O_\pm^{(0),\text{bare}} &= \int d\omega d\omega' Z_\pm(\omega,\omega') O_\pm(\omega') \\ &= \left(\int d\omega Z_\pm(\omega,\omega') \right) \int d\omega O_\pm(\omega). \end{aligned} \tag{3.16}$$

As mentioned above, the last integral over the renormalized operators $O_\pm(\omega)$ is ultraviolet divergent from the region $\omega \to \infty$, and we define

$$\int d\omega O_\pm(\omega) = Z'_\pm O_\pm^{(0)} = \left(1 - \frac{1}{2\varepsilon}\Gamma'_\pm + \cdots\right) O_\pm^{(0)}. \tag{3.17}$$

Combining Eqs. (3.14), (3.16), and (3.17), we find that

$$\begin{aligned} Z_0 &= Z'_\pm \int d\omega Z_\pm(\omega,\omega'), \\ \Gamma_0 &= \Gamma'_\pm + \int d\omega \Gamma_\pm(\omega,\omega'). \end{aligned} \tag{3.18}$$

With our explicit expressions in Eq. (3.13), we obtain

$$Z'_\pm = 1 \pm C_F \frac{\alpha_s}{4\pi\varepsilon}, \quad \Gamma'_\pm = \mp C_F \frac{\alpha_s}{2\pi}. \tag{3.19}$$

To see in detail how this works, we calculate the ultraviolet divergences of the matrix elements $\langle 0 | O_\pm^{(0),\text{bare}} | \omega' \rangle$ to one-loop order, leaving the integration over k_+ until the end. Since now we are dealing with local operators, the only one-particle irreducible diagram is a vertex diagram analogous to that in Fig. 2(a). Its contribution is

$$2C_F\alpha_s\gamma_\pm u\mu^{2\varepsilon}\left\{\pm\int_{\omega'}^{\infty}d\omega\,\omega\int\frac{d^{2-2\varepsilon}k_\perp}{(2\pi)^{2-2\varepsilon}}\frac{1}{(\vec{k}_\perp^2+\omega^2-i0)[\vec{k}_\perp^2+\omega(\omega-\omega')-i0]}+\frac{1}{\omega'}\int_0^{\omega'}d\omega\,\omega\int\frac{d^{2-2\varepsilon}k_\perp}{(2\pi)^{2-2\varepsilon}}\frac{k_\pm}{(\vec{k}_\perp^2+\omega^2-i0)(\vec{k}_\perp^2-i0)}\right\},\quad(3.20)$$

where in the numerator of the second integral we have to substitute $k_-=\vec{k}_\perp^2/\omega$. This expression shows the origin of the two types of ultraviolet divergences. The $k_\perp$ integral in the first term is convergent, but there is a logarithmic divergence in the remaining integral over ω. In the second term, the ω integral is cut off in the ultraviolet; however, the $k_\perp$ integral diverges for the minus projection. Keeping only the poles in $1/\varepsilon$, we obtain

$$C_F\frac{\alpha_s}{2\pi}\gamma_\pm u\left\{\pm\omega'^{2\varepsilon}\int_{\omega'}^{\infty}d\omega\,\omega^{-1-2\varepsilon}+(1\mp1)\frac{1}{2\varepsilon}+\cdots\right\}=C_F\frac{\alpha_s}{4\pi\varepsilon}\gamma_\pm u.\quad(3.21)$$

Adding to this the contributions from the wave-function renormalization of the external lines, we recover the expressions for Z_0 and Γ_0 given in Eqs. (3.14) and (3.15). The contribution in Eq. (3.21) arising from the logarithmic divergence of the ω integral is removed by the factor $Z'_\pm$ defined in Eq. (3.17), whereas all other contributions are removed by the renormalization of the operators $O_\pm(\omega)$.

IV. QCD SUM RULES

The first moments of the wave functions $\varphi_\pm(\omega)$ are known exactly from the equations of motion, as shown in Eq. (2.19). However, they only set the scale of ω. In order to obtain information about the shape of the wave functions, we need to consider some of the higher moments. In this section, we use QCD sum rules to investigate the parameters λ_E^2 and λ_H^2, which according to Eq. (2.24) determine the second moments of $\varphi_\pm(\omega)$.

Let us consider the operators

$$O_E=Q^*i\vec{\alpha}\cdot\vec{E}q=v_\nu v^\alpha Q^*G^{\mu\nu}\sigma_{\mu\alpha}q,$$

$$O_H=-Q^*\vec{\sigma}\cdot\vec{H}q=(\tfrac{1}{2}g_\nu^\alpha-v_\nu v^\alpha)Q^*G^{\mu\nu}\sigma_{\mu\alpha}q,\quad(4.1)$$

whose matrix elements give $f\lambda_E^2$ and $f\lambda_H^2$, respectively. To obtain the sum rule for λ_E^2 (the sum rule for λ_H^2 is obtained in a similar way), we investigate the correlator

$$\langle 0|T\{O_E(x),\overline{q}\,\tfrac{1}{2}(1+\not{v})Q(0)\}|0\rangle=\frac{1+\not{v}}{2}\theta(v\cdot x)\delta(\vec{x}_\perp)\Pi_E(v\cdot x),\quad(4.2)$$

where $\vec{x}_\perp$ contains the components of x orthogonal to v. The operator $\overline{q}\,\frac{1}{2}(1+\not{v})Q$ has the quantum numbers of the ground-state meson. Next, we analytically continue $\Pi_E(v\cdot x)$ to $v\cdot x=-i\tau$.[6] The correlator can be expressed via its spectral density $\rho_E(\omega)$:

$$\Pi_E(\tau)=\int_0^\infty \mathrm{d}\varepsilon\,\rho_E(\varepsilon)e^{-\varepsilon\tau},\quad(4.3)$$

which contains the contribution $f^2\lambda_E^2\delta(\varepsilon-\overline{\Lambda})$ from the ground-state meson, as well as contributions from excited states, which we will refer to as the continuum. Hence, the phenomenological expression for the correlator is

$$\Pi_E(\tau)=f^2\lambda_E^2e^{-\overline{\Lambda}\tau}+\Pi_E^{\mathrm{cont}}(\tau).\quad(4.4)$$

For sufficiently small values of τ, the correlator can be calculated in QCD using the operator product expansion [53]. The theoretical spectral density, $\rho_E^{\mathrm{th}}(\varepsilon)$, contains perturbative as well as nonperturbative contributions, where the latter are proportional to vacuum condensates of local, gauge-invariant operators. The continuum contributions in Eq. (4.4) are usually modeled by the theoretical spectral density above a threshold ε_c, which is called the continuum threshold. Equating the two expressions for $\Pi_E(\tau)$ obtained in this way, we derive the sum rule

$$f^2\lambda_E^2e^{-\overline{\Lambda}\tau}=\int_0^{\varepsilon_c}d\varepsilon\,\rho_E^{\mathrm{th}}(\varepsilon)e^{-\varepsilon\tau}.\quad(4.5)$$

A similar sum rule holds for the product $f^2\lambda_H^2$.

The leading contributions in the operator product expansion of the correlators are shown in Fig. 4. We include the leading perturbative contribution, as well as the contributions of the quark condensate $\langle\overline{q}q\rangle$ (dimension $d=3$), the gluon condensate $\langle G^2\rangle=\langle G_{a\mu\nu}G_a^{\mu\nu}\rangle$ $(d=4)$, and the mixed quark-gluon condensate $\langle\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle\equiv m_0^2\langle\overline{q}q\rangle$ $(d=5)$. In the calculation of the nonperturbative contributions, it is convenient to use the fixed-point gauge $x_\mu A^\mu(x)=0$; then the heavy quark does not interact with gluons. After a straightforward calculation, we obtain the sum rules

[6]The procedure of analytic continuation of the coordinate-space correlator to imaginary time is equivalent to the Borel transformation of the corresponding momentum-space correlator. Thus, our sum rules coincide with the usual Laplace sum rules.

