Yang-Mills theory in a modified axial gauge

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The Yang-Mills functional integral is studied in an axial variant of 't Hooft's maximal Abelian gauge. In this gauge Gauss' law can be completely resolved resulting in a description in terms of unconstrained variables. Compared to previous work along this line starting with the work of Goldstone and Jackiw one ends up here with half as many integration variables, in addition to a field living in the Cartan subgroup of the gauge group and in $D-1$ dimensions. The latter is of particular relevance for the infrared behavior of the theory. Keeping only this variable we calculate the Wilson loop and find an area law. [S0556-2821(97)00404-9]

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I. INTRODUCTION

It is a common belief that the fundamental interactions are described by gauge theories. This is, in particular, true for strong interactions, which are assumed to be described by QCD. This theory has been tested in the high energy regime, where perturbation theory is applicable due to asymptotic freedom. On the other hand, low energy hadron physics requires a nonperturbative treatment of QCD. This regime, which is ultimately related to the confinement problem, is much less understood. Perturbative calculations indicate that the confinement phenomenon is due to the non-Abelian nature of Yang-Mills theory. Furthermore, one-loop calculations show that the perturbative Yang-Mills vacuum is unstable [1] and various models of the Yang-Mills vacuum have been designed as, for example, the Copenhagen vacuum [2], the instanton liquid model [3], which extends the instanton gas picture [4], the dual superconductor [5], or the stochastic vacuum [6]. The various models aim at different aspects of strong interaction; e.g., the instanton models seem to be suited to explain spontaneous breaking of chiral symmetry [7], while the Copenhagen vacuum, stochastic vacuum, or the dual superconductor focus on the color confinement.

A rigorous approach to strong coupling Yang-Mills theory is provided by lattice Monte Carlo calculations [8], which have been developed to a high level of sophistication. This approach has given much insight into the nature of the Yang-Mills vacuum. The great successes of lattice calculations are in low energy hadron physics (where confinement is perhaps not of much relevance) [9]. However, a complete understanding of the Yang-Mills theory will probably not be provided by the lattice simulations alone but requires also analytic tools. For some applications of lattice QCD a separation of scales is required and input from perturbation theory is needed (see, e.g., [10]). Also the interpretation of the lattice results sometimes requires or at least is facilitated by modeling properties of the Yang-Mills-vacuum-like correlation functions, which in turn are fed by the lattice calculations (see, e.g., [11]).

Several analytic approaches have been proposed to explore the nonperturbative features of strong coupling Yang-Mills theory, e.g., the strong coupling lattice expansion [8] or the small volume expansion [12,13]. A crucial point in all analytical approaches to Yang-Mills theory is gauge fixing, which cannot be performed in a unique way due to the existence of Gribov copies [14].

Most analytic approaches to Yang-Mills theory are based on the Weyl gauge $A_0=0$, where Gauss' law has to be inforced as a constraint to guarantee local gauge invariance [17]. Violation of Gauss' law generates color charges during the time evolution and, by this leaking of color, confinement is lost. This fact has recently been emphasized in Ref. [18], where explicit projection on gauge-invariant states has been performed in the construction of the path integral representation of the Yang-Mills transition amplitude. Not surprisingly projection onto gauge-invariant states requires a compact integration measure (the Haar measure of the gauge group), reminiscent to the lattice approach. (In fact, the approach of Ref. [18] can be obtained from the lattice formulation by taking the continuum limit in the spatial directions only.)

Several approaches have been advocated, which explicitly resolved the Gauss' law constraint by changing variables, resulting in a description in terms of a reduced number of unconstraint variables. These approaches are based on the Schrödinger functional formulation of Yang-Mills theory [17]. References [19,20] use variants of the unitary gauge, while in Ref. [21] the Coulomb gauge $\nabla A=0$ was used. References [19,20] basically end up in a description in terms of gauge-invariant variables (further approaches along these lines are proposed in [22,23]).

Recently alternative descriptions of Yang-Mills theory in terms of gauge-invariant variables constructed either from the magnetic [24] or electric [25] fields have been proposed. In Ref. [26] the long wavelength (strong coupling) limit of the formulation of Ref. [24] has been studied, exploiting methods from the description of collective excitations of atomic nuclei. Let us also mention an early attempt [27] where the self-dual sector of $D=4$ Yang-Mills theory has

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been formulated in terms of gauge-invariant variables. In a similar fashion in Refs. [28,29] the $D=3$ Yang-Mills functional integral has been expressed in terms of the gauge-invariant variables introduced in [24].

Recently, QCD on a spatial torus has been considered in the Weyl gauge $A_0=0$ in the canonical quantization approach [31]. Using a (partial) axial gauge, the resolution of Gauss' law has been achieved by applying unitary gauge transformations, which rely on quantum field operators. This results in a Schrödinger description in terms of unconstraint variables, where the resulting Hamiltonian is nonlocal. It is fair to say that at the moment we have little experience with solving functional Schrödinger equations in quantum field theories; see, e.g., [32]. Furthermore, even in the functional (operator) approach matrix elements are given by $(D-1)$-dimensional functional integrals. Therefore it might be more convenient to use the (D-dimensional) functional integral representation from the very beginning. In the present paper I perform a resolution of Gauss' law in the functional integral representation of Yang-Mills theory. For this purpose I will use a variant of 't Hooft's maximum Abelian gauge [15], which is the analogue of the gauge used in [31] to time-dependent classical fields. It is the hope that the functional integral formulation will facilitate in finding appropriate approximation schemes. Furthermore, the functional integral approach provides more direct acccess to the topological properties of the Yang-Mills vacuum and to numerical simulations, exploiting Monte Carlo techniques.

The balance of the paper is as follows. In order to set notation and conventions, in Sec. II the functional integral description of Yang-Mills theory is briefly reviewed and some relevant features are discussed. In Secs. III and IV we fix the gauge and resolve the Gauss' law constraint. The Faddeev-Popov determinant is evaluated in Sec. V. In Sec. VI the Wilson loop is evaluated, thereby including only the dominant infrared unconstrained degrees of freedom. In Sec. VII the electric field variables are integrated out, resulting in a theory in unconstrained degrees of freedom of the gauge potential. A short summary and some concluding remarks are given in Sec. VIII. Some calculations are relegated to the Appendixes.

II. HAMILTONIAN FORMULATION OF GAUGE THEORIES

Below we briefly summarize the essential ingredients of the path integral quantization of Yang-Mills theory. Special emphasis is put on the implementation of gauge invariance.

We consider the gauge group $G=\mathrm{SU}(N)$ with anti-Hermitian generators T^a satisfying the commutation relation

$$[T^a,T^b]=f^{abc}T^c, \tag{2.1}$$

where f^{abc} are the structure constants. We choose the standard normalization

$$\mathrm{tr}(T^aT^b)=-\tfrac{1}{2}\,\delta^{ab}. \tag{2.2}$$

Later on we will also make use of the generators in the adjoint representation defined by

$$(\hat{T}^a)_{bc}=-f^{abc}, \tag{2.3}$$

which satisfy the same commutation relation (2.1). Throughout the paper we shall indicate the adjoint representation by the caret. For a quantity χ^a living in the gauge group we define the fundamental and adjoint representations, respectively, by

$$\chi=\chi^aT^a, \quad \hat{\chi}=\chi^a\hat{T}^a. \tag{2.4}$$

We also use the Cartan decomposition of the gauge group

$$G=H\otimes G/H, \tag{2.5}$$

where $H=U(1)^{N-1}$ denotes the Cartan subgroup of the gauge group and G/H is the corresponding coset space. Furthermore, we use the indices with index zero, $a_0,b_0,\ldots$, to denote a generator of the Cartan subgroup $T^{a_0}\in H$, $[T^{a_0},T^{b_0}]=0$, while the indices $\underline{a},\underline{b},\ldots$ are reserved for generators of the coset space, G/H. Accordingly the gauge potential $A_i(x)$ is decomposed as

$$A_i=A_i^n+A_i^{\mathrm{ch}}, \tag{2.6}$$

where $A_i^n=A_i^{a_0}T^{a_0}$ is the gauge potential of the Cartan subgroup H and A_i^{ch} lives in (the algebra of) the coset space G/H. With respect to (color) charges of the Cartan subgroup H, A_i^n is neutral, while A_i^{ch} is charged.

We also introduce the covariant derivative by

$$D_\mu=\partial_\mu+A_\mu \tag{2.7}$$

and the field strength tensor

$$F_{\mu\nu}=[D_\mu,D_\nu]=F_{\mu\nu}^aT^a,$$
$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+f^{abc}A_\mu^bA_\nu^c. \tag{2.8}$$

Under a gauge transformation $\Omega(x)\in\mathrm{SU}(N)$ the gauge potential transforms as

$$A_\mu\to A_\mu^\Omega=\Omega(D_\mu\Omega^\dagger)=\Omega A_\mu\Omega^\dagger+\Omega(\partial_\mu\Omega^\dagger). \tag{2.9}$$

In the Hamilton formulation of Yang-Mills theory, which is based on the Weyl gauge

$$A_0(x)=0, \tag{2.10}$$

the dynamical variables are the spatial components of the gauge potential A_i^a. We shall use spatially periodic boundary conditions for the field variables

$$A_i^a(x+L_k\mathbf{e}^k)=A_i(x), \tag{2.11}$$

where $\mathbf{e}^k$ denotes a three-dimensional (spatial) unit vector, so that we consider Yang-Mills theory on a three-dimensional torus. We have not yet specified the boundary condition in the time direction.

