

Baryon magnetic moments and proton spin: A model with collective quark rotation

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We analyze the baryon magnetic moments in a model that relates them to the parton spins Δu , Δd , Δs , and includes a contribution from orbital angular momentum. The specific assumption is the existence of a three-quark correlation (such as a flux string) that rotates with angular momentum $\langle L_z \rangle$ around the proton spin axis. A fit to the baryon magnetic moments, constrained by the measured values of the axial vector coupling constants $a^{(3)} = F + D$, $a^{(8)} = 3F - D$, yields $\langle S_z \rangle = 0.08 \pm 0.13$, $\langle L_z \rangle = 0.39 \pm 0.09$, where the error is a theoretical estimate. A second fit, under slightly different assumptions, gives $\langle L_z \rangle = 0.37 \pm 0.09$, with no constraint on $\langle S_z \rangle$. The model provides a consistent description of axial vector couplings, magnetic moments, and the quark polarization $\langle S_z \rangle$ measured in deep inelastic scattering. The fits suggest that a significant part of the angular momentum of the proton may reside in a collective rotation of the constituent quarks. [S0556-2821(97)03503-0]

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I. INTRODUCTION

The question of the angular momentum composition of the proton, first raised in the context of the quark parton model in 1974 [1], has developed into a burning issue, following experiments on polarized deep inelastic scattering and progress in the theoretical understanding of QCD. Within the quark parton model, the contribution of polarized quarks and antiquarks to the spin of a polarized proton ($J_z = 1/2$) is [1]

$$\langle S_z \rangle = \frac{1}{2}(\Delta u + \Delta d + \Delta s) \equiv \frac{1}{2} \Delta \Sigma, \quad (1)$$

with

$$\Delta \Sigma = (3F - D) + \delta_{\text{EJ}}.$$

Here Δq is the net polarization of quarks of flavor q , $\Delta q = \int dx [\{q_+(x) - q_-(x)\} + \{\bar{q}_+(x) - \bar{q}_-(x)\}]$, F and D are the axial vector coupling constants of β decay ($F = 0.462 \pm 0.01$, $D = 0.794 \pm 0.01$ [2]), and δ_{EJ} is the ‘‘defect’’ in the Ellis-Jaffe sum rule [3]:

$$\delta_{\text{EJ}} = \int g_1^p(x) dx - \left(\frac{1}{2}F - \frac{1}{18}D\right). \quad (2)$$

In QCD, the expression for $\Delta \Sigma$ is modified by perturbative gluon corrections [4] and by a contribution from the gluon anomaly in the singlet axial vector current [5], and reads

$$\Delta \Sigma = (3F - D) + \delta_{\text{EJ}}(Q^2) + \delta_{\text{anomaly}}, \quad (3)$$

where, to lowest order in α_s/π ,

$$\delta_{\text{EJ}}(Q^2) = \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)^{-1} \int g_1^p(x, Q^2) dx - \left(\frac{1}{2}F - \frac{1}{18}D\right), \quad (4)$$

$$\delta_{\text{anomaly}} = n_f \frac{\alpha_s}{2\pi} \Delta G. \quad (5)$$

Here ΔG is the net gluon polarization, $\Delta G = \int dx [G_+(x) - G_-(x)]$, and $n_f = 3$ is the number of light quark flavors. A number of authors [6] have analyzed the data [7] on the structure functions $g_1^{p,n}$, and have reached the conclusion that, barring a large correction from the anomalous term δ_{anomaly} , $\Delta \Sigma$ lies in the interval

$$\Delta \Sigma \approx (0.1, \dots, 0.3). \quad (6)$$

Thus the polarization of the quarks and antiquarks accounts for only 10–30 % of the spin of the proton, a typical solution for the spin decomposition being $\Delta u = 0.83 \pm 0.03$, $\Delta d = -0.43 \pm 0.03$, and $\Delta s = -0.10 \pm 0.03$ [8].