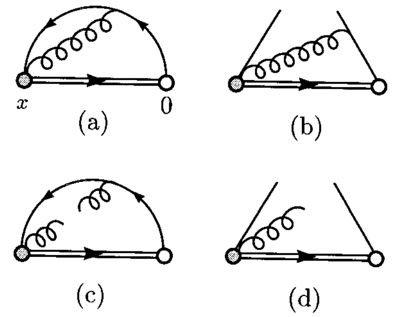

FIG. 4. Nonvanishing diagrams for the correlators Π_E and Π_H. The higher-dimensional current operators O_E and O_H are represented by a gray circle; the interpolating current is represented by a white circle.

$$f^2\lambda_E^2 e^{-\bar{\Lambda}\tau} = -N_c C_F \frac{\alpha_s}{\pi^3\tau^5}\,\delta_4(\varepsilon_c\tau) - \frac{m_0^2\langle\bar{q}q\rangle}{16},$$

$$f^2\lambda_H^2 e^{-\bar{\Lambda}\tau} = -N_c C_F \frac{\alpha_s}{2\pi^3\tau^5}\,\delta_4(\varepsilon_c\tau) - C_F\frac{3\alpha_s}{4\pi\tau^2}\,\langle\bar{q}q\rangle\,\delta_1(\varepsilon_c\tau) + \frac{\alpha_s\langle G^2\rangle}{16\pi\tau}\delta_0(\varepsilon_c\tau) - \frac{m_0^2\langle\bar{q}q\rangle}{16}, \qquad (4.6)$$

where

$$\delta_n(x) = \theta(x)\left(1 - e^{-x}\sum_{m=0}^{n}\frac{x^m}{m!}\right). \qquad (4.7)$$

The fact that the sum rule for λ_E^2 does not contain contributions from the quark and gluon condensates is a consequence of the fact that, in the fixed-point gauge, the light quark interacts only with the magnetic components of the gluon field [54]. In Appendix B, we also list the contributions of higher-dimensional quark-gluon condensates ($d=7$) to the sum rules. For the range of τ values considered below, we expect that the contributions of these condensates are very small. Since their values are moreover unknown, they will be neglected in the numerical analysis.

It is convenient to divide the sum rules in Eq. (4.6) by the sum rule for the meson decay constant derived in [43,18,20]:

$$f^2 e^{-\bar{\Lambda}\tau} = \frac{N_c}{2\pi^2\tau^3}\delta_2(\varepsilon_c\tau) - \frac{\langle\bar{q}q\rangle}{4}\left(1 - \frac{m_0^2\tau^2}{16}\right). \qquad (4.8)$$

This procedure leads to expressions for λ_E^2 and λ_H^2 as a function of τ, the continuum threshold ε_c, and the vacuum condensates. For our analysis we use the standard values [53]

$$\langle\bar{q}q\rangle = -(0.23\ \text{GeV})^3,$$

$$\alpha_s\langle G^2\rangle = 0.04\ \text{GeV}^4,$$

$$m_0^2 = 0.8\ \text{GeV}^2, \qquad (4.9)$$

as well as $\alpha_s = 0.4$. The value of the continuum threshold extracted from the analysis of the sum rule (4.8) is $\varepsilon_c = 1.00 \pm 0.15$ GeV [18,20]. Moreover, stability of this sum rule requires that the parameter τ be in the range $0.3\ \text{GeV} < 1/\tau < 0.5\ \text{GeV}$, which is called the stability window. Our numerical results for λ_E^2 and λ_H^2 as a function of $1/\tau$ are shown in Fig. 5. We observe a sizable dependence of the results on the parameter τ, as it is not unexpected for the matrix elements of higher-dimensional local operators such as O_E and O_H. Nevertheless, taking an average over the stability window determined from the sum rule (4.8), we obtain as a rough estimate:

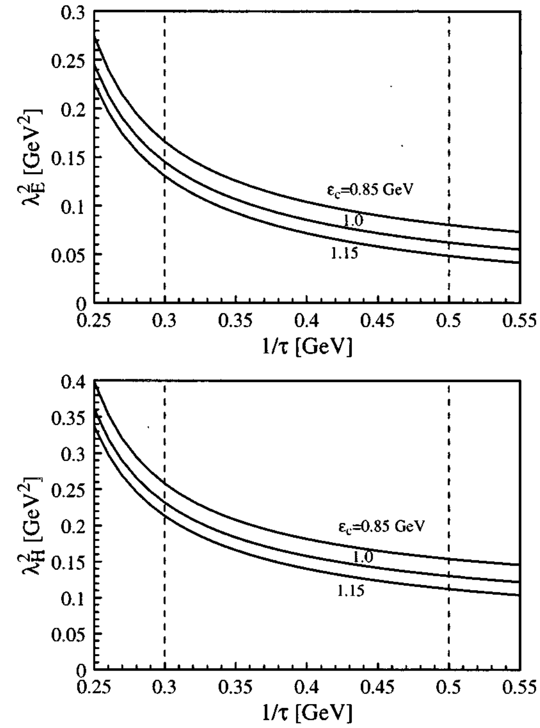

FIG. 5. Sum-rule results for λ_E^2 (upper plot) and λ_H^2 (lower plot) as a function of $1/\tau$, for three values of the continuum threshold ε_c. The stability window lies in between the dashed lines.

$$\lambda_E^2 = (0.11 \pm 0.06)\ \text{GeV}^2, \quad \lambda_H^2 = (0.18 \pm 0.07)\ \text{GeV}^2. \qquad (4.10)$$

In units of $\bar{\Lambda} \approx 0.55$ GeV [40,49], this implies $\lambda_E^2/\bar{\Lambda}^2 = 0.36 \pm 0.20$ and $\lambda_H^2/\bar{\Lambda}^2 = 0.60 \pm 0.23$.

According to Eq. (2.24), the parameters λ_E^2 and λ_H^2 determined, together with $\bar{\Lambda}^2$, the second moments of the meson wave functions. In order to learn more about the shape of the wave functions, we shall now go a step further and construct sum rules for the functions $\varphi_\pm(\omega)$ themselves. To this end, we start from the correlator[7]

[7] Expanding this correlator in powers of t, one recovers, according to Eqs. (2.13) and (2.15), the sum rules for the moments $f^2\langle\omega^n\rangle_\pm$. To lowest order, this gives the sum rule for the decay constant f considered in [42,18,20]. The sum rule for $f^2\langle\omega\rangle_+$ has been considered in [55]; however, the mixed-condensate contribution had the wrong sign. Note that, because of Eq. (2.19), the sum rules for $f^2\langle\omega\rangle_\pm$ can be obtained by taking derivatives with respect to τ in the sum rule for f^2. Taking linear combinations of the sum rules for $f^2\langle\omega^2\rangle_\pm$, we recover the sum rules for λ_E^2 and λ_H^2.

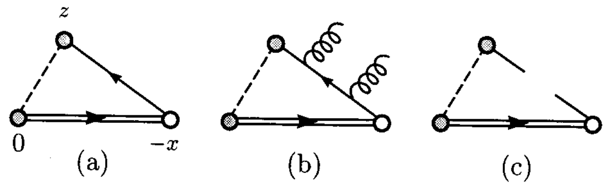

FIG. 6. Leading diagrams for the correlators of the bilocal operators $\widetilde{O}_{\pm}(t)$ (gray circles) with the local current $\overline{q}Q$ (white circle).

$$\langle 0|T\{\widetilde{O}_{\pm}(t),\overline{q}\tfrac{1}{2}(1+\not{v})Q(-x)\}|0\rangle$$
$$=\gamma_{\pm}\frac{1+\not{v}}{2}\theta(v\cdot x)\delta(\vec{x}_{\perp})\widetilde{\Pi}_{\pm}(v\cdot x,t), \qquad (4.11)$$

where $\widetilde{O}_{\pm}(t)$ has been defined in Eq. (2.7), and analytically continue $\widetilde{\Pi}_{\pm}(v\cdot x,t)$ to $v\cdot x=-i\tau$. Using the light-quark propagator in coordinate space [54], it is straightforward to obtain the contribution of the diagrams in Figs. 6(a) and 6(b). The result is

$$\widetilde{\Pi}_{+}^{(1)}(\tau,t)=\frac{N_c}{2\pi^2\tau(\tau+2it)^2}\left(1-\frac{\pi\alpha_s\langle G^2\rangle\tau^2t^2}{48N_c}\right),$$
$$\widetilde{\Pi}_{-}^{(1)}(\tau,t)=\frac{N_c}{2\pi^2\tau^2(\tau+2it)}\left(1-\frac{\pi\alpha_s\langle G^2\rangle\tau^2t^2}{48N_c}\right). \qquad (4.12)$$

The contribution with the cut light-quark line shown in Fig. 6(c) involves the trilocal, non-collinear quark condensate [56]. This object is discussed in detail in Appendix B. Keeping for simplicity only the leading term in the operator product expansion, we obtain

$$\widetilde{\Pi}_{\pm}^{(2)}(\tau,t)=-\tfrac{1}{4}\langle\overline{q}q\rangle\int d\nu\widetilde{f}_S(\nu)e^{-\nu\tau(\tau+2it)}, \qquad (4.13)$$

where $\widetilde{f}_S(\nu)$ describes the distribution of quarks with virtuality ν in the vacuum. Unfortunately, little is known about the shape of this function. A simple ansatz is discussed in Appendix B.

