

ρ' 's in analyzing e^+e^- annihilation: Mark III, LASS, and ARGUS data

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The results of an analysis of some recent data on the reactions $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$, $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ with the subtracted $\omega\pi^0$ events, $e^+e^- \rightarrow \omega\pi^0$, $e^+e^- \rightarrow \eta\pi^+\pi^-$, $e^+e^- \rightarrow \pi^+\pi^-$, $K^-p \rightarrow \pi^+\pi^-\Lambda$, the decays $J/\psi \rightarrow \pi^+\pi^-\pi^0$, $\tau^- \rightarrow \nu_\tau\pi^+\pi^-\pi^0$, and $\tau^- \rightarrow \nu_\tau\omega\pi^-$, upon taking into account both the strong energy dependence of the partial widths on energy and the previously neglected mixing of the ρ -type resonances, are presented. The above effects are shown to exert an essential influence on the specific values of masses and coupling constants of heavy resonances and, hence, are necessary to be accounted for in establishing their true nature. [S0556-2821(97)00803-5]

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I. INTRODUCTION

The potential interest in the energy range between 1 and 2 GeV in e^+e^- annihilation is determined by the presence there of both the higher excitations of the ground state vector nonet and the possible existence of exotic non- $q\bar{q}$ states possessing the quantum numbers allowed in the $q\bar{q}$ quark model. Earlier attempts at an interpretation of the observed resonance structures as manifestations of the production of the states with hidden exotics [1] were based on naive pure Breit-Wigner model of the production amplitude with the supposed fixed partial widths and with the neglect of the complicated dynamics of reactions. In the meantime the above energy range is characterized by the opening of a large number of the multiparticle hadronic decay modes. Their influence on the dynamics of the amplitudes is caused by the fact that the signals with the hidden exotics themselves can be imitated by the decays of the higher excitations of the ground state vector nonet proceeding via two-step processes with those hadronic states as intermediate ones [2]. The fast growth of the partial widths with energy side by side with their large magnitudes becomes essential. This results in an appreciable mixing via the common decay channels between both the heavy resonances and between them and the states from the ground state nonet.

Taking into account the crucial role of heavy ρ' resonances in the problem of the identification of the states with hidden exotics, we consistently take into account in the present paper the above-mentioned effects of the mixing and of the fast energy growth of the partial widths. To this end the production of such resonances is analyzed in the $I=1$ channel of the e^+e^- annihilation for the final states $\pi^+\pi^-$ [3,4], $\omega\pi^0$ [5,6], $\pi^+\pi^-\pi^+\pi^-$ [5,6], $\pi^+\pi^-\pi^0\pi^0$ with the subtracted $\omega\pi^0$ events [5,6], $\eta\pi^+\pi^-$ [7], in the decays $J/\psi \rightarrow \pi^+\pi^-\pi^0$ [8], $\tau^- \rightarrow \nu_\tau\pi^+\pi^-\pi^0$, $\tau^- \rightarrow \nu_\tau\omega\pi^-$ [9], and in the reaction $K^-p \rightarrow \pi^+\pi^-\Lambda$ [10]. These effects were ignored in earlier works [11,12], devoted to the analysis of the e^+e^- annihilation data and in the papers [8,10] dealing with the reactions $J/\psi \rightarrow \pi^+\pi^-\pi^0$ and $K^-p \rightarrow \pi^+\pi^-\Lambda$.

This paper is organized as follows. Section II contains the expressions for the relevant reaction cross section, for the mass spectrum of the $\pi^+\pi^-$ pair in the decay $J/\psi \rightarrow \pi^+\pi^-\pi^0$, and for the spectral functions in the τ leptonic decays. The results of the analysis of some recent data [3–10] in the framework of the approach when the resonances $\rho(770)$, ρ'_1 , ρ'_2 , and their mixing are taken into account are given in Sec. III. Section IV is devoted to the conclusions drawn from the data analysis.

II. EXPRESSIONS FOR THE CROSS SECTIONS AND MASS SPECTRUM

Let us give the expressions for the cross sections of the reactions of interest taking into account the mixing of the resonances $\rho(770)$, ρ'_1 , and ρ'_2 in the framework of the field theory-inspired approach [13] based on the summation of the loop corrections to the propagators of the unmixed states. The virtue of this approach is that corresponding amplitudes obey the unitarity requirements. First, consider the final states with the simple reaction dynamics, $e^+e^- \rightarrow \pi^+\pi^-$, $\omega\pi^0$, and $e^+e^- \rightarrow \eta\pi^+\pi^-$, leaving for a while dynamically more involved final states $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$ with subtracted $\omega\pi^0$ events. One has

$$\begin{aligned} \sigma(e^+e^- \rightarrow \rho + \rho'_1 + \rho'_2 \rightarrow f) \\ = \frac{4\pi\alpha^2}{s^{3/2}} \left| \left(\frac{m_\rho^2}{f_\rho}, \frac{m_{\rho'_1}^2}{f_{\rho'_1}}, \frac{m_{\rho'_2}^2}{f_{\rho'_2}} \right) G^{-1}(s) \begin{pmatrix} g_{\rho f} \\ g_{\rho'_1 f} \\ g_{\rho'_2 f} \end{pmatrix} \right|^2 P_f, \end{aligned} \quad (2.1)$$

where $f = \pi^+\pi^-$, $\omega\pi^0$, and $\eta\pi^+\pi^-$, s is the total center-of-mass energy squared, $\alpha = 1/137$. For a purpose of uniformity of the expression, Eq. (2.1) in the case of the $\pi^+\pi^-$ channel, the contribution of the $\rho\omega$ mixing is omitted for a while. It will be restored later on. The leptonic widths on the mass shell of the unmixed states are expressed through the leptonic coupling constants f_{ρ_i} as usual:

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$$\Gamma_{\rho_i e^+ e^-} = \frac{4\pi\alpha^2}{3f_{\rho_i}^2} m_{\rho_i}. \quad (2.2)$$

The factor P_f for the mentioned final states reads, respectively,

$$P_f \equiv P_f(s) = \frac{2}{3s} q_{\pi\pi}^3, \frac{1}{3} q_{\omega\pi}^3, \frac{1}{3} \langle q_{\rho\eta}^3 \rangle \times \frac{2}{3}. \quad (2.3)$$