Let $|C\rangle$ denote an eigenstate of $A_i(x)$, i.e., $A_i(x)|C\rangle=C_i(x)|C\rangle$, where $C_i(x)$ is a classical field function. The gauge-invariant transition amplitude between static initial and final field configurations $A_i(x_0=0,\mathbf{x})=C_i'(\mathbf{x})$ and $A_i(x_0=T,\mathbf{x})=C_i''(\mathbf{x})$ is defined by [33,18,34]

$$Z[C'',C']=\langle C''|e^{-HT}P|C'\rangle, \tag{2.12}$$

where

$$H=\int d^3x\left(\frac{g^2}{2}E_i^a(x)E_i^a(x)+\frac{1}{2g^2}B_i^a(x)B_i^a(x)\right) \tag{2.13}$$

is the Yang-Mills Hamiltonian with bare coupling constant g, electric field $E_k^a(x)=\delta/i\,\delta A_k^a(x)$, and magnetic field $B_k^a(x)=\frac{1}{2}\epsilon_{kij}F_{ij}^a(x)$. Furthermore, P is the projector onto gauge-invariant states,

$$P|C\rangle=\sum_n e^{-in\Theta}\int_G \mathcal{D}\mu(\Omega_n)|C^{\Omega_n}\rangle. \tag{2.14}$$

Here Θ is the vacuum angle [17] and the functional integration with respect to the Haar measure $\mu(\Omega)$ of the gauge group extends over all time-independent gauge transformations $\Omega_n(\mathbf{x})$ with winding number n. For a gauge transformation $\Omega(\mathbf{x})$ the winding number is defined by

$$n[\Omega]=\frac{1}{24\pi^2}\int d^3x\,\epsilon_{ijk}\mathrm{tr}(R_iR_jR_k),\quad R_k=\Omega\partial_k\Omega^\dagger. \tag{2.15}$$

As usual we assume here that the gauge function $\Omega(\mathbf{x})$ approaches a unique value Ω_∞ for $|\mathbf{x}|\to\infty$, so that R^3 can be compactified to S^3 and $n[\Omega]$ is a topological invariant.

For many purposes it is sufficient to consider the partition function

$$Z=\int \mathcal{D}C_i\langle C|e^{-HT}P|C\rangle, \tag{2.16}$$

which can be easily reduced to the standard form

$$Z=\sum_k e^{-E_kT}, \tag{2.17}$$

with E_k being the energy eigenvalues. Using the completeness of the eigenstates $|k\rangle$ of $H(H|k\rangle=E_k|k\rangle)$ and $P^2=P$ it can be rewritten as

$$Z=\int \mathcal{D}C_i\sum_k \Psi_k(C)e^{-E_kT}\Psi_k^*(C), \tag{2.18}$$

where

$$\Psi_k(C)=\langle C|P|k\rangle \tag{2.19}$$

are the gauge-''invariant'' energy eigenfunctionals, which under a gauge transformation Ω_n with winding number n transform as

$$\Psi_k(C^{\Omega_n})=e^{-in\Theta}\Psi_k(C), \tag{2.20}$$

as is easily inferred from the explicit form of the projector (2.14). Assuming proper normalization of the energy eigenfunctionals $\Psi_k(C)$, i.e.,

$$\int \mathcal{D}C_i\Psi_k^*(C)\Psi_l(C)=\delta_{kl}, \tag{2.21}$$

Eq. (2.18) reduces to the standard form (2.17).

In Ref. [35] it was explicitly shown that the gauge-invariant partition function (2.16) is given by the standard functional integral representation

$$Z=\int \mathcal{D}A_\mu\delta_{\mathrm{GF}}e^{-S_{\mathrm{YM}}[A]+i\Theta\nu[S]}, \tag{2.22}$$

where

$$S_{\mathrm{YM}}[A]=\frac{1}{4g^2}\int d^4xF_{\mu\nu}^a(x)F_{\mu\nu}^a(x) \tag{2.23}$$

is the usual Yang-Mills action and

$$\nu[A]=\frac{1}{32\pi^2}\int d^4xF_{\mu\nu}^aF_{\mu\nu}^{a*} \tag{2.24}$$

is the Pontryagin index with $F_{\mu\nu}^*=\frac{1}{2}\epsilon_{\mu\nu\kappa\lambda}F_{\kappa\lambda}$ being the dual field strength. The functional integration runs over all temporally periodic gauge field configurations $A_\mu(x_0=T)=A_\mu(x_0=0)$ and it is understood that the gauge fixing is included by the Faddeev-Popov method as indicated in Eq. (2.22) by δ_{GF}.

At first sight one may wonder that Eq. (2.22) reproduces the gauge-invariant partition function (2.16) inspite of the missing Haar measure.[1] However, as explicitly shown in [35] the Haar measure arises from the Faddeev-Popov determinant. Similar investigations have been previously performed in Ref. [33].

The equivalence proof between Eqs. (2.16) and (2.22) [35] relies only on the gauge invariance of the Hamiltonian and holds therefore also true when fermions are included. In this case the partition function is given by

$$Z=\int \mathcal{D}q\mathcal{D}\overline{q}\int \mathcal{D}A_\mu\exp\Big(\int \overline{q}i\not{\partial}q+\overline{q}\not{A}q - S_{\mathrm{YM}}[A]+i\Theta\nu[A]\Big), \tag{2.25}$$

where the fermion fields satisfy antiperiodic boundary conditions $q(x_0=T)=-q(x_0=0)$. For later convenience we rewrite the partition function as

$$Z=\int \mathcal{D}q\mathcal{D}\overline{q}\exp\Big(\int \overline{q}i\not{\partial}q\Big)Z_{\mathrm{YM}}[J], \tag{2.26}$$

where

$$Z_{\mathrm{YM}}[J]=\int \mathcal{D}A_\mu\exp\Big(-S_{\mathrm{YM}}[A]+\int J_\mu A_\mu+i\Theta\nu[A]\Big) \tag{2.27}$$

is formally the partition function of a gauge field coupled to an external color current $J_\mu^a\equiv\overline{q}(\lambda^q/2)\gamma_\mu q$. Equation (2.27) defines the Lagrange representation, which is fully covariant. For subsequent considerations it is more convenient to use the Hamilton formulation which arises from Eq. (2.27) by

[1] In Ref. [18] it was claimed that the conventional functional integral representation (2.22) falls short of guaranteeing gauge invariance in the nonperturbative regime.

linearizing the $(F_{oi})^2$ term by means of an integration over the electric field variable $E_i^a(x)$ which in view of Eq. (2.11) has to satisfy the spatially periodic boundary condition $E_i(x+L\mathbf{e}_i)=E_i(x)$. Then the A_0 field can be integrated out, yielding the Gauss' law constraint

$$\int \mathcal{D}A_0 \exp\left(i\int d^4x A_0^a(x)\Gamma^a(x)\right)=\delta(\Gamma^a), \quad (2.28)$$

where

$$\Gamma(A,E)=\partial_i E_i+[A_i,E_i]+J_0\equiv[D_i,E_i]+J_0 \quad (2.29)$$

is the generator of infinitesimal gauge transformations. Equation (2.27) then becomes the Hamilton functional integral representation of the partition function of Yang-Mills theory in the presence of an external source $J^\mu=(J^0,J^i)$, which after continuing to Minkowski space and assuming $\Theta=0$ reads[2]

$$Z[C'',C',J]=\int \mathcal{D}(A_i,E_i)\prod_{\vec{x}}\delta(f^a(A,E))\delta(\Gamma^a(A,E)) \times\prod_{x_0}\mathrm{Det}\mathcal{M}^{ab}(x_0)\exp\left\{\frac{i}{g^2}\int d^4x[E_i\partial_0 A_i-\tfrac{1}{2} \times(E_i^aE_i^a+B_i^aB_i^a)-A_iJ_i]\right\}. \quad (2.30)$$

Here, $\mathcal{D}(A_i,E_i)$ denotes the (flat) functional integral measure over the gauge potential A_i^a and the electric field E_i^a. Furthermore, $f^a(A,E)=0$ is the gauge-fixing constraint and $\mathrm{Det}\mathcal{M}^{ab}(x,y)$, where $x_0=y_0$, is the Faddeev-Popov determinant.

In the following two sections we will explicitly resolve the Gauss' law constraint $\delta(\Gamma^a)$ and the gauge constraint $\delta(f^a)$ in Eq. (2.30), leaving a functional integral over unconstrained, gauge-fixed variables.

