Relative strength of W exchange and factorization contributions in hadronic decays of charmed baryons

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The nonleptonic decays $\Lambda_c^+ \to \Delta^{++}K^-, \Sigma^{*0}\pi^+$, and $\Xi^{*0}K^+$ and $\Omega_c^0 \to \Xi^0\overline{K^0}$, $\Omega^-\pi^+$, and $\Xi^{*0}\overline{K^0}$ are studied. The dominant contribution for the former decays comes from the W exchange and for the latter decays the W emission gives the dominant contribution. These decays are especially suitable to determine directly the scale of W exchange and W emission from the experimental data. We obtain $\alpha(\Omega_c^0 \to \Xi^0\overline{K^0} \approx 0.35$. [S0556-2821(97)02901-9]

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With the coming of data on two body hadronic decays of charmed baryons [1–3] the study of such decays is gaining considerable attention [4–10]. The meager data available at present are already begining to distinguish between the various theoretical models. The study of hadronic two-body decays of charmed baryons is complicated due to various competing mechanisms, such as factorization, pole terms, and W-exchange terms, each of which has uncertanities of its own. It is thus desirable to know the relative strength or importance of these mechanisms. The purpose of this paper is to study a class of charmed baryon decays which throws light on the relative strength of W-exchange term and the factorization term.

The W emission for these decays leads to factorization, i.e., the contribution to nonleptonic decays comes from the coupling of the weak transition $B_c \to B$ with the pion current. For this case the relevant matrix elements are

$$\langle B|\overline{q}\gamma_\mu(1+\gamma_5)c|B_c\rangle$$

where $q=s$, d, or u quark. On the other hand, the W exchange in the nonrelativistic limit gives the effective Hamiltonian equation [11]

$$H_W^{\mathrm{PC}} = \frac{G_F}{\sqrt{2}} V_{du} V_{cs} \sum_{i\neq j} \alpha_i^+ \gamma_j^- (1-\sigma_i\cdot\sigma_j)\,\delta^3(r), \qquad (1)$$

where α_i^+ converts a d quark into a u quark and γ_j^- converts a c quark into an s quark. The W exchange is relevant when one considers the baryon-pole contribution (Born terms) to the p-wave decay amplitude. Such a contribution involves the matrix elements of the form $\langle B|H_W^{\mathrm{PC}}|B_c\rangle$ which can be evaluated in the nonrelativistic quark model (NQM) by using Eq. (1). It may also be mentioned that the equal time commutator which contributes to the s-wave decay amplitude also involves these matrix elements [4,10].

We should point out that Eq. (1) severely restricts the state B or B_c in $\langle B|H_W|B_c\rangle$. First we note that in the leading nonrelativistic limit, only H_W^{PC} as given in Eq. (1) survives where it is implicit that an overlap integral in momentum space for ground state (s wave) wave functions is set equal to unity due to the normalization condition. Thus one can ignore resonances $J^P=\frac{1}{2}^-$, $\frac{3}{2}^-$ or other orbital excitations as they would require at least one power of momentum in H_W in order to connect them with the relevant ground state in the overlap integral, i.e., one has to go to order v/c. Similarly, in order to connect radial excitations with the corresponding ground state, one would need terms of order $(v/c)^2$; otherwise the overlap integral would be zero due to orthogonality of the wave functions. Thus we are left with the matrix elements involving s-wave ground state of the form $\langle 8,\frac{1}{2}^+|H_W^{\mathrm{PC}}|\overline{3},\frac{1}{2}^+\rangle$ and $\langle 10,\frac{3}{2}^+|H_W^{\mathrm{PC}}|\overline{3},\frac{1}{2}^+\rangle$ for the s-channel baryon pole and $\langle 10,\frac{3}{2}^+|H_W^{\mathrm{PC}}|6,\frac{1}{2}^+\rangle$ and $\langle 10,\frac{3}{2}^+|H_W^{\mathrm{PC}}|6,\frac{3}{2}^+\rangle$ for the u channel baryon pole contributions to Λ_c^+ decay. Now the spin wave functions of $|\overline{3},\frac{1}{2}^+\rangle$, $|6,\frac{1}{2}^+\rangle$ and $|6,\frac{3}{2}^+\rangle$ are respectively $\chi_{MA}=(1/\sqrt{2})|(\uparrow\downarrow-\downarrow\uparrow)\uparrow\rangle$, $\chi_{MS}=(1/\sqrt{6})|-(\uparrow\downarrow+\downarrow\uparrow)\uparrow+2\uparrow\uparrow\downarrow\rangle$, $\chi_S=(1/\sqrt{3})|\uparrow\uparrow\downarrow+(\uparrow\downarrow+\downarrow\uparrow)\uparrow\rangle$. It is easy to see that $(1-\sigma_i\cdot\sigma_j)\chi_S=0$ for $i,j=1,3$ or $2,3$ (3 refers to the c quark) since the two particle spin state in each case is a spin triplet for which $\sigma_i\cdot\sigma_j=1$, e.g., $(1-\sigma_2\cdot\sigma_3)\chi_S=(1/\sqrt{3})(1-\sigma_2\cdot\sigma_3)|\uparrow(\uparrow\downarrow+\downarrow\uparrow)+\downarrow\uparrow\uparrow\rangle=0$. Further $(1-\sigma_i\cdot\sigma_j)$ acting on χ_{MA} and χ_{MS} cannot generate χ_S. In fact