II. BARYON MAGNETIC MOMENTS

In Ref. [1], a tentative attempt was made to relate the nucleon magnetic moments to the spin structure of the proton, encoded in the parameters Δu , Δd , and Δs . This idea has recently been generalized to the full baryon octet in two papers [9,10] that have investigated the following ansatz for the magnetic moments:

$$\mu(p) = \mu_u \delta u + \mu_d \delta d + \mu_s \delta s, \quad (7)$$

$$\mu(n) = \mu_u \delta d + \mu_d \delta u + \mu_s \delta s,$$

$$\mu(\Sigma^+) = \mu_u \delta u + \mu_d \delta s + \mu_s \delta d,$$

$$\mu(\Sigma^-) = \mu_u \delta s + \mu_d \delta u + \mu_s \delta d,$$

$$\mu(\Xi^-) = \mu_u \delta s + \mu_d \delta d + \mu_s \delta u,$$

$$\mu(\Xi^0) = \mu_u \delta d + \mu_d \delta s + \mu_s \delta u,$$

$$\begin{aligned} \mu(\Lambda^0) &= \frac{1}{6}(\delta u + 4\delta d + \delta s)(\mu_u + \mu_d) \\ &\quad + \frac{1}{6}(4\delta u - 2\delta d + 4\delta s)\mu_s. \end{aligned}$$

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TABLE I. Fit to baryon magnetic moments in model (A). Magnetic moments are in nucleon magnetons and the ± 0.1 is a fictive theoretical error.

	Magnetic moments	Model 0 $S_z = \frac{1}{2}, L_z = 0$	Model AI S_z free, $L_z = 0$	Model AII $S_z + L_z = \frac{1}{2}$	Model AIII S_z, L_z free
$\mu(p)$	2.79 ± 0.1 ± 0.00000006	2.67	2.68	2.74	2.74
$\mu(n)$	-1.91 ± 0.1 ± 0.00000005	-1.92	-1.84	-1.78	-1.79
$\mu(\Sigma^+)$	2.46 ± 0.1 ± 0.01	2.54	2.58	2.52	2.52
$\mu(\Sigma^-)$	-1.16 ± 0.1 ± 0.025	-1.14	-1.21	-1.20	-1.20
$\mu(\Xi^-)$	-0.65 ± 0.1 ± 0.0025	-0.48	-0.60	-0.60	-0.60
$\mu(\Xi^0)$	-1.25 ± 0.1 ± 0.014	-1.40	-1.34	-1.38	-1.39
$\mu(\Lambda)$	-0.61 ± 0.1 ± 0.004	-0.61	-0.60	-0.60	-0.61
Input		$\Delta u = \frac{4}{3}$ $\Delta d = -\frac{1}{3}$ $\Delta s = 0$	$\mu_u = -2\mu_d$ $\mu_s = \frac{3}{5}\mu_d$ $G_A = 1.26$	$\mu_u = -2\mu_d$ $\mu_s = \frac{3}{5}\mu_d$ $G_A = 1.26$	$\mu_u = -2\mu_d$ $\mu_s = \frac{3}{5}\mu_d$ $G_A = 1.26$ $a^{(8)} = 0.60$
χ^2/N_{DOF}		1.82	1.12	1.105	1.095
Fitted parameters		$\mu_u = 1.75 \pm 0.06$ $\mu_d = -1.01 \pm 0.06$ $\mu_s = -0.61 \pm 0.05$	$\mu_u = 2.17 \pm 0.09$ $S_z = 0.14 \pm 0.12$ $a^{(8)} = 0.85 \pm 0.06$ exp: 0.60 ± 0.02	$\mu_u = 2.17 \pm 0.09$ $S_z = 0.11 \pm 0.14$ $a^{(8)} = 0.60 \pm 0.10$ exp: 0.60 ± 0.02	$\mu_u = 2.17 \pm 0.08$ $S_z = 0.08 \pm 0.13$ $L_z = 0.39 \pm 0.09$

The baryon magnetic moments are linear combinations of δu , δd , and δs , defined by $\delta q = \int dx [\{q_+(x) - q_-(x)\} - \{\bar{q}_+(x) - \bar{q}_-(x)\}]$, which differs from Δq in the sign of the antiquark contribution. We consider two hypotheses for the relation between δq and Δq .