III. GAUGE FIXING AND PARTIAL RESOLUTION OF GAUSS' LAW

Gauss' law (2.29) $\Gamma^a=0$ has the generic form

$$\boldsymbol{\nabla}\mathbf{E}^a=\rho^a, \quad \rho^a=-[A_i,E_i]^a-J_0^a, \quad (3.1)$$

where ρ^a is the total color charge density. Applying Gauss' integration theorem it follows

$$\oint_{\partial M} d\boldsymbol{\Sigma}\mathbf{E}^a=Q^a, \quad Q^a=\int d^3x\rho^a. \quad (3.2)$$

For periodic electric fields the electric flux through the surface of the box, $\oint d\boldsymbol{\Sigma}\mathbf{E}$, vanishes. Consequently periodic boundary conditions to $E_i^a(x)$ can only tolerate a vanishing total charge:

$$Q^a=0. \quad (3.3)$$

For the resolution of Gauss' law a proper choice of gauge fixing is crucial. In the past a complete resolution of Gauss' law has been achieved in the gauge $\epsilon^{aik}E_i^a=0$ for SU(2) in Ref. [19] and an extension to SU(3) was considered in Ref. [20]. There have been also attempts of a complete resolution of Gauss' law in the Coulomb gauge [21]. The Coulomb gauge, which is singled out in QED by the absence of radiation of static charges, has proved, however, to be inconvenient in non-Abelian gauge theories, in particular for an explicit resolution of Gauss' law. In this respect axial types of gauges are much more convenient as was already realized in Refs. [36,37] and recently discussed in detail in Ref. [31], where an explicit resolution of Gauss' law in the canonical quantization approach has been performed. Below we will perform an analogous resolution of Gauss' law in the functional integral approach. For this purpose it is convenient to choose the three-axis as the preferred direction of the axial gauge and divide the Gauss' law generator into parts parallel and perpenticular to the three-axis:

$$\Gamma(x)=\hat{D}_3E_3+\Gamma_\perp, \quad \Gamma_\perp=\hat{D}_\perp E_\perp+J_0. \quad (3.4)$$

If $\hat{D}_3$ were regular, the Gauss' law $\Gamma=0$ could be easily resolved, leading to an elimination of E_3. Unfortunately, as we will explicitly see below, on the torus $\hat{D}_3$ has always zero modes, independently of the used gauge. In fact, since $\hat{D}_3$ transforms gauge covariantly, its eigenvalues are independent of the gauge. Nevertheless, we can exploit the gauge freedom to cast $\hat{D}_3$ in as simple a form as possible. From this point of view the axial gauge $A_3=0$ would be preferable. However, this gauge condition conflicts with the periodic boundary condition. This can be easily seen by considering the Polyakov line operator

$$\mathcal{P}_3(x)=P\exp\left(\oint dx_3' A_3(\overline{x},x_3')\right), \quad (3.5)$$

where P denotes path ordering and the integration runs from a point[3] $x=(\overline{x},x_3)$ along the three-axis to the point $x=(\overline{x},x_3+L)$. Because of the periodic boundary condition on A_3, the integration in Eq. (3.5) runs over a closed loop but nevertheless due to the path ordering $\mathcal{P}_3(x)$ depends on the starting point x. Under a gauge transformation this quantity transforms as

[2]In fact in the derivation of the path integral representation (2.27) the Hamilton formulation (2.30) arises in an intermediate step of the calculation. In the present case the Hamilton and Lagrange form are obviously completely equivalent. But in more general cases (e.g., in theories with momentum-dependent masses) the Hamilton form is obviously the more fundamental representation and the Lagrange form may even not exist. Furthermore, the path integral derivation shows that, while the integral over the gauge field configurations has to be taken with temporally periodic boundary conditions $A_i(x^0=T,\vec{x})=A_i(x^0=0,\vec{x})$, the integration over the electric fields is not constrained by any temporal boundary condition.

[3]Here and in the following we use the convention $(x)=(x^0,\mathbf{x})=(\overline{x},x_3)$.

$$\mathcal{P}_3(x)\to\mathcal{P}_3^{\Omega}(x)=\Omega(x)\mathcal{P}_3(x)\Omega^{\dagger}(x), \tag{3.6}$$

and one can obviously choose a gauge in which $\mathcal{P}_3(x)$ is diagonal:

$$\mathcal{P}_3^{\Omega}(x)=e^{a_3(x)L}, \quad a_3(x)=a_3^{c_0}T^{c_0}. \tag{3.7}$$

However, it is impossible to gauge transform[4] $\mathcal{P}_3(x)$ to $\mathcal{P}_3(x)=1$.

For the resolution of Gauss' law it is convenient to follow Ref. [31] and use the gauge

$$A_3^{\mathrm{ch}}(x)=0, \quad \text{or} \quad A_3^{\overline{a}}(x)=0. \tag{3.8}$$

This condition, of course, does not fix the gauge completely but allows still for arbitrary Abelian gauge transformations $\omega(x)\in H$. We will later make use of this freedom. Let us also mention that the gauge transformation necessary to bring a given gauge field $A_i(x)$ into the form (3.8) requires in general also topologically nontrivial gauge transformations.

In the gauge (3.8) the operator $\hat{D}_3$ is block diagonal with respect to the color neutral and charged components:

$$\hat{D}_3^{ab}=\begin{pmatrix}\partial_3\delta^{a_0b_0} & 0\\ 0 & \hat{D}_3^{\overline{a}\overline{b}}\end{pmatrix}, \tag{3.9}$$

since $f^{a_0b_0c}=0$. Hence in this gauge the neutral part of the Gauss' law generator simplifies to

$$\Gamma^{a_0}(x)=\partial_3E_3^{a_0}+\Gamma_{\perp}^{a_0}. \tag{3.10}$$

On the space of periodic functions $\xi_n(x)=e^{i\omega_nx_3}$, $\omega_n=2\pi n/L$ the operator ∂_3 has a zero eigenvalue ($n=0$) corresponding to a x_3-independent eigenfunction. For simplicity of notation we have set here $L=L_3$. The corresponding projection of E_3^n onto this zero mode,

$$e_3(\overline{x})=\frac{1}{L}\int_0^L dx_3E_3^n(\overline{x},x_3), \tag{3.11}$$

does not enter $\Gamma^n(x)$, Eq. (3.4), and is hence not restricted by Gauss' law.

Since the eigenfunctions of ∂_3 belonging to zero and nonzero eigenvalues are orthogonal in the Hilbert space of periodic functions, the neutral part of the Gauss' law constraint separates in the two independent constraints corresponding to the subspaces of the zero and nonzero eigenvalues. Defining

$$\gamma_{\perp}=\frac{1}{L}\int_0^L dx_3\Gamma_{\perp}^n, \quad \Gamma'^n_{\perp}=\Gamma_{\perp}^n-\gamma_{\perp}, \tag{3.12}$$

we have

$$\delta(\Gamma^n)=\delta(\partial_3E'^n_3+\Gamma'^n_{\perp})\,\delta(\gamma_{\perp})=\delta(\partial_3E'^n_3+\Gamma_{\perp}^n)\,\delta(\gamma_{\perp}), \tag{3.13}$$

where

$$E_3'=E_3-e_3 \tag{3.14}$$

lives entirely in the subspace of eigenfunctions with nonzero eigenvalues of ∂_3. The constraint of the first δ function can be easily resolved. Defining by ∂_3' the operator resulting from ∂_3 when the zero eigenvalue is removed, we obtain (with $\partial_3E'^n_3=\partial_3'E'^n_3$)

$$\delta(\partial_3'E'^n_3+\Gamma_{\perp}^n)=\frac{1}{\det\partial_3'}\,\delta\left(E'^n_3+\frac{1}{\partial_3'}\Gamma_{\perp}^n\right). \tag{3.15}$$

Hence the neutral part of Gauss' law eliminates the variable E'^n_3. In addition we now exploit the residual invariance under Abelian gauge transformations to remove also the corresponding conjugate field variable,

$$A_3'=A_3-a_3, \quad a_3=\frac{1}{L}\int dx_3A_3^n, \tag{3.16}$$

by imposing the gauge condition

$$\partial_3A_3^n(x)=0. \tag{3.17}$$

Since $\partial_3A'^n_3=\partial_3'A'^n_3$, this gauge implies $A'^n_3(x)=0$ and hence leaves from $A_3^n(x)$ only the x_3-independent part $a_3(\overline{x})$.

By construction [see Eqs. (3.16) and (3.11)] the reduced Abelian fields $a_3(\overline{x})$ and $e_3(\overline{x})$ are canonically conjugated variables. Note also that the change of variables from A_3^n to (A'^n_3,a_3) [and correspondingly from E_3^n to (E'^n_3,e_3)] does not yield any nontrivial Jacobian since A'^n_3 and a_3 are orthogonal coordinates in the sense that they belong to orthogonal subspaces of the Hilbert space of periodic eigenfunctions of $i\partial_3$.

