CP violation and CKM phases from angular distributions for B_s decays into admixtures of CP eigenstates

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We investigate the time evolutions of angular distributions for B_s decays into final states that are admixtures of CP-even and CP-odd configurations. A sizable lifetime difference between the B_s mass eigenstates allows a probe of CP violation in time-dependent untagged angular distributions. Interference effects between different final state configurations of $B_s \to D_s^{*+} D_s^{*-}$, $J/\psi\phi$ determine the Wolfenstein parameter η from *untagged* data samples, or, if one uses $|V_{ub}|/|V_{cb}|$ as additional input, the notoriously difficult to measure CKM angle γ. Another determination of γ is possible by using isospin symmetry of strong interactions to relate *untagged* data samples of $B_s \to K^{*+}K^{*-}$ and $B_s \to K^{*0}\overline{K^{*0}}$. We note that the *untagged* angular distribution for $B_s \to \rho^0\phi$ provides interesting information about electroweak penguin diagrams. [S0556-2821(97)00401-3]

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I. INTRODUCTION

Within the standard model [1] one expects [2] a large mass difference $\Delta m \equiv m_H - m_L > 0$ between the physical mixing eigenstates B_s^H (''heavy'') and B_s^L (''light'') of the neutral B_s meson system leading to very rapid $\Delta m t$ oscillations in data samples of tagged B_s decays. In order to measure these oscillations, an excellent vertex resolution system is required which is a formidable experimental task. However, in a recent paper [3] it has been shown that it may not be necessary to trace these rapid $\Delta m t$ oscillations in order to obtain insight into the fundamental mechanism of CP violation. The point is that the time evolution of *untagged* nonleptonic B_s decays, where one does not distinguish between initially present B_s and $\overline{B_s}$ mesons, depends only on combinations of the two exponents $\exp(-\Gamma_L t)$ and $\exp(-\Gamma_H t)$ and not on the rapid oscillatory $\Delta m t$ terms. Since the width difference $\Delta\Gamma \equiv \Gamma_H - \Gamma_L$ of the B_s system is predicted to be of the order 20% of the average B_s width [4], interesting CP-violating effects may show up in untagged rates [3].

In the present paper we restrict ourselves to quasi-two-body modes $B_s \to X_1 X_2$ into final states that are admixtures of CP-even and CP-odd configurations. The different case where the final states are not admixtures of CP eigenstates but can be classified instead by their parity eigenvalues is discussed in [5], where we present an analysis of angular correlations for B_s decays governed by $\bar{b} \to \bar{c} u \bar{s}$ quark-level transitions. If both X_1 and X_2 carry spin and continue to decay through CP-conserving interactions, valuable information can be obtained from the angular distributions of their decay products. Examples for such transitions are $B_s \to D_s^{*+}(\to D_s^+\gamma) D_s^{*-}(\to D_s^-\gamma)$ and $B_s \to J/\psi(\to l^+ l^-)\phi(\to K^+K^-)$ which allow a determination of the Wolfenstein parameter η [6] from the time dependences of their *untagged* angular distributions as we will demonstrate in a later part of this paper. Of course, the formalism developed here applies also to final states where the $D_s^{*\pm}$ mesons are substituted by higher resonances, such as $B_s \to D_{s1}(2536)^+ D_{s1}(2536)^-$. For many detector configurations, such higher resonances may be preferable over $D_s^{*\pm}$, because of their significant branching fractions into all charged final states and because of additional mass constraints of their daughter resonances.

If we use the CKM factor

$$R_b \equiv \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|} \tag{1}$$

with $\lambda = \sin\theta_C = 0.22$ as an additional input, which is constrained by present experimental data to lie within the range $R_b = 0.36 \pm 0.08$ [7–9], η fixes the angle γ in the usual ''nonsquashed'' unitarity triangle [10] of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [11] through

$$\sin\gamma = \frac{\eta}{R_b}. \tag{2}$$

Using the isospin symmetry of strong interactions to relate the $\bar{b} \to \bar{s}$ QCD penguin contributions to $B_s \to K^{*+}(\to \pi K) K^{*-}(\to \pi K)$ and $B_s \to K^{*0}(\to \pi K)\overline{K^{*0}}(\to \pi K)$, another determination of γ is possible by measuring the corresponding *untagged* angular distributions. This approach is another highlight of our paper. The formulas describing $B_s \to K^{*+}K^{*-}$ apply also to $B_s \to \rho^0\phi$ if we make an appropriate replacement of variables providing a fertile ground for obtaining information about the physics of electroweak penguin diagrams.

This paper is organized as follows. In Sec. II we calculate the time dependences of the observables of the angular dis-

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tributions for B_s decays into final state configurations that are admixtures of different CP eigenstates. The general formulas derived in Sec. II simplify considerably if the unmixed $B_s \to X_1 X_2$ amplitude is dominated by a single CKM amplitude. This important special case is the subject of Sec. III and applies to an excellent accuracy to the decays $B_s \to D_s^{*+} D_s^{*-}$ and $B_s \to J/\psi\phi$ which are analyzed in Sec. IV. There we demonstrate that *untagged* data samples of these modes allow a determination of the Wolfenstein parameter η, which fixes the CKM angle γ if R_b is known. In Sec. V we present another method to determine γ from *untagged* $B_s \to K^{*+} K^{*-}$ and $B_s \to K^{*0} \overline{K^{*0}}$ decays. The formulas derived there are also useful to obtain information about electroweak penguin diagrams from *untagged* $B_s \to \rho^0 \phi$ events. Finally, in Sec. VI the main results of our paper are summarized.

