# **Diffractive production of** *bb* **in proton-antiproton collisions at the Fermilab Tevatron**

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We show that the cross section of the diffractive production of  $b\overline{b}$  can be described as the sum of two contributions: the first is proportional to the probability of finding a small size  $b\overline{b}$  color dipole in the fast hadron wave function before the interaction with a target, while the second is the  $b\bar{b}$  production after or during the interaction with the target. The formulas are presented as well as the discussion of the interrelation between these two contributions and the Ingelman-Schlein and coherent diffraction mechanisms. The main prediction is that the coherent diffraction mechanism dominates, at least at the Fermilab Tevatron energies, giving the unique possibility to study it experimentally.  $[$ S0556-2821(97)01805-5 $]$ 

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#### **I. INTRODUCTION**

The main goal of this paper is to consider the possibility of measuring the inelastic cross section in the diffractive kinematic region and to discuss the diffractive production of *bb* pairs as a way to extract the value of the gluon structure function  $[x_{\text{Bj}}G(x_{\text{Bj}}, Q^2)]$  in the region of small  $x_{\text{Bj}}$ . New data from the DESY  $ep$  collider HERA [1] show a rapid increase of  $F_2(x_{\text{Bi}}, Q^2)$  in the region of small  $x_{\text{Bi}}$  $(x_{\text{Bi}}<10^{-2})$ , which could be interpreted as a manifestation of the growth of the gluon structure function at small  $x_{\text{Bi}}$ . However, the data on  $F_2$  do not allow the extraction of the value of the gluon structure function within good accuracy. At present we know the gluon structure function with an accuracy up to a factor  $2$  (see Fig. 1, which shows the gluon structure in three parametrizations, Gluck-Reya-Vogt 1994  $(GRV94)$  [2], Martin-Roberts-Stirling set A  $[MRS(A)]$  [3], and CTEQ [4], at different values of  $Q^2$  as function of  $x_{\text{Bi}}$ . Data on photoproduction of  $J/\psi$  seem to favor the MRS(A) parametrization  $[5]$ . This question, however, is still open.

We will argue that the large rapidity coverage collider detectors at the fermilab Tevatron offer an unique oportunity to measure the gluon structure function at 2  $\text{GeV}^2$  $\leq Q^2 \leq m_b^2 + p_t^2$ , where  $m_b$  is the *b*-quark mass and  $p_t$  is its transverse momentum, at  $10^{-4} < x < 10^{-2}$ , using the process of the diffractive dissociation of proton into  $b\overline{b}$  pairs. This process lego plot and amplitude are pictured in Figs. 2 and 3, respectively. It is clear from these figures that this process is a typical large rapidity gap (LRG) process, suggested by Bjorken  $[6]$ . As pointed out by Bjorken and as we demonstrate below, such a process can be described as the exchange of a ''hard'' Pomeron, which could be rewritten through the gluon structure function due to the intimate relation between inelastic and elastic processes given by the optical theorem  $(Fig. 4)$  (see Refs.  $[7,8]$  for more details).

We will show that the cross section of the diffraction  $dissection$  (DD) can be described as the sum of two different contributions.<sup>1</sup>

 $(1)$  The first is proportional to the probability of finding a (1) The first is proportional to the probability of finding a<br> *bb* color dipole with a small size, of the order of  $r_t^2$  $\propto 1/(m_b^2 + p_t^2)$ , in the fast hadron wave function before the interaction with the target. This dipole scatters with the target and produces the measured final state of the DD process. This mechanism has a normal partonic interpretation and, in the Bjorken frame for the projectile (in other words, in the the Bjorken frame for the projectile (in other words, in the frame where the  $\overline{b}b$  color dipole is at rest), it looks as a measurement of the partonic content of the Pomeron and corresponds to the Ingelman-Schlein (IS) hypothesis of the Pomeron structure function [9].

meron structure function [9].<br>
(2) The second is the production of  $\overline{b}b$  pairs after or during the interaction with a target. We will show that this mechanism corresponds to so-called coherent diffraction  $(CD)$  (see Ref. [10]), and we will demonstrate that the mea-(CD) (see Ref. [10]), and we will demonstrate that the measurement of the  $\overline{bb}$  diffraction will allow, thanks to the different dependences on the transverse momenta of produced quarks for both mechanisms, the separation of the CD contribution from the IS one.

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<sup>&</sup>lt;sup>1</sup>In what follows we use at large the parton picture of interactions. It is easier to discuss diffractive processes in this picture in the frame where the antiproton is at rest (the fixed target frame). Of course, all results will be given in a relativistic invariant way.



FIG. 1. Gluon structure function  $xG(x,Q^2)$  in different parameterizations: GRV [2] (a), MRS(A') [3] (b), and CTEQ [4] (c).