Next we perform the Fourier transform of the correlator in Eq. (4.11) with respect to t, which leads to

$$\langle 0|T\{O_{\pm}(\omega),\overline{q}\tfrac{1}{2}(1+\not{v})Q(-x)\}|0\rangle$$
$$=\gamma_{\pm}\frac{1+\not{v}}{2}\theta(v\cdot x)\delta(\vec{x}_{\perp})\Pi_{\pm}(v\cdot x,\omega). \qquad (4.14)$$

The leading perturbative terms in Eq. (4.12) give

$$\Pi_{+}^{(1)}(\tau,\omega)=\frac{N_c}{8\pi^2\tau}\omega e^{-\omega\tau/2},$$
$$\Pi_{-}^{(1)}(\tau,\omega)=\frac{N_c}{4\pi^2\tau^2}e^{-\omega\tau/2}. \qquad (4.15)$$

The contributions proportional to the gluon condensate lead to singular behavior of the form $\delta(\omega)$ and $\delta'(\omega)$. These terms would acquire a finite width if the nonlocality of the gluon condensate would be taken into account [57]. Below, we shall neglect the gluon condensate. The Fourier transform of the quark-condensate contribution in Eq. (4.13) is given by

$$\Pi_{\pm}^{(2)}(\tau,\omega)=-\frac{\langle\overline{q}q\rangle}{8\tau}\widetilde{f}_S\left(\frac{\omega}{2\tau}\right)e^{-\omega\tau/2}, \qquad (4.16)$$

i.e., it is directly determined by the virtuality distribution of quarks in the vacuum.

In order to perform the continuum subtraction for the perturbative contributions, we calculate the inverse Laplace transforms of the expressions in Eq. (4.15), which give the corresponding spectral densities. We finally arrive at the sum rules

$$f^2\varphi_{+}(\omega)e^{-\overline{\Lambda}\tau}=\frac{N_c}{8\pi^2\tau}\omega e^{-\omega\tau/2}\delta_0\left[\left(\varepsilon_c-\frac{\omega}{2}\right)\tau\right]-\frac{\langle\overline{q}q\rangle}{8\tau}\widetilde{f}_S\left(\frac{\omega}{2\tau}\right)e^{-\omega\tau/2},$$
$$f^2\varphi_{-}(\omega)e^{-\overline{\Lambda}\tau}=\frac{N_c}{4\pi^2\tau^2}e^{-\omega\tau/2}\delta_1\left[\left(\varepsilon_c-\frac{\omega}{2}\right)\tau\right]-\frac{\langle\overline{q}q\rangle}{8\tau}\widetilde{f}_S\left(\frac{\omega}{2\tau}\right)e^{-\omega\tau/2}. \qquad (4.17)$$

The contributions from the nonlocal quark condensate fall off quickly for both $\omega\to 0$ and $\omega\to\infty$. The first statement follows from a general property of the function $\widetilde{f}_S$ (see Appendix B). However, the precise functional form of these contributions is unknown. The leading perturbative contributions vanish for $\omega>2\varepsilon_c$. They suggest the model shapes

$$\varphi_{+}(\omega)=\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}, \quad \varphi_{-}(\omega)=\frac{1}{\omega_0}e^{-\omega/\omega_0}, \qquad (4.18)$$

which do indeed exhibit the correct behavior for $\omega\to 0$ [cf. Eq. (2.33)]. These functions are shown in Fig. 7. The parameter $\omega_0=\frac{2}{3}\overline{\Lambda}$ is fixed by Eq. (2.19). In this simple model, we obtain for the second moments $\langle\omega^2\rangle_{+}=3\langle\omega^2\rangle_{-}=\frac{8}{3}\overline{\Lambda}^2$. This corresponds to $\lambda_E^2=\lambda_H^2=\frac{2}{3}\overline{\Lambda}^2$, which does not contradict our sum-rule estimates obtained earlier in this section.

V. ASYMPTOTICS OF FORM FACTORS

We shall now use the results obtained in the previous sections to analyze, in a model-independent way, the asymptotic behavior at large recoil of the form factors describing the current matrix elements between two heavy mesons. The contribution of the quark-antiquark wave functions to the Isgur-Wise form factor is depicted by the diagrams in Figs. 8(a) and 8(b). To deal with two heavy mesons moving at different velocities v and v', it is convenient to choose the Breit frame, in which the two mesons move in opposite directions with rapidities $\pm\vartheta/2$, so that

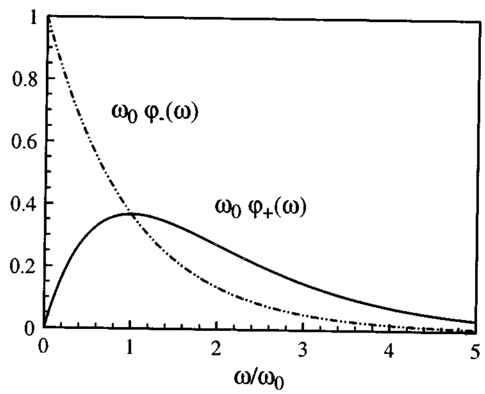

FIG. 7. Model wave functions $\varphi_\pm(\omega)$ defined in Eq. (4.18).

$$v^\mu=\left(\cosh\frac{\vartheta}{2},0,0,\sinh\frac{\vartheta}{2}\right),$$

$$v'^\mu=\left(\cosh\frac{\vartheta}{2},0,0,-\sinh\frac{\vartheta}{2}\right), \tag{5.1}$$

and $v\cdot v'=\cosh\vartheta$. In terms of the light-cone vectors $n_\pm^\mu$, we have

$$v^\mu+v'^\mu=\cosh\frac{\vartheta}{2}\,(n_+^\mu+n_-^\mu),$$

$$v^\mu-v'^\mu=-\sinh\frac{\vartheta}{2}\,(n_+^\mu-n_-^\mu). \tag{5.2}$$

It follows that $v_+=e^{\vartheta/2}$ and $v_-=e^{-\vartheta/2}$, but $v'_-=e^{\vartheta/2}$ and $v'_+=e^{-\vartheta/2}$. Similarly, in the large-recoil limit ($\vartheta\gg 0$) the light quark in the initial meson has large $p_+=\omega e^{\vartheta/2}$ and small p_-, whereas the light quark in the final meson has large $p'_-=\omega' e^{\vartheta/2}$ and small p'_+. Thus, the roles of the plus/minus directions for the final meson are opposite to those for the initial meson. The virtuality of the gluon and of the heavy quark are both large: $k_g^2=(p'-p)^2\simeq-\omega\omega' e^{\vartheta}$, and $v\cdot k_Q\simeq v\cdot p'\simeq\frac{1}{2}\omega' e^{\vartheta}$ [Fig. 8(a)] or $v'\cdot k_Q\simeq v'\cdot p\simeq\frac{1}{2}\omega e^{\vartheta}$ [Fig. 8(b)], respectively.

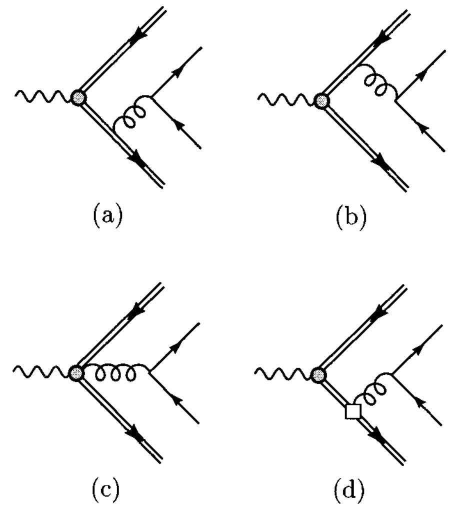

FIG. 8. Hard-gluon exchange contributions to heavy-meson form factors. The external current is represented by the wave line attached to the gray circle. The white square in (d) represents an insertion of $1/m_Q$-suppressed operators from the effective Lagrangian of the HQET.