II. CALCULATION OF THE TIME EVOLUTIONS

A characteristic feature of the angular distributions for the decays $B_s \to X_1 X_2$ specified above is that they depend in general on real or imaginary parts of the following bilinear combinations of decay amplitudes:

$$A_{\tilde{f}}^*(t) A_f(t). \tag{3}$$

Here we have introduced the notation

$$\begin{aligned} A_f(t) &\equiv A[B_s(t) \to (X_1 X_2)_f] = \langle (X_1 X_2)_f | H_{\text{eff}} | B_s(t) \rangle, \\ A_{\tilde{f}}(t) &\equiv A[B_s(t) \to (X_1 X_2)_{\tilde{f}}] = \langle (X_1 X_2)_{\tilde{f}} | H_{\text{eff}} | B_s(t) \rangle, \end{aligned} \tag{4}$$

for the transition amplitudes of initially, i.e., at $t=0$, present B_s mesons decaying into the final state configurations f and $\tilde{f}$ of $X_1 X_2$ that are both CP eigenstates satisfying

$$\begin{aligned} (\mathcal{CP}) | (X_1 X_2)_f \rangle &= \eta_{CP}^f | (X_1 X_2)_f \rangle, \\ (\mathcal{CP}) | (X_1 X_2)_{\tilde{f}} \rangle &= \eta_{CP}^{\tilde{f}} | (X_1 X_2)_{\tilde{f}} \rangle, \end{aligned} \tag{5}$$

with $\eta_{CP}^f, \eta_{CP}^{\tilde{f}} \in \{-1,+1\}$. Here f and $\tilde{f}$ are labels that define the relative polarizations of the two hadrons X_1 and X_2. The tilde is useful for discussing the case where different configurations of $X_1 X_2$ with the *same CP* eigenvalue are present. To make this point more transparent, consider the mode $B_s \to J/\psi\phi$ which has been analyzed in terms of the linear polarization amplitudes [12] $A_0(t)$, $A_\parallel(t)$, and $A_\perp(t)$ in [13]. Whereas $A_\perp(t)$ describes a CP-odd final state configuration, both $A_0(t)$ and $A_\parallel(t)$ correspond to CP eigenvalue $+1$, i.e., to $A_f(t)$ and $A_{\tilde{f}}(t)$ in our notation (4) with $\eta_{CP}^{\tilde{f}} = \eta_{CP}^f = +1$.

The amplitudes describing decays of initially present $\overline{B_s}$ mesons are given by

$$\begin{aligned} \overline{A}_f(t) &\equiv A[\overline{B_s}(t) \to (X_1 X_2)_f] = \langle (X_1 X_2)_f | H_{\text{eff}} | \overline{B_s}(t) \rangle, \\ \overline{A}_{\tilde{f}}(t) &\equiv A[\overline{B_s}(t) \to (X_1 X_2)_{\tilde{f}}] = \langle (X_1 X_2)_{\tilde{f}} | H_{\text{eff}} | \overline{B_s}(t) \rangle. \end{aligned} \tag{6}$$

Both in these expressions and in Eq. (4) the operator

$$H_{\text{eff}} = H_{\text{eff}}(\Delta B = -1) + H_{\text{eff}}(\Delta B = +1) \tag{7}$$

denotes an appropriate low energy effective Hamiltonian with

$$H_{\text{eff}}(\Delta B = +1) = H_{\text{eff}}(\Delta B = -1)^\dagger \tag{8}$$

and

$$\begin{aligned} &H_{\text{eff}}(\Delta B = -1) \\ &= \frac{G_F}{\sqrt{2}} \sum_{j=u,c} v_j^{(r)} \mathcal{Q}^j \\ &\equiv \frac{G_F}{\sqrt{2}} \sum_{j=u,c} v_j^{(r)} \left\{ \sum_{k=1}^{2} Q_k^j C_k(\mu) + \sum_{k=3}^{10} Q_k C_k(\mu) \right\}, \end{aligned} \tag{9}$$

where $v_j^{(r)} \equiv V_{jr}^* V_{jb}$ is a CKM factor that is different for $b \to d$ and $b \to s$ transitions corresponding to $r=d$ and $r=s$, respectively. The four-quark operators Q_k can be divided into current-current operators ($k \in \{1,2\}$), QCD penguin operators ($k \in \{3, \ldots, 6\}$) and electroweak penguin operators ($k \in \{7, \ldots, 10\}$), with index r implicit. Note that these operators create s and d quarks for $r=s$ and $r=d$, respectively. The Wilson coefficients $C_k(\mu)$ of these operators, where $\mu = O(m_b)$ is a renormalization scale, can be calculated in renormalization group improved perturbation theory. The reader is referred to a nice recent review [14] for the details of such calculations. There numerical results for the relevant Wilson coefficients are summarized and the four-quark operators Q_k are given explicitly.

Applying the well-known formalism describing B_s-$\overline{B_s}$ mixing [3,15], a straightforward calculation yields the following expression for the time dependence of the bilinear combination of decay amplitudes given in Eq. (3):

$$\begin{aligned} A_{\tilde{f}}^*(t) A_f(t) = {} & \langle (X_1 X_2)_{\tilde{f}} | H_{\text{eff}} | B_s \rangle^* \langle (X_1 X_2)_f | H_{\text{eff}} | B_s \rangle \\ & \times [|g_+(t)|^2 + \eta_{CP}^{\tilde{f}} \xi_{\tilde{f}}^* g_+(t) g_-^*(t) \\ & + \eta_{CP}^f \xi_f g_+^*(t) g_-(t) + \eta_{CP}^{\tilde{f}} \eta_{CP}^f \xi_{\tilde{f}}^* \xi_f |g_-(t)|^2], \end{aligned} \tag{10}$$

where

$$|g_\pm(t)|^2 = \frac{1}{4} [e^{-\Gamma_L t} + e^{-\Gamma_H t} \pm 2 e^{-\Gamma t} \cos(\Delta m t)], \tag{11}$$

$$g_+(t) g_-^*(t) = \frac{1}{4} [e^{-\Gamma_L t} - e^{-\Gamma_H t} - 2 i e^{-\Gamma t} \sin(\Delta m t)], \tag{12}$$

with $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$. The observables ξ_f and $\xi_{\tilde{f}}$, which contain essentially all the information needed to evaluate the time dependence of Eq. (10), are related to hadronic matrix elements of the combinations $\mathcal{Q}^j$ of four-quark operators and Wilson coefficients appearing in the low energy effective Hamiltonian (9) through

$$\xi_f = e^{-i\phi_M^{(s)}} \frac{\sum_{j=u,c} v_j^{(r)} \langle (X_1X_2)_f | \mathcal{Q}^j | \overline{B_s} \rangle}{\sum_{j=u,c} v_j^{(r)*} \langle (X_1X_2)_f | \mathcal{Q}^j | \overline{B_s} \rangle}, \tag{13}$$

where $\phi_M^{(s)} \equiv 2\arg(V_{ts}^* V_{tb})$ is the B_s-$\overline{B_s}$ mixing phase. In order to evaluate $\xi_{\tilde f}$, we have simply to replace f in Eq. (13) by $\tilde f$. Note that we have neglected the extremely small CP-violating effects in the B_s-$\overline{B_s}$ oscillations in order to derive Eqs. (10)–(13) [3]. We shall come back to Eq. (13) in a moment. Let us consider the CP-conjugate processes first. The expression corresponding to Eq. (10) for initially present $\overline{B_s}$ mesons is very similar to that equation and can be written as

$$\begin{aligned}\overline{A^*_{\tilde f}}(t)\overline{A_f}(t) = {} & \langle (X_1X_2)_{\tilde f} | H_{\rm eff} | B_s \rangle^* \langle (X_1X_2)_f | H_{\rm eff} | B_s \rangle \\ & \times [|g_-(t)|^2 + \eta_{CP}^{\tilde f} \xi^*_{\tilde f} g^*_+(t) g_-(t) \\ & + \eta_{CP}^f \xi_f g_+(t) g^*_-(t) + \eta_{CP}^{\tilde f} \eta_{CP}^f \xi^*_{\tilde f} \xi_f |g_+(t)|^2].\end{aligned} \tag{14}$$