We would like to stress that the above two contributions are closely related to the classic diffractive dissociation picture suggested by Good and Walker  $[11]$  25 years ago. Indeed, there are two different possibilities for the dissociation of a hadron *h* into a pair of hadrons  $(h_1$  and  $h_2)$ : First, the beam particle (*h*) interacts with the target and dissociates into a pair of hadrons  $(h_1$  and  $h_2)$ ; second, the beam particle dissociates first and one of the produced particle interacts with the target (see Ref.  $[12]$  for details).

However, we will argue that the diffractive production of the heavy quark system originates from the small distances where we can develop a theoretical approach based on perturbative QCD  $(PQCD)$ . The PQCD approach allows us to furbative QCD (PQCD). The PQCD approach allows us to calculate the diffractive dissociation process of  $\overline{b}b$  in such detail that is beyond our reach in ''soft'' high energy phenomenology.

### **II. NOTATION AND KINEMATICS**

(1)  $y = \frac{1}{2} \ln(E + p_L)/(E - p_L)$  is the rapidity of a particle with energy  $E$  and longitudinal momentum (along the beam direction)  $p<sub>L</sub>$ . For the rapidity in the center-of-mass frame, we use the notation *y*\*.

 $(2)$   $P_1$  and  $P_2$  are the momenta of colliding protons and antiprotons (in the c.m. frame  $P_1 = P_2$ ):

$$
P_1 = \left\{ \sqrt{\frac{s}{2}} \left( 1 + \frac{2m^2}{s} \right), \sqrt{\frac{s}{2}}, 0, 0 \right\},
$$
  

$$
P_2 = \left\{ \sqrt{\frac{s}{2}} \left( 1 + \frac{2m^2}{s} \right), -\sqrt{\frac{s}{2}}, 0, 0 \right\}. \tag{1}
$$

(3)  $y_1$  and  $y_2$  are the rapidities of produced *b* and  $\overline{b}$ quarks,  $p_{1t}$  and  $p_{2t}$  are their transverse momenta and,  $m_b$  is the *b* mass.

 $(4)$  *M*<sup>2</sup> is the mass of the produced *b***b**<sup> $\overline{b}$  pair. *m* is the mass</sup> of the proton or antiproton. *s* is the squared energy of the reaction in the c.m. frame, and it is equal to  $s=(P_1+P_2)^2$ .

 $(5)$   $\Delta y = y_1 - y_2$  is the difference of rapidities between the (5)  $\Delta y = y_1 - y_2$ <br>produced *b* and *b*.

 $y = (y_1 + y_2)/2$  is the mean rapitity of the  $\overline{b}b$  system.



FIG. 2. Lego plot of  $b\overline{b}$  diffractive production in  $p\overline{p}$  collisions.

(7)  $m_{it}^2 = m_b^2 + p_{it}^2$ , where  $i = 1,2$ .

 $(8)$  For the purpose of obtaining the kinematic relation in the simplest way we use the Sudakov decomposition  $[13]$  of the momenta of all particles, namely,

$$
p_{i\mu} = \alpha_i P_{1\mu} + \beta_i P_{2\mu} + p_{it\mu} \,, \tag{2}
$$

where vector  $\vec{p}_{it}$  is orthogonal to  $P_{1\mu}$  and  $P_{2\mu}$ .

At high energy  $p_{i\mu}^2 = \alpha_i \beta_i s - p_{it}^2$  and the rapidity of particle *i* is equal to

$$
y_i^* = \frac{1}{2} \ln \frac{\alpha_i}{\beta_i}.
$$
 (3)

 $(9)$  Using Eqs.  $(1)$ ,  $(2)$ , and  $(3)$  we obtain, for produced *b* quarks,

$$
\alpha_1 = \frac{m_{1t}}{\sqrt{s}} e^{y_1^*}, \quad \beta_1 = \frac{m_{1t}}{\sqrt{s}} e^{-y_1^*},
$$
  

$$
\alpha_2 = \frac{m_{2t}}{\sqrt{s}} e^{y_2^*}, \quad \beta_2 = \frac{m_{2t}}{\sqrt{s}} e^{-y_2^*}
$$
(4)

and

$$
M^{2} = 2m_{1t}m_{2t}\cosh(\Delta y) + m_{1t}^{2} + m_{2t}^{2}.
$$
 (5)

 $(10)$  Let us introduce  $x_1$ , the energy fraction of hadron 1 carried by gluon *k* in Fig. 3 and the energy fraction of hadron 2 carried by the Pomeron with momentum  $q$  (gluon ''ladder'' in Fig. 3). We show below that  $x_1$  and  $x_2$  will be the arguments of the gluon structure functions in the cross section expression. Directly from Fig. 3 one can see that

$$
x_1 = \alpha_1 + \alpha_2 + \alpha_q, \quad x_2 = \beta_1 + \beta_2 + \beta_k, \tag{6}
$$

where  $(x_1, \beta_k)$  and  $(\alpha_q, x_2)$  are the longitudinal components of the four-momenta of gluon 1 and the Pomeron, respectively.