The gauge condition (3.17) has also the advantage that it enormously simplifies the operator (3.9),

$$\hat{D}_3^{\overline{a}\overline{b}}=\delta^{\overline{a}\overline{b}}\partial_3+\hat{a}_3^{\overline{a}\overline{b}}(\overline{x})=:\hat{d}_3^{\overline{a}\overline{b}}, \tag{3.18}$$

which enters the charged part of Gauss' law (3.4),

$$\Gamma^{\overline{a}}=\hat{d}_3^{\overline{a}\overline{b}}E_3^{\overline{b}}+\Gamma_{\perp}^{\overline{a}}. \tag{3.19}$$

[The evaluation of the eigenvalues and hence the inversion of $\hat{d}_3$ become trivial since $a_3(\overline{x})$ is independent of x_3; see below.] Let us also mention that Eqs. (3.8) and (3.17) define a variant of 't Hooft's maximal Abelian gauge [15], which preserves invariance under x_3-independent Abelian gauge transformations.

For the time being, let us assume that $\hat{d}_3^{\overline{a}\overline{b}}$ has no zero eigenvalue in the charged subspace. [We will later see that the system dynamically avoids configurations $a_3(\overline{x})=0$, giving rise to zero modes of $\hat{d}_3$.] The charged part of the Gauss' law can now be used to eliminate the charged part of E_3 by using

[4]This can be also easily seen in the lattice formulation. Starting at $x_3=0$ one can bring the links $U_3(x)=\exp[-aA_3(x)]$ to the gauge $U_3(x)=1$ except for the last link terminating at $x_3=L$, which cannot be gauged away due to the periodic boundary condition.

$$\delta(\Gamma^{\overline{a}})=\frac{1}{\det\hat{d}_3}\,\delta(E_3^{\overline{a}}+(\hat{d}_3^{-1})^{\overline{a}\,\overline{b}}\Gamma_\perp^{\overline{b}}). \tag{3.20}$$

Later we will observe that the corresponding functional determinant $\det\hat{d}_3^{\overline{a}\,\overline{b}}$ will be canceled by the Faddeev-Popov determinant.

We can use now the two constraints (3.15) and (3.20) arising from Gauss' law to integrate out explicitly the electric field variables $E'^{c_0}_3, E_3^{\overline{a}}$ leaving from E_3 only the x_3-independent neutral part e_3, Eq. (3.11). Furthermore, the two gauge constraints (3.8) and (3.17) eliminate the gauge variables $A_3^{\rm ch}$ and A'^n_3, respectively, leaving from the gauge potential A_3 only the neutral x_3-independent part $a_3(\overline{x})$. Since the changes of variables from $E_3^{c_0}$ to $E'^{c_0}_3, e_3$ and analogously from $A_3^{c_0}$ to $A'^{c_0}_3, a_3$ are trivial; i.e., the corresponding Jacobians equal 1, we then obtain, from Eq. (2.30),

$$\begin{aligned}Z[J]=&\int\mathcal{D}(A_\perp,a_3,E_\perp,e_3)\prod_x\delta(\overline{f^{c_0}}(A))\delta(\gamma_\perp^n)\\&\times\prod_{x^0}\mathrm{Det}\mathcal{M}^{ab}[\mathrm{Det}(\partial_3')\mathrm{Det}\hat{d}_3^{\overline{a}\,\overline{b}}]^{-1}\\&\times\exp\left\{\frac{i}{g^2}\left[L\int d^3x[e_3\partial_0a_3-\tfrac{1}{2}e_3(\overline{x})e_3(\overline{x})]\right]\right.\\&+\int d^4xE_\perp^a\partial_0A_\perp^a-\frac{1}{2}\int d^4x\\&\left.\times\{[\hat{d}_3^{-1}\Gamma_\perp^{\rm ch}]^2+(\partial_3'^{-1}\Gamma'^n_\perp)^2+E_\perp E_\perp+B^2\}\right\}.\end{aligned} \tag{3.21}$$

Here $\delta(\overline{f^{c_0}}(A))$ denotes the gauge condition necessary to fix the residual invariance under x_3-independent Abelian gauge transformations, which is left by the constraints (3.8) and (3.17). This residual gauge will be fixed in the following section when we resolve the residual Gauss' law $\gamma_\perp=0$.

IV. RESOLUTION OF THE RESIDUAL GAUSS' LAW

The residual Gauss' law constraint

$$\gamma_\perp=\frac{1}{L}\int dx_3(\nabla_\perp\cdot E_\perp^n+[A_\perp,E_\perp]^n+J_0^n) \tag{4.1}$$

can be used to remove the x_3-independent part of $E_\perp^n$ which is longitudinal in the 1-2 plane ($i=1,2$) defined by

$$\mathbf{e}_\perp:=\nabla_\perp\frac{1}{\Delta_\perp'}\frac{1}{L}\int dx_3\nabla_\perp\mathbf{E}_\perp^n\equiv l\mathbf{E}_\perp^n, \tag{4.2}$$

where $\mathbf{\Delta}_\perp'$ is the two-dimensional Laplacian, $\nabla_\perp\cdot\nabla_\perp$, in the Hilbert space of periodic functions with the zero mode omitted. Its inverse is defined in the space of periodic functions by the Green's function

$$\begin{aligned}G^{(2)}(x_\perp'',x_\perp')&=\langle x_\perp''|\frac{1}{-\mathbf{\Delta}_\perp'}|x'\rangle\\&=\frac{1}{(2\pi)^2}\sum_{\mathbf{n}_\perp\neq0}\frac{1}{\mathbf{n}_\perp^2}e^{i\mathbf{n}_\perp\cdot(\mathbf{x}_\perp''-\mathbf{x}_\perp')2\pi/L},\\&\mathbf{n}_\perp=(n_1,n_2),\end{aligned} \tag{4.3}$$

which obviously satisfies periodic boundary conditions. Note that the longitudinal projector l defined by Eq. (4.2) is in fact an orthogonal projector, $l\cdot l=l$. This follows from the relation

$$\int d^2x_\perp''\langle x|\triangle_\perp|x''\rangle\langle x''|\frac{1}{\triangle_\perp'}|x'\rangle=\delta^{(2)}(x_\perp-x_\perp')-\frac{1}{L^2}, \tag{4.4}$$

where

$$\delta^{(2)}(x_\perp,y_\perp)=\frac{1}{L^2}\sum_{\mathbf{k}_\perp}e^{i\mathbf{k}_\perp(\mathbf{x}_\perp-\mathbf{y}_\perp)},\quad \mathbf{k}_\perp=\left(\frac{2\pi}{L_1}n_1,\frac{2\pi}{L_2}n_2\right), \tag{4.5}$$

is the two-dimensional periodic δ function $[\delta^{(2)}(\mathbf{x}_\perp+\mathbf{e}_iL_i,\mathbf{y}_\perp)=\delta^{(2)}(\mathbf{x}_\perp,\mathbf{y}_\perp)]$ and the last term in Eq. (4.4) arises from the fact that in $\triangle_\perp'$, Eq. (4.3), the zero mode $n_1=n_2=0$, is excluded. This term, however, does not contribute when l acts on vector fields $V_i(x)$ periodic in x_1 and x_2. In fact, from the definition of the longitudinal field $\mathbf{e}_\perp$, Eq. (4.2), we find, by using Eq. (4.4),

$$\nabla_\perp\cdot\mathbf{e}_\perp=\frac{1}{L}\int dx_3\nabla_\perp E_\perp^n-\frac{1}{LL_1L_2}\int d^3x\nabla_\perp\cdot\mathbf{E}_\perp^n, \tag{4.6}$$

where the last term vanishes for periodic electric fields, so that we obtain

$$\nabla_\perp e_\perp=\frac{1}{L}\int dx_3\nabla_\perp E_\perp^n. \tag{4.7}$$

The residual Gauss' law (4.1) then simplifies to

$$\gamma_\perp=\nabla_\perp\cdot\mathbf{e}_\perp-\rho^{(2)}=0, \tag{4.8}$$

where

$$\rho^{(2)}=-\frac{1}{L}\int dx_3([A_\perp,E_\perp]^n+J_0^n). \tag{4.9}$$

Since, by definition, $\mathbf{e}_\perp$, Eq. (4.2), is a curl-free, two-dimensional vector field, $\nabla_\perp\times\mathbf{e}_\perp=0$, it has the representation

$$\mathbf{e}_\perp=-\nabla_\perp\varphi(x_\perp), \tag{4.10}$$

where the scalar potential $\varphi(x)$ follows from the residual Gauss' law (4.8):

$$\varphi(x_\perp)=\int d^2y_\perp G^{(2)}(x_\perp,y_\perp)\rho^{(2)}(y_\perp). \tag{4.11}$$

In fact, inserting Eqs. (4.11) into (4.10) and taking the divergence we find, with the help of Eq. (4.4),

$$\nabla_\perp \cdot \mathbf{e}_\perp = \rho^{(2)} - \overline{\rho}, \quad \overline{\rho} = \frac{1}{L_1 L_2}\int d^2x_\perp \rho^{(2)} = \frac{1}{L L_1 L_2} Q^n, \tag{4.12}$$

where Q^n is the total charge (in the Cartan subgroup), which according to Eq. (3.3) has to vanish for periodic E_i^a fields, so that $\overline{\rho}=0$ and Eq. (4.10) solves, in fact, Eq. (4.8).