For large ϑ, the contribution of the diagram in Fig. 8(a) to the matrix element in Eq. (1.4), which defines the Isgur-Wise function, is

$$2\pi\alpha_s\frac{C_F}{N_c}f^2e^{-2\vartheta}\int\frac{d\omega d\omega'}{\omega\omega'^2}\overline{u(v')}[\varphi_+(\omega')\gamma_+e^{\vartheta/2}+\varphi_-(\omega')\gamma_-e^{-\vartheta/2}]\not{v}[\varphi_+(\omega)\gamma_-e^{\vartheta/2}+\varphi_-(\omega)\gamma_+e^{-\vartheta/2}]u(v), \tag{5.3}$$

where $\not{v}=\frac{1}{2}(\gamma_-e^{\vartheta/2}+\gamma_+e^{-\vartheta/2})$. The factor $1/N_c$ arises from the normalization of the color wave functions of the meson states, which is such that the matrix element in Eq. (1.1) is normalized to the decay constant f. Using the relations in Eq. (2.8) together with Eq. (5.2), we find that the leading contribution contains the product $\varphi_+(\omega')\varphi_-(\omega)$, i.e., the subleading-twist wave function is taken on the side where the gluon exchange occurs. Adding the contribution of the diagram in Fig. 8(b), we arrive at

$$\xi(\cosh\vartheta)=16\pi\alpha_s\frac{C_F}{N_c}f^2\langle\omega^{-2}\rangle_+\langle\omega^{-1}\rangle_-e^{-2\vartheta}. \tag{5.4}$$

Based on our assumptions about the behavior of the wave functions for $\omega\to 0$ [cf. Eq. (2.33)], we expect that both $\langle\omega^{-2}\rangle_+$ and $\langle\omega^{-1}\rangle_-$ diverge logarithmically at low ω. This divergence is cut off by the transverse momenta and virtualities of the light quarks in the mesons, similar to the case of the π-ρ form factor in QCD [29]. This infrared sensitivity results in an additional enhancement of the form factor, as can be seen by replacing q^2 by $q^2-\Lambda^2$ in the gluon propagator (where Λ is of the order of a typical hadronic scale), which leads to the replacement:

$$\langle\omega^{-2}\rangle_+\langle\omega^{-1}\rangle_-\to\int\frac{d\omega d\omega'}{\omega\omega'+\Lambda^2e^{-\vartheta}}\,\varphi'_+(\omega)\varphi_-(\omega')$$
$$=\varphi'_+(0)\varphi_-(0)\frac{\vartheta^2}{2}+O(\vartheta). \tag{5.5}$$

Unfortunately, however, the subleading terms of order ϑ cannot be calculated without knowing details of the infrared cutoff.

Consider now the form factors ξ_i and χ_i defined in Eqs. (1.5)–(1.9), which appear at order $1/m_Q$ in the heavy-quark expansion. Since their contributions to the meson form factors are suppressed by a power of Λ/m_Q with respect to the contribution of the Isgur-Wise function, they can only become important if they have a slower falloff at large ϑ. Therefore, it is sufficient to retain only those contributions where the leading-twist wave function appears on both sides of the diagrams. Then the light-quark helicity is conserved, and the gluon polarization is orthogonal to the v-v' plane. Let us first focus on the functions ξ_i. The only way to get the leading-twist wave function on both sides of the diagram is to attach the gluon to the current operator, as shown by the diagram in Fig. 8(c). Simplifying the resulting spinor product using Eqs. (2.8) and (5.2), we obtain

$$4\pi\alpha_s \frac{C_F}{N_c} f^2 e^{-\vartheta}\overline{u(v')}\gamma^\mu u(v)\int \frac{d\omega d\omega'}{\omega\omega'}\varphi_+(\omega)\varphi_+(\omega'), \tag{5.6}$$

which means that only the function ξ_3 receives a leading-twist contribution. It is given by

$$\xi_3(\cosh\vartheta) = 4\pi\alpha_s \frac{C_F}{N_c} f^2 \langle \omega^{-1}\rangle_+^2 e^{-\vartheta}. \tag{5.7}$$

Here the integral is infrared convergent, as in the case of the pion from factor in QCD [25–31]. Next consider the functions χ_i. The conservation of the light-quark helicity implies that there must be an odd number of γ matrices in the matrix element. Indeed, only the function χ_2 in Eq. (1.9) receives a leading contribution. Calculating the diagram in Fig. 8(d), we find

$$-8\pi\alpha_s \frac{C_F}{N_c} f^2 e^{-2\vartheta}\int \frac{d\omega d\omega'}{\omega\omega'^2}\varphi_+(\omega)\varphi_+(\omega')$$
$$\times\overline{u(v')}(\gamma^\mu k_g^\nu - \gamma^\nu k_g^\mu)u(v), \tag{5.8}$$

where $k_g = \omega' v' - \omega v$ is the gluon momentum. Since, by definition, the indices μ and ν are restricted to the subspace orthogonal to v, only the first term in k_g has to be kept. Taking into account the definition of χ_2 in Eqs. (1.8) and (1.9), we obtain

$$\chi_2(\cosh\vartheta) = -\xi_3(\cosh\vartheta)e^{-\vartheta}. \tag{5.9}$$

Note that in the expressions for the meson form factors χ_2 is multiplied by $\cosh\vartheta$ [9,16], so that its contributions are of the same order as the contributions of ξ_3.

We can compare our asymptotic results for the leading and subleading Isgur-Wise functions in Eqs. (5.4), (5.7) and (5.9) with the large-ϑ limit of the two-loop QCD sum-rule expressions for these functions, which have been obtained in [21,24]. We find that the results of the sum-rule calculations do indeed reproduce the correct asymptotic behavior; in particular, the relation (5.9) between χ_2 and ξ_3 is satisfied. Moreover, the sum rules allow us to determine the normalization factors appearing in the expressions for ξ in Eq. (5.4) and for ξ_3 in Eq. (5.7). This is explained in detail in Appendix C. For later convenience, we also present the expressions obtained using the model wave functions (4.18). They are

$$\xi(v\cdot v') \approx 3\pi\alpha_s \frac{f^2}{\overline{\Lambda}^3}\frac{\ln^2(v\cdot v')}{(v\cdot v')^2},$$

$$\xi_3(v\cdot v') \approx 2\pi\alpha_s \frac{f^2}{\overline{\Lambda}^2}\frac{1}{v\cdot v'}. \tag{5.10}$$

Let us discuss the applicability regions for these asymptotic results. QCD sum rules suggest that there are ‘‘soft’’ contributions to the Isgur-Wise function which fall of like $1/(v\cdot v')^2$ [17–20]. If this is correct, the asymptotic behavior given by Eqs. (5.4) and (5.5) would dominate only if $\ln(v\cdot v')\gg 1$ and $\alpha_s\ln^2(v\cdot v')\gg 1$. If the ‘‘soft’’ contributions vanish faster than $1/(v\cdot v')^2$, the second requirement is removed. The fact that $\ln(v\cdot v')$ is, in most practical applications, not a large parameter implies that the asymptotic result for the Isgur-Wise function may be considered as a rough estimate only. To reach the asymptotic regime would require $v\cdot v' = O(100)$. QCD sum rules also suggest that the ‘‘soft’’ contributions to ξ_3 fall off like $1/(v\cdot v')^2$ [22–24], meaning that the leading hard contribution given in Eq. (5.7) is enhanced by a power of $v\cdot v'$. As a consequence, our predictions for the functions ξ_3 and χ_2 are much more accurate than for the Isgur-Wise function. The asymptotic behavior should set in when $\alpha_s v\cdot v' \gg 1$, which requires $v\cdot v' \sim O(10)$.

An important aspect of physics is still missing from our discussion of the asymptotic behavior of meson form factors at large recoil. Since the quarks receive a large acceleration during the transition process, they emit gluon bremsstrahlung, which leads to an additional damping of the transition amplitudes (Sudakov form factor). Because the mesons are colorless, the double logarithms of the type $[\alpha_s\ln^2(v\cdot v')]^n$ cancel in the expressions for the meson form factors; however, single logarithms of $v\cdot v'$ remain, which are enhanced by logarithms of the heavy-quark mass. They arise from the emission of gluon bremsstrahlung with energies in the range $\mu < E_g < m_Q$ ($\mu \ll m_Q$, see below). Thus, in perturbation theory there are large double-logarithmic contributions of the type $[\alpha_s\ln(m_Q/\mu)\ln(v\cdot v')]^n$ to the form factors. The situation is similar to the case of the contributions to the pion form factor coming from the region $x\to 0$, where almost all of the pion momentum is carried by one quark [58].