In the general case the *tagged* angular distribution for a given decay $B_s(t) \to X_1X_2$ can be written as [16]

$$f(\theta,\varphi,\psi;t) = \sum_k b^{(k)}(t) g^{(k)}(\theta,\varphi,\psi), \tag{15}$$

where we have denoted the angles describing the kinematics of the decay products of X_1 and X_2 generically by θ, φ, and ψ. Note that we have to deal in general with an arbitrary number of such angles. For quasi-two-body modes $B_s(t) \to X_1X_2$ into final states that are admixtures of CP-even and CP-odd configurations, the observables $b^{(k)}(t)$ describing the time evolution of the angular distribution (15) can be expressed in terms of real or imaginary parts of bilinear combinations of decay amplitudes having the same structure as Eq. (10). The angular distribution for the *tagged* CP-conjugate decay $\overline{B_s}(t) \to X_1X_2$ on the other hand is given by

$$\overline{f(\theta,\varphi,\psi;t)} = \sum_k \overline{b^{(k)}}(t) g^{(k)}(\theta,\varphi,\psi), \tag{16}$$

where the observables $\overline{b^{(k)}}(t)$ are related correspondingly to real or imaginary parts of bilinear combinations like Eq. (14). Since the states X_1X_2 resulting from the B_s and $\overline{B_s}$ decays are equal, we use the same generic angles θ, φ, and ψ to describe the angular distributions of their decay products. Within our formalism the effects of CP transformations relating $B_s(t) \to (X_1X_2)_{f,\tilde f}$ and $\overline{B_s}(t) \to (X_1X_2)_{f,\tilde f}$ are taken into account already by the CP eigenvalues $\eta_{CP}^{\tilde f}$ and η_{CP}^f appearing in Eqs. (10) and (14) and do not affect $g^{(k)}(\theta,\varphi,\psi)$. Therefore the same functions $g^{(k)}(\theta,\varphi,\psi)$ are present in Eqs. (15) and (16).

The main focus of this paper are *untagged* rates, where one does not distinguish between initially present B_s and $\overline{B_s}$ mesons. Such studies are obviously much more efficient from an experimental point of view than tagged analyses. In the distant future it will become feasible to collect also *tagged* B_s data samples and to resolve the rapid oscillatory $\Delta m t$–terms. Then Eqs. (10) and (14) describing the corresponding observables should turn out to be very useful.

Combining Eqs. (15) and (16) we find that the *untagged* angular distribution takes the form

$$\begin{aligned}[f(\theta,\varphi,\psi;t)] & \equiv \overline{f(\theta,\varphi,\psi;t)} + f(\theta,\varphi,\psi;t) \\ & = \sum_k [\overline{b^{(k)}}(t) + b^{(k)}(t)] g^{(k)}(\theta,\varphi,\psi).\end{aligned} \tag{17}$$

As we will see in a moment, interesting CP-violating effects show up in this untagged rate, if the width difference $\Delta\Gamma$ is sizable. The time evolution of the relevant observables $[\overline{b^{(k)}}(t) + b^{(k)}(t)]$ behaves as the real or imaginary parts of

$$\begin{aligned}[A^*_{\tilde f}(t) A_f(t)] & \equiv \overline{A^*_{\tilde f}}(t)\overline{A_f}(t) + A^*_{\tilde f}(t) A_f(t) \\ & = \frac{1}{2} \langle (X_1X_2)_{\tilde f} | H_{\rm eff} | B_s \rangle^* \langle (X_1X_2)_f | H_{\rm eff} | B_s \rangle \\ & \quad \times [(1 + \eta_{CP}^{\tilde f} \eta_{CP}^f \xi^*_{\tilde f} \xi_f)(e^{-\Gamma_L t} + e^{-\Gamma_H t}) \\ & \quad + (\eta_{CP}^{\tilde f} \xi^*_{\tilde f} + \eta_{CP}^f \xi_f)(e^{-\Gamma_L t} - e^{-\Gamma_H t})].\end{aligned} \tag{18}$$

In order to calculate this equation, we have combined Eq. (10) with Eq. (14) and have moreover taken into account explicitly the time-dependences of Eqs. (11) and (12). We can distinguish between the following special cases.

$\tilde f = f$:

$$\begin{aligned}[|A_f(t)|^2] = {} & \frac{1}{2} |\langle (X_1X_2)_f | H_{\rm eff} | B_s \rangle|^2 \\ & \times [(1 + |\xi_f|^2)(e^{-\Gamma_L t} + e^{-\Gamma_H t}) \\ & + 2\,\eta_{CP}^f\, \mathrm{Re}(\xi_f)(e^{-\Gamma_L t} - e^{-\Gamma_H t})].\end{aligned} \tag{19}$$

$\tilde f \neq f$ and $\eta_{CP}^{\tilde f} = \eta_{CP}^f$:

$$\begin{aligned}[A^*_{\tilde f}(t) A_f(t)] = {} & \frac{1}{2} \langle (X_1X_2)_{\tilde f} | H_{\rm eff} | B_s \rangle^* \langle (X_1X_2)_f | H_{\rm eff} | B_s \rangle \\ & \times [(1 + \xi^*_{\tilde f} \xi_f)(e^{-\Gamma_L t} + e^{-\Gamma_H t}) \\ & + \eta_{CP}^f (\xi^*_{\tilde f} + \xi_f)(e^{-\Gamma_L t} - e^{-\Gamma_H t})].\end{aligned} \tag{20}$$

$\tilde f \neq f$ and $\eta_{CP}^{\tilde f} = -\eta_{CP}^f$:

$$\begin{aligned}[A^*_{\tilde f}(t) A_f(t)] = {} & \frac{1}{2} \langle (X_1X_2)_{\tilde f} | H_{\rm eff} | B_s \rangle^* \langle (X_1X_2)_f | H_{\rm eff} | B_s \rangle \\ & \times [(1 - \xi^*_{\tilde f} \xi_f)(e^{-\Gamma_L t} + e^{-\Gamma_H t}) \\ & - \eta_{CP}^f (\xi^*_{\tilde f} - \xi_f)(e^{-\Gamma_L t} - e^{-\Gamma_H t})].\end{aligned} \tag{21}$$

As advertised, the rapidly oscillating $\Delta m t$ terms cancel in the untagged combinations described by Eq. (18). While the time dependence of Eq. (19) was given in [3], the explicit time dependences of Eqs. (20) and (21) have not been given previously. They play an important role for the untagged angular distribution (17).