For nonvanishing total charge $Q^n \neq 0$ Gauss' law requires one to abandon the periodic boundary condition to the electric fields and the second term in Eq. (4.6) no longer vanishes. Even in this case Eq. (4.8) is still solved by Eqs. (4.10) and (4.11). Therefore the resolution of the residual part of Gauss' law leads to the elimination of the longitudinal part $\mathbf{e}_\perp$ of the neutral vector field $\mathbf{E}_\perp^n$ and we are left with the transversal part

$$\mathbf{E}'_\perp = \mathbf{E}_\perp - \mathbf{e}_\perp \tag{4.13}$$

as the dynamical quantity.

Note that only the charged parts $\mathbf{A}_\perp^{\mathrm{ch}}$ and $\mathbf{E}_\perp^{\mathrm{ch}}$ enter $\rho^{(2)}$, Eq. (4.9), and thus $\mathbf{e}_\perp$. Furthermore, $\mathbf{e}_\perp$ and $\mathbf{E}'^n_\perp$ live in orthogonal subspaces of the Hilbert space of periodic functions in x_3. Therefore the change of variables from $(\mathbf{E}_\perp^n, \mathbf{E}_\perp^{\mathrm{ch}})$ to $(\mathbf{E}_\perp^n = \mathbf{E}'^n_\perp + \mathbf{e}_\perp, \mathbf{E}_\perp^{\mathrm{ch}})$ does not give rise to any nontrivial Jacobian. Let us also emphasize that after resolution of Gauss' law (4.8), $\mathbf{e}_\perp$ is not an integration variable but a function of $\mathbf{E}_\perp^{\mathrm{ch}}, \mathbf{A}_\perp^{\mathrm{ch}}$ and independent of the remaining integration variables $\mathbf{E}'^n_\perp$, $\mathbf{A}_\perp^n$, etc.

We can exploit now the residual invariance under x_3-independent Abelian gauge transformations, left by the constraints (3.8) and (3.17), to remove the field

$$\mathbf{a}_\perp(\overline{x}) = (l\mathbf{A}_\perp^n)(\overline{x}), \tag{4.14}$$

canonically conjugated to $\mathbf{e}_\perp$. Since by definition of the longitudinal projector l, Eq. (4.2), this field is curl free, $\nabla_\perp \times \mathbf{a}_\perp = 0$, and, for periodic fields $\mathbf{A}_\perp(x)$, satisfies the relation [cf. Eq. (4.7)]

$$\nabla_\perp \cdot \mathbf{a}_\perp(\overline{x}) = \frac{1}{L}\int dx_3 \nabla_\perp \cdot \mathbf{A}_\perp^n, \tag{4.15}$$

it suffices to require the gauge

$$\frac{1}{L}\int dx_3 \nabla_\perp \cdot \mathbf{A}_\perp^n(\overline{x}, x_3) = 0 \tag{4.16}$$

to make $\mathbf{a}_\perp$ vanishing:

$$\mathbf{a}_\perp = 0. \tag{4.17}$$

In the following we will denote by $\mathbf{A}'_\perp$ the field satisfying the gauge condition (4.16), i.e.,

$$\mathbf{A}'_\perp = \mathbf{A}_\perp - \mathbf{a}_\perp. \tag{4.18}$$

Since $\mathbf{e}_\perp$ and $\mathbf{a}_\perp$ live in the Cartan subgroup, we can trivially extend Eqs. (4.13) and (4.18) to the charged field components, where they read

$$\mathbf{E}'^{\mathrm{ch}}_\perp = \mathbf{E}_\perp^{\mathrm{ch}}, \quad \mathbf{A}'^{\mathrm{ch}}_\perp = \mathbf{A}_\perp^{\mathrm{ch}}. \tag{4.19}$$

We can then express $\mathbf{e}_\perp$ defined by Eqs. (4.10) and (4.11) as

$$\mathbf{e}_\perp(\overline{x}) = \nabla_\perp^x \frac{1}{L}\int d^3y G^{(2)}(x_\perp, y_\perp)([\mathbf{A}'_\perp, \mathbf{E}'_\perp]^n + J_0^n)(x^0, \vec{y}). \tag{4.20}$$

(Note that only the charge fields $\mathbf{E}_\perp^{\mathrm{ch}}, \mathbf{A}_\perp^{\mathrm{ch}}$ enter the commutator.)

Similarly we can express the Gauss law generator $\Gamma_\perp$, Eq. (3.4), in terms of the new variables

$$\Gamma_\perp^n = \nabla_\perp \cdot (\mathbf{E}'^n_\perp + \mathbf{e}_\perp) + [\mathbf{A}'_\perp, \mathbf{E}'_\perp]^n + J_0^n,$$

$$\Gamma_\perp^{\mathrm{ch}} = \nabla_\perp \cdot \mathbf{E}'^{\mathrm{ch}}_\perp + [\mathbf{A}'_\perp, \mathbf{E}'_\perp + \mathbf{e}_\perp] + J_0^{\mathrm{ch}}. \tag{4.21}$$

Furthermore, since $\mathbf{e}_\perp(\overline{x})$, Eq. (4.20), is independent of x_3, i.e., $\partial_3 e_\perp = \partial'_3 e_3 = 0$, it drops out from

$$\partial_3'^{-1}\Gamma_\perp^n \equiv \frac{1}{\partial_3'^2}\partial'_3 \Gamma_\perp^n, \tag{4.22}$$

and the neutral part of the Gauss' law generator (4.21) can be replaced by

$$\Gamma_\perp^{c_0} = \nabla_\perp \cdot \mathbf{E}'^{c_0}_\perp + [\mathbf{A}'_\perp, \mathbf{E}'_\perp]^{c_0} + J_0^{c_0}. \tag{4.23}$$

Recall that the change of integration variables from $\mathbf{E}_\perp^n \to (\mathbf{E}'^n_\perp, \mathbf{e}_\perp)$ yields a trivial Jacobian equal to 1 since $\mathbf{E}'^n_\perp$ and $\mathbf{e}_\perp$ are orthogonal components of $\mathbf{E}_\perp^n$ in the Hilbert space of periodic functions in x_3. The same is true for the change of variables from $\mathbf{A}_\perp^n$ to $(\mathbf{A}'^n_\perp, \mathbf{a}_\perp)$. Therefore, after complete resolution of Gauss' law and implementation of the gauge-fixing contraints, we are left with the following functional integral representation of Yang-Mills theory:

$$\begin{aligned} Z = \int \mathcal{D}(E'_\perp, e_3, A'_\perp, a_3) \prod_{x_0} \mathrm{Det}\mathcal{M}^{ab} \mathrm{Det}^{-1}(\hat{d}'_3) \\ \times \exp\left\{ \frac{i}{g^2}\left[L\int d^3\overline{x}(e_3 \partial_0 a_3 - \tfrac{1}{2} e_3 e_3) \right.\right. \\ + \int d^4x E'_\perp \partial_0 A'_\perp + \int (a_3 J_3 + A'_\perp J'_\perp) \\ \left.\left. - \tfrac{1}{2}\int \{(\hat{d}'^{-1}_3 \Gamma_\perp)^2 + E'_\perp E'_\perp + e_\perp^2 + [B(A')]^2\} \right] \right\}, \end{aligned} \tag{4.24}$$

where the residual Abelian gauge constraint (4.16) has been used to replace the perpendicular field $\mathbf{A}_\perp$ by its two-dimensional transversal part $\mathbf{A}'_\perp$; see Eq. (4.18). Furthermore, the magnetic field $B(A')$ is defined in terms of the reduced field variables due to the implementation of Gauss' law by

$$F_{\underline{i3}} = [D'_{\underline{i}}, d_3], \quad d_3 = \partial_3 + a_3, \quad d'_3 = \partial'_3 + a_3,$$

$$F_{\underline{ij}} = [D'_{\underline{i}}, D'_j], \quad D'_i = \partial_i + A'_i. \tag{4.25}$$

Let us also emphasize that there are no cross terms between the reduced electric field $\mathbf{E}'_\perp$ and the static electric field $\mathbf{e}_\perp$. This is a consequence of $\int dx_3 \nabla_\perp \mathbf{E}'_\perp = 0$, which holds due to the periodicity of the fields.

It remains to calculate the Faddeev-Popov determinant which is done in the next section.