(A) Antiquarks in a polarized baryon reside entirely in a cloud of spin-zero mesons. In this case, antiquarks have no net polarization, i.e., $\bar{q}_+ - \bar{q}_- = 0$, so that $\delta q = \Delta q$. Models of this type have been discussed, for instance, by Cheng and Li [11].

(B) Antiquarks in a polarized baryon are generated entirely by the perturbative splitting of gluons $g \rightarrow q\bar{q}$. In such a case, it is reasonable to expect $\bar{u}_+ - \bar{u}_- \approx \bar{d}_+ - \bar{d}_- \approx \bar{s}_+ - \bar{s}_- \approx s_+ - s_-$. The corresponding relation between δq and Δq is $\delta u = \Delta u - \Delta s$, $\delta d = \Delta d - \Delta s$, and $\delta s = 0$ (see, e.g., Ref. [10]).

Below, we give the results of fits to the baryon magnetic moments based on each of the above two hypotheses.

Fit (A). Assumption (A) implies that Eqs. (7) may be rewritten with δq replaced by Δq . Such an approximation was considered by Karl [9], who concluded that the data could be fitted with values of Δu , Δd , and Δs similar to those deduced from polarized deep inelastic scattering, and that the fit was superior to that given by the conventional quark model characterised by $\Delta u = 4/3$, $\Delta d = -1/3$, and $\Delta s = 0$. Our own results for model (A) are shown in Table I. As in Ref. [9], each magnetic moment was assigned a theoretical uncertainty of $\pm 0.1\mu_N$. This (arbitrary) choice ensures that the various magnetic moments have approximately equal weight and that the fits have a χ^2 of about one unit per

degree of freedom. The conventional quark model result is given under the appellation ‘‘model 0.’’ Note that this model necessarily implies a nucleon axial vector coupling $G_A \equiv a^{(3)} = F + D = \Delta u - \Delta d = 5/3$, in conflict with the measured value 1.26. Notice also that the fit deviates markedly from the expectation $\mu_u = -2\mu_d$. By contrast, the column labeled ‘‘model AI’’ gives the result of a fit to Eqs. (7) in which Δu and Δd are constrained to give the correct value of G_A , i.e., $G_A = 1.26$. Additionally, we take $\mu_u = -2\mu_d$ and $\mu_s = 3/5\mu_d$ (the latter assumption agrees with the fitted value in Ref. [9], and also with the usual constituent quark model estimate $m_d/m_s = 0.6$). It is convenient to rewrite Δu , Δd , and Δs as

$$\begin{aligned} \Delta u &= \frac{2}{3}S_z + \frac{1}{2}G_A + \frac{1}{6}a^{(8)}, & \Delta d &= \frac{2}{3}S_z - \frac{1}{2}G_A + \frac{1}{6}a^{(8)}, \\ \Delta s &= \frac{2}{3}S_z - \frac{1}{3}a^{(8)}, \end{aligned} \quad (8)$$

so that the magnetic moments in Eq. (7) can be treated as functions of three parameters μ_u , $S_z = \frac{1}{2}(\Delta u + \Delta d + \Delta s)$, and $a^{(8)} = \Delta u + \Delta d - 2\Delta s$. The results of the fit are

$$\begin{aligned} \mu_u &= 2.39 \pm 0.06, & S_z &= 0.14 \pm 0.12, \\ a^{(8)} &= 0.85 \pm 0.06 \text{ (model AI)}. \end{aligned} \quad (9)$$

For the central value of μ_u , the allowed domain of the parameters S_z and $a^{(8)}$ is shown in Fig. 1 (ellipse labeled $L_z = 0$). While the value of S_z is in good agreement with the determinations from high energy scattering, there is a clear discrepancy between the value of $a^{(8)}$ obtained from the fit and its experimental value $a^{(8)} = 3F - D \approx 0.60$.