The main property of high energy scattering is the fact that  $\alpha_q \ll \alpha_1$  *and/or*  $\alpha_2$  and  $\beta_k \ll \beta_1$  *and/or*  $\beta_2$  (see, for example, Ref. [14]). Therefore, we can easily derive from Eq.  $(6)$ , assuming  $m_{1t} = m_{2t}$ :

$$
x_1 = \frac{2m_{1t}}{\sqrt{s}} e^{Y^*} \cosh\left(\frac{\Delta y}{2}\right), \quad x_2 = \frac{2m_{1t}}{\sqrt{s}} e^{-Y^*} \cosh\left(\frac{\Delta y}{2}\right). \tag{7}
$$



FIG. 3. Amplitude of  $b\overline{b}$  diffractive production.

 $(11)$  Throughout the paper we will choose a frame where the antiproton (see Figs. 2 and 3) is essentially at rest and where all momenta  $(l_i)$  of fast particles look as follows:

$$
l_i = (l_{i+}, l_{i-}, \vec{l}_{it}) = \left( l_{i+}, \frac{m^2 + l_i^2}{l_{i+}}, \vec{l}_t \right), \tag{8}
$$

where  $l_{i+} = l_{i0} + l_{i3}$  and  $l_{i-} = l_{i0} - l_{i3}$ .

 $(12)$   $xG(x, Q^2)$  everywhere in the paper is the gluon structure function.

### **III. VALUE OF THE CROSS SECTION IN THE GENERALIZED PARTON MODEL**

From Figs. 3 and 4 we can see that the value of the cross section of our process

$$
p(P_1) + \overline{p}(P_2) \to b(y_1, p_{1t}) + \overline{b}(y_2, p_{2t}) + X + [LRG(Y)]
$$
  
+  $\overline{p}(P_2 - q)$  (9)

is equal to



FIG. 4. Optical theorem.



FIG. 5. Feyman diagrams for  $b\overline{b}$  diffractive production by a colorless gluon probe.

$$
\left. \frac{d\sigma}{dYdq_t^2 \ d\Delta y \ dp_t^2} \right|_{q_t^2 = 0} = [x_1 G(x_1, \mu^2)]
$$
\n
$$
\times \left. \frac{d\sigma^G}{dYdq_t^2 \ d\Delta y \ dp_t^2} \right|_{q_t^2 = 0},
$$
\n(10)

where  $\sigma$ <sup>G</sup> is the reaction cross section,

$$
G(x_1, k_t^2) + \overline{p}(P_2) \to b(y_1, p_{1t}) + \overline{b}(y_2, p_{2t}) + [LRG(Y)] + \overline{p}(P_2 - q).
$$
 (11)

The physical meaning of Eq.  $(10)$  is very simple:  $x_1 G(x_1, \mu^2)$  is the probability of finding a gluon with the fraction of energy  $x_1$  inside of the proton and  $\sigma$ <sup>G</sup> is the cross section of its interaction with the antiproton. In the spirit of the factorization theorem  $[15]$ , we introduce the factorization scale  $\mu^2$ , the maximal value of  $k_t^2$  at which we still can neglect the dependence of  $\sigma$ <sup>G</sup> on  $k_t^2$ .

To simplify the color algebra, we adopt throughout the paper the colorless probe approach, replacing the gluon with the transverse momentum  $k_t$  and the fraction of energy  $x_1$  by a colorless probe with the same kinematics. The physical motivation is clear and based on the factorization equation [Eq.  $(10)$ ]. Indeed, we can measure the gluon structure function using a colorless probe such as the graviton or heavy Higgs boson. The properties of such a probe have been studied in details in Ref.  $[16]$ .

The cross section for the reaction of Eq.  $(11)$  can be easily The cross section for the reaction of Eq. (11) can be easily calculated. It is clear that we have two mechanisms for  $\overline{bb}$ production by the colorless probe which we will discuss in the rest target rest frame (antiproton in Fig.  $2$ ).

rest target rest frame (antiproton in Fig. 2).<br>(1) The first mechanism is the following. There is a  $\overline{bb}$ component in the wave function of the fast probe before its component in the wave function of the fast probe before its interaction with the target. This  $\overline{b}b$  pair is a color dipole with sufficiently small transverse size of the order of  $r_t^2$  $\propto 1/(m_b^2 + p_t^2)$ , which scatters with the target, producing the measured final state.



FIG. 6. Cross section for  $b\overline{b}$  diffractive production in the Ingelman-Schlein approach [9].

(2) The second mechanism is the production of the  $\overline{b}b$ pair after or during the interaction with the target.

These two mechanisms correspond to the two sets of Feyman diagrams pictured in Figs.  $5(a)$  and  $5(b)$ , respectively.