III. DOMINANCE OF A SINGLE CKM AMPLITUDE

If we look at expression (13), we observe that ξ_f and $\xi_{\tilde{f}}$ suffer in general from large hadronic uncertainties. However, if the unmixed $B_s \to X_1 X_2$ amplitude is dominated by a single CKM amplitude proportional to a CKM factor $v_j^{(r)}$ the unknown hadronic matrix elements cancel in Eq. (13) and both $\xi_{\tilde{f}}$ and ξ_f take the simple form

$$\xi_{\tilde{f}} = \xi_f = e^{2i\phi_j^{(r)}}, \tag{22}$$

where $\phi_j^{(r)} \equiv [\arg(V_{jr}^* V_{jb}) - \arg(V_{ts}^* V_{tb})]$ is a CP-violating weak phase consisting of the corresponding decay and B_s-$\overline{B_s}$ mixing phase. Consequently, in that very important special case, Eq. (18) simplifies to

$$[A_{\tilde{f}}^*(t) A_f(t)] = \frac{1}{2} |\langle (X_1X_2)_{\tilde{f}} | H_{\rm eff} | B_s\rangle \langle (X_1X_2)_f | H_{\rm eff} | B_s\rangle| \times e^{i(\delta_f - \delta_{\tilde{f}})} [(1+\eta_{CP}^{\tilde{f}} \eta_{CP}^f)(e^{-\Gamma_L t} + e^{-\Gamma_H t}) + (\eta_{CP}^{\tilde{f}}\, e^{-2i\phi_j^{(r)}} + \eta_{CP}^f\, e^{2i\phi_j^{(r)}})(e^{-\Gamma_L t} - e^{-\Gamma_H t})], \tag{23}$$

where δ_f and $\delta_{\tilde{f}}$ are *CP-conserving* strong phases. They are induced through strong final state interaction processes and are defined by

$$\langle (X_1X_2)_f | H_{\rm eff} | B_s\rangle = e^{+i\delta_f}\, e^{-i\phi_j^{(r)}}, \tag{24}$$

$$\langle (X_1X_2)_{\tilde{f}} | H_{\rm eff} | B_s\rangle^* = e^{-i\delta_{\tilde{f}}}\, e^{+i\phi_j^{(r)}}. \tag{25}$$

Note that the structure of Eqs. (24) and (25), which is essentially due to the fact that the unmixed $B_s \to X_1 X_2$ amplitude is dominated by a single weak amplitude, implies that the weak phase factors $e^{-i\phi_j^{(r)}}$ and $e^{+i\phi_j^{(r)}}$ cancelled each other in Eq. (23) and that only the strong phases play a role as an overall phase in this equation. We would like to emphasize that such a simple behavior is not present in the general case where more than one weak amplitude is present.

The time evolution of Eq. (23) depends only on $\cos 2\phi_j^{(r)}$ and $\sin 2\phi_j^{(r)}$, since we have only to deal with the following two cases:

$\eta_{CP}^{\tilde{f}} = \eta_{CP}^f$:

$$[A_{\tilde{f}}^*(t) A_f(t)] = |\langle (X_1X_2)_{\tilde{f}} | H_{\rm eff} | B_s\rangle \langle (X_1X_2)_f | H_{\rm eff} | B_s\rangle| \times e^{i(\delta_f - \delta_{\tilde{f}})} [(e^{-\Gamma_L t} + e^{-\Gamma_H t}) + \eta_{CP}^f (e^{-\Gamma_L t} - e^{-\Gamma_H t}) \cos 2\phi_j^{(r)}]. \tag{26}$$

$\eta_{CP}^{\tilde{f}} = -\eta_{CP}^f$:

$$[A_{\tilde{f}}^*(t) A_f(t)] = |\langle (X_1X_2)_{\tilde{f}} | H_{\rm eff} | B_s\rangle \langle (X_1X_2)_f | H_{\rm eff} | B_s\rangle| \times e^{i(\delta_f - \delta_{\tilde{f}})}\, i\,\eta_{CP}^f (e^{-\Gamma_L t} - e^{-\Gamma_H t}) \sin 2\phi_j^{(r)}. \tag{27}$$

Whereas the structure of Eq. (26), in particular the $\cos 2\phi_j^{(r)}$ term, has already been discussed for $\tilde{f} = f$ in [3], to the best of our knowledge it has not been pointed out so far that untagged data samples of angular distributions for certain nonleptonic B_s decays allow also a determination of $\sin 2\phi_j^{(r)}$ with the help of Eq. (27). These $\sin 2\phi_j^{(r)}$ terms play an important role if the weak phase $\phi_j^{(r)}$ is small. The point is that $\sin 2\phi_j^{(r)}$ is proportional to $\phi_j^{(r)}$ in that case, while $\cos 2\phi_j^{(r)} = 1 + O(\phi_j^{(r)2})$. Consequently we obtain up to terms of $O(\phi_j^{(r)2})$:

$\eta_{CP}^{\tilde{f}} = \eta_{CP}^f = +1$:

$$[A_{\tilde{f}}^*(t) A_f(t)] = 2|\langle (X_1X_2)_{\tilde{f}} | H_{\rm eff} | B_s\rangle \langle (X_1X_2)_f | H_{\rm eff} | B_s\rangle| \times e^{i(\delta_f - \delta_{\tilde{f}})} e^{-\Gamma_L t}. \tag{28}$$

$\eta_{CP}^{\tilde{f}} = \eta_{CP}^f = -1$:

$$[A_{\tilde{f}}^*(t) A_f(t)] = 2|\langle (X_1X_2)_{\tilde{f}} | H_{\rm eff} | B_s\rangle \langle (X_1X_2)_f | H_{\rm eff} | B_s\rangle| \times e^{i(\delta_f - \delta_{\tilde{f}})} e^{-\Gamma_H t}. \tag{29}$$

$\eta_{CP}^{\tilde{f}} = -\eta_{CP}^f$:

$$[A_{\tilde{f}}^*(t) A_f(t)] = 2\, i\, \eta_{CP}^f |\langle (X_1X_2)_{\tilde{f}} | H_{\rm eff} | B_s\rangle \langle (X_1X_2)_f | H_{\rm eff} | B_s\rangle| \times e^{i(\delta_f - \delta_{\tilde{f}})} (e^{-\Gamma_L t} - e^{-\Gamma_H t}) \phi_j^{(r)}. \tag{30}$$

We observe that only the mixed combination (30) is sensitive, i.e., proportional, to the small phase $\phi_j^{(r)}$ and allows an extraction of this quantity. These considerations have an interesting phenomenological application as we will see in the following section.