$$(1-\sigma_2\cdot\sigma_3)\chi_{MA}=\sqrt{2}|\uparrow(\downarrow\uparrow-\uparrow\downarrow)\rangle,$$

$$(1-\sigma_2\cdot\sigma_3)\chi_{MS}=\sqrt{6}|\uparrow(\downarrow\uparrow-\uparrow\downarrow)\rangle$$

[and similarly for $(1-\sigma_1\cdot\sigma_3)$], which when projected on to χ_S, gives zero. Thus all the above matrix elements are zero except $\langle 8,\frac{1}{2}^+|H_W^{\mathrm{PC}}|\overline{3},\frac{1}{2}^+\rangle$, which gives rise to Σ^+ pole in s channel for Λ_c^+ decay.

For the purpose already stated, we study the nonleptonic decays of Λ_c^+ and Ω_c^0 into B^*P, where P represents the pseudoscalar octet and B^* is a member of spin $\frac{3}{2}^+$ decuplet. In addition we also discuss the decay $\Omega_c^0 \to \Xi^0\overline{K^0}$. These decays are interesting because for the decays $\Lambda_c^+ \to \Delta^{++}K^-, \Sigma^{*0}\pi^+, \Xi^{*0}K^+$, as already seen the dominant contribution comes from the s-channel baryon pole

Σ^+, whereas for the decays $\Omega_c^0 \to \Omega^- \pi^+, \Xi^{*0}\overline{K^0}, \Xi^0\overline{K^0}$, the factorization contributes. Thus in these decays, it is possible to determine the scale of W-exchange and W-emission contributions directly from the experimental data. It may be noted that Ω_c^0 is the only stable baryon against strong and electromagnetic interactions in the sextet representation of charmed baryons.

For the decay of the type $B_c(p') \to B^*(p) + P(q)$, the decay amplitude can be expressed as

$$T = \frac{1}{(2\pi)^{9/2}} \sqrt{\frac{m' m^*}{2 q_0 p_0 p_0'}} \frac{q_\lambda}{F_P} \overline{u_\lambda}(p)[C + D\gamma_5] u(p') \quad (2)$$

where u_λ is the Rarita-Schwinger spinor, u is the Dirac spinor, and $q = p - p'$. It is clear that C is the parity-conserving (p wave) amplitude and D is the parity-violating (d wave) amplitude. F_P is the pseudoscalar meson decay constant (F_π = 132 MeV), which is introduced here so as to make the amplitudes C and D dimensionless. The decay rate is given by

$$\Gamma = \frac{1}{6\pi} \frac{m' |p|^3}{m^{*2} F_P^2} [(p_0 + m^*)|C|^2 + (p_0 - m^*)|D|^2]. \quad (3)$$

The polarization α is given by

$$\alpha = \frac{2|p| \mathrm{ReCD}^*}{(p_0 + m^*)|C|^2 + (p_0 - m^*)|D|^2}. \quad (4)$$

Since neither the factorization nor the baryon pole can generate d-wave transition [at least in the naive quark model (NQM)], the amplitude $D = 0$. Hence $\alpha = 0$. This is in accordance with the two-particle nonleptonic decays of Ω^-, for which the experimental value of α is zero.