V. EVALUATION OF THE FADDEEV-POPOV DETERMINANT

For the above chosen gauge the Faddeev-Popov determinant is straightforwardly evaluated. The two Abelian gauge-fixing conditions (3.17) and (4.16) are independent of each other; i.e., they belong to orthogonal subspaces of the Hilbert space of periodic functions in $x_3 \in [0,L]$. Both conditions can therefore be absorbed into a single gauge constraint for the neutral component of the gauge field:

$$f^{a_0}(x) = \partial_3 A_3^{a_0}(\overline{x},x_3) + \nabla_\perp \frac{1}{L}\int_0^L dx_3 A_\perp^{a_0}(\overline{x},x_3). \quad (5.1)$$

Furthermore, the gauge (3.8) defines a color-charged gauge functional

$$f^{\overline{a}} = A_3^{\overline{a}}. \quad (5.2)$$

For the above gauge functionals (5.1) and (5.2) the Faddeev-Popov kernel $\mathcal{M}^{ab}(x,y)$ becomes ($x_0 = y_0$)

$$\mathcal{M}^{ab_0}(x,y) = \hat{D}_3^{ab_0}(x)\nabla_3^y \delta^{(3)}(\mathbf{x}-\mathbf{y}) + \frac{1}{L}\hat{D}_\perp^{ab_0}(x)\nabla_\perp^y \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp),$$

$$\mathcal{M}^{a\overline{b}}(x,y) = \hat{D}_3^{a\overline{b}}(x)\,\delta^{(3)}(\mathbf{x}-\mathbf{y}). \quad (5.3)$$

This expressions hold so far for arbitrary gauge field configurations. We need, however, these expressions only on the gauge manifold, i.e., for those field configurations which fulfill the above chosen gauge constraints. Using $f^{ab_0c_0} = 0$, which implies $\hat{a}_3^{ab_0} = 0$, the Faddeev-Popov kernel reduces at the chosen gauge orbits to

$$\mathcal{M}^{ab}(x,y) \equiv \begin{pmatrix} \mathcal{M}^{a_0b_0} & \mathcal{M}^{a_0\overline{b}} \\ \mathcal{M}^{\overline{a}b_0} & \mathcal{M}^{\overline{a}\overline{b}} \end{pmatrix} = \begin{pmatrix} -\delta^{a_0b_0}\left(\nabla_3^x\nabla_3^x\delta^{(3)}(\mathbf{x}-\mathbf{y}) + \frac{1}{L}\nabla_\perp^x\nabla_\perp^x\delta^{(2)}(\mathbf{x}_\perp-\mathbf{y}_\perp)\right) & 0 \\ -\frac{1}{L}\hat{\mathbf{A}}_\perp^{\overline{a}b_0}(x)\cdot\nabla_\perp^x\delta^{(2)}(\mathbf{x}_\perp-\mathbf{y}_\perp) & \hat{d}_3^{\overline{a}\overline{b}}(x)\,\delta^{(3)}(\mathbf{x}-\mathbf{y}) \end{pmatrix}. \quad (5.4)$$

Since this matrix has triangle form, we find for the Faddeev-Popov determinant, finally,

$$\mathrm{Det}\mathcal{M}^{ab}(x,y) = \mathrm{Det}\left[-\delta^{a_0b_0}\left(\nabla_3^x\nabla_3^x\delta^{(3)}(\mathbf{x}-\mathbf{y}) + \frac{1}{L}\nabla_\perp^x\nabla_\perp^x\delta^{(2)}(\mathbf{x}_\perp-\mathbf{y}_\perp)\right)\right]\mathrm{Det}[\hat{d}_3^{\overline{a}\overline{b}}\delta^{(3)}(\mathbf{x}-\mathbf{y})]. \quad (5.5)$$

It factorizes into contributions arising from the Cartan subgroup (first factor) and the coset space. The former one is an irrevelant constant and will be dropped in the following. The contribution from the coset space can be easily calculated since the eigenvalues of $\hat{d}_3^{\overline{a}\overline{b}}$ are analytically known; see Appendix A. But for the moment we do not need the explicit form of Det $\hat{d}_3$.

A glance at Eq. (5.5) shows that (the nontrivial part of) the Faddeev-Popov determinant cancels precisely the determinant $(\mathrm{Det}\,\hat{d}_3)^{-1}$ arising from the resolution of Gauss' law. Consequently Eq. (4.24) reduces to

$$Z = \int \mathcal{D}(\mathbf{E}'_\perp, e_3, \mathbf{A}'_\perp, a_3)\exp\left\{\frac{i}{g^2}\left[L\int d^3\overline{x}(e_3\partial_0 a_3 - \tfrac{1}{2}e_3e_3) + \int d^4x\,\mathbf{E}'_\perp\partial_0\mathbf{A}'_\perp - \tfrac{1}{2}\int d^4x \times\{(\hat{d}'^{-1}_3\Gamma_\perp)^2 + \mathbf{E}'_\perp\mathbf{E}'_\perp + \mathbf{e}_\perp^2 + [\mathbf{B}(A')]^2\}\right]\right\}. \quad (5.6)$$

This is the desired functional integral representation of Yang-Mills theory in unconstrained, gauge-fixed variables, resulting from a complete resolution of Gauss' law. Note that in the unconstrained theory the functional integration over the canonical variables is performed with a flat integration measure. (There is no preexponential factor, e.g., a functional determinant, which could be interpreted as the nontrivial measure.) This is obviously a general feature of Yang-Mills theory in unconstrained variables (provided one chooses a gauge condition which is canonically conjugated to the Gauss' law constraint) and could, perhaps, have been anticipated in view of the fact that the Faddeev-Popov kernel is given by $\mathcal{M}^{ab}(x,y) = \{f^a(x), \Gamma^b(y)\}$, where $\{,\}$ denotes the Poisson brackets.

The cancellation of the Faddeev-Popov determinant against the determinant arising from the resolution of Gauss' law was also obtained in Ref. [19], where the gauge was fixed by demanding that the antisymmetric part of the matrix E_i^a vanish. In that case Gauss' law requires the vanishing of the antisymmetric part of A_i^a and one ends up with a func-

tional integral over the symmetric parts of A_i^a and E_i^a where unfortunately the remaining electric field variables cannot explicitly been integrated out. In this respect the present approach has the advantage over Refs. [19,20] in that the remaining unconstrained electric field variables in Eq. (5.6) can still be integrated out in closed form. This will be done in Sec. VII.

Before concluding this section let us notice that *assuming* a flat integration measure the functional integral representation (5.6) could have also been derived by starting from the Yang-Mills Hamilton operator in unconstrained variables obtained in Ref. [31] in the canonical operator approach and following the standard procedure [39]. In this sense the present functional integral derivation of the unconstrained Yang-Mills theory (5.6) is equivalent to the canonical operator approach of Ref. [31]. We believe, however, that the functional integral representation derived in the present paper, Eq. (5.6), is more flexible than the operator approach when it comes to an approximate solution of the theory.

Finally a comment on the gauge fixing is in order. We have fixed the gauge in such a way to remove the components of the gauge field $A_i(x)$ which are canonically conjugate to those components of the electric field $E_i(x)$ which are eliminated by Gauss' law. This has led to the gauge conditions (3.8), (4.16), and (3.17), which eliminate $A_3^{\rm ch}, \mathbf{a}_\perp$ and make $A_3^n = a_3^n$ independent of x_3. These gauge constraints do, however, not yet fix the gauge completely but leave a residual gauge invariance which consists of (i) (global) permutations of the color indices of the fundamental representations, i.e., elements of the Weyl (sub)group S_N of the gauge group SU(N), (ii) global Abelian gauge transformations, and (iii) displacement transformations $\Omega = e^{-\boldsymbol{\alpha x}}$, with $\boldsymbol{\alpha}$ an arbitrary but fixed c-number three-vector. These residual gauge symmetries were also found in Ref. [31]. For completeness we work out the emergence of these residual gauge symmetries in the present functional integral approach in Appendix C.

VI. WILSON LOOP

Below we evaluate the potential between two static color charges or, equivalently, the Wilson loop[5]

$$W(C) = \left\langle \mathrm{Tr} P \exp\left(- \oint dx_\mu A_\mu(x) \right) \right\rangle \tag{6.1}$$

in the gauge-fixed theory defined by Eq. (5.6). For simplicity we consider a planar rectangular Wilson loop C, which by Lorentz invariance can be placed into the 0-3 plane. One should note here, however, that the present approach (5.6) has not been formulated in a manifestly Lorentz covariant way, although all Green functions (calculated in the full theory) will respect Lorentz invariance. As a consequence the quality of approximations will depend in general on the chosen Lorentz frame.