Let us start from the first one which looks normal from the partonic point of view in the sense that, in the Bjorken frame for the probe, it looks like the measurement of the partonic content of the Pomeron and corresponds to the Ingelman-Schlein hypothesis of the Pomeron structure function [9]. For the set of the Feyman diagrams of Fig.  $5(a)$ , the tion [9]. For the set of the Feyman diagrams of Fig. 5(a), the amplitude of  $\overline{bb}$  production can be written as a product of amplitude of *bb* production can be written as a product of two factors: (i) the wave function of a  $\overline{bb}$  pair in a virtual gluon  $\Psi_{\lambda_1 \lambda_2}^{G^*}$  and (ii) the rescattering amplitude of the quarkantiquark pair on the target  $T_{\lambda_1 \lambda_2}$ , where  $\lambda_i$  is the quark polarization. Following the conventions of Ref.  $[17]$ , we have

$$
M_f = \sqrt{N_c} \int \frac{d^2 p'_t}{16\pi^3} \int_0^1 dz'
$$
  
 
$$
\times \Psi_{\lambda_1 \lambda_2}^{G^*}(p'_t, z') T_{\lambda_1 \lambda_2}(p'_t, z'; p_t, z), \qquad (12)
$$

where  $p_t$  is the transverse momentum of the produced quark and *z* is the fraction of energy carried by the *b* quark with respect to the energy of the gluon. It is easily found from Eqs.  $(4)$  and  $(6)$  that

$$
z = \frac{\alpha_1}{x_1} = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \alpha_q} = \frac{e^{\Delta y/2}}{e^{\Delta y/2} + e^{-\Delta y/2}},
$$
(13)

where  $\alpha_q$  can be found from the equation  $(P_2 - q)^2 = m^2$  and it is equal to  $\alpha_q = q^2/(1-x_2)s \ll \alpha_1 + \alpha_2$  at large *s*. In deriving Eq.  $(13)$  we have also assumed that  $\vec{p}_{1t} = -\vec{p}_{2t} + \vec{k}_t + \vec{q}_t \rightarrow -\vec{p}_{2t}$ .

The virtual gluon breaks into a quark-antiquark pair with a large lifetime which is equal to  $\tau_{G^*}$ . In a leading ln (1/*x*) approximation of PQCD, which we will use here, the time of interaction is much smaller than  $\tau_{G^*}$ , and during this time, the exchange of gluons does not change the fraction of energy carried by a quark and/or antiquark. It is instructive to recall the argument of why this is so. According to the unrecall the argument of why this is so. According to the uncertainty principle, the lifetime of the  $\overline{bb}$  fluctuation ( $\tau_{G*}$ ) is



FIG. 7. Behavior of average  $\langle \gamma \rangle$  for the GRV parametrization of the gluon structure function.

$$
\tau_{G^*} \sim \frac{1}{\Delta E} = \left| \frac{1}{k_{-} - p_{1-} - p_{2-}} \right| = \frac{x_1 z (1 - z) P_{1+}}{m_t^2 + z (1 - z) k_t^2}.
$$
 (14)

An estimate of the interaction time can be obtained from the typical time for the emission of a gluon with momentum *l*, from the quark  $p_1$ , say. Then

$$
\tau_i \sim \left| \frac{1}{p'_{1-} - p_{1-} - l_-} \right| = \left| \frac{x_1 P_{1+}}{m_t^2 / z' - m_t^2 / z - l_t^2 / \alpha_l} \right|, \quad (15)
$$

where  $\alpha_l = l_+ / x_1 p_{1+}$  and  $z' = z + \alpha_l$ . In the leading ln(1/*x*) approximation of PQCD, we have  $\alpha_l \ll z$  and, hence,

$$
\tau_i \approx \frac{\alpha_i x_1 P_{1+}}{l_t^2} \ll \tau_{G^*}.
$$
\n(16)

Therefore the interaction only changes the transverse momenta of quarks (see Fig. 3). The vertices also do not depend on the type of the diagram since the exchange of gluons preserves helicity at high energy. Finally, the amplitude *T* can be reduced to the form  $[17]$ 

$$
T_{\lambda_1 \lambda_2} = 16\pi^3 \int \{ 2\,\delta(\vec{k}'_t - \vec{k}_t) - \delta(\vec{k}'_t - \vec{k}_t - \vec{l}_t) -\delta(\vec{k}'_t - \vec{k}_t + \vec{l}_t) \} \cdot \delta(z - z') \phi(l_t, x) \frac{d^2 l_t dl_+}{16\pi^3 l_t^4},\tag{17}
$$

where the function  $\phi$  corresponds to the "ladder" diagram (see Fig. 3) and only weakly (logarithmically) depends on  $l_t$ . Here  $l_+$  is the large component of vector  $l_{\mu}$ , which we have introduced in the previous section. The difference in signs between the terms in Eq.  $(17)$  reflects the different color charge of quark and antiquark.