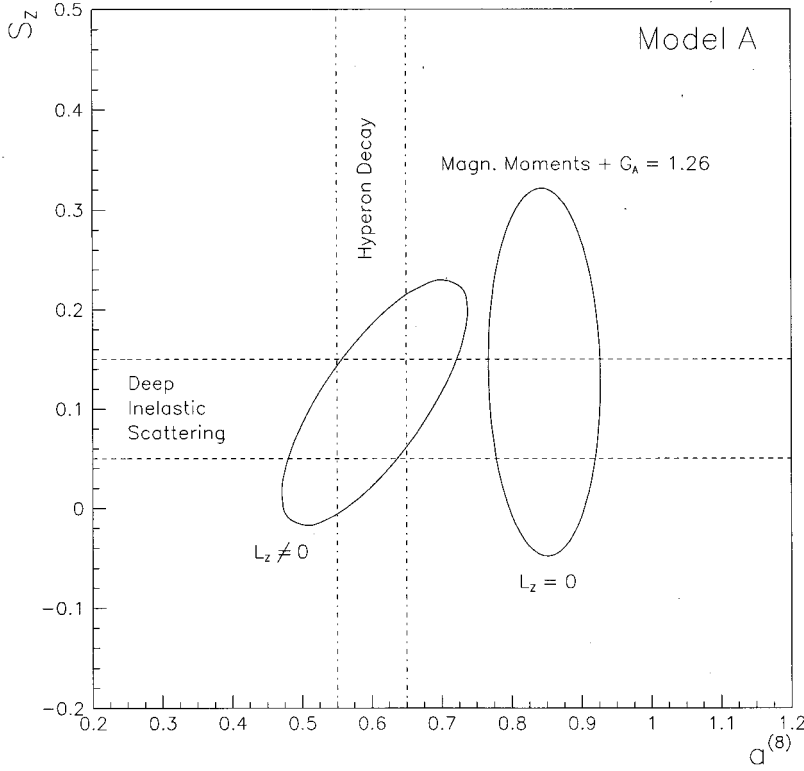


FIG. 1. Fit to baryon magnetic moments in model (A), compared with value of $a^{(8)}$ from hyperon decay, and S_z from polarized deep inelastic scattering (bands correspond to $a^{(8)} = 0.60 \pm 0.05$, $S_z = 0.10 \pm 0.05$). The ellipses labeled $L_z = 0$ and $L_z \neq 0$ correspond to the solutions AI and AII in Table I.

Fit (B). We now repeat the analysis of the magnetic moments using the ansatz (B). Written in terms of Δq , Eqs. (7) now involve only the combinations $a^{(3)} = \Delta u - \Delta d = G_A$ and $a^{(8)} = \Delta u + \Delta d - 2\Delta s$, and are independent of the combination $a^{(0)} = \Delta u + \Delta d + \Delta s = 2S_z$. Accordingly, the fit, using $G_A = 1.26$ as input, determines only the two parameters

$$\mu_u = 2.40 \pm 0.06, \quad a^{(8)} = 0.82 \pm 0.05 \quad (\text{model BI}), \quad (10)$$

no constraint being obtained on S_z . The allowed domain of these two parameters is shown in Fig. 2 by the ellipse labeled $L_z = 0$. The value of $a^{(8)}$ in Eq. (10) is very similar to the value in Fit (A), Eq. (9). In both cases, however, the value of $a^{(8)}$ deviates significantly from the value measured in hyperon decay.

III. ROTATING PROTON

In an attempt to resolve the above discrepancy, we have constructed a model containing orbital angular momentum. The total angular momentum of a polarized proton can be resolved as $J_z = S_z + L_z + \Delta G = \frac{1}{2}$. We consider here the effects of an orbital angular momentum $\langle L_z \rangle$ associated with the motion of three constituent quarks in the baryon. As pointed out in [1], such orbital motion will produce a correction to the magnetic moments, dependent on the way in which the angular momentum $\langle L_z \rangle$ is shared between the constituents. Our central hypothesis is that the quarks in a baryon are held together by a flux string in a ‘‘Mercedes-star’’ configuration. In the plane transverse to the proton spin axis, the quarks will tend to be situated at the corners of an equilateral triangle (Fig. 3). Let us imagine that this correlated three-quark structure rotates collectively around the z

axis, with total orbital angular momentum $\langle L_z \rangle$. For a baryon containing constituents q_1 , q_2 , and q_3 with masses m_1 , m_2 , and m_3 , the orbital angular momentum carried by the quark q_i is $[m_i / (m_1 + m_2 + m_3)] \langle L_z \rangle$ [we assume rotation about the geometrical center of the triangle, thereby maintaining SU(3) symmetry in the baryon spatial wave function]. With this simple ansatz, we obtain the following corrections to the seven baryon magnetic moments listed in Eq. (7):