The present approach obviously singles out the zero- and three-axes. (It exactly integrates out the A_0 field and eliminates most of the degrees of freedom of A_3 and E_3 by gauge fixing and resolution of Gauss' law, respectively.) We therefore expect that the Wilson loop is most efficiently evaluated when placed in the 0-3 plane. Then the $A'_\perp$ field will not explicitly enter the Wilson loop. Therefore we will ignore it together with its conjugate variable $E'_\perp$ since we anyhow expect the dominant infrared behaviour to be governed by the $a_3(\overline{x}), e_3(\overline{x})$ fields.[6] The generating functional of axial-gauge-fixed Yang-Mills theory (5.6) reduces then to

$$Z[J] = \int \mathcal{D}(a_3, e_3) \exp\left\{ \frac{i}{g^2} \left[L \int d^3 \overline{x(} e_3 \partial_0 a_3 - \tfrac{1}{2} e_3 e_3) + \int d^4 a_3 J_3 - \tfrac{1}{2} \int d^4 x \left(J_0 \frac{1}{-\hat{d}'_3 \hat{d}'_3} J_0 + (\mathbf{e}_\perp^{(0)})^2 \right) \right] \right\}, \tag{6.2}$$

where

$$\mathbf{e}_\perp^{(0)} = \mathbf{e}_\perp|_{E_\perp = 0} = -\nabla_\perp \frac{1}{L} \int d^3 y G^{(2)}(x_\perp, y_\perp) J_0^n(y). \tag{6.3}$$

The last two terms in Eq. (6.2) describe the interaction between static charges J_0. The last term can be cast into the form

$$\int (e_\perp^{(0)})^2 = -\int dx_0 d^2 x_\perp d^2 y_\perp \overline{J_0^{c_0}}(x_0, x_\perp) \times G^{(2)}(x_\perp, y_\perp) \overline{J_0^{c_0}}(x_0, y_\perp), \tag{6.4}$$

where a partial integration has been performed and

$$\overline{J} = \frac{1}{L} \int dx_3 J_0(x). \tag{6.5}$$

This quantity obviously vanishes in the thermodynamic (infinite volume) limit $L \to \infty$ for any localized charge distribution $J_0(x)$. To illustrate the meaning of this term let us consider two opposite Abelian charges $(q, -q)$ separated by a distance R. If we place these two charges on a line parallel to the three-axis, e.g.,

$$J_0^{c_0}(x) = q^{c_0} \delta(x_1) \delta(x_2) \left[\delta\left(x_3 - \frac{R}{2} \right) - \delta\left(x_3 + \frac{R}{2} \right) \right], \tag{6.6}$$

then obviously $\overline{J_0(\overline{x})} = 0$ and this term does not contribute. But it does contribute when we place the charges in the $x-y$ plane: e.g.,

[5]Note that in our convention $A_\mu(x)$ is anti-Hermitian.

[6]As already mentioned above the gauge adopted in the present paper is a variant of 't Hooft's maximum Abelian gauge [15]. In these gauges one expects a dominance of the Abelian field components, since non-Abelian components are supposed to become massive and hence irrelevant at low energies. In fact, lattice calculations performed in the maximum Abelian gauge [16] show that about 95% of the string tension comes from Abelian field configurations [30], which is referred to as Abelian dominance.

$$J_0^{c_0}(x)=q^{c_0}\delta(x_3)\,\delta(x_2)\left[\,\delta\left(x_1-\frac{R}{2}\right)-\delta\left(x_1+\frac{R}{2}\right)\right]. \tag{6.7}$$

If we take, for simplicity, the thermodynamic limit $L_1,L_2\to\infty$ of $G^{(2)}(x_\perp,y_\perp)$, Eq. (4.3),

$$G^{(2)}(\mathbf{x}_\perp,\mathbf{y}_\perp)=\ln|\mathbf{x}_\perp-\mathbf{y}_\perp|, \tag{6.8}$$

we receive from Eq. (6.4), besides an infinite constant, a logarithmically increasing potential. However, we do not expect that Eq. (6.2), which discards all perpendicular degrees of freedom $A'_\perp,E'_\perp$ can give a realistic description of the interaction between two charges sitting in the 1-2 plane. As discussed before the present approach singles out the three-axis and in fact preserves the rotational symmetry around the three-axis. Let us therefore consider the axial symmetric charge distribution (6.6). In this case $e_\perp^{(0)}=0$, and from the second to last term in Eq. (6.2) we obtain the static interaction potential

$$V=\frac{1}{2g^2}[\,\delta(0)]^2q^2G^{(1)}(R,0), \tag{6.9}$$

where $G^{(1)}$ is the Green's function of $-\partial'^2_3$. If we again take the thermodynamic limit $L\to\infty$

$$G^{(1)}(x_3,y_3)=\langle x_3|\frac{1}{-\partial_3^2}|y_3\rangle=\tfrac{1}{2}\,|x_3-y_3|, \tag{6.10}$$

we obtain a linearly raising potential

$$V=\sigma R, \tag{6.11}$$

with a string tension

$$\sigma=\frac{q^2}{g^2}[\,\delta(0)]^2. \tag{6.12}$$

Here it is understood that $\delta(0)$ is regularized in an appropriate way. The above obtained interaction potential is in agreement with the findings of the canonical quantization approach [31]; see also Ref. [42].

The emergence of the linear confinement potential in the three-direction should come as no surprise since, except for the dummy x_1,x_2 dependence, Eq. (6.2) represents the generating functional for $(1+1)$-dimensional Yang-Mills theory, which is known to confine. In fact, if we ignore the $e_\perp^{(0)}$ term [which, as seen above, vanishes in the thermodynamic limit $L_3\to\infty$ for any localized charge distribution $J_0(x)$ and furthermore depends only on the ''dummy'' coordinates x_1,x_2] and linearize the term quadratic in J_0 by means of a field $a_0(x)$, and furthermore perform the integration over e_3, we obtain

$$Z[J]=\int\mathcal{D}(a_0,a_3)\exp\left\{\frac{i}{g^2}\int d^2x_\perp\left[\int dx_0dx_3[\,\tfrac{1}{2}\,(\partial_0a_3)^2+\,(\hat d_3a_0)^2+a_0J_0+a_3J_3]\right]\right\}. \tag{6.13}$$

Here the first two terms in the bracket combine to $f^2_{\overline{\mu\nu}}$, where

$$f_{\overline{\mu\nu}}=\partial_{\overline{\mu}}a_{\overline{\nu}}-\partial_{\overline{\nu}}a_{\overline{\mu}},\quad \overline{\mu,\nu}=0,3, \tag{6.14}$$

and additionally a_3 satisfies, by construction [see Eqs. (3.8) and (3.16)], the gauge

$$a_3^{\rm ch}=0,\quad \partial_3a_3^n=0. \tag{6.15}$$

In $D=1+1$ the corresponding Faddeev-Popov determinant is an irrelevant constant. Thus Eq. (6.13) represents in fact the properly gauge-fixed generating functional of two-dimensional Yang-Mills theory, except for the parametric x_1,x_2 dependence of the fields. Equation (6.13) can be regarded as the strong coupling limit of Yang-Mills theory. This interpretation is consistent with the result of Ref. [38] where a strong coupling expansion of Yang-Mills theory was performed and the leading order was found to be given by $D=2$ Yang-Mills theory. This result is also confirmed in the field strength approach [29].

It is now straightforward to evaluate in the reduced, two-dimensional Yang-Mills theory (6.13) a Wilson loop in the 0-3 plane, which is most easily done in Euclidean space. One finds the area law in agreement with the linear rising potential between static charges as found above.

VII. ELIMINATION OF THE ELECTRIC FIELDS

In the gauged-fixed Yang-Mills theory, where the Gauss' law constraint has been fully resolved, the electric field variables occur still only quadratically in the exponent, so that these variables can be integrated out. The integral over e_3 is trivial. To perform the integral over $E'_\perp$ it is convenient to introduce a more compact notation. We define the kernel[7]

$$K^{ab}(x,y)=\begin{pmatrix}K^{a_0b_0} & 0\\ 0 & K^{\overline{a}\overline{b}}\end{pmatrix}=\begin{pmatrix}-\partial'^2_3\delta^{a_0b_0} & 0\\ 0 & -(\hat d_3\hat d_3)^{\overline{a}\overline{b}}\end{pmatrix}\delta^{(4)}(x-y). \tag{7.1}$$

Furthermore, we define

$$[E'_\perp(x)+e_\perp(\overline{x})]^c=\int d^4yP^{cc'}(x,y)E_\perp^{c'}(y)+e_\perp^{(0)c}, \tag{7.2}$$

where $e_\perp^{(0)}$ is defined by Eq. (6.3) and

$$P_{\underline{ij}}^{cc'}=\delta^{cc'}\delta_{\underline{ij}}\delta^{(4)}(x,y)-\delta^{cc_0}\left(\nabla_{\underline{i}}\frac{1}{\underline{\triangle}'_\perp}\right)_x\frac{1}{L}\int dx_3\hat A'^{c_0\overline{b}}_{\underline{j}}(x)\,\delta^{\overline{bc}'}\delta^{(4)}(x-y). \tag{7.3}$$

This quantity fulfills the relations

[7]Note that, up to an irrelevant constant, Det K gives the square of the Faddeev-Popov determinant (5.5).

$$\nabla'_{\underline{i}} P^{\overline{a}\overline{b}}_{\underline{i}\underline{j}}(x,y) = \nabla'_{\underline{j}} \delta^{\overline{a}\overline{b}} \delta^{(4)}(x-y),$$

$$\nabla'_{\underline{i}} P^{a_0 b}_{\underline{i}\underline{j}}(x,y) = \frac{1}{L}\int dx_3 \hat{D}'^{a_0 b_0}_{\underline{j}}(x)\,\delta^{(4)}(x-y). \quad (7.4)$$