Since Λ_c^+ belongs to the triplet representation of SU(3), the matrix element

$$\langle B^* | \overline{q} \gamma_\mu (1 + \gamma_5) c | \Lambda_c^+ \rangle = 0. \quad (5)$$

Thus the factorization does not contribute to the decays $\Lambda_c^+ \to \Delta^{++} K^-$, $\Sigma^{*0}\pi^+$, $\Xi^{*0}K^+$. The dominant contributation to these decays comes from the Σ^+ pole i.e., from the chain $\Lambda_c^+ \to \Sigma^+ \to \Delta^{++}K^-$, $\Sigma^{*0}\pi^+$, or $\Xi^{*0}K^+$. Hence the decay amplitude C for these decays is given by

$$C = (\sqrt{6}, -1, -\sqrt{2}) g^* \frac{\langle \Sigma^+ | H_W^{\mathrm{PC}} | \Lambda_c^+ \rangle}{m_{\Lambda_c} - m_\Sigma} \quad (6)$$

where the factors multiplying g^* are given by SU(3). It follows that

$$\frac{\Gamma(\Lambda_c^+ \to \Xi^{*0} K^+)}{\Gamma(\Lambda_c^+ \to \Delta^{++} K^-)} = \frac{1}{3} \quad \text{(phase space factor)}$$
$$\approx 0.07 \quad (7)$$

and

$$\frac{\Gamma(\Lambda_c^+ \to \Sigma^{*0} \pi^+)}{\Gamma(\Lambda_c^+ \to \Delta^{++} K^-)} \approx \frac{1}{6} \left(\frac{F_K}{F_\pi} \right)^2 \quad \text{(phase space factor)}$$
$$\approx 0.23. \quad (8)$$

The experimental branching ratios are given by [1] and [3]:

$$B(\Lambda_c^+ \to \Delta^{++} K^-) = (0.7 \pm 0.4)\%, \quad (9a)$$

$$B(\Lambda_c^+ \to \Xi^{*0} K^+) = (0.2 \pm 0.1)\%. \quad (9b)$$

The improved experimental data can test Eqs. (7) and (8), which are independent of the scale.

We now calculate the decay rate $\Gamma(\Lambda_c^+ \to \Delta^{++} K^-)$. For this evaluation we need g^* and the matrix element $\langle \Sigma^+ | H_W^{\mathrm{PC}} | \Lambda_c^+ \rangle$. From SU(3) and the experimental value for the decay width $\Gamma(\Delta^{++} \to p\pi^+) \approx 120$ MeV , we find $\sqrt{6} g^* \approx 2.09$. On the other hand, the weak matrix element $\langle \Sigma^+ | H_W^{\mathrm{PC}} | \Lambda_c^+ \rangle$ on using Eq. (1) is given by [11]

$$\langle \Sigma^+ | H_W^{\mathrm{PC}} | \Lambda_c^+ \rangle = \left[\frac{G_F}{\sqrt{2}} V_{du} V_{cs} \right] \sqrt{2} d' \quad (10)$$

where [12]

$$d' = \langle \Psi_0 | \delta^3(r) | \Psi_0 \rangle = \frac{3(m_\Delta - m_N) m_{cu}^2}{8\pi\alpha_s} \approx 5 \times 10^{-3} \ \mathrm{GeV}^3. \quad (11)$$

For the numerical value of d' we have used $\alpha_s = 0.5$, $m_u = 0.340$ MeV, $m_c = 1520$ MeV, $m_{cu} = m_c m_u/(m_c + m_u) \approx 278$ MeV and $m_\Delta - m_N \approx 293$ MeV. From Eqs. (3), (6), (10), and (11), using the above value for g^*, we find

$$\Gamma(\Lambda_c^+ \to \Delta^{++} K^-) \approx 0.46 \times 10^{11} \ \mathrm{s}^{-1} \quad (12)$$

so that

$$B(\Lambda_c^+ \to \Delta^{++} K^-) \approx 0.9 \times 10^{-2} \quad (13)$$

to be compared with the experimental value given in Eq. (9a).