$$\begin{aligned} \mu(p) &= \dots + \left[2\mu_u \left(\frac{1}{3} \right) + \mu_d \left(\frac{1}{3} \right) \right] \langle L_z \rangle, \\ \mu(n) &= \dots + \left[\mu_u \left(\frac{1}{3} \right) + 2\mu_d \left(\frac{1}{3} \right) \right] \langle L_z \rangle, \\ \mu(\Sigma^+) &= \dots + \left[2\mu_u \left(\frac{\lambda}{1+2\lambda} \right) + \mu_s \left(\frac{1}{1+2\lambda} \right) \right] \langle L_z \rangle, \\ \mu(\Sigma^-) &= \dots + \left[2\mu_d \left(\frac{\lambda}{1+2\lambda} \right) + \mu_s \left(\frac{1}{1+2\lambda} \right) \right] \langle L_z \rangle, \\ \mu(\Xi^-) &= \dots + \left[\mu_d \left(\frac{\lambda}{2+\lambda} \right) + 2\mu_s \left(\frac{1}{2+\lambda} \right) \right] \langle L_z \rangle, \\ \mu(\Xi^0) &= \dots + \left[\mu_u \left(\frac{\lambda}{2+\lambda} \right) + 2\mu_s \left(\frac{1}{2+\lambda} \right) \right] \langle L_z \rangle, \\ \mu(\Lambda^0) &= \dots + \left[\mu_u \left(\frac{\lambda}{1+2\lambda} \right) + \mu_d \left(\frac{\lambda}{1+2\lambda} \right) \right. \\ &\quad \left. + \mu_s \left(\frac{1}{1+2\lambda} \right) \right] \langle L_z \rangle, \end{aligned} \quad (11)$$

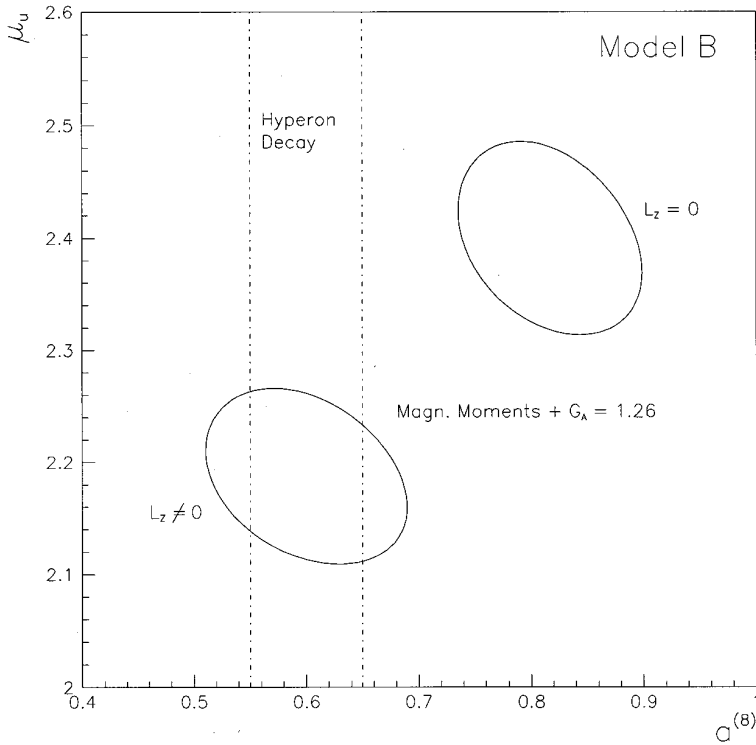


FIG. 2. Fit to baryon magnetic moments in model (B), compared with value of $a^{(8)}$ from hyperon decay (band corresponds to $a^{(8)} = 0.60 \pm 0.05$). The ellipses labeled $L_z = 0$ and $L_z \neq 0$ correspond to the solutions BI and BIII in Table II.

where $\lambda = m_d/m_s$ is taken to be 0.6, and the ellipses represent the spin contribution given in Eq. (7).