Because of the explicit dependence on the heavy-quark mass, these large logarithms are not contained in the form factors of the HQET, which are renormalized at a scale $\mu \ll m_Q$.[8] However, they appear when we relate the form factors of the HQET to physical meson form factors using a perturbative matching procedure. In this relation, there appear short-distance coefficient functions $C_n(m_Q/\mu, v\cdot v')$, which can be calculated in renormalization-group improved perturbation theory (see [9] for a review). In the case of transitions between two heavy hadrons, the reparametrization invariance [59] of the HQET ensures that the coefficients multiplying the subleading functions χ_2 and ξ_3 are the same

[8]In practice, the scale μ should be chosen such that there are no large logarithms contained in the form factors of the HQET, but yet large enough for perturbation theory to be valid.

as the coefficients multiplying the leading-order Isgur-Wise function ξ [60]. Indeed, in leading logarithmic approximation (which is sufficient to control the large logarithms mentioned above), all HQET form factors are multiplied by a universal coefficient

$$C(m_Q/\mu, v\cdot v') = \left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right)^{a(v\cdot v')}, \tag{5.11}$$

where ($v\cdot v' = \cosh\vartheta$) [15,61,62]

$$a(v\cdot v') = \frac{2C_F}{\beta_0}(\vartheta\coth\vartheta - 1) = \frac{2C_F}{\beta_0}[\vartheta - 1 + O(e^{-\vartheta})]. \tag{5.12}$$

For large recoil, we find that

$$C \to \left(\frac{e}{2v\cdot v'}\right)^{\eta} = e^{-\eta(\vartheta-1)}, \tag{5.13}$$

where

$$\eta = \frac{2C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}. \tag{5.14}$$

This expression sums the large Sudakov logarithms correctly to all orders in perturbation theory. The effect of this bremsstrahlung correction is an additional powerlike suppression of the physical meson form factors for large values of $v\cdot v'$. Using $\mu \approx 1$ GeV, we find that for B-meson decays the power η is given by $\eta \approx 0.2$–0.3, i.e., the overall effect of bremsstrahlung emission is rather small.

Using the results obtained in this section, it is straightforward to derive the asymptotic behavior of all form factors describing current-induced transitions between any two pseudoscalar or vector mesons containing a heavy quark. The relevant formulas, which relate the meson form factors to the Isgur-Wise functions, can be found, e.g., in [9,16,52]. Here we restrict ourselves to the results obtained for the matrix elements of the vector current $V^\mu = \bar{b}\gamma^\mu b$ between B-meson states. We find[9]

$$\begin{aligned}
\langle B(v')|V^\mu|B(v)\rangle &= h_+(v+v')^\mu,\\
\langle B^*(e',v')|V^\mu|B(v)\rangle &= h_V \epsilon^{\mu\nu\alpha\beta} e'^*_\nu v'_\alpha v_\beta,\\
\langle B^*_L(v')|V^\mu|B^*_L(v)\rangle &= h_L(v+v')^\mu,\\
\langle B^*_T(e',v')|V^\mu|B^*_T(e,v)\rangle &= -h_T\, e\cdot e'^*(v+v')^\mu,\\
\langle B^*_T(e',v')|V^\mu|B^*_L(v)\rangle &= \sinh\vartheta\, h_{TL} e'^{*\mu},
\end{aligned} \tag{5.15}$$

where L and T refer to longitudinal (i.e., in the v-v' plane) and transverse (i.e., orthogonal to that plane) polarization states. We find that, asymptotically,

$$\begin{aligned}
h_+ &= C\left(\xi - \frac{4}{m_b}\cosh\vartheta\,\chi_2\right) = C\left(\xi + \frac{2}{m_b}\xi_3\right),\\
h_V &= C\left[\xi - \frac{1}{m_b}(\xi_3 + 2\cosh\vartheta\,\chi_2)\right] = C\xi,\\
h_L &= C\left(\xi + \frac{4}{m_b}\cosh\vartheta\,\chi_2\right) = C\left(\xi - \frac{2}{m_b}\xi_3\right),\\
h_T &= C\xi,\\
h_{TL} &= C\left[\xi + \frac{1}{m_b}(\xi_3 + 2\cosh\vartheta\,\chi_2)\right] = C\xi,
\end{aligned} \tag{5.16}$$

with C given in Eq. (5.13). The most striking feature of these results is the fact that the form factor for two longitudinally polarized B^* mesons, which is positive for $\cosh\vartheta \ll m_b/\Lambda$, becomes negative for $\cosh\vartheta \gg m_b/\Lambda$, and hence has a zero at some intermediate value $\cosh\vartheta \sim m_b/\Lambda$. Since for spacelike (negative) values of q^2, corresponding to $v\cdot v' > 1$, all form factors are real, the existence of this zero is an exact statement not affected by subleading corrections. We should stress that this observation is not specific to heavy-light mesons. The form factor of, say, longitudinally polarized ρ mesons also has a zero at some negative value of q^2 [29]. For timelike (positive) values of q^2, on the other hand, it is the form factor of pseudoscalar B mesons which has a zero, and this zero is situated inside the physical region of the production of $B\bar{B}$ pairs in e^+e^- collisions. Strictly speaking, because form factors at timelike values of q^2, corresponding to $v\cdot v' < -1$, are complex, this zero is not absolutely exact. However, in our approximation the imaginary part is negligible.[10]

The model-independent results obtained in this section can be checked in the simple model where a meson is built out of two heavy quarks with masses m and μ such that $m \gg \mu \gg \Lambda$. This is discussed in detail in Appendix D. The same model has been considered by Brodsky and Ji [45], who observed for the first time the zero of the pseudoscalar form factor in the physical region of large positive q^2. However, their claim that the form factor of longitudinally polarized vector mesons would have the same behavior is incorrect. Our analysis shows that this form factor has a zero in the region of spacelike momentum transfer, i.e., for large negative q^2.

VI. APPLICATIONS

We finally apply our results to calculate the cross section for the reaction $e^+e^- \to B^{(*)}\bar{B}^{(*)}$ in the region $s \gg 4m_B^2$. Recall from the introduction that the part of the electromagnetic current that couples to light quarks does not give a leading contribution in the asymptotic regime. Hence, it is justified to use the relations in Eq. (5.16) for the relevant form factors. As usual, we define

[9]To obtain the conventional relativistic normalization of meson states, the right-hand sides in these equations have to be multiplied by $m_{B^{(*)}}$.

[10]Note also that any mechanism that leads to a common phase factor of the form factors, such as final-state interactions, does not spoil the existence of the zero.

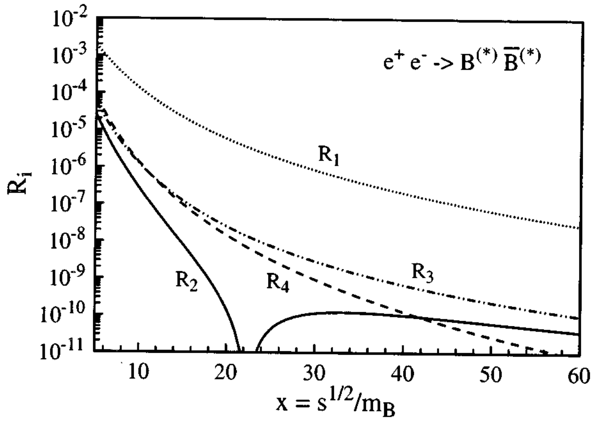

FIG. 9. Cross-section ratios R_i for $B\bar{B}$ pair production in e^+e^- collisions at large x. We use the form factors in Eq. (6.3) with $\Lambda=550$ MeV and $\alpha_s f^2/\Lambda^3=0.06$.

$$R_X=\frac{\sigma(e^+e^-\to X)}{\sigma(e^+e^-\to\mu^+\mu^-)}. \tag{6.1}$$

Using crossing symmetry and the matrix elements given in Eq. (5.15), we obtain for $x=\sqrt{s}/m_B\gg 1$:

$$R_1\equiv R_{BB^*+B_T^*B_L^*}=\frac{z_b^2}{2}x^2|C|^2|\xi|^2,$$

$$R_2\equiv R_{BB}=\frac{z_b^2}{4}|C|^2\left|\xi+\frac{2}{m_b}\xi_3\right|^2,$$

$$R_3\equiv R_{B_L^*B_L^*}=\frac{z_b^2}{4}|C|^2\left|\xi-\frac{2}{m_b}\xi_3\right|^2,$$

$$R_4\equiv R_{B_T^*B_T^*}=\frac{z_b^2}{2}|C|^2|\xi|^2, \tag{6.2}$$

where the form factors are functions of $v\cdot v'\simeq-\frac{1}{2}x^2$, and $|C|=(e/x^2)^{-\eta}$ with η given in Eq. (5.14). Here $z_b=-\frac{1}{3}$ is the electric charge of the b quark. As an illustration, we show in Fig. 9 our predictions for the various $B^{(*)}\overline{B^{(*)}}$ production cross-sections as a function of x obtained using the model wave functions in Eq. (4.18), for which [cf. Eq. (5.10)]