The multiplier $2/3$ in the case of $\eta\pi^+\pi^-$ arising in the simplest quark model relates the $\rho\eta$ and $\omega\pi^0$ production amplitudes, provided the pseudoscalar mixing angle is taken to be $\theta_\rho = -11^\circ$, and

$$\langle q_{\rho\eta}^3 \rangle = \int_{(2m_\pi)^2}^{(\sqrt{s}-2m_\eta)^2} dm^2 \rho_{\pi\pi}(m) q^3(\sqrt{s}, m, m_\eta). \quad (2.4)$$

Hereafter, the function $\rho_{\pi\pi}(m)$ is aimed to account for the finite width of the intermediate $\rho(770)$ meson and looks as

$$\rho_{\pi\pi}(m) = \frac{\frac{1}{\pi} m \Gamma_\rho(m)}{(m^2 - m_\rho^2)^2 + [m \Gamma_\rho(m)]^2}, \quad (2.5)$$

where $\Gamma_\rho(m)$ is the width of the ρ meson determined mainly by the $\pi^+\pi^-$ decay, while

$$q_{ij} \equiv q(M, m_i, m_j) = \frac{1}{2M} \sqrt{[M^2 - (m_i - m_j)^2][M^2 - (m_i + m_j)^2]} \quad (2.6)$$

is the magnitude of the momentum of either particle i or j , in the rest frame of the decaying particle.

Let us make some remarks about the way of accounting for the $\pi^+\pi^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^0\pi^0$ decay modes. The details of the mechanisms of the decays $\rho'_{1,2} \rightarrow \pi^+\pi^-\pi^+\pi^-$ and $\rho'_{1,2} \rightarrow \pi^+\pi^-\pi^0\pi^0$ with subtracted $\omega\pi^0$ events are still poorly understood. The 4π mode is known to originate from the $\rho\pi\pi$ states. Guided by the isotopic invariance, one can express the amplitudes of production of the specific charge combinations through the amplitudes M_I with the given isospin I of the final pion pair. Then, the relation between the $I=0$ and $I=2$ amplitudes results from the absence of $\rho^0\pi^0\pi^0$ [5,6] which, in turn, permits one to write

$$M(\rho' \rightarrow \rho^0 \pi^+ \pi^-) = M_2, \\ M(\rho' \rightarrow \rho^\mp \pi^\pm \pi^0) = \frac{1}{2}(M_2 + M_1). \quad (2.7)$$

The amplitude M_2 will be further taken into account as a pointlike vertex $\rho'_{1,2} \rightarrow \rho^0 \pi^+ \pi^-$. Such an approximation seems to be justifiable since the possible $a_1(1260)\pi$ and $h_1(1170)\pi$ intermediate states containing the axial vector mesons have two partial waves in their decay into $\rho\pi$, thus resulting in a structureless angular distribution of final pions. An analogous form is assumed for the vertex $\rho^0(770) \rightarrow \rho^0(770)\pi^+\pi^-$ for the s -channel ρ meson laying off its mass shell. Taking into account the vector current

conservation, the relation $g_{\rho^0\rho^0\pi^+\pi^-} = 2g_{\rho^0\rho\pi\pi}^2$ can be considered as a guide for corresponding coupling constant [14]. The amplitude M_1 corresponds to the decay $\rho' \rightarrow \rho^+\rho^-$. Having in mind all these remarks, one can write the expressions for the cross sections. One has

$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-}(s) = \frac{(4\pi\alpha)^2}{s^{3/2}} \left| \left(\frac{m_\rho^2}{f_\rho}, \frac{m_{\rho'_1}^2}{f_{\rho'_1}}, \frac{m_{\rho'_2}^2}{f_{\rho'_2}} \right) G^{-1}(s) \right. \\ \left. \times \begin{pmatrix} 2g_{\rho\pi\pi}^2 \\ g_{\rho'_1\rho^0\pi^+\pi^-} \\ g_{\rho'_2\rho^0\pi^+\pi^-} \end{pmatrix} \right|^2 W_{\pi^+\pi^-\pi^+\pi^-}(s) \quad (2.8)$$

in the case of the final state $\pi^+\pi^-\pi^+\pi^-$, and

$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0}(s) = \frac{(4\pi\alpha)^2}{s^{3/2}} \left\{ \frac{1}{2} \left| \left(\frac{m_\rho^2}{f_\rho}, \frac{m_{\rho'_1}^2}{f_{\rho'_1}}, \frac{m_{\rho'_2}^2}{f_{\rho'_2}} \right) G^{-1}(s) \right. \right. \\ \times \begin{pmatrix} 2g_{\rho\pi\pi}^2 \\ g_{\rho'_1\rho^0\pi^+\pi^-} \\ g_{\rho'_2\rho^0\pi^+\pi^-} \end{pmatrix} \left. \right|^2 W_{\pi^+\pi^-\pi^+\pi^-}(s) \\ + \left| \left(\frac{m_\rho^2}{f_\rho}, \frac{m_{\rho'_1}^2}{f_{\rho'_1}}, \frac{m_{\rho'_2}^2}{f_{\rho'_2}} \right) G^{-1}(s) \right. \\ \left. \times \begin{pmatrix} g_{\rho\pi\pi} \\ g_{\rho'_1\rho^+\rho^-} \\ g_{\rho'_2\rho^+\rho^-} \end{pmatrix} \right|^2 W_{\pi^+\pi^-\pi^0\pi^0}(s) \right\} \quad (2.9)$$

in the case of the final state $\pi^+\pi^-\pi^0\pi^0$ with subtracted $\omega\pi^0$ events. Note that in view of universality of the ρ coupling, the relation $g_{\rho^0\rho^+\rho^-} = g_{\rho^0\pi^+\pi^-}$ holds. The final state factors in the above expressions are, respectively,

$$W_{\pi^+\pi^-\pi^+\pi^-}(s) = \frac{1}{(2\pi)^3 4s} \int_{(2m_\pi)^2}^{(\sqrt{s}-2m_\pi)^2} dm_1^2 \rho_{\pi\pi}(m_1) \int_{(2m_\pi)^2}^{(\sqrt{s}-m_1)^2} \frac{dm_2^2}{2m_2} \\ \times \left(1 + \frac{q^2(\sqrt{s}, m_1, m_2)}{3m_1^2} \right) q(\sqrt{s}, m_1, m_2) q(m_2, m_\pi, m_\pi),$$

$$W_{\pi^+\pi^-\pi^0\pi^0}(s) = \frac{1}{2\pi s} \int_{(2m_\pi)^2}^{(\sqrt{s}-2m_\pi)^2} dm_1^2 \rho_{\pi\pi}(m_1) \int_{(2m_\pi)^2}^{(\sqrt{s}-m_1)^2} \\ \times dm_2^2 \rho_{\pi\pi}(m_2) q^3(\sqrt{s}, m_1, m_1). \quad (2.10)$$