IV. THE "GOLD-PLATED" TRANSITIONS $B_s \to D_s^{*+} D_s^{*-}$ AND $B_s \to J/\psi\phi$ TO EXTRACT THE WOLFENSTEIN PARAMETER η

Concerning the dominance of a single CKM amplitude, in analogy to $B_d \to J/\psi K_S$ measuring $\sin 2\beta$ to excellent accuracy [17] (β is another angle of the unitarity triangle [10]), the "gold-plated" modes are B_s decays caused by $b \to \overline{c} c s$ quark-level transitions. The corresponding exclusive modes relevant for our discussion are $B_s \to D_s^{*+}(\to D_s^+ \gamma) D_s^{*-}(\to D_s^- \gamma)$ and $B_s \to J/\psi(\to l^+ l^-)\phi(\to K^+ K^-)$. They are dominated to an excellent accuracy by the CKM amplitudes proportional to $v_c^{(s)} = V_{cs}^* V_{cb}$. Therefore the corresponding weak phase $\phi_j^{(r)}$ defined after Eq. (22) is related to elements of the CKM matrix [11] through

$$\phi_c^{(s)}=[\arg(V_{cs}^*V_{cb})-\arg(V_{ts}^*V_{tb})]. \tag{31}$$

At leading order in the Wolfenstein expansion [6] this phase vanishes. In order to obtain a nonvanishing result, we have to take into account higher order terms in the Wolfenstein parameter $\lambda=\sin\theta_C=0.22$ (for a treatment of such terms see, e.g., [6,8]) yielding [18,19]

$$\phi_c^{(s)}=\lambda^2\eta\sim 0.015. \tag{32}$$

Consequently the small weak phase $\phi_c^{(s)}$ measures simply the CKM parameter η [6,18,19].

Another interesting interpretation of Eq. (31) is the fact that it is related to one angle in a rather squashed (and therefore ''unpopular'') unitarity triangle [20]. Other useful expressions for Eq. (31) can be found in [21]. If we use the CKM factor R_b defined by Eq. (1) as an additional input, η fixes the notoriously difficult to measure angle γ of the unitarity triangle [21]. That input allows, however, also a determination of γ (or of the Wolfenstein parameter η) from the mixing-induced CP-violating asymmetry arising in $B_d\to J/\psi K_S$ measuring $\sin 2\beta$. Comparing these two results for γ (or η), an interesting test whether the phases in B_d-$\overline{B}_d$ and B_s-$\overline{B}_s$ mixing are indeed described by the standard model can be performed.

The extraction of the weak phase Eq. (32) from $B_s\to J/\psi\phi$, $D_s^{*+}D_s^{*-}$, etc., is not as clean as that of β from $B_d\to J/\psi K_S$. The reason is that although the contributions to the unmixed amplitudes proportional to $V_{ub}^*V_{us}$ are similarly suppressed in both cases, their importance is enhanced by the smallness of $\phi_c^{(s)}$ versus β [22].

Given that $\phi_c^{(s)}$ is small, we see that Eqs. (28)–(30) apply to an excellent approximation to the exclusive channels $B_s\to D_s^{*+}(\to D_s^+\gamma)D_s^{*-}(\to D_s^-\gamma)$ and $B_s\to J/\psi(\to l^+l^-)\phi(\to K^+K^-)$, i.e., to $X_1X_2\in\{D_s^{*+}D_s^{*-},J/\psi\phi\}$. Whereas the angular distribution of the latter process has been derived in [13], a followup note [23] not only examines the angular distributions for both processes but also discusses an efficient method for determining the relevant observables—the *moment analysis* [24]—and predicts these observables, thereby allowing comparisons with future experimental data.

The combination (30) enters the *untagged* angular distribution in the form

$$\begin{aligned}\mathrm{Im}\{[A_{\tilde f}^*(t)A_f(t)]\}&=-2|\langle(X_1X_2)_{\tilde f}|H_{\rm eff}|B_s\rangle\langle(X_1X_2)_f|H_{\rm eff}|B_s\rangle|\\&\quad\times\cos(\delta_f-\delta_{\tilde f})(e^{-\Gamma_Lt}-e^{-\Gamma_Ht})\phi_c^{(s)},\end{aligned} \tag{33}$$

where $\tilde f\in\{\|,0\}$ and $f=\perp$ denote linear polarization states [12,13]. In order to determine the weak phase $\phi_c^{(s)}$ from Eq. (33), we have to know both $|\langle(X_1X_2)_{\tilde f}|H_{\rm eff}|B_s\rangle|$, $|\langle(X_1X_2)_f|H_{\rm eff}|B_s\rangle|$ and the strong phase differences $\delta_f-\delta_{\tilde f}$. Whereas the former quantities can be determined straightforwardly from

$$[|A_f(t)|^2]=2|\langle(X_1X_2)_f|H_{\rm eff}|B_s\rangle|^2e^{-\Gamma_Lt}\quad(f\in\{\|,0\}), \tag{34}$$

$$[|A_\perp(t)|^2]=2|\langle(X_1X_2)_\perp|H_{\rm eff}|B_s\rangle|^2e^{-\Gamma_Ht}, \tag{35}$$

the latter ones can be obtained by combining the ratio of Eq. (33) for $\tilde f=\|$ and $\tilde f=0$ given by

$$\frac{\mathrm{Im}\{[A_\|^*(t)A_\perp(t)]\}}{\mathrm{Im}\{[A_0^*(t)A_\perp(t)]\}}=\frac{|\langle(X_1X_2)_\||H_{\rm eff}|B_s\rangle|}{|\langle(X_1X_2)_0|H_{\rm eff}|B_s\rangle|}\frac{\cos(\delta_\perp-\delta_\|)}{\cos(\delta_\perp-\delta_0)} \tag{36}$$

with the term of the untagged angular distribution corresponding to [13,23]

$$\begin{aligned}\mathrm{Re}\{[A_0^*(t)A_\|(t)]\}&=2|\langle(X_1X_2)_0|H_{\rm eff}|B_s\rangle\langle(X_1X_2)_\||H_{\rm eff}|B_s\rangle|\\&\quad\times\cos(\delta_\|-\delta_0)e^{-\Gamma_Lt}.\end{aligned} \tag{37}$$

Consequently the angular distributions for the *untagged* $B_s\to D_s^{*+}(\to D_s^+\gamma)D_s^{*-}(\to D_s^-\gamma)$ and $B_s\to J/\psi(\to l^+l^-)\phi(\to K^+K^-)$ modes allow a determination of the weak phase $\phi_c^{(s)}$.