$$\xi(x)\approx 48\pi\frac{\alpha_s f^2}{\Lambda^3}\frac{\ln^2 x}{x^4},$$

$$\xi_3(x)\approx -4\pi\frac{\alpha_s f^2}{\Lambda^2}\frac{1}{x^2}. \tag{6.3}$$

For simplicity, we have neglected bremsstrahlung effects setting $C=1$ in Eq. (6.2). Because of the kinematic enhancement factor x^2, the ratio R_1 generally dominates at large x. This means that mostly BB^* and $B_T^*B_L^*$ pairs are produced, with $R\sim 1/s^3$ and angular distribution $1+\cos^2\theta$ [63,64]. More interesting from the point of view of the present work are the three other ratios. The BB production cross section vanishes at some value $x_0\sim\sqrt{m_b/\Lambda}$, since $\xi\sim\ln^2x/x^4$ and $\xi_3\sim-\Lambda/x^2$. In our simple model, we find that

$$x_0\approx\sqrt{\frac{6m_b}{\Lambda}}\ln x_0\approx 25. \tag{6.4}$$

We stress again that the accuracy of this prediction is not high, because of our poor knowledge of the asymptotic behavior of the Isgur-Wise function. (In particular, if there is a soft contribution $\sim 1/x^4$ to ξ that still dominates at such large values of x^-, then the turnover point is delayed until $x_0^{-1}\sim\sqrt{m_b/\Lambda\alpha_s}$.) For $1\ll x\ll x_0$, the ratios R_2, R_3, and R_4 are all of the same magnitude and scale like $1/x^8$. This situation had been studied previously in the context of the HQET [63,64]. For $x\gg x_0$, however, another pattern sets in. Then the contribution of ξ_3 to the ratios R_2 and R_3 dominates over the contribution from the Isgur-Wise function ξ, so that R_2 and R_3 scale like $1/x^4$ and dominate over R_4. In principle, at very large x the ratios R_2 and R_3 should even dominate over R_1, which scales like $1/x^6$. However, because of the double-logarithmic enhancement of the Isgur-Wise function this would require enormous values $x>x_1$, where in our model x_1 is given by

$$x_1\approx\frac{6m_b}{\Lambda}\ln^2 x_1\approx 3500. \tag{6.5}$$

In the ultra-asymptotic region $x\gg x_1$, mostly BB and $B_L^*B_L^*$ pairs would be produced, with $R\sim 1/s^2$ and angular distribution $\sin^2\vartheta$.

These qualitative features, which are independent of the particular choice adopted for the meson wave functions, are clearly exhibited in Fig. 9. Unfortunately, however, the cross sections for $B^{(*)}\overline{B^{(*)}}$ production at large x are so small that they will most likely be irrelevant to experiments. The situation is somewhat more favorable in the case of the pair production of charm mesons. We can apply our results to this case by performing obvious substitutions ($m_b\to m_c$, $z_b\to z_c=\frac{2}{3}$, etc.) in the above formulas. We then find that $x_0\approx 8$ in the case of charm pair production, corresponding to moderate energies of order 15 GeV. The second turnover point is, however, still too high ($x_1\approx 650$) to be of any interest. The resulting cross sections are shown in Fig. 10.

In summary, we have applied methods developed for hard exclusive QCD processes to calculate the asymptotic behavior of heavy-meson form factors at large recoil. We find that this behavior is determined by the leading- and subleading-twist meson wave functions. For $1\ll|v\cdot v'|\ll m_Q/\Lambda$, the form factors are dominated by the Isgur-Wise function ξ, which is determined by the interference between the wave functions of leading and subleading twist. At $|v\cdot v'|\gg m_Q/\Lambda$, they are dominated by the two functions ξ_3 and χ_2 arising at order $1/m_Q$ in the heavy-quark expansion, which are determined by the leading-twist wave function alone. The sum of these contributions describes the form factors in the whole region $|v\cdot v'|\gg 1$. Central objects of our study are the meson wave functions $\varphi_\pm(\omega)$, which are defined in terms of the Fourier transforms of the matrix element of bilocal operators on the light cone. We have derived

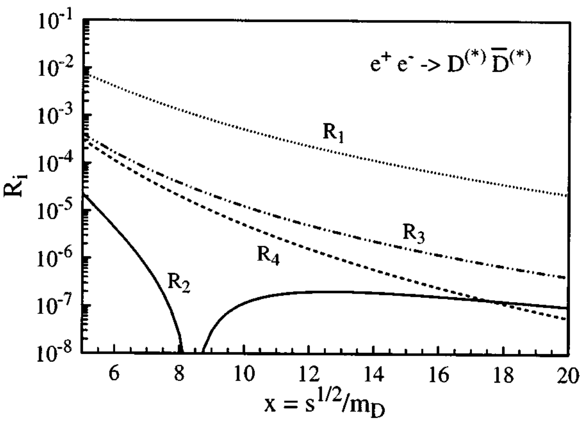

FIG. 10. Cross-section ratios R_i for $D\bar{D}$ pair production in e^+e^- collisions at large x.

the (Brodsky-Lepage) evolution equations obeyed by these wave functions, and we have investigated the properties of the wave functions (such as their moments) using QCD sum rules. Finally, we have discussed as an application the implications of our results for the production of heavy-meson pairs in e^+e^- collisions.

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APPENDIX A: COVARIANT TRACE FORMALISM

The most convenient way to calculate the matrix elements of operators between the physical pseudoscalar and vector $(q\bar{Q})$ meson states (rather than the spin-$\frac{1}{2}$ mesons obtained when the heavy-quark spin is switched off) is provided by the covariant tensor formalism introduced in [15]. In the HQET, the spin wave function of the ground-state meson doublet is described by the 4×4 Dirac matrix

$$\mathcal{M}(v)=\frac{1+\not{v}}{2}\begin{cases}-i\gamma_5, & \text{pseudoscalar meson } M(v),\\ \not{e}, & \text{vector meson } M^*(e,v),\end{cases} \tag{A1}$$

where v is the meson velocity, and e is the polarization vector of the vector meson $(e\cdot v=0)$. The matrix $\mathcal{M}(v)$ simply contains the appropriate spin-parity projections of the spinor product $u_q(v)\bar{v}_Q(v)$ [14]. It satisfies

$$\not{v}\mathcal{M}(v)=\mathcal{M}(v)=-\mathcal{M}(v)\not{v}. \tag{A2}$$

Operator matrix elements between meson states can be represented by traces over these wave functions. Consider first the matrix elements of heavy-light operators of the type $\bar{Q}_v\Gamma\mathbf{O}(iD)q$, where Γ is an arbitrary Dirac matrix, and $\mathbf{O}(iD)$ is a differential operator acting on the light-quark field, between a meson state and the vacuum. Their representation is

$$\langle\, 0|\bar{Q}_v\Gamma\mathbf{O}(iD)q|M(v)\rangle=\mathrm{Tr}\{O(v)\mathcal{M}(v)\Gamma\}, \tag{A3}$$

where $O(v)$ is the most general matrix with the same transformation properties (under the Lorentz group and heavy-quark symmetry) as the operator $\mathbf{O}$. In the spinor formalism adopted in our paper, the same matrix element would read

$$\langle\, 0|Q_v^*\mathbf{O}(iD)q|M(v)\rangle=O(v)u(v) \tag{A4}$$

with the same matrix $O(v)$. The covariant decomposition of this matrix determines the number of reduced matrix elements (generalized Isgur-Wise form factors) that appear in the heavy-quark expansion. As an example, we give the expressions in the trace formalism which correspond to the definitions in Eqs. (1.1), (2.18), and (2.20):

$$\begin{aligned}\langle\, 0|\bar{Q}_v\Gamma q|M(v)\rangle&=f\ \mathrm{Tr}\{\mathcal{M}(v)\Gamma\},\\ \langle\, 0|\bar{Q}_v\Gamma iD^\mu q|M(v)\rangle&=\tfrac{1}{3}f\bar{\Lambda}\mathrm{Tr}\{(4v^\mu-\gamma^\mu)\mathcal{M}(v)\Gamma\},\\ \langle\, 0|\bar{Q}_v\Gamma iD^\mu iD^\nu q|M(v)\rangle&=f\ \mathrm{Tr}\{\Theta^{\mu\nu}(v)\mathcal{M}(v)\Gamma\},\end{aligned} \tag{A5}$$

where $\Theta^{\mu\nu}(v)$ is given in Eq. (2.21).

In the end of Sec. II, we need the generalization of Eq. (2.3) in the trace formalism. It reads

$$\begin{aligned}&\langle\, 0|\bar{Q}_v(0)\Gamma E(0,z)q(z)|M(v)\rangle\\ &\quad=f\ \mathrm{Tr}\left\{\left[\tilde{\varphi}_+(t)+\frac{1}{2t}[\tilde{\varphi}_-(t)-\tilde{\varphi}_+(t)]\not{z}\right]\mathcal{M}(v)\Gamma\right\}.\end{aligned} \tag{A6}$$

Evaluating the trace for various choices of Γ, we recover the results given in Eqs. (2.26)–(2.31).