We have fitted the seven magnetic moments under the same assumptions employed in models (A) and (B) (namely, $a^{(3)} = \Delta u - \Delta d = 1.26$, $\mu_u = -2\mu_d$, $\mu_s = \frac{3}{5}\mu_d$), using $\langle L_z \rangle$ as an additional parameter. In a first variation of model (A), the parameter $\langle L_z \rangle$ was fixed such that $\langle L_z \rangle + \langle S_z \rangle = \frac{1}{2}$. This represents the extreme hypothesis that the “missing” angular momentum of the proton is precisely accounted for by the orbital angular momentum of the correlated structure depicted in Fig. 3. This model then contains the same free parameters as model AI, namely, μ_u , S_z , and $a^{(8)}$. A fit to the magnetic moments (see Table I) yields

$$\begin{aligned} \mu_u &= 2.17 \pm 0.09, & S_z &= 0.11 \pm 0.14, \\ a^{(8)} &= 0.60 \pm 0.10 \quad (\text{model AII}). \end{aligned} \quad (12)$$

The quality of the fit is essentially the same as in model AI, but there is a dramatic improvement in the value of $a^{(8)}$, the result of the fit coinciding with the measured value. This improvement is evident from Fig. 1, which shows that with the inclusion of L_z there is a convergence of the data on magnetic moments, axial vector couplings, and polarized deep inelastic scattering. Within the framework of ansatz (A), we can also consider $\langle S_z \rangle$ and $\langle L_z \rangle$ as independent free parameters, using the experimental value of $a^{(8)}$ as input. A three-parameter fit to the magnetic moments then yields

$$\begin{aligned} \mu_u &= 2.17 \pm 0.08, & \langle S_z \rangle &= 0.08 \pm 0.13, \\ \langle L_z \rangle &= 0.39 \pm 0.09 \quad (\text{model AIII}). \end{aligned} \quad (13)$$

If the effects of orbital angular momentum given by Eqs. (10) are incorporated into model (B), we obtain the results

indicated in columns BII and BIII in Table II. A three-parameter fit in terms of μ_u , L_z , and $a^{(8)}$ yields

$$\begin{aligned} \mu_u &= 2.10 \pm 0.19, & \langle L_z \rangle &= 0.54 \pm 0.37, \\ a^{(8)} &= 0.49 \pm 0.23 \quad (\text{model BII}). \end{aligned} \quad (14)$$

On the other hand, if $a^{(8)} = 0.6$ is used as input, we find

$$\mu_u = 2.19 \pm 0.08, \quad L_z = 0.37 \pm 0.09, \quad (\text{model BIII}). \quad (15)$$

The fits in model (B) have a χ^2 that is inferior to that of model (A). The improved convergence of magnetic moment and axial vector coupling data in the presence of orbital an-

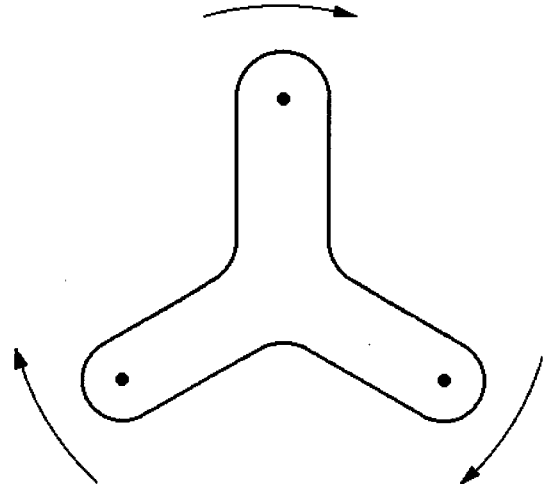


FIG. 3. Flux string connecting three constituent quarks, rotating collectively around the proton spin axis.

TABLE II. Fit to baryon magnetic moments in model (B). Magnetic moments are in nucleon magnetons and the ± 0.1 is a fictive theoretical error.