The rather complicated extraction of the strong phase differences $\delta_f-\delta_{\tilde f}$ outlined above, which is needed to accomplish this task, can, however, be simplified considerably by making an additional assumption. In the case of the color-allowed channel $B_s\to D_s^{*+}D_s^{*-}$ the *factorization hypothesis* [25,26], which can be justified to some extent within the $1/N_C$ expansion [27], predicts rather reliably that the strong phase shifts are $0\bmod\pi$. This prediction for the strong phases can be tested experimentally by investigating the angular correlations for the SU(3)-related modes $B_{u,d}\to D_s^{*+}\overline{D^*_{u,d}}$. Since $B_s\to J/\psi\phi$ is, on the other hand, a color-suppressed transition, the validity of the factorization approach is very doubtful in this case [28]. However, flavor SU(3) symmetry of strong interactions is probably a good working assumption and can be used to determine the hadronization dynamics of $B_s\to J/\psi\phi$, in particular the strong phase differences $\delta_f-\delta_{\tilde f}$, from an analysis of the SU(3)-related $B\to J/\psi K^*$ modes [23,24]. These strategies should be very helpful to constrain $\phi_c^{(s)}$ with more limited statistics.

Whereas one expects $\Gamma_H<\Gamma_L$ and a small value of $\phi_c^{(s)}$ within the standard model, that need not to be the case in many scenarios for ''new physics'' beyond the standard model (see, e.g., [29]). The untagged data samples described by Eqs. (26) and (27) allow then only the extraction of $\cos 2\phi_c^{(s)}$ and $\sin 2\phi_c^{(s)}$ up to some discrete ambiguities. In particular they do not allow the determination of the sign of $\Delta\Gamma$ which could give us hints to physics beyond the standard model. This feature is simply due to the fact that we cannot decide which decay width is Γ_L and Γ_H, respectively, since we do not know the sign of $\Delta\Gamma$. Using, however, in addition the time dependences of *tagged* data samples, $\sin 2\phi_c^{(s)}$ can be extracted and the discrete ambiguities are resolved. With the help of the observables corresponding to Eq. (27) even the sign of $\Delta\Gamma$ can then be extracted, which was missed in a recent work [29]. In general, the ambiguities encountered in studies of untagged data samples are resolved by incorporating the additional information available from Δmt oscillations.

V. A DETERMINATION OF γ USING UNTAGGED DATA SAMPLES OF $B_s \to K^{*+}K^{*-}$ AND $B_s \to K^{*0}\overline{K^{*0}}$

After our discussion of some exclusive $\bar{b} \to \bar{c}c\bar{s}$ transitions and a brief excursion to "new physics" in the previous section let us now consider the $\bar{b} \to \bar{u}u\bar{s}$ decay $B_s \to K^{*+}(\to \pi K)K^{*-}(\to \pi K)$ and investigate what can be learned from *untagged* measurements of its angular distribution. Because of the special CKM structure of the $\bar{b} \to \bar{s}$ penguins [30], their contributions to $B_s \to K^{*+}K^{*-}$ can be written in the form

$$P'_f = -|P'_f| e^{i\delta^f_{P'}} e^{i\pi}, \tag{38}$$

where f denotes final state configurations of $K^{*+}K^{*-}$ with CP eigenvalue η^f_{CP} [see Eq. (5)], $\delta^f_{P'}$ are CP-conserving strong phases, the CP-violating weak phase has the numerical value of π and the minus sign is due to our definition of meson states which is similar to the conventions applied in [31].

The penguin contributions include not only penguins with internal top-quark exchanges, but also those with internal up and charm quarks [30]. Rescattering processes are included by definition in the penguin amplitude P'_f. For example, the process $B_s \to \{D_s^{*+} D_s^{*-}\} \to K^{*+}K^{*-}$ (see, e.g., [32]) is related to penguin topologies with charm quarks running in the loops as can be seen easily by drawing the corresponding Feynman diagrams. Although such rescattering processes may affect $|P'_f|$ and $\delta^f_{P'}$ they do not modify the weak phase in Eq. (38).

On the other hand, the contributions of the current-current operators appearing in the low energy effective Hamiltonian (7), which are color allowed in the case of $B_s \to K^{*+}K^{*-}$, have the structure

$$T'_f = -|T'_f| e^{i\delta^f_{T'}} e^{i\gamma}, \tag{39}$$

where $\delta^f_{T'}$ is again a CP-conserving strong phase. Consequently, combining these considerations, we obtain the following transition matrix element for $B_s \to (X_1X_2)_f$ with $X_1X_2 = K^{*+}K^{*-}$:

$$\langle (X_1X_2)_f | H_{\rm eff} | B_s \rangle = |P'_f| e^{i\delta^f_{P'}} [1 - r_f\, e^{i\gamma}], \tag{40}$$

where

$$r_f \equiv \frac{|T'_f|}{|P'_f|} e^{i(\delta^f_{T'} - \delta^f_{P'})}. \tag{41}$$

Hence the quantity ξ_f defined through Eq. (13) is given by

$$\xi_f = \frac{1 - r_f\, e^{-i\gamma}}{1 - r_f\, e^{+i\gamma}}. \tag{42}$$

Following the plausible hierarchy of decay amplitudes introduced in [31], we expect that penguins play, in analogy to $B_s \to K^+K^-$ [33,34], the dominant role in $B_s \to K^{*+}K^{*-}$.