The matrix of inverse propagators looks as

$$G(s) = \begin{pmatrix} D_\rho & -\Pi_{\rho\rho'_1} & -\Pi_{\rho\rho'_2} \\ -\Pi_{\rho\rho'_1} & D_{\rho'_1} & -\Pi_{\rho'_1\rho'_2} \\ -\Pi_{\rho\rho'_2} & -\Pi_{\rho'_1\rho'_2} & D_{\rho'_2} \end{pmatrix}. \quad (2.11)$$

It contains the inverse propagators of the unmixed states $\rho_i = \rho(770), \rho'_1, \rho'_2$,

$$D_{\rho_i} \equiv D_{\rho_i}(s) = m_{\rho_i}^2 - s - i\sqrt{s}\Gamma_{\rho_i}(s), \quad (2.12)$$

where

$$\begin{aligned} \Gamma_{\rho_i}(s) &= \frac{g_{\rho_i\pi\pi}^2}{6\pi s} q_{\pi\pi}^3 + \frac{g_{\rho_i\omega\pi}^2}{12\pi} \left(q_{\omega\pi}^3 + q_{K^*K}^3 + \frac{2}{3} \langle q_{\rho\eta}^3 \rangle \right) \\ &+ \frac{3}{2} g_{\rho_i\rho^0\pi^+\pi^-} W_{\pi^+\pi^-\pi^+\pi^-}(s) \\ &+ g_{\rho_i\rho^+\rho^-} W_{\pi^+\pi^-\pi^0\pi^0}(s) \end{aligned} \quad (2.13)$$

are the energy-dependent widths, and the nondiagonal polarization operators

$$\Pi_{\rho_i\rho_j} = \text{Re}\Pi_{\rho_i\rho_j} + i\text{Im}\Pi_{\rho_i\rho_j}$$

describing the mixing. Their real parts are still unknown and further will be assumed to be some constants while the imaginary parts are given by the unitarity relation as

$$\begin{aligned} \text{Im}\Pi_{\rho_i\rho_j} &= \sqrt{s} \left[\frac{g_{\rho_i\pi\pi} g_{\rho_j\pi\pi}}{6\pi s} q_{\pi\pi}^3 + \frac{g_{\rho_i\omega\pi} g_{\rho_j\omega\pi}}{12\pi} \right. \\ &\times \left(q_{\omega\pi}^3 + q_{K^*K}^3 + \frac{2}{3} \langle q_{\rho\eta}^3 \rangle \right) \\ &+ \frac{3}{2} g_{\rho_i\rho^0\pi^+\pi^-} g_{\rho_j\rho^0\pi^+\pi^-} W_{\pi^+\pi^-\pi^+\pi^-}(s) \\ &\left. + g_{\rho_i\rho^+\rho^-} g_{\rho_j\rho^+\rho^-} W_{\pi^+\pi^-\pi^0\pi^0}(s) \right]. \end{aligned} \quad (2.14)$$

The states vector+pseudoscalar in Eqs. (2.13) and (2.14) are taken into account assuming the quark model relations between their couplings with the ρ_i resonances. However, the possibility of the violation of these relations will be included below. The $K\bar{K}$ final states in the decays of the $\rho'_{1,2}$ have relatively small branching ratios [15] and hence are neglected. In the meantime the decay $\rho(770) \rightarrow K\bar{K}$ at $\sqrt{s} > 1$ GeV is included assuming the quark model relation for the off-mass-shell $\rho(770)$ couplings.

To describe the Mark III data [8] on the $\pi^+\pi^-$ mass spectrum in the decay $J/\psi \rightarrow \pi^+\pi^-\pi^0$, taking into account the cut in the cosine of the angle of the momentum of outgoing pions in the $\pi^+\pi^-$ rest system relative to the ρ^0 momentum in the center-of-mass (c.m.s.), $|\cos\theta_1| \leq 0.2$, one can use the expression

$$\begin{aligned} \frac{d\Gamma}{dm}(J/\psi \rightarrow \pi^+\pi^-\pi^0) &= \frac{1}{6(2\pi)^3} [q(m_{J/\psi}, m, m_\pi) q(m, m_\pi, m_\pi)]^3 \\ &\times \int_{-0.2}^{0.2} d\cos\theta_1 \sin^2\theta_1 \cdot |A(m^2) + A(m_+^2) + A(m_-^2)|^2 \end{aligned} \quad (2.15)$$

with

$$A(m^2) = (F_1, F_2, F_3) G^{-1}(m^2) \begin{pmatrix} g_{\rho\pi\pi} \\ g_{\rho'_1\pi\pi} \\ g_{\rho'_2\pi\pi} \end{pmatrix}, \quad (2.16)$$

where

$$\begin{aligned} m_\pm^2 &= \frac{1}{2} (m_{J/\psi}^2 + 3m_\pi^2 - m^2) \\ &\pm 2m_{J/\psi} q(m_{J/\psi}, m, m_\pi) q(m, m_\pi, m_\pi) \cos\theta_1 / m, \end{aligned} \quad (2.17)$$

$F_{1,2,3}$ are the production amplitudes of the $\rho(770)$, ρ'_1 , and ρ'_2 resonances in the J/ψ decays whose explicit form will be specified below, m being an invariant mass of the $\pi^+\pi^-$ pair.