Let us now consider the decay $\Omega_c^0 \to \Xi^0 \overline{K^0}$. For this decay, the factorization gives for the s-wave and p-wave amplitudes A and B, the contributions

$$A_{\mathrm{fact}} = -\left[\frac{G_F}{\sqrt{2}} V_{ud} V_{cs} \right] C_2 (m_{\Omega_c} - m_\Xi) g_V^{\Omega_c - \Xi}(q^2) F_K,$$

$$B_{\mathrm{fact}} = \left[\frac{G_F}{\sqrt{2}} V_{ud} V_{cs} \right] C_2 (m_{\Omega_c} + m_\Xi) g_A^{\Omega_c - \Xi}(q^2) F_K \quad (14)$$

where

$$C_2 = \frac{C_+ - C_-}{2}, \quad C_+ = 0.7 \ \text{and} \ C_- = 1.93.$$

In the NQM [13], one obtains

$$g_V^{\Omega_c-\Xi}(0)\equiv g_V^{\Omega_c-\Xi}=-\frac{1}{\sqrt{3}},$$
$$g_A^{\Omega_c-\Xi}(0)\equiv g_A^{\Omega_c-\Xi}=\frac{1}{3\sqrt{3}}. \tag{15}$$

For p wave, Ξ_c^0 and Ξ'^0_c baryon poles also contribute. This contribution is given by

$$B_{\rm pole}=\left[g_{\Omega_c K^0 \Xi_c^0}\frac{\langle \Xi^0|H_W^{\rm PC}|\Xi_c^0\rangle}{(m_{\Xi_c^0}-m_{\Xi^0})}+g_{\Omega_c K^0 \Xi_c'^0}\frac{\langle \Xi^0|H_W^{\rm PC}|\Xi_c'^0\rangle}{(m_{\Xi_c'^0}-m_{\Xi^0})}\right]. \tag{16}$$

Now PCAC (partial conservation of axial vector current) gives

$$g_{\Omega_c K^0 \Xi_c^0}=(m_{\Omega_c}+m_\Xi)\frac{g_A}{F_K}, \tag{17}$$

$$g_{\Omega_c K^0 \Xi_c'^0}=(m_{\Omega_c}+m_{\Xi'})\frac{g_A'}{F_K}. \tag{18}$$

In the NQM, one obtains [13]

$$g_A=-\frac{2\sqrt{2}}{\sqrt{3}},\quad g_A'=\frac{2\sqrt{2}}{3}. \tag{19}$$

Also using Eq. (1), we find

$$\langle \Xi^0|H_W^{\rm PC}|\Xi_c^0\rangle=\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}(-\sqrt{2}d'),$$

$$\langle \Xi^0|H_W^{\rm PC}|\Xi_c'^0\rangle=\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}(-\sqrt{6}d'). \tag{20}$$

Hence in the symmetry limit, i.e., $m_{\Xi_c^0}=m_{\Xi'^0_c}$, $B_{\rm pole}$ vanishes. Thus we ignore the pole contributation. Neglecting the q^2 dependence ($q^2=-m_K^2$) of the form factors $g_V^{\Omega_c-\Xi}$ and $g_A^{\Omega_c-\Xi}$ we obtain, from Eq. (14), using $m_{\Omega_c}=2701$ MeV,

$$\Gamma(\Omega_c^0\to\Xi^0\overline{K^0})\approx 0.54\times 10^{11}\ {\rm s}^{-1},\quad \alpha=0.35. \tag{21}$$

Note that the value of α is independent of the scale.