Substituting Eq.  $(17)$  into Eq.  $(12)$ , we obtain

$$
M_f = \sqrt{N_c} \int \Delta \Psi_{\lambda_1 \lambda_2}^{G^*}(p_t, l_t, z) \phi(l_t, x) \frac{d^2 l_t dl_+}{16 \pi^3 l_t^4}, \quad (18)
$$

where

$$
\Delta \Psi^{G^*}(p_t, l_t, z) = 2\Psi^{G^*}(p_t, z) - \Psi^{G^*}(p_t - l_t, z) - \Psi^{G^*}(p_t + l_t, z).
$$
 (19)

The function  $\Psi$  has been found to be (see, for example, Ref. [17])

$$
\Psi_{\pm}^{G^*}(p_t, z) = -g \frac{\overline{u}_{\lambda_1}(p_1) \overline{\gamma} \cdot \overline{\epsilon}^{G^*} v_{\lambda_2}(p_2)}{\sqrt{z(1-z)}[k^2 - (m_b^2 + p_t^2)/z(1-z)]}
$$

$$
= -g \frac{1}{a^2 + p_t^2} \{ \delta_{\lambda_1 - \lambda_2}[\lambda_1(1-2z) \pm 1]
$$

$$
\times [\overline{\epsilon}_{\pm}^{G^*} \cdot \overline{p}_t \pm m_b \delta_{\lambda_1 \lambda_2}]\}, \tag{20}
$$

where  $\alpha_s = g^2/4\pi$ ,  $\vec{\epsilon}_{\pm}^{G^*}$  is the circular polarization vector of the gluon  $[\vec{\epsilon}_{\pm}^{G^*}=(1/\sqrt{2}), (0,1,\pm 1,0)],$  and  $a^2=m_b^2$  $+k^{2}z(1-z)$ . We have used formulas from Refs. [18] and  $[16]$  in the above calculations.



FIG. 8.  $[x_2G(x_2, k_t^2)]^2/k_t^2$  versus  $\ln(k_t^2/Q_0^2)$  in different parametrizations: GRV [2] (a), MRS(A') [3] (b), and CTEQ [4] (c).

Considering  $l_t^2 \ll m_{bt}^2$ , we obtain

$$
\Delta \Psi_{\pm}^{G^*}(p_t, l_t, z) = -2g \cdot l_t^2 \left\{ 4 \frac{a^2}{(a^2 + p_t^2)^3} \delta_{\lambda_1 \lambda_2} \right.\n \times \left[ \lambda_1 (1 - 2z) \pm 1 \right] \vec{\epsilon}_{\pm}^{G^*} \cdot \vec{p}_t\n + m_b \frac{a^2 - p_t^2}{(a^2 + p_t^2)^3} \pm \delta_{\lambda_1 - \lambda_2} \right\}.
$$
\n(21)

In the leading logarithm approximation in  $ln(1/x)$  and  $\ln (a^2/\Lambda^2)$  [19,17]

$$
\int \phi(l_t, x) \frac{d^2 l_t dl_+}{16\pi^3 l_t^2} = i \frac{4\pi^2 T_R \alpha_S}{N_c} (s + k^2) x G(x, a^2 + p_t^2),
$$
\n(22)

where  $T_R/N_c$  arises from averaging over colors  $(T_R=1/2)$ .

Collecting all previous equations, we can calculate the cross section:

$$
\frac{d\sigma^{G}}{dYdq_{t}^{2} d\Delta y \ dp_{t}^{2}}\Big|_{q_{t}^{2}=0} = \frac{\Sigma_{\lambda_{1}\lambda_{2}} M_{f}^{2}}{16\pi s^{2}} \frac{dz}{d\Delta y}
$$
\n
$$
= \frac{dz}{d\Delta y} \alpha \alpha_{S}^{2} \frac{16\pi^{2}}{9} \Big\{ \big[ z^{2} + (1-z)^{2} \big] p_{t}^{2} \big\}
$$
\n
$$
\times \bigg( \frac{a^{2}}{(a^{2} + p_{t}^{2})^{3}} \bigg)^{2} + \frac{1}{4} m_{b}^{2} \bigg( \frac{a^{2} - p_{t}^{2}}{(a^{2} + p_{t}^{2})^{3}} \bigg)^{2} \Big\}
$$
\n
$$
\times [xG(x, a^{2} + p_{t}^{2})]^{2}.
$$
\n(23)

Finally, we can rewrite Eq.  $(23)$  in the form  $(N_c=3)$ 



FIG. 9. (a) Cross section for *bb* diffractive production for coherent diffraction (CD) [Eq. (29)] and the Ingelman-Schlein (IS) diffraction [Eq. (25)]. (b) Cross section for *bb* for coherent diffraction (CD) in different parametrizations for the gluon structure function.

$$
\frac{d\sigma^{G}}{dYdq_{t}^{2} d\Delta y dp_{t}^{2}}\Big|_{q_{t}^{2}=0}
$$
\n
$$
=\frac{16\pi^{2} \alpha_{S}^{3}}{9} \frac{1}{4 \cosh^{2}(\Delta y/2)}\Big[\frac{\cosh(\Delta y)}{2 \cosh^{2}(\Delta y/2)}p_{t}^{2}\Big] + \frac{m_{b}^{2}}{4}\Big(1-\frac{p_{t}^{2}}{a^{2}}\Big)^{2}\Big\{\Big(\frac{a^{2}}{(a^{2}+p_{t}^{2})^{3}}\Big)^{2}\Big\}
$$
\n
$$
\times [xG(x,a^{2}+p_{t}^{2})]^{2}.
$$
\n(24)