The ARGUS data [9] on the decays $\tau^- \rightarrow \nu_\tau \pi^+ \pi^- \pi^- \pi^0$ and $\tau^- \rightarrow \nu_\tau \omega \pi^-$ are analyzed, respectively, in terms of the spectral function $v_1(m)$ [16]:

$$\begin{aligned} v_1(m) &= \frac{d\Gamma_{4\pi\nu}(m)}{dm} \frac{16\pi^2 m_\tau^3}{G_F^2 \cos^2\theta_C} \frac{1}{m(m_\tau^2 - m^2)^2 (m_\tau^2 + 2m^2)} \\ &= \frac{m^2}{(2\pi\alpha)^2} \left[\frac{1}{2} \sigma_{e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-}(m^2) \right. \\ &\quad \left. + \sigma_{e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0}(m^2) \right] \end{aligned} \quad (2.18)$$

[17], and

$$v_{1\omega\pi}(m) = \frac{m^2}{(2\pi\alpha)^2} \sigma_{e^+e^- \rightarrow \omega\pi^0}(m^2), \quad (2.19)$$

with m being an invariant mass of corresponding hadronic final state.

Finally, to fit the LASS data [10] on the modulus of the $\pi^+\pi^-$ production amplitude in the reaction $K^- p \rightarrow \pi^+\pi^-\Lambda$, one can use the expression analogous to Eq. (2.1) in the case of $f = \pi^+\pi^-$, but without the factor $s^{-3/2}$ pertinent to the one-photon e^+e^- annihilation and with the proper relative production amplitudes instead of $m_{\rho'_i}^2/f_{\rho_i}$ as is exemplified in Eq. (2.16).

III. RESULTS AND DISCUSSION

Let us describe briefly the procedure of the fit to the data. Since the data on the e^+e^- annihilation

$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-, \quad (3.1)$$

$$e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0, \quad (3.2)$$

with subtracted $\omega\pi^0$ events

$$e^+e^- \rightarrow \omega\pi^0, \quad (3.3)$$

$$e^+e^- \rightarrow \pi^+\pi^-\eta, \quad (3.4)$$

$$e^+e^- \rightarrow \pi^+\pi^-, \quad (3.5)$$

on the decays

$$J/\psi \rightarrow \pi^+\pi^-\pi^0, \quad (3.6)$$

$$\tau^- \rightarrow \nu_\tau\pi^+\pi^-\pi^0, \quad (3.7)$$

$$\tau^- \rightarrow \nu_\tau\omega\pi^-, \quad (3.8)$$

and on the reaction

$$K^-p \rightarrow \pi^+\pi^-\Lambda \quad (3.9)$$

are gathered in diverse experiments and still possess large uncertainties, the fit is carried out for each channel separately by means of the χ^2 minimization. The fitted parameters for the reactions in e^+e^- annihilation and for the τ leptonic decays are

$$m_{\rho'_{1,2}}, g_{\rho'_{1,2}\pi^+\pi^-}, g_{\rho'_{1,2}\omega\pi}, g_{\rho'_{1,2}\rho^0\pi^+\pi^-}, g_{\rho'_{1,2}\rho^+\rho^-}, f_{\rho'_{1,2}}, \text{Re}\Pi_{\rho'_1\rho'_2}, \quad (3.10)$$

while for the reactions (3.6) and (3.9) one should take the relative production amplitudes F_1 , F_2 , and F_3 [see Eq. (2.15)] instead of the leptonic couplings, because the production mechanisms in these reactions are different. The real parts of nondiagonal polarization operators $\text{Re}\Pi_{\rho\rho'_1}$ and $\text{Re}\Pi_{\rho\rho'_2}$ are set to zero. Indeed, in a sharp distinction with the imaginary parts $\text{Im}\Pi_{\rho_i\rho_j}$ fixed by the unitarity relation, the real parts $\text{Re}\Pi_{\rho_i\rho_j}$ cannot be evaluated at present and should be treated as free parameters. However, one must have in mind that nonzero $\text{Re}\Pi_{\rho\rho'_{1,2}} \neq 0$ result in an appreciable mass shift of the $\rho(770)$ resonance,

$$\delta m_\rho \simeq -\frac{1}{2m_\rho} \text{Re} \left[\frac{\Pi_{\rho\rho'_{1,2}}^2(m_\rho)}{m_{\rho'_{1,2}}^2 - m_\rho^2 - im_\rho[\Gamma_{\rho'_{1,2}}(m_\rho) - \Gamma_\rho(m_\rho)]} \right] \quad (3.11)$$

[18,19]. Since the minimization fixes only the combination $m_\rho + \delta m_\rho$, it is natural to assume that the dominant contribution to the mass renormalization equation (3.11) coming from $(\text{Re}\Pi_{\rho\rho'_{1,2}})^2$ is already subtracted, so that the mass of the ρ meson minimizing the χ^2 function differs from the actual position of the ρ peak by the magnitudes quadratic in $\text{Im}\Pi_{\rho\rho'_{1,2}}$. In practice, it manifests itself in that one seeks the minimum of the χ^2 given by the values of m_ρ which are near 770 MeV. Then, the minimization procedure automatically chooses (with rather large errors, of course) the values of

$\text{Re}\Pi_{\rho\rho'_{1,2}}$ lying close to zero. By this reason, having in mind the existing accuracy of the data, it is natural to set these parameters to zero from the very start. These considerations justifiable in the case of the $\rho(770)$ resonance whose $q\bar{q}$ nature is firmly established, however, cannot be applied to the case of heavy $\rho'_{1,2}$ resonances. The nature of these resonances is, in fact, not yet established, and one cannot exclude that they may contain an appreciable portion of exotics like $q\bar{q}g$, $q\bar{q}q\bar{q}$, etc. [20]. Hence, it is a matter of principle to extract from the existing data the values of masses and coupling constants of bare unmixed states in order to compare them with current predictions. By this reason, $\text{Re}\Pi_{\rho'_1\rho'_2}$ is considered to be a free parameter.