	Magnetic moments	Model 0 $S_z = \frac{1}{2}$ $L_z = 0$	Model BI S_z undetermined $L_z = 0$	Model BII S_z undetermined L_z free	Model BIII S_z undetermined L_z free
$\mu(p)$	2.79 ± 0.1 ± 0.00000006	2.67	2.76	2.81	2.80
$\mu(n)$	-1.91 ± 0.1 ± 0.00000005	-1.92	-1.78	-1.73	-1.74
$\mu(\Sigma^+)$	2.46 ± 0.1 ± 0.01	2.54	2.65	2.54	2.59
$\mu(\Sigma^-)$	-1.16 ± 0.1 ± 0.025	-1.14	-1.09	-1.14	-1.13
$\mu(\Xi^-)$	-0.65 ± 0.1 ± 0.0025	-0.48	-0.49	-0.54	-0.53
$\mu(\Xi^0)$	-1.25 ± 0.1 ± 0.014	-1.40	-1.28	-1.36	-1.33
$\mu(\Lambda)$	-0.61 ± 0.1 ± 0.004	-0.61	-0.52	-0.57	-0.55
Input		$\Delta u = \frac{4}{3}$ $\Delta d = -\frac{1}{3}$ $\Delta s = 0$	$\mu_u = -2\mu_d$ $\mu_s = \frac{3}{5}\mu_d$ $G_A = 1.26$	$\mu_u = -2\mu_d$ $\mu_s = \frac{3}{5}\mu_d$ $G_A = 1.26$	$\mu_u = -2\mu_d$ $\mu_s = \frac{3}{5}\mu_d$ $G_A = 1.26$ $a^{(8)} = 0.60$
χ^2/N_{DOF}		1.82	1.99	1.72	1.43
Fitted parameters		$\mu_u = 1.75 \pm 0.06$ $\mu_d = -1.01 \pm 0.06$ $\mu_s = -0.61 \pm 0.05$	$\mu_u = 2.40 \pm 0.06$ $a^{(8)} = 0.82 \pm 0.05$ exp: 0.60 ± 0.02	$\mu_u = 2.10 \pm 0.19$ $L_z = 0.54 \pm 0.37$ $a^{(8)} = 0.49 \pm 0.23$ exp: 0.60 ± 0.02	$\mu_u = 2.19 \pm 0.08$ $L_z = 0.37 \pm 0.09$

gular momentum is evident from Fig. 2. Also noteworthy is the similarity in the fitted value of $\langle L_z \rangle$ in models (A) and (B), Eqs. (13) and (15). It is certainly intriguing that the value of $\langle L_z \rangle$ derived from fits to the static properties of baryons (magnetic moments and axial vector couplings) has the correct sign and approximately the correct magnitude to explain the ‘‘spin deficit’’ of the nucleon revealed by high energy scattering.

IV. CONCLUSION

It would appear from the above that the quark parton model defined by the parton spins Δu , Δd , and Δs can provide a consistent description of axial vector couplings, baryon magnetic moments, and the spin structure functions, provided we supplement the spin angular momentum with a collective orbital angular momentum as symbolized in Fig. 3. The role of the rotating flux string in achieving this agreement draws renewed attention to flux-string models of the baryon (see, e.g., [12] and references therein). Such models have been invoked in the past to explain states in the baryon spectrum [such as the Roper resonance $N(1440)$] that have not been easy to accommodate in the traditional three-quark

picture [13]. The idea that the nucleon may contain $L \neq 0$ components in its wave function (‘‘configuration mixing’’) has also been entertained before [14]. The possibility of rotation as a source of hadron spin has been emphasized by Chou and Yang [15]. The specific structure introduced in the present paper may be expected, naively, to produce rotational levels with energy $E_{\text{rot}} = J(J+1)/(2I)$, where I is the moment of inertia of the three-quark correlation. Assuming this structure to consist of three constituent quarks in close contact, each with radius 0.2–0.3 fm [16], the excitation energy is 0.5–1.0 GeV. It remains to be seen whether the spectrum of baryonic levels will show evidence for states associated with stringlike configurations, beyond those that are expected from the shell model with three independently moving quarks. Direct experimental tests for rotating constituents in the nucleon have been proposed in [17], and some tentative evidence from hadronic reactions has been reported [18].

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