For the cross section of the diffractive dissociation, we have [after a sum over gluon polarization and correct averaging over color  $(N_c=3)$ 

$$
\begin{aligned}\n\left| \frac{d\sigma}{dYdq_t^2 \ d\Delta y \ dp_t^2} \right|_{q_t^2 = 0} \\
= \left[ x_1 G(x_1, \mu^2) \right] \frac{16\pi^2 \alpha_S^3}{9} \frac{1}{4 \cosh^2(\Delta y/2)} \\
\times \left[ \frac{\cosh(\Delta y)}{2 \cosh^2(\Delta y/2)} p_t^2 + \frac{m_b^2}{4} \left( 1 - \frac{p_t^2}{a^2} \right)^2 \right] \\
\times \left\{ \frac{a^2}{(a^2 + p_t^2)^3} \right\}^2 \left[ x_2 G(x_2, a^2 + p_t^2) \right]^2.\n\end{aligned} \tag{25}
$$

From the expression for *a*, we can set the factorization scale, since our cross section ceases to depend on  $k_t^2$  if  $k_t^2 \le 4m_{bt}^2$ . Therefore the reasonable choice is  $\mu^2 = 4m_{bt}^2$ . We can neglect the scale dependence in our cross section and put  $a^2 = m_b^2$ . All ingredients of Eq. (25) are clearly seen in Fig. 6.

Taking into account the running QCD coupling constant, we have to replace  $\alpha_S^3$  in Eq. (25) by  $\alpha_S(\mu^2)\alpha_S^2(a^2+p_t^2)$ .

Now let us consider the diagrams of Fig.  $5(b)$ . They cor-Now let us consider the diagrams of Fig. 5(b). They correspond to the possibility of producing a  $\overline{bb}$  pair inside of the Pomeron. Indeed, the Pomeron is not a pointlike particle; gluons inside it live a sufficiently long time, and during this gluons inside it live a sufficiently long time, and during this time they can create a  $\overline{b}b$  pair, which rescatters with the proton by exchange of only one gluon. Indeed, the lifetime of the  $G_lG_l$  pair in the diagram of Fig. 5(b) is equal to  $\tau_l = x_1 s / [k_t^2 + l_t^2 / x_l (1 - z_l)]$ , where  $z_l$  is the energy fraction of gluon *l*. This time is much larger than the time  $\tau_b$  that a of gluon *l*. This time is much larger  $\overline{bb}$  pair lives  $(\tau_b = x_1 s/M^2 \ll \tau_l)$ .

As has been discussed many times (see, for example, Refs.  $[16,20,21]$ , we can safely calculate the diagram of Fig. 5(b) by closing the contour of integration over  $\beta_l$  on the propagator marked by cross in Fig.  $5(b)$ . We anticipate that  $l_t < k_t$  and that the vertex of emission of the gluon 1' is proportional to  $l_{\mu t}$  (see Ref. [14]). The interaction of the gluon with transverse momentum  $k_t + l_t$  with the target is calculated using Eq.  $(18)$  with

$$
\Delta \Psi(p_t, l_t, z) = \Psi(p_t + l_t, z) - \Psi(p_t - l_t, z). \tag{26}
$$

Substituting Eq.  $(20)$  into Eq.  $(26)$ , one obtains, after integration over the azimuthal angle of vector  $l_t$ ,

$$
\Delta \Psi(p_t, l_t, z) = -2g l_t^2 \vec{p}_t \left\{ \delta_{\lambda_1 - \lambda_2} [(1 - 2z)\lambda_1 \pm 1] \right\}
$$

$$
\times \frac{a^2}{(a^2 + p_t^2)^2} - \delta_{\lambda_1 \lambda_2} m_b \lambda_1 \frac{1}{(a^2 + p_t^2)^2} \right\}. \tag{27}
$$

Using Eqs.  $(22)$  and  $(23)$ , we obtain  $(N_c=3)$ 

$$
\frac{d\sigma^{G*}[\text{CD}]}{dY \, dq_t^2 \, d\Delta y \, dp_t^2}\Big|_{q_t^2 = 0}
$$
\n
$$
= \frac{4\pi^2 \alpha_S^3}{9} \frac{1}{4 \cosh^2(\Delta y/2)} \Big[ \frac{\cosh(\Delta y)}{2 \cosh^2(\Delta y/2)} a^4 + m_b^2 p_t^2 \Big] \frac{1}{k_t^2} \Big\{ \frac{1}{(a^2 + p_t^2)^2} \Big\}^2 \Big[ x_2 G(x_2, k_t^2) \Big]^2. \tag{28}
$$