The parameters of the $\rho(770)$ are chosen from the fitting to the pion form factor from the threshold to 1 GeV upon taking into account both the $\rho\omega$ mixing and the mixing of the $\rho(770)$ with the $\rho'_{1,2}$ resonances originating from their common decay modes. To allow for the $\rho\omega$ mixing, one should add the term

$$\frac{m_\omega^2 \Pi_{\rho\omega}}{f_\omega D_\rho D_\omega} g_{\rho\pi\pi}$$

to the expression in between the modulus sign in Eq. (2.1) in the case of the $\pi^+\pi^-$ channel, where

$$D_\omega \equiv D_\omega(s) = m_\omega^2 - s - i\sqrt{s}\Gamma_\omega(s),$$

$$\Gamma_\omega(s) = \frac{g_{\omega\rho\pi}^2}{4\pi} W_{3\pi}(s) \quad (3.12)$$

are, respectively, the inverse propagator of the ω meson and its width determined mainly by the $\pi^+\pi^-\pi^0$ decay mode while $W_{3\pi}(s)$ stands for the phase-space volume of the final 3π state (see its expression in, e.g., [18]). Here, the real part of the polarization operator of the $\rho\omega$ transition is taken in the form

$$\text{Re}\Pi_{\rho\omega} = 2m_\omega \delta_{\rho\omega} + \frac{4\pi\alpha m_\rho^2 m_\omega^2}{f_\rho f_\omega} (1/m_\omega^2 - 1/s), \quad (3.13)$$

$\delta_{\rho\omega}$ is the amplitude of the $\rho\omega$ transition as measured at the ω mass while the last term is aimed to take into account the fast-varying one-photon contribution. The expression for imaginary part $\text{Im}\Pi_{\rho\omega}$ is given in [21]. Note that $g_{\rho\omega\pi} = g_{\omega\rho\pi} = 14.3 \text{ GeV}^{-1}$ [5]. The $\rho(770)$ parameters obtained from fitting the $e^+e^- \rightarrow \pi^+\pi^-$ channel are $m_\rho = 774 \pm 10 \text{ MeV}$, $g_{\rho\pi\pi} = 5.9 \pm 0.2$, $f_\rho = 5.1 \pm 0.2$, $\delta_{\rho\omega} = 2.4 \pm 1.4 \text{ MeV}$. The error bars are determined from the function χ^2 . Notice that the $\rho\omega$ mixing in the reaction (3.6) is neglected, since the Mark III data show no $\rho\omega$ interference pattern and hence the fit is insensitive to additional free parameters characterizing the $\omega\pi$ coupling to the J/ψ . As for the reaction (3.9), the amplitude of the $\rho\omega$ transition $\delta_{\rho\omega}$ is fixed to be 2.4 MeV, while the relative ω production amplitude is varied. The fit of the LASS data [10] turns out to be insensitive to the specific value of this amplitude.

It seems to be rather natural that the function of 13 variables, Eq. (3.10), χ^2 possesses a number of local minima characterized by the parameters considerably differing (by

TABLE I. The magnitudes of the parameters of the ρ'_1 giving the best description of the data on the reaction equations (3.1)–(3.9). The error bars are determined from the function χ^2 . The parameter $F_{\rho'_1} = F_2/F_1$ is the relative production amplitude of ρ'_1 .

Eq.	$m_{\rho'_1}$ (GeV)	$g_{\rho'_1\pi^+\pi^-}$	$g_{\rho'_1\omega\pi}$ (GeV $^{-1}$)	$g_{\rho'_1\rho^0\pi^+\pi^-}$	$g_{\rho'_1\rho^+\rho^-}$	$F_{\rho'_1}$
(3.1)	1.35 ± 0.05	$-9^{+10}_{-11} \times 10^{-1}$	$14.9^{+3.6}_{-2.6}$	< 25	< 72	$2.1^{+0.5}_{-0.4}$
(3.2)	$1.40^{+0.22}_{-0.14}$	< 18	$16.6^{+4.6}_{-3.2}$	< 210	7 ± 3	$2.4^{+0.6}_{-0.5}$
(3.3)	~ 1.4	$< 72 \times 10^{-1}$	< 10	< 210	Undetermined	$< 66 \times 10^{-1}$
(3.4)	$1.46^{+0.30}_{-0.40}$	< 39	19^{+11}_{-6}	< 240	< 114	3.7 ± 1.0
(3.5)	$1.37^{+0.09}_{-0.07}$	-1.0 ± 0.3	$16.6^{+2.2}_{-1.5}$	< 150	< 45	2.3 ± 0.2
(3.6)	$1.57^{+0.25}_{-0.19}$	$(-17^{+12}_{-13}) \times 10^{-1}$	21^{+3}_{-7}	< 660	< 57	$12^{+8}_{-2} \times 10^{-1}$
(3.7)	~ 1.4	< 15	< 110	< 240	Undetermined	$< 16 \times 10^{-1}$
(3.9)	$1.36^{+0.18}_{-0.16}$	$< 57 \times 10^{-1}$	$13.7^{+4.3}_{-3.2}$	< 540	< 48	2.1 ± 0.5

more than three standard deviations) from channel to channel, Eqs. (3.1)–(3.9). The final choice is implemented under the demand of possibility of the simultaneous fit to all the channels in the framework of a uniform approach. The results are presented in Tables I and II and in Figs. 1–9. Taking into account yet large uncertainties of the data, one can conclude that the magnitudes of the parameters obtained from the fitting of diverse channels do not contradict one another.

Let us dwell on the role played by the coupling constant $g_{\rho'_{1,2}\rho^+\rho^-}$ in the analysis of the reaction equation (3.2) with the subtracted $\omega\pi^0$ events. The current data on this channel are contradictory (see Fig. 2). In the meantime, the ND [5] and DM2 [6] data on the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$, Fig. 1, are consistent in the region of overlap. So it would be quite natural to take the parameters of resonances extracted from fitting the reaction equation (3.1) and to vary them within the error bars in order to describe both the data [5] with $\sqrt{s} \leq 1.4$ GeV and the nonoverlapping data [6] with $\sqrt{s} > 1.7$ GeV. As it appears, the better description is achieved under introduction of a nonzero $g_{\rho'_{1,2}\rho^+\rho^-} = 7 \pm 3$ in the case of the channel $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$; in the meantime the analysis of the remaining channels gives the magnitudes of this coupling constant which do not contradict zero (see Table I) [22]. If the value of $g_{\rho'_{1,2}\rho^+\rho^-} = 0$ were fixed from the very start, the central value of the leptonic width $\Gamma_{\rho'_1 ee} = 12.9^{+4.5}_{-4.3}$ keV extracted from the reaction equation (3.2) would deviate by more than the factor of 2 from

the value $\Gamma_{\rho'_1 ee} = 5.2^{+2.2}_{-1.9}$ keV obtained from the reaction equation (3.1), though not going from the double standard deviation. The final resolution could be possible only after gathering new consistent data. Note that the contribution of the $\rho^0\rho^-$ state in the decay $\tau^- \rightarrow \nu_\tau\pi^+\pi^-\pi^-\pi^0$ is calculated to be small in accord with the ARGUS data [23].