To evaluate the time evolution of the observables of the untagged angular distribution corresponding to real or imaginary parts of Eq. (18), we need $1 \pm \xi^*_{\tilde f}\xi_f$ and $\xi^*_{\tilde f} \pm \xi_f$ which are given by

$$1 + \xi^*_{\tilde f}\xi_f = \frac{2}{N_{f,\tilde f}} [1 - (r^*_{\tilde f} + r_f)\cos\gamma + r^*_{\tilde f} r_f], \tag{43}$$

$$1 - \xi^*_{\tilde f}\xi_f = i\,\frac{2}{N_{f,\tilde f}} (r^*_{\tilde f} - r_f)\sin\gamma \tag{44}$$

and

$$\xi^*_{\tilde f} + \xi_f = \frac{2}{N_{f,\tilde f}} [1 - (r^*_{\tilde f} + r_f)\cos\gamma + r^*_{\tilde f}\; r_f \cos 2\gamma], \tag{45}$$

$$\xi^*_{\tilde f} - \xi_f = -i\,\frac{2}{N_{f,\tilde f}} [r^*_{\tilde f} + r_f - 2r^*_{\tilde f}\; r_f \cos\gamma]\sin\gamma, \tag{46}$$

respectively, where

$$N_{f,\tilde f} \equiv 1 - r^*_{\tilde f}\, e^{-i\gamma} - r_f\, e^{i\gamma} + r^*_{\tilde f}\; r_f. \tag{47}$$

These combinations of $\xi^*_{\tilde f}$ and ξ_f are multiplied in Eq. (18) by

$$\begin{aligned}\langle (X_1X_2)_{\tilde f} | H_{\rm eff} | B_s\rangle^* \langle (X_1X_2)_f | H_{\rm eff} | B_s \rangle \\ = |P'_{\tilde f}||P'_f| e^{i(\delta^f_{P'} - \delta^{\tilde f}_{P'})} N_{f,\tilde f}.\end{aligned} \tag{48}$$

Here we have used expression (40) to calculate this product of hadronic matrix elements, which—in contrast to the case where a single CKM amplitude dominates [see the cautious remark after Eq. (25)]—depends also on the weak phase γ through $N_{f,\tilde f}$. However, these factors cancel in Eq. (18) so that we finally arrive at the following set of equations describing $B_s \to (K^{*+}K^{*-})_f$:

$\eta^{\tilde f}_{CP} = \eta^f_{CP} = +1$:

$$\begin{aligned}[A^*_{\tilde f}(t)A_f(t)] = 2\,|P'_{\tilde f}||P'_f| e^{i(\delta^f_{P'} - \delta^{\tilde f}_{P'})} \\ \times [\{1 - (r^*_{\tilde f} + r_f)\cos\gamma + r^*_{\tilde f}\; r_f \cos^2\gamma\} \\ \times e^{-\Gamma_L t} + r^*_{\tilde f}\; r_f \sin^2\gamma\; e^{-\Gamma_H t}].\end{aligned} \tag{49}$$

$\eta^{\tilde f}_{CP} = \eta^f_{CP} = -1$:

$$\begin{aligned}[A^*_{\tilde f}(t)A_f(t)] = 2\,|P'_{\tilde f}||P'_f| e^{i(\delta^f_{P'} - \delta^{\tilde f}_{P'})} \\ \times [\{1 - (r^*_{\tilde f} + r_f)\cos\gamma + r^*_{\tilde f}\; r_f \cos^2\gamma\} \\ \times e^{-\Gamma_H t} + r^*_{\tilde f}\; r_f \sin^2\gamma\; e^{-\Gamma_L t}].\end{aligned} \tag{50}$$

$\eta^{\tilde{f}}_{CP} = -\eta^{f}_{CP} = +1$:

$$[A^*_{\tilde{f}}(t)A_f(t)] = i\,2|P'_{\tilde{f}}||P'_f|e^{i(\delta^f_{P'}-\delta^{\tilde{f}}_{P'})}\times[r^*_{\tilde{f}}\,e^{-\Gamma_H t}-r_f e^{-\Gamma_L t}+r^*_{\tilde{f}}\,r_f\times(e^{-\Gamma_L t}-e^{-\Gamma_H t})\cos\gamma]\sin\gamma. \quad (51)$$

The structure of these equations, which are valid exactly, is much more complicated than that of Eqs. (28)–(30) where a single CKM amplitude dominates to an excellent accuracy. Note that a measurement of either the $e^{-\Gamma_H t}$ or $e^{-\Gamma_L t}$ terms in Eqs. (49) and (50), respectively, or of nonvanishing observables corresponding to Eq. (51) would give unambiguous evidence for a nonvanishing value of $\sin\gamma$.

A determination of γ is possible if one measures in addition the time-dependent *untagged* angular distribution for $B_s \to K^{*0}\overline{K^{*0}}$ which is a pure penguin-induced $\bar{b}\to\bar{s}d\bar{d}$ transition. Its time evolution can be obtained from Eqs. (49)–(51) by setting $r_{\tilde{f}} = r_f = 0$ and depends only on the hadronization dynamics of the penguin operators.

There are two classes of penguin topologies as we have already noted briefly after Eq. (9): QCD and electroweak penguins originating from strong and electroweak interactions, respectively. In contrast to naive expectations, the contributions of electroweak penguin operators may play an important role in certain nonleptonic B-meson decays because of the presence of the *heavy* top-quark [35,36] (see also [37–40]). However, in the case of the $B_s \to K^*K^*$ transitions considered in this section, these contributions are color suppressed and play only a minor role compared to those of the dominant QCD penguin operators.

If we neglect these electroweak penguin contributions, which has not been done in the formulas given above and should be a good approximation in our case, and use furthermore the SU(2) isospin symmetry of strong interactions, the $B_s \to K^{*0}\overline{K^{*0}}$ observables can be related to the $B_s \to K^{*+}K^{*-}$ case. In terms of linear polarization states [12], these observables fix $|P'_0|$, $|P'_\parallel|$, $|P'_\perp|$, and $\cos(\delta^0_{P'}-\delta^\parallel_{P'})$. Since the overall normalizations of the untagged $B_s \to K^{*+}K^{*-}$ observables can be determined this way, the $e^{-\Gamma_L t}$ and $e^{-\Gamma_H t}$ pieces of the observables $[|A_0(t)|^2]$, $[|A_\parallel(t)|^2]$ and $\mathrm{Re}\{[A^*_0(t)A_\parallel(t)]\}$ [see Eq. (49)] allow another extraction of the CKM angle γ. The remaining observables can be used to resolve possible discrete ambiguities. Needless to say, also the quantities r_f and the QCD penguin amplitudes P_f are of particular interest since they provide insights into the hadronization dynamics of the QCD penguins. A detailed analysis of the decays $B_s \to K^{*+}K^{*-}$ and $B_s \to K^{*0}\overline{K^{*0}}$ is presented in [41], where also the angular distributions are given explicitly.