For the decay $\Omega_c^0\to\Omega^-\pi^+$, neither s channel nor u channel pole contributes. The factorization contribution is given by

$$D_{\rm fact}=\left[\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}\right]C_1[D_{1V}^{\Omega_c-\Omega}(q^2)+\cdots]F_\pi^2,$$

$$C_{\rm fact}=\left[\frac{G_F}{\sqrt{2}}V_{ud}V_{cs}\right]C_1[D_{1A}^{\Omega_c-\Omega}(q^2)+\cdots]F_\pi^2,$$

$$C_1=\frac{C_+ + C_-}{2}, \tag{22}$$

where the ellipses denote the contribution of other form factors. In the NQM, one obtains [13]

$$\langle\Omega^-|\overline{s\gamma_\mu}\gamma_5 c|\Omega_c^0\rangle\approx D_{1A}^{\Omega_c-\Omega}(0)=D_{1A}^{\Omega_c-\Omega}=-\frac{2\sqrt{2}}{3}, \tag{23}$$

$$\langle\Omega^-|\overline{s\gamma_\mu}c|\Omega_c^0\rangle\approx D_V^{\Omega_c-\Omega}(0)=0. \tag{24}$$

For the decay $\Omega_c^0\to\Xi^{*0}\overline{K^0}$, replace F_π by F_K, C_1 by C_2, m_Ω by m_{Ξ^*}, and D_{1A} by $-2\sqrt{2}/3\sqrt{3}$. The u-channel poles can contribute to this decay, but since

$$\langle \Xi^{*0}|H_W^{\rm PC}|\Xi_c^0,\Xi_c'^0\rangle=0$$

as obtained from Eq. (1), their contribution is zero. Hence

$$\frac{\Gamma(\Omega_c^0\to\Xi^{*0}\overline{K^0})}{\Gamma(\Omega_c^0\to\Omega^-\pi^+)}=\frac{1}{3}\left(\frac{F_K}{F_\pi}\right)^2\left(\frac{C_2}{C_1}\right)^2\ \text{(phase space factor)}$$
$$\approx 0.13. \tag{25}$$

On the other hand, we obtain

$$\frac{\Gamma(\Omega_c^0\to\Xi^{*0}\overline{K^0})}{\Gamma(\Omega_c^0\to\Xi^0\overline{K^0})}\approx 0.6, \tag{26}$$

$$\frac{\Gamma(\Omega_c^0\to\Omega^-\pi^+)}{\Gamma(\Omega_c^0\to\Xi^0\overline{K^0})}\approx 4. \tag{27}$$

To conclude, the W emission in the decay $\Omega_c^0\to\Xi^0\overline{K^0}$ gives $\alpha=0.35$ independent of the scale. The scale can be fixed from its decay width when Ω_c^0 lifetime is measured experimentally with accuracy. The recent experimental value [14] for the lifetime is $\tau_{\Omega_c^0}=55^{+13}_{-11}({\rm stat})^{+18}_{-23}({\rm syst})\times 10^{-15}$ s. Taking $\tau_{\Omega_c^0}=(50-100)\times 10^{-15}$ s, we find the following branching ratios $B(\Omega_c^0\to\Xi^0\overline{K^0})\approx(0.3-0.5)\times 10^{-2}$ and $B(\Omega_c^0\to\Omega^-\pi^+)\approx(1.2-2)\times 10^{-2}$. However, in the factorization contribution, the form of the form factors can change this contribution. These decays are especially suitable for studying these effects. On the other hand, the W exchange gives the dominant contribution to the decays $\Lambda_c^+\to\Delta^{++}K^-,\Sigma^{*0}\pi^+,\Xi^{*0}K^+$. Here we find

$$B(\Lambda_c^+\to\Delta^{++}K^-)=0.9\times 10^2$$

and

$$B(\Lambda_c^+\to\Xi^{*0}K^+)/B(\Lambda_c^+\to\Delta^{++}K^-)=0.07.$$

More accurate experimental data on these decays will improve our understanding of W-exchange contribution to the nonleptonic decays of charmed baryons.

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