The bare masses of the resonances $\rho'_{1,2}$ are seen to be considerably higher than the actual position of the peaks or structures in cross sections and mass spectra. This is explained by two reasons. First, the fast growth of the partial widths with energy results in the shift

$$\delta m_{\rho'_{1,2}} \sim -\Gamma(s) \frac{d\Gamma}{d\sqrt{s}}(\sqrt{s} = m_{\rho'_{1,2}}) \quad (3.14)$$

towards the lower values from the bare masses. Second, there exists the shift due to the mixing [18] of the upper of two states,

$$\delta m_{\rho'_{1,2}} \simeq \frac{1}{2m_{\rho'_{1,2}}} \times \text{Re} \left[\frac{\Pi_{\rho\rho'_{1,2}}^2(m_{\rho'_{1,2}})}{m_{\rho'_{1,2}}^2 - m_\rho^2 - im_{\rho'_{1,2}}[\Gamma_{\rho'_{1,2}}(m_{\rho'_{1,2}}) - \Gamma_\rho(m_{\rho'_{1,2}})]} \right], \quad (3.15)$$

TABLE II. The same as in Table I but for the ρ'_2 . To avoid the introduction of additional free parameters in the case of the reaction equations (3.6) and (3.7), $\text{Re}\Pi_{\rho'_1\rho'_2}$ is fixed to zero, while the slope of the mass dependence of the relative production amplitude in the decay equation (3.6) (see the body of the paper) is varied. Note that the LASS data do not put any constraint on the ρ'_2 parameters.

Eq.	$m_{\rho'_2}$ (GeV)	$g_{\rho'_2\pi^+\pi^-}$	$g_{\rho'_2\omega\pi}$ (GeV $^{-1}$)	$g_{\rho'_2\rho^0\pi^+\pi^-}$	$g_{\rho'_2\rho^+\rho^-}$	$F_{\rho'_2}$	$\text{Re}\Pi_{\rho'_1\rho'_2}$ (GeV 2)
(3.1)	$1.851^{+0.027}_{-0.024}$	$(18 \pm 11) \times 10^{-1}$	$-6.1^{+0.7}_{-0.8}$	-222 ± 9	< 24	2.9 ± 0.1	$< 3 \times 10^{-1}$
(3.2)	$1.79^{+0.11}_{-0.07}$	< 20	-10 ± 3	-184^{+23}_{-32}	< 15	2.9 ± 0.4	$< 5 \times 10^{-1}$
(3.3)	1.71 ± 0.09	$< 33 \times 10^{-1}$	-6.0 ± 1.2	-188 ± 22	< 48	2.9 ± 0.4	$< 12 \times 10^{-1}$
(3.4)	$1.91^{+1.00}_{-0.37}$	< 42	< 45	< 960	< 165	$< 28 \times 10^{-1}$	$< 36 \times 10^{-1}$
(3.5)	$1.90^{+0.17}_{-0.13}$	$< 18 \times 10^{-1}$	< 18	-63^{+19}_{-55}	< 30	2.8 ± 0.8	$< 6 \times 10^{-1}$
(3.6)	$2.08^{+0.16}_{-0.30}$	$< 33 \times 10^{-1}$	-12^{+7}_{-4}	-180^{+190}_{-130}	< 144	$-18^{+2}_{-6} \times 10^{-1}$	$\equiv 0$
(3.7)	$1.86^{+0.26}_{-0.16}$	< 15	-7^{+5}_{-4}	-210^{+90}_{-100}	< 75	4.3 ± 0.6	$\equiv 0$

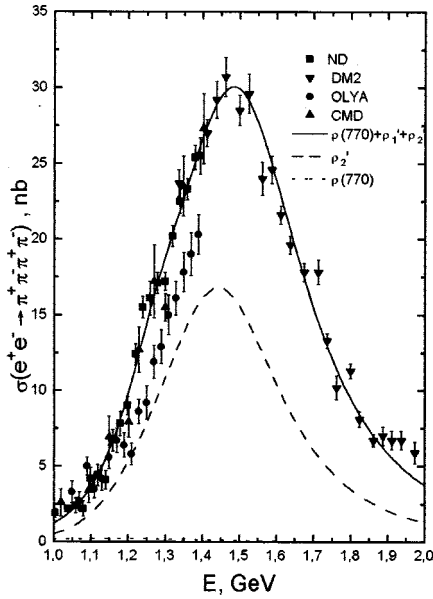


FIG. 1. The result of the description of the reaction equation (3.1). The data are: ND [5], DM2 [6], CMD [24], and OLYA [25].

which is negative, since the nondiagonal polarization operator of the $\rho\rho'_{1,2}$ mixing dominates over the $\rho'_1\rho'_2$ mixing and possesses, in the present case, the imaginary part much greater than the real part. In the meantime, the corresponding mass shift of the $\rho(770)$ in the $\pi^+\pi^-$ channel due to the mixing with higher states, see Eq. (3.11), turns out to be rather small, $\delta m_\rho \approx 4$ MeV, since $\text{Im}\Pi_{\rho\rho'_{1,2}}$ is small at $\sqrt{s} \approx m_\rho$. Plotting the contributions of the unmixed states in Figs. 1–4 shows that the dominant contribution to the mass shifts comes from the fast growth of the partial widths, because the resonance peaks without the mixing being taken into account are also displaced considerably.

The magnitude of the real part $\text{Re}\Pi_{\rho'_1\rho'_2}$ does not contradict zero. However, χ^2 is minimized with nonzero values of

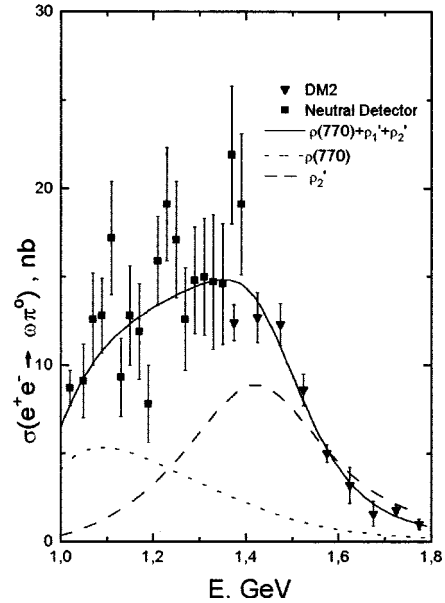


FIG. 3. The result of the description of the reaction equation (3.3). The data are: ND [5], DM2 [6].

this parameter though with large errors. This means that the quality of the data is still insufficient to establish $\text{Re}\Pi_{\rho'_1\rho'_2} \neq 0$. Note also that a better description of the channel $\pi^+\pi^-\eta$ is achieved upon introducing the suppression factor $y_\eta = 0.7 \pm 0.2$ of the coupling constant $g_{\rho'_1\rho\eta}$ as compared to the value $\sqrt{2/3}g_{\rho'_1\omega\pi}$ given by the simplest quark model.