The trace formalism is readily extended to more complicated cases, such as transition matrix elements between two meson states. For instance, the expressions corresponding to the definitions in Eq. (1.4) and (1.5) read

$$\begin{aligned}\langle M(v')|\bar{Q}_v\Gamma Q_{v'}|M(v)\rangle&=\xi(v\cdot v')\mathrm{Tr}\{\mathcal{M}(v)\Gamma\bar{\mathcal{M}}(v')\},\\ \langle M(v')|(iD^{\mu\dagger}\bar{Q}_v)\Gamma Q_{v'}|M(v)\rangle&\\ =\mathrm{Tr}\{\xi^\mu(v,v')\mathcal{M}(v)\Gamma\bar{\mathcal{M}}(v')\},&\end{aligned} \tag{A7}$$

where the covariant decomposition of $\xi^\mu(v,v')$ is given in Eq. (1.6). Note that as a consequence of the fact that in this paper we work with $(q\bar{Q})$ rather than $(Q\bar{q})$ mesons, the trace formalism is slightly different from the one usually employed in the literature [9,16,52]. Crossing symmetry implies that the form factors for $(q\bar{Q})$ mesons are related to those for $(Q\bar{q})$ mesons by Hermitean conjugation followed by the substitutions $v\to -v$ and $v'\to -v'$. We have defined the invariant functions in the HQET (ξ, ξ_3, χ_i, etc.) in such a way that the resulting expressions for the physical matrix elements look the same as in the conventional formalism.

APPENDIX B: NONLOCAL CONDENSATES

The contributions of higher-order nonperturbative corrections to the sum rules for λ_E^2 and λ_H^2 in Eq. (4.6) can be included by introducing two functions, $f^{(1)}(x^2)$ and $f^{(2)}(x^2)$, which parametrize the following nonlocal condensates [56]:

$$f^{(1)}(x^2)=\frac{\langle\overline{q(0)}E(0,x)\sigma_{\mu\nu}G^{\mu\nu}(x)q(x)\rangle}{\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle}=1+\frac{Q_1-Q_2-2Q_3}{m_0^2\langle\bar{q}q\rangle}\,\frac{x^2}{8}+\cdots,$$

$$f^{(2)}(x^2)=\frac{4x^\alpha x_\beta}{x^2}\,\frac{\langle\overline{q(0)}E(0,x)\sigma_{\mu\alpha}G^{\mu\beta}(x)q(x)\rangle}{\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle}=1+\frac{2Q_1-Q_2-3Q_3}{m_0^2\langle\bar{q}q\rangle}\,\frac{x^2}{12}+\cdots. \qquad \text{(B1)}$$

The quantities Q_i form a basis of dimension-seven quark-gluon condensates and are defined as ($\widetilde{G}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}$)

$$Q_1=\langle\bar{q}G_{\mu\nu}G^{\mu\nu}q\rangle,$$
$$Q_2=i\langle\bar{q}G_{\mu\nu}\widetilde{G}^{\mu\nu}\gamma_5q\rangle,$$
$$Q_3=i\langle\bar{q}\sigma_{\mu\nu}G^{\mu\lambda}G^{\nu}{}_{\lambda}q\rangle,$$
$$Q_4=\langle\bar{q}\sigma_{\mu\nu}(D^\mu D_\alpha G^{\nu\alpha})q\rangle. \qquad \text{(B2)}$$

If these corrections are included, the sum rules (4.6) are modified in the following way:

$$f^2\lambda_E^2e^{-\bar{\Lambda}\tau}=-N_cC_F\frac{\alpha_s}{\pi^3\tau^5}\,\delta_4(\varepsilon_c\tau)-\frac{m_0^2\langle\bar{q}q\rangle}{16}f^{(2)}(-\tau^2),$$

$$f^2\lambda_H^2e^{-\bar{\Lambda}\tau}=-N_cC_F\frac{\alpha_s}{2\pi^3\tau^5}\,\delta_4(\varepsilon_c\tau)-C_F\frac{3\alpha_s}{4\pi\tau^2}\langle\bar{q}q\rangle\,\delta_1(\varepsilon_c\tau)+\frac{\alpha_s\langle G^2\rangle}{16\pi\tau}\delta_0(\varepsilon_c\tau)-\frac{m_0^2\langle\bar{q}q\rangle}{16}\times[2f^{(1)}(-\tau^2)-f^{(2)}(-\tau^2)]. \qquad \text{(B3)}$$

Next we give some details of the calculation of the quark-condensate contributions to the QCD sum rules for the meson wave functions. The contribution with the cut light-quark line shown in Fig. 6(c) involves a trilocal object, the noncollinear quark condensate [56]:

$$\langle\overline{q_\beta}(y)E(y,0)E(0,x)q_\alpha(x)\rangle=\frac{\langle\bar{q}q\rangle}{4}\bigg[f_S(x,y)+\frac{m_0^2}{48}[\not{x},\not{y}]f_T(x,y)-\frac{i}{4}[\not{x}f_V(x,y)-\not{y}f_V(y,x)]\bigg]_{\alpha\beta}. \qquad \text{(B4)}$$

Neglecting the function $f_V(x,y)$, whose operator product expansion contains even-dimensional quark condensates with $d\geqslant6$ and whose contribution to heavy-meson sum rules is negligible [18,41–43], we obtain

$$\Pi_\pm^{(2)}(v\cdot x,t)=-\tfrac{1}{4}\langle\bar{q}q\rangle[f_S(z,-x)\pm\tfrac{1}{24}m_0^2v\cdot xtf_T(z,-x)], \qquad \text{(B5)}$$

where $z^2=0$, $x^2=(v\cdot x)^2$, and $z\cdot x=v\cdot xt$.

The noncollinear condensate in Eq. (B4) can be expanded in x at fixed y. One finds [56]

$$f_S(x,y)=f_S[(y-x)^2]+\frac{4}{3}\,\frac{(xy)^2-x^2y^2}{y^2}\bigg[f_S'(y^2)+\frac{y^2}{2}f_S''(y^2)-\frac{m_0^2}{16}f^{(1)}(y^2)\bigg]+O(x^3),$$

$$f_T(x,y)=\frac{16}{m_0^2}\bigg\{f_S'(y^2)-\frac{xy}{y^2}\bigg[f_S'(y^2)+2y^2f_S''(y^2)-\frac{m_0^2}{16}f^{(2)}(y^2)\bigg]\bigg\}+O(x^2), \qquad \text{(B6)}$$

where the function $f_S(x^2)$ parametrizes the bilocal quark condensate and is given by [43,55]

$$f_S(x^2)=\frac{\langle\overline{q(0)}E(0,x)q(x)\rangle}{\langle\bar{q}q\rangle}=1+\frac{m_0^2x^2}{16}+\frac{6Q_1-3Q_2-6Q_3+2Q_4}{\langle\bar{q}q\rangle}\,\frac{x^4}{1152}+O(x^6). \qquad \text{(B7)}$$

A convenient representation of the bilocal quark condensate is [44]

$$f_S(x^2)=\int d\nu\widetilde{f}_S(\nu)e^{\nu x^2}, \qquad \text{(B8)}$$

where

$$\int d\nu\widetilde{f}_S(\nu)=1,\quad \int d\nu\nu\widetilde{f}_S(\nu)=\frac{m_0^2}{16}, \qquad \text{(B9)}$$

and so on. The function $\widetilde{f}_S(\nu)$ can be interpreted as the distribution quarks with virtuality ν in the QCD vacuum. The local expansion in Eq. (B7) corresponds to the expansion

$$\widetilde{f}_S(\nu)=\delta(\nu)-\frac{m_0^2}{16}\delta'(\nu)+\cdots. \qquad \text{(B10)}$$

Because the factor $E(0,x)$ in Eq. (B7) can be interpreted as the heavy-quark propagator, the asymptotic behavior of $f_S(x^2)$ at large $-x^2$ is

$$f_S(x^2) \sim e^{-\bar{\Lambda}\sqrt{-x^2}}. \tag{B11}$$

This fixes the behavior of $\tilde{f}_S(\nu)$ for $\nu \to 0$. A simple ansatz for the distribution function, which satisfies this constraint, was proposed in [65]:

$$\tilde{f}_S(\nu) = N \exp\left(-\frac{\bar{\Lambda}^2}{4\nu} - \sigma\nu \right), \tag{B12}$$

where N and σ are fixed by the conditions (B9).