Another interesting application of Eq. (49) is associated with the decays $B_s \to K^+K^-$ and $B_s \to K^0\overline{K^0}$. Using again the SU(2) isospin symmetry of strong interactions to relate their QCD penguin contributions (electroweak penguin contributions are once more color suppressed and are hence very small), the time-dependent *untagged* rates for these modes evolve as

$$[|A(t)|^2] = 2\,|P'|^2[(1-2\,|r|\cos\rho\,\cos\gamma+|r|^2\cos^2\gamma)\times e^{-\Gamma_L t}+|r|^2\,\sin^2\gamma\,e^{-\Gamma_H t}] \quad (52)$$

and

$$[|A(t)|^2] = 2\,|P'|^2 e^{-\Gamma_L t}, \quad (53)$$

respectively, where we have used

$$r \equiv |r|e^{i\rho}. \quad (54)$$

Here ρ is a CP-conserving strong phase and $|r| = |T'|/|P'|$. In general, there are a lot fewer observables in ''pseudoscalar-pseudoscalar'' cases than in ''vector-vector'' cases. In particular there is no observable corresponding to $\mathrm{Re}\{[A^*_0(t)A_\parallel(t)]\}$. We therefore need some additional input in order to extract γ from Eq. (52). That is provided by the SU(3) flavor symmetry of strong interactions. If we neglect the color-suppressed current-current contributions to $B^+ \to \pi^+\pi^0$, which are expected to be suppressed relative to the color-allowed contributions by a factor of ~ 0.2, this symmetry yields [31]

$$|T'| \approx \lambda\,\frac{f_K}{f_\pi}\sqrt{2}\,|A(B^+\to\pi^+\pi^0)|, \quad (55)$$

where λ is the Wolfenstein parameter [6], f_K and f_π are the K- and π-meson decay constants, respectively, and $A(B^+\to\pi^+\pi^0)$ denotes the appropriately normalized $B^+\to\pi^+\pi^0$ decay amplitude. Since $|P'|$ is known from $B_s \to K^0\overline{K^0}$, the quantity $|r|$ can be estimated with the help of Eq. (55) and allows the extraction of γ from the part of Eq. (52) evolving with the exponent $e^{-\Gamma_H t}$. Using in addition the piece evolving with $e^{-\Gamma_L t}$ the strong phase ρ can also be determined up to certain discrete ambiguities. Since one expects $|r|\sim 0.2$ [31,33,34], it may be difficult to measure the $e^{-\Gamma_H t}$ contribution to Eq. (52) which is proportional to $|r|^2$. The value of γ and the observable r estimated that way could be used as an input to determine electroweak penguin amplitudes by measuring in addition the branching ratios $\mathcal{B}(B^+\to\pi^0K^+)$, $\mathcal{B}(B^-\to\pi^0K^-)$, and $\mathcal{B}(B^+\to\pi^+K^0) = \mathcal{B}(B^-\to\pi^-\overline{K^0})$ as has been proposed in [33].

Let us finally note that Eqs. (49)–(51) apply also to the mode $B_s \to \rho^0\phi$, if we perform the replacements

$$|P'_f| \to |P'^{\,\mathrm{EW}}_f|, \qquad \delta^f_{P'} \to \delta^f_{\mathrm{EWP}'}, \qquad r_f \to \frac{|C'_f|}{|P'^{\,\mathrm{EW}}_f|}\exp[i(\delta^f_{C'}-\delta^f_{\mathrm{EWP}'})], \quad (56)$$

where C'_f denotes color-suppressed contributions of the current-current operators and $|P'^{\,\mathrm{EW}}_f|, \delta^f_{\mathrm{EWP}'}$ are related to color-allowed contributions of electroweak penguin operators. Similar to the situation arising in $B_s \to \pi^0\phi$, which has been discussed in [36] (see also [38–40]), we expect that this decay is dominated by electroweak penguins. Consequently its *untagged* angular distribution may inform us about the physics of the corresponding operators. In respect of controlling electroweak penguins in a quantitative way by using SU(3) relations among $B\to\pi K$ decay amplitudes [33], the CKM angle γ is a central input. Therefore the new strategies to extract this angle in a rather clean way from *untagged* B_s data samples presented in Secs. IV and V are also very helpful to accomplish this ambitious task.

VI. SUMMARY

We have calculated the time evolutions of angular distributions for B_s decays into final states that are admixtures of different CP eigenstates. Interestingly, due to the expected perceptible B_s-$\overline{B_s}$ lifetime difference, the corresponding observables may allow the extraction of CKM phases even in the *untagged* case where one does not distinguish between initially present B_s and $\overline{B_s}$ mesons. As we have demonstrated in this paper, such studies of the exclusive $\bar{b}\to\bar{c}c\bar{s}$ modes $B_s\to D_s^{*+}D_s^{*-}$ and $B_s\to J/\psi\phi$, which are dominated to an excellent approximation by a single CKM amplitude, allow a determination of the Wolfenstein parameter η thereby fixing the height of the usual unitarity triangle. Using the CKM factor $R_b\propto|V_{ub}|/|V_{cb}|$ as an additional input, γ can be determined both from η and from mixing-induced CP violation in $B_d\to J/\psi K_S$ measuring sin2β. A comparison of these two results for γ determined from B_s and B_d decays, respectively, would allow an interesting test whether the corresponding mixing phases are described by the standard model.

If we apply the SU(2) isospin symmetry of strong interactions to relate the QCD penguin contributions to the $\bar{b}\to\bar{u}u\bar{s}$ mode $B_s\to K^{*+}K^{*-}$ and to the $\bar{b}\to\bar{s}d\bar{d}$ transition $B_s\to K^{*0}\overline{K^{*0}}$, which should play the dominant role there, another extraction of γ is possible from *untagged* measurements of their angular distributions. Substituting the relevant variables appropriately, the results derived for $B_s\to K^{*+}K^{*-}$ apply also to $B_s\to\rho^0\phi$ which is expected to be dominated by electroweak penguin operators.

We will come back to these decays in separate forthcoming publications [23,41]. The case of B_s decays into final states that are not admixtures of different CP eigenstates but only of different parity eigenstates is outlined in [5]. There we discuss how angular correlations for untagged B_s decays governed by $\bar{b}\to\bar{c}u\bar{s}$ quark-level transitions allow also a determination of the CKM angle γ.

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