The fit to the $\pi^+\pi^-$ mass spectrum in the decay $J/\psi \rightarrow \pi^+\pi^-\pi^0$ with the relative production amplitudes independent of the $\pi^+\pi^-$ -invariant mass gives all the resonance parameters but $g_{\rho'_1\pi^+\pi^-} = -2.8^{+0.5}_{-0.4}$ in agreement with other channels. This latter value strongly deviates from $-0.9^{+1.0}_{-1.1}$ and -1.0 ± 0.3 extracted from the channels equations (3.1) and (3.5), respectively. Hence, we tried to include

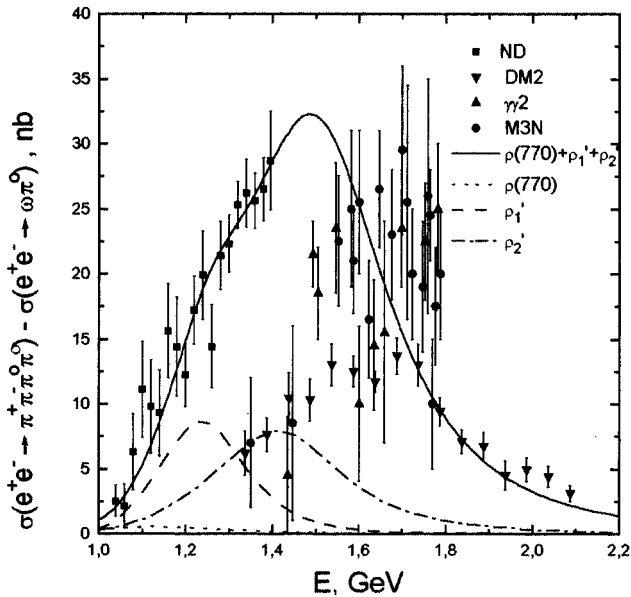


FIG. 2. The result of the description of the reaction equation (3.2). The data are: ND [5], DM2 [6], $\gamma\gamma 2$, and M3N [26].

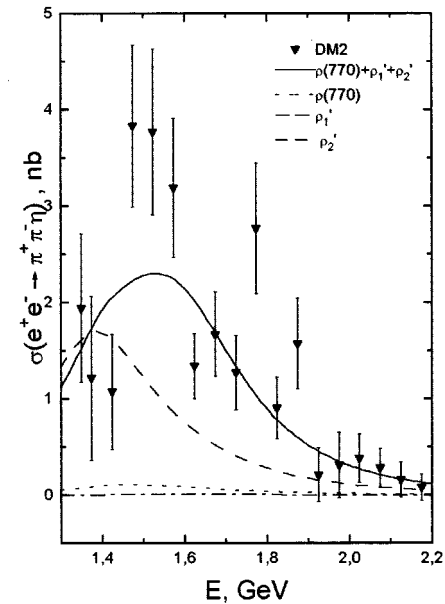


FIG. 4. The result of the description of the reaction equation (3.4). The data are from Ref. [7].

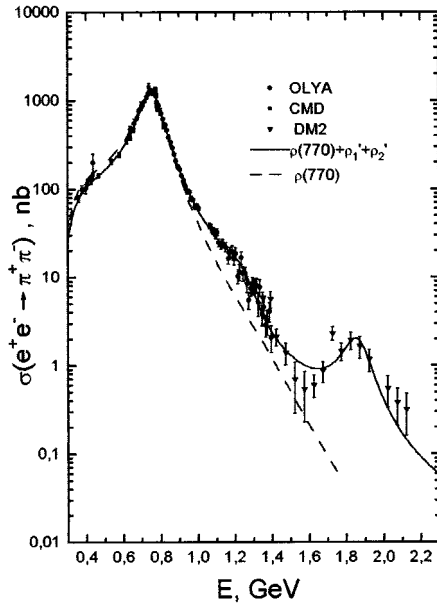


FIG. 5. The result of the description of the reaction equation (3.5). The data are: OLYA and CMD [3], DM2 [4].

the mass dependence in the simplest linear form $F_i(m^2) = a_i + b_i(m^2 - m_i^2)$. This is equivalent to the introduction of a background which generates the structure at $\sqrt{s} \sim 1.3$ GeV. As a result, the agreement with other channels is achieved but at the expense of poor determination of the ρ_1' mass. It is this variant that is included in Table I. Note that the slopes of the mass dependence $b_{\rho_{1,2}'}$ are compatible with zero while $b_{\rho(770)}$ is not. Specifically, the relative $\rho(770)$ production amplitude normalized to unity at $m = m_\rho$ is $F_\rho(m^2) = 1 + 20_{-5}^{+3} \times 10^{-2} \text{ GeV}^{-2} (m^2 - m_\rho^2)$.

As for the τ decays, we fit only the spectral function for the mode $\tau^- \rightarrow \nu_\tau \pi^+ \pi^- \pi^- \pi^0$. The obtained parameters coincide, within the error bars, with the parameters extracted

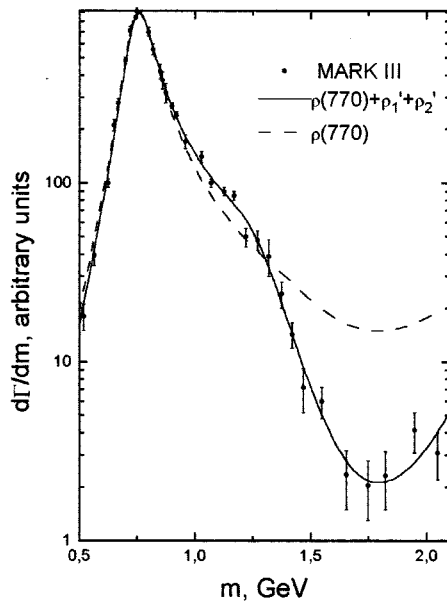


FIG. 6. The result of the description of the $\pi^+ \pi^-$ mass spectrum [8] in the decay equation (3.6).