APPENDIX C: FORM-FACTOR ASYMPTOTICS FROM QCD SUM RULES

It is instructive to compare our asymptotic results for the leading and subleading Isgur-Wise functions with the large-ϑ limit of the two-loop (order-α_s) QCD sum-rule expressions for these functions, which have been obtained in [21,24]. For very large recoil, the three-point correlators considered in these sum rules factorize into the convolution of the two-point correlators (4.14) with hard-scattering amplitudes. In this limit, only diagrams with a gluon exchange between a heavy quark and the light quark remain. We find that the results of the sum-rule calculations do indeed reproduce the correct asymptotic behavior; in particular, the relation (5.9) between χ_2 and ξ_3 is satisfied. For the normalization factors appearing in the expressions for ξ in Eq. (5.4) and for ξ_3 in Eq. (5.7), we obtain, from [21,24],

$$f^2 \varphi'_+(0)\varphi_-(0) = \frac{N_c^2}{4\pi^4 f^2 \tau^3} e^{\bar{\Lambda}\tau} \delta_2(\varepsilon_c \tau),$$

$$f^2 \langle \omega^{-1} \rangle_+^2 = \frac{N_c^2}{\pi^4 f^2 \tau^4} e^{\bar{\Lambda}\tau} \delta_3(\varepsilon_c \tau). \tag{C1}$$

We have retained the leading perturbative contributions only, since the relevant nonlocal condensates have not yet been calculated to order α_s. Note, in particular, that the leading quark-condensate contribution to the sum rule for the Isgur-Wise function is constant and seems to dominate for large recoil. However, once the nonlocality of the quark condensate is taken into account, one finds that this contribution actually vanishes quickly at large recoil [17,18].

It is straightforward to reproduce the expressions in Eq. (C1) starting from the sum-rule results for the wave functions $\varphi_\pm(\omega)$ obtained in Sec. IV. The sum rule for the product $\varphi_+(\omega)\varphi_\pm(\omega')$ at equal Borel parameters has the form

$$f^4 \varphi_+(\omega)\varphi_\pm(\omega') e^{-\bar{\Lambda}\tau} = \int d\varepsilon\, d\varepsilon' \rho_+(\omega,\varepsilon)\rho_\pm(\omega',\varepsilon') \times e^{-(\varepsilon+\varepsilon')\tau/2}, \tag{C2}$$

where $\rho_\pm(\omega,\varepsilon)$ are the spectral densities of the correlators (4.14), and the integral is taken over the complement of the continuum region. The precise form of the result will depend on the particular way in which the continuum subtraction is performed. The spectral densities for three-point correlators depend on two variables, ε and ε'; see Fig. 11. The "square model" of the continuum subtraction amounts to cutting off the integrals over these variables at the threshold ε_c. This leads to an exact factorization of the integrals in Eq. (C2). Using the appropriate products of the sum rules for the wave functions given in Eq. (4.17), and retaining the leading perturbative contributions only, we then obtain

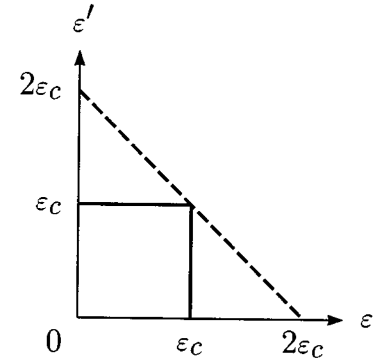

FIG. 11. Square model (solid line) and triangle model (dashed line) for the continuum subtraction in QCD sum rules for three-point correlation functions.

$$f^2 \varphi'_+(0)\varphi_-(0) = \frac{N_c^2}{4\pi^4 f^2 \tau^3} e^{\bar{\Lambda}\tau} \delta_0(\tfrac{1}{2}\varepsilon_c\tau)\,\delta_1(\tfrac{1}{2}\varepsilon_c\tau),$$

$$f^2 \langle \omega^{-1} \rangle_+^2 = \frac{N_c^2}{\pi^4 f^2 \tau^4} e^{\bar{\Lambda}\tau} [\delta_1(\tfrac{1}{2}\varepsilon_c\tau)]^2. \tag{C3}$$

On the other hand, it is well known that for small recoil the square model of the continuum subtraction is inconsistent, as it leads to an unphysical infinite slope of the Isgur-Wise function at $v \cdot v' = 1$ [18]. This deficiency is removed by using the "triangle model," where $0 < \varepsilon + \varepsilon' < 2\varepsilon_c$, while the difference $\varepsilon - \varepsilon'$ is unconstrained [18,19]. The triangle model was adopted in the calculations in [21,24]. If we use it to evaluate Eq. (C2), we indeed recover Eq. (C1). This is a strong check of both, the present approach and the two-loop calculations performed in [21,24]. However, in this case the resulting sum rule for the product of the wave functions is no longer exactly factorizable, meaning that the triangle model is not fully consistent at large recoil, and so the square model is preferrable. Comparing Eq. (C1) with (C3), we observe that both results agree in the limit $\varepsilon_c \to \infty$, when the choice of the continuum model becomes irrelevant ($\delta_n \to 1$).

The sum rule for the Isgur-Wise function [18] has a built-in model of the infrared cutoff, and therefore it allows us to estimate the subleading $O(\vartheta)$ term to the Isgur-Wise function in Eq. (5.5). The result is

$$\xi(\cosh\vartheta) \sim e^{-2\vartheta}\{\vartheta^2 + [4L(\varepsilon_c\tau) - 5]\vartheta + \cdots\}, \tag{C4}$$

where

$$L(x_c) = \frac{\int_0^{x_c} dx\, x^2 e^{-x} \ln 2x}{\int_0^{x_c} dx\, x^2 e^{-x}}. \tag{C5}$$

In the relevant region of values of $\varepsilon_c \tau \approx 2.5$, we find that $(4L-5) \approx -0.7$, meaning that the subleading term is negative and has a coefficient of order unity.

APPENDIX D: STATIC QUARK MODEL

Our model-independent results in Eqs. (5.4), (5.7), (5.9), and (5.15) can be checked in the simple model where a meson is composed of two heavy quarks with masses m and μ, such that $m \gg \mu \gg \Lambda$ [47]. In this case, $\varphi_+(\omega) = \varphi_-(\omega) = \delta(\omega - \mu)$, and $\langle \omega^n \rangle_\pm = \mu^n$. Note that formulas (2.19) and (2.24) for the lowest moments are based on the equation of motion for a massless quark, and are thus no longer applicable.

Let us consider the pseudoscalar form factor h_+ in Eq. (5.15). It is convenient to calculate it from the relation $(\cosh\vartheta + 1) h_+ = \langle B(v') | v_\mu V^\mu | B(v) \rangle$. A simple evaluation of the diagrams in Figs. 8(a) and 8(b) gives

$$h_+ = 2\pi\alpha_s \frac{C_F}{N_c} \frac{f^2}{\mu^2} e^{-2\vartheta} \mathrm{Tr}[\gamma_\mu \Phi \gamma^\mu S \not{v} \overline{\Phi'} + \Phi \not{v} S' \gamma_\mu \overline{\Phi'} \gamma^\mu], \tag{D1}$$

where $\Phi = \gamma_5 (1 - \not{v})$ and $\overline{\Phi'} = -(1 - \not{v}')\gamma_5$ are the spin structures arising for pseudoscalar mesons. The heavy-quark propagators in Figs. 8(a) and 8(b) are given by

$$S = \frac{m(1-\not{v}) + \mu \not{v}'}{-m\mu e^{\vartheta}} \tag{D2}$$

and $S' = S(v \leftrightarrow v')$. If we retain the leading term proportional to m in the numerator of Eq. (D2), then the gluon is longitudinally polarized; the two diagrams contribute equally, and we recover the contribution of the function ξ in Eq. (5.4) to the form factor h_+ in Eq. (5.16). If, on the other hand, we retain the term with $\not{v}'$, we lose a factor μ/m but gain a factor e^{ϑ} from the trace. Then only the first diagram contributes; the gluon is transversely polarized, and we recover the contribution of the function ξ_3 in Eq. (5.7) to the form factor h_+. In this model, the form factor of pseudoscalar mesons has a zero at $q^2 = 2m^3/\mu$. The calculation can be repeated for vector mesons using the spin structures $\Phi = \not{e}(1 - \not{v})$ and $\overline{\Phi'} = (1 - \not{v}')\not{e}'$. In particular, we find that the form factor of longitudinally polarized vector mesons has a zero at $q^2 = -2m^3/\mu$. Other form factors can be calculated in a similar way; our results agree with [47].

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