Notice that extra factor  $1/k_t^2$  in Eq. (28) comes from the fact that  $\Delta \Psi$  of Eq. (27) does not depend on the polarization of the gluon with transverse momentum  $k_t$ , which is proportional to  $k_t$  and cancels one of the gluon propagators in Eq.  $(23)$ . We would like to recall that in the previous calculation we assumed that  $l_t < k_t$  and that this inequality establishes the scale in the gluon structure function in Eq.  $(28)$ . The answer for the cross section of the coherent diffraction has the form

$$
\begin{aligned}\n\left| \frac{d\sigma[\text{CD}]}{dY \, dq_t^2 \, d\Delta y \, dp_t^2} \right|_{q_t^2 = 0} \\
&= \int_{-\infty}^{\infty} \frac{dk_t^2}{k_t^4} \frac{\partial x_1 G(x_1, k_t^2)}{\partial \ln k_t^2} \\
&\times [x_2 G(x_2, k_t^2)]^2 \frac{4\pi^2 \alpha_s^3(k_t^2)}{9} \frac{1}{4 \cosh^2(\Delta y/2)} \\
&\times \left[ \frac{\cosh(\Delta y)}{2 \cosh^2(\Delta y/2)} a^4 + m_b^2 p_t^2 \right] \frac{1}{(a^2 + p_t^2)^4}.\n\end{aligned} \tag{29}
$$

This equation gives the contribution for so-called coherent diffraction  $(CD)$  [10]: The greatest contribution to the integral comes from the region of sufficiently small  $k<sub>t</sub>$  due to the factor  $k_t^4$  in the dominator and for the proton  $k_t \propto 1/R_p$ , where  $R_p$  is the proton radius. It means that we cannot trust our perturbative calculation for the CD contribution. However, if we calculate the integral

$$
\int_{t_0}^{t_0} \frac{dk_t^2}{k_t^4} \frac{\partial x_1 G(x_1, k_t^2)}{\partial \ln k_t^2} [x_2 G(x_2, k_t^2)]^2, \tag{30}
$$

using the current parametrization of the gluon structure function, we can find out that the typical  $k_t^2$ , which is essential in the integral, is not very small, but about  $1-2 \text{ GeV}^2$ . To understand this fact we have to remember that the gluon structure function behaves as  $(k_t^2)^{\langle \gamma \rangle}$  (at least in a semiclassical approach) and the value of  $\langle \gamma \rangle$  calculated in the current parametrization for the gluon structure function turns out to be rather large in the region of  $k^2 \approx 1-2 \text{ GeV}^2$  (see Fig. 7). One can see that if  $\langle \gamma \rangle > 0.5$ , the integral converges on the upper limit or, in other words, the small distances start to be important.

To check this statement we calculate the integrand of Eq. (30) as a function of  $\ln(k_t^2/Q_0^2)$  with  $\partial x_1 G(x_1, k_t^2)/\partial \ln k_t^2 = 1$ . Here  $Q_0^2$ =0.34 GeV<sup>2</sup> is the initial virtuality in the GRV parametrization. The result of the calculation is plotted in Fig.  $8(a)$  for the GRV, in Fig.  $8(b)$  for the MRS( $A'$ ), and in Fig.  $8(c)$  for the CTEQ parametrizations. We see a definite maxi-

mum in the  $\ln k_t^2/Q_0^2$  dependence around  $k_t^2 \approx 1 - 1.5 \text{ GeV}^2$ , which becomes more pronounced at smaller values of  $x_2$ . It means that we can safely use the perturbative QCD approach to calculate the CD contribution.

In numerical esstimates of Eq.  $(29)$ , we use the Gribov-Lipatov-Altarelli-Parisi  $(GLAP)$  equation  $[24]$  to calculate  $\partial x_1 G(x_1, k_t^2)/\partial \ln k_t^2$ : namely,

$$
\frac{\partial x_1 G(x_1, k_t^2)}{\partial \ln k_t^2} = \frac{\alpha_S(k_t^2)}{2\pi} \left\{ \frac{4}{3} \int_x^1 \frac{dz}{z} [z^2 + 2(1-z)] \sum_i \frac{x}{z} q_i \right\}
$$

$$
\times \left( \frac{x}{z}, k_t^2 \right) + 6 \int_x^1 \frac{dz}{z} [z^2(1-z) + 1 - z] \frac{x}{z} G
$$

$$
\times \left( \frac{x}{z}, k_t^2 \right) + 6 \int_x^1 \frac{z dz}{1-z} \left[ \frac{x}{z} G \left( \frac{x}{z}, k_t^2 \right) \right]
$$

$$
-x G(x k_t^2) + 6 \left[ \frac{11}{12} - \frac{N_f}{18} \right] x G(x, k_t^2) \right\},
$$
(31)

where  $N_f$  is the number of flavors and  $N_c = 3$  is the number of colors. The running coupling QCD constant  $\alpha_S(k_t^2) = 4 \pi / [(11 - \frac{2}{3}N_f) \ln k_t^2 / \Lambda^2].$ 

It is worthwhile mentioning that in the case of diffractive dissociation in deep inelastic scattering the smallest value of  $k_t$  is  $k_t^2 = Q^2$  and Eq. (29) gives the contribution of the order of  $1/Q^2$ . In other words, the coherent diffraction in this case is a high twist contribution, while the  $(IS)$  diffraction [see Eq.  $(25)$  occurs in the leading twist. This result has been obtained in Ref. [22].