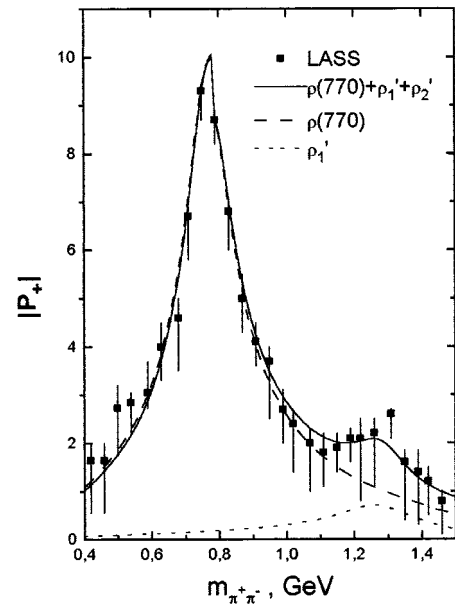


FIG. 7. The result of the description of the data [10] on the modulus of the p -wave amplitude of the reaction equation (3.9).

from the fitting of the cross section of the reaction $e^+e^- \rightarrow \pi^+ \pi^- \pi^- \pi^-$. The ρ_1' resonance is not needed for explanation of the mass behavior of the spectral function $v_1(m)$. The result is shown in Fig. 8. With these parameters we calculate the spectral function $v_{1\omega\pi}$ for the decay $\tau^- \rightarrow \nu_\tau \omega \pi^-$. The result is shown in Fig. 9.

Note that the Blatt-Weiskopf range parameters which are sometimes introduced into the expressions for the partial widths of the $\rho(770)$ to make acceptable their fast growth with energy, are chosen to be zero by the χ^2 minimization. Hence, corresponding factors are omitted in the expressions for the partial widths. In the meantime, the inclusion of such

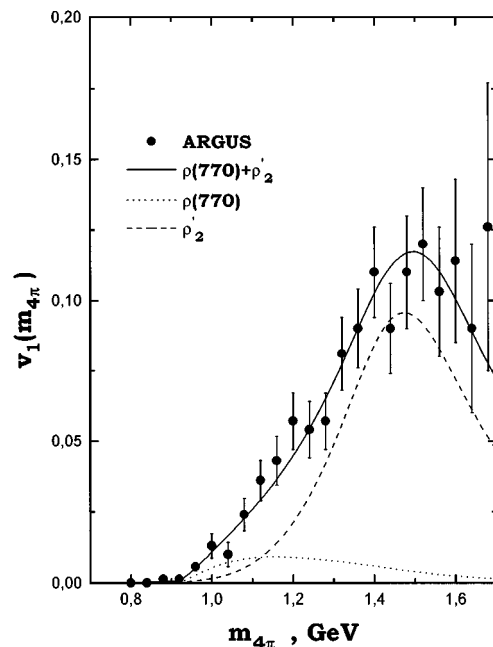


FIG. 8. The spectral function for the decay $\tau^- \rightarrow \nu_\tau \pi^+ \pi^- \pi^- \pi^0$. The data are from [9].

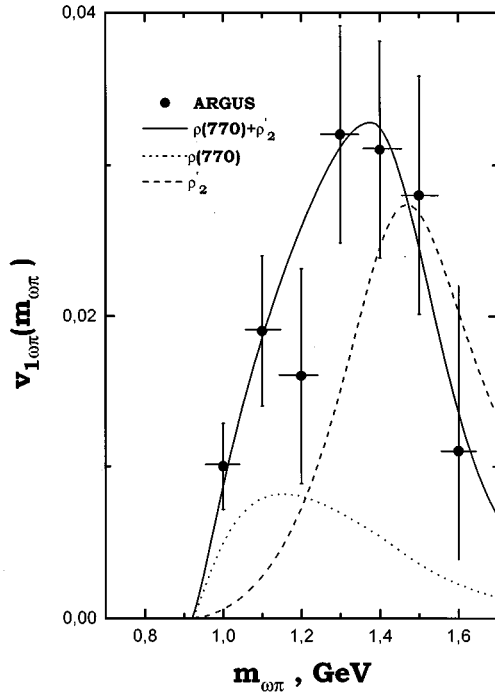


FIG. 9. The spectral function for the decay $\tau^- \rightarrow \nu_\tau \omega \pi^-$. The data are from [9].

factors in the case of heavy $\rho'_{1,2}$ resonances is unnecessary because the energies of the present interest are in the mass range of these states.

IV. CONCLUSION

The main conclusion from the present analysis is that the inclusion of both the mixing of heavy isovector resonances and the energy dependence of their partial widths is com-

pletely necessary when describing the data on the reaction equations (3.1)–(3.9). The possibility of simultaneous fit of the existing data and the specific magnitudes of the extracted parameters necessary for the comparison with current models, are crucially affected by these effects. It should be emphasized that the fits to the data on the reactions, Eqs. (3.1)–(3.4), (3.7), and (3.8) do not at all demand the presence of the resonance ρ'_1 . The large magnitudes of the coupling constant $g_{\rho'_1 \omega \pi}$ for these reactions given in Table I are already pointed out to be chosen under the demand of the possibility of simultaneous description of all variety of data including the $\pi^+ \pi^-$ production reactions (3.5), (3.6), and (3.9). If one discards these latter reactions, the variants of the fits exist giving the couplings of the ρ'_1 resonance compatible with zero.

The novel dynamical feature revealed in the present analysis is the possible nonzero magnitude of the coupling constants $g_{\rho'_{1,2} \rho^+ \rho^-}$. The threshold region in the reaction $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$ with the subtracted $\omega \pi^0$ events is especially promising for improvement of the quality of extraction of above coupling constants. The real part of the polarization operator $\text{Re} \Pi_{\rho'_1 \rho'_2}$ is determined poorly from the current data. The study of the $e^+ e^-$ annihilation channel equations (3.1)–(3.5) with good statistics and on the same facility is urgent for measuring these important dynamical parameters required for establishing the nature of heavy isovector resonances and for the final elucidation of the situation.

ACKNOWLEDGMENTS

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