It should be stressed that there is no interference between the Ingelman-Schlein and the coherent diffraction contributions. Indeed, the interference term vanishes as a result of integration over the azimuthal angle of  $p_t$  and summation over gluon polarizations, as one can see comparing Eqs.  $(21)$ and  $(27)$  [23].

#### **IV. NUMERICAL ESTIMATES**

Setting  $x_1=0.1$ , we can estimate  $x_2 \approx 0.6 \times 10^{-3}$ . As far as the value of the cross section is concerned, we get, at  $\Delta y = 0$  and  $p_t = 0$ , the value (at  $\alpha_s = 0.25$ )

$$
\left. \frac{d\sigma}{dY\ dq_t^2\ d\Delta y\ dp_t^2} \right|_{q_t^2 = 0} \approx 0.1 \times 10^{-3} \frac{\text{mbarn}}{\text{GeV}^4}.
$$

Here we took  $x_2G(x_2, m_{bt}^2) = 20$ , which is the value in the GRV parametrization.

The result of a detailed calculation is presented in Fig. 10, below. To test the sensitivity of our result to high order QCD corrections, we plotted the cross section for the coherent diffraction for two cases: fixed and running QCD coupling constants. The difference is rather large, but it does not change the main conclusion: The coherent diffraction gives a much larger cross section than the Ingelman-Schlein contribution of Eq.  $(25)$  [IS in Fig. 9(a)]. Therefore the measurement of



FIG. 10. Integrated cross section for the coherent diffraction [Eq.  $(32)$ ] versus  $p_t^{\min}$  in different parametrizations of the gluon structure function.

the diffractive dissociation in the  $b\overline{b}$  system gives the unique opportunity to study CD unlike deep inelastic processes where CD is suppressed.

One can see from Fig.  $9(b)$  that the value of the differential cross section crucially depends on the parametrization of the gluon structure function with the difference about a factor of 2. This difference encourages us to claim that the measurement of the *bb* diffractive production could provide the selection of the parametrization and give an important contribution to the extraction of the value of the gluon structure function from experiment.

We also calculate the integrated cross section defined as

$$
\frac{d\sigma}{dY} = \int_{p_t^{\min}}^{\infty} dp_t^2 \int_{-\infty}^{+\infty} d\Delta y \int_0^{\infty} dq_t^2 \frac{d\sigma}{dY d\Delta y \, dq_t^2 \, dp_t^2},\tag{32}
$$

The value of  $p_t^{\min}$  can be taken from the experimental values obtained by the Tevatron Collider experiments. In Ref.  $[25]$ analysis techniques are used to separate muons coming from different sources and, in particular, from  $b \rightarrow \mu + \nu + c$  processes for  $p_t^{\min} \ge 5$  GeV. We assumed, in the integration over  $q_t^2$ , an exponential behavior of  $d\sigma$  with respect to  $q_t^2$ :

$$
\frac{d\sigma}{dq_t^2} = \frac{d\sigma}{dq_t^2}\bigg|_{q_t^2 = 0} e^{-bq_t^2}.
$$
 (33)

We take the slope  $b=4.9 \text{ GeV}^{-2}$  as has been measured at HERA [23]. In our estimates we took  $\alpha_s$ =0.25, which corresponds to the value of the running QCD coupling constant  $\left[\alpha_S(k^2)\right]$  at  $k^2 \sim m_b^2$ .

Figure 10 shows that the value of the integrated cross section is not very small and can be measured by the Tevatron detectors in the next run. One can also see that the differences between the estimates in different parametrizations is rather large. It is about a factor of 2–3 between the highest values of the cross section in the GRV parametrization and the lowest one in the  $MRS(A')$  parametrization.

Finally, we would like to stress that the Tevatron provides an unique possibility to look inside the microscopic mechanism of diffractive dissociation by measuring the coherent diffraction which gives a small contribution to the deep inelastic proccesses. The formulas written in this paper give us the basis for the Monte Carlo simulation of diffractive events in three-dimensional phase space  $(\eta, \phi, p_t)$ , which we are going to present in further publications. This Monte Carlo will provide more detailed estimates of the experimental possibilies at the Tevatron and, we hope, will encourage future experiments on large rapidity gap physics. We firmly believe that the diffractive dissociation opens a new window to study such difficult questions as the Pomeron structure, the matching between hard and soft processes, and the search for new collective phenomena in QCD related to the high density parton system.

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