In this notation we have

$$\begin{aligned}
&\int d^4x\left[\left(\frac{1}{\partial'_3}\Gamma^n_\perp\right)^2+\left(\frac{1}{\hat{d}_3}\Gamma^{\rm ch}_\perp\right)^2\right]\\
&\quad=\int d^4x[\Gamma_\perp K^{-1}\Gamma_\perp]\\
&\quad=\int d^4x\{[\hat{\mathbf{D}}'_\perp\cdot(\mathbf{E}'_\perp+\mathbf{e}_\perp)+\rho]K^{-1}[\hat{\mathbf{D}}'_\perp\cdot(\mathbf{E}'_\perp+\mathbf{e}_\perp)+\rho]\}\\
&\quad=\int d^4x[\hat{\mathbf{D}}'_\perp\cdot(P\mathbf{E}'_\perp+\mathbf{e}^{(0)}_\perp)+J_0]\\
&\qquad\times K^{-1}[\hat{\mathbf{D}}'_\perp\cdot(P\mathbf{E}'_\perp+\mathbf{e}^{(0)}_\perp)+J_0]\\
&\quad=\int d^4x[(\mathbf{E}'_\perp P^T+\mathbf{e}^{(0)}_\perp)\cdot(-\hat{\mathbf{D}}'_\perp)+J_0]\\
&\qquad\times K^{-1}[\hat{\mathbf{D}}'_\perp\cdot(P\mathbf{E}'_\perp+\mathbf{e}^{(0)}_\perp)+J_0]. \quad (7.5)
\end{aligned}$$

Here, we have also introduced the transposed projector P^T by

$$(P\mathbf{E}'_\perp)^c(x)=(\mathbf{E}'_\perp P^T)^c(x). \quad (7.6)$$

In this notation we can also write

$$\begin{aligned}
\int (\mathbf{E}'_\perp\mathbf{E}'_\perp+\mathbf{e}^2_\perp)&=\int(\mathbf{E}'_\perp+\mathbf{e}_\perp)^2\\
&=\int(P\mathbf{E}'_\perp+\mathbf{e}^{(0)}_\perp)^2\\
&=\int[\mathbf{E}'_\perp P^T P\mathbf{E}'_\perp+2\mathbf{e}^{(0)}_\perp P\mathbf{E}'_\perp+(\mathbf{e}^{(0)}_\perp)^2]. \quad (7.7)
\end{aligned}$$

In Eq. (7.5) and below it is understood that in $\hat{\mathbf{D}}'_\perp=\boldsymbol{\nabla}_\perp+\hat{\mathbf{A}}'_\perp$ the two-dimensional gradient operator $\nabla_\perp$ is replaced by the corresponding operator $\nabla'_\perp$ with the zero mode excluded:

$$\langle x_\perp|\boldsymbol{\nabla}'_\perp|x'\rangle = i\sum_{\mathbf{n}_\perp\neq 0}\mathbf{n}_\perp e^{i2\pi\mathbf{n}_\perp(\mathbf{x}-\mathbf{x}')/L},\quad \mathbf{n}_\perp=(n_1,n_2). \quad (7.8)$$

This is admissible since $\nabla_\perp\cdot\mathbf{E}_\perp=\nabla'_\perp\cdot\mathbf{E}_\perp$. The integral over $\mathbf{E}'_\perp$ can then easily be carried out, yielding, for the transition amplitude,[8]

$$\begin{aligned}
Z[J]=\int\mathcal{D}(A'_\perp,a_3)\prod_{x_0}\ &\mathrm{Det}^{-1/2}H\\
&\times\exp\Bigg\{\frac{i}{g^2}\Bigg[L\tfrac{1}{2}\int d^3\overline{x}[\overline{\partial_0 a_3(\overline{x})}]^2\\
&-L\int d^3\overline{x}\,\overline{a_3(\overline{x})J_3(\overline{x})}\\
&+\tfrac{1}{2}\int d^4x[e^{(0)}_\perp-\partial_0 A'_\perp\\
&+(\mathbf{D}'_\perp\mathbf{e}^{(0)}_\perp+J_0)K^{-1}D_\perp]PH^{-1}P^T\\
&\times[e^{(0)}_\perp-\partial_0 A'_\perp-D'_\perp K^{-1}(\mathbf{D}_\perp\mathbf{e}^{(0)}_\perp+J_0)]\\
&+\int d^4x[\mathbf{e}^{(0)}_\perp\partial_0\mathbf{A}'_\perp-\tfrac{1}{2}(\mathbf{e}^{(0)}_\perp)^2]\\
&-\tfrac{1}{2}\int d^4x(\hat{\mathbf{D}}'_\perp\mathbf{e}^{(0)}_\perp+J_0)\frac{1}{K}(\hat{\mathbf{D}}'_\perp\mathbf{e}^{(0)}_\perp+J_0)\\
&-\tfrac{1}{2}\int d^4x[B(A')]^2\Bigg]\Bigg\}. \quad (7.9)
\end{aligned}$$

Here, we have introduced the kernel

$$H=P^TP+P^T\left(-\hat{D}'_\perp\frac{1}{K}\hat{D}'_\perp\right)P, \quad (7.10)$$

which is defined in the space of the spatially periodic two-dimensional vector functions.

It is instructive to consider the limiting case $A'_\perp=0$, where the kernel (7.10) reduces to

$$H=1-\nabla_\perp K^{-1}\nabla_\perp. \quad (7.11)$$

In this case the terms of the action (7.9) containing the static source J_0 reduce to

$$\begin{aligned}
&\left[J_0\frac{1}{K}\hat{D}'_\perp PH^{-1}P^T(-\hat{D}'_\perp)\frac{1}{K}J_0-J_0\frac{1}{K}J_0\right]_{A'_\perp=0}\\
&\qquad=J_0\frac{-1}{K-\Delta'_\perp}J_0, \quad (7.12)
\end{aligned}$$

and in the limit $a_3=0$ we recover the familiar Coulomb law

$$\left[J_0\frac{-1}{K-\Delta'_\perp}J_0\right]_{a_3=0}=J_0\frac{1}{\Delta'}J_0,\quad \Delta'=\Delta'_\perp+\partial_3^2. \quad (7.13)$$

Note that in the continuum limit the zero eigenvalue of Δ' disappears and $\Delta'\to\Delta$. In Yang-Mills theory the infrared singular behavior of the static Coulomb law is avoided by the presence of the Abelian field $a_3(\overline{x})$. The infrared behavior of Yang-Mills theory, and in particular the confinement mechanism, should therefore be essentially determined by this fluctuating field. This is in agreement with the findings of the previous section.

In Appendix B it is shown that, for $A'_\perp=0$,

[8]Note that in Eq. (7.9) from $\hat{\mathbf{D}}'_\perp\cdot\mathbf{e}^{(0)}_\perp=\nabla_\perp\cdot\mathbf{e}^{(0)}_\perp+\hat{\mathbf{A}}'_\perp\cdot\mathbf{e}^{(0)}_\perp$ the term $\nabla_\perp e^{(0)}_\perp$ can be dropped since this quantity does not depend on x_3 and hence vanishes when acted on with $K^{-1}\sim(\partial'^2_3)^{-1}$.

$$\mathrm{Det}^{-1/2}H=\mathrm{Det}^{1/2}K\mathrm{Det}^{-1/2}(K-\Delta'_{\perp}). \tag{7.14}$$

Here

$$\mathrm{Det}^{1/2}K=\mathrm{const}\times\mathrm{Det}^{1/2}(-\hat{d}_3\hat{d}_3)=\mathrm{const}\times J \tag{7.15}$$

is precisely the Faddeev-Popov determinant (5.5) which, as shown in Appendix A, up to an irrelevant constant coincides with the Haar measure of SU(N):

$$J=\prod_{k>l}\frac{1}{L^2}\sin^2 L\frac{\alpha_k(\overline{x})-\alpha_l(\overline{x})}{2},\quad \sum_{k=1}^{N}\alpha_k(\overline{x})=0, \tag{7.16}$$

where $\alpha_k(\overline{x})$ are the diagonal elements of $i\,a_3(\overline{x})$ (cf. also Ref. [35]).

Note that after imposing Gauss' law we have obtained here the Haar measure of the gauge group for the functional integral over the gauge field $a_3(\overline{x})$ although we started from the usual functional integral representation with flat integration measure but gauge fixed by the Faddeev-Popov method. This result is a manifestation of the observation [35] that the standard functional integral repesentation of Yang-Mills theory with the gauge fixed by the Faddeev-Popov method fully respects gauge invariance even in the nonperturbative regime and that, in particular, the Haar measure of the gauge group (necessary for a projection onto gauge-invariant states) naturally arises from the Faddeev-Popov determinant.

The Haar measure and hence $\mathrm{Det}^{-1/2}\,H$ vanish for degenerate field configurations $a_3(\overline{x})$, for which two diagonal elements coincide, i.e., $[a_3(\overline{x})]_{kk}=[a_3(\overline{x})]_{ll}$ for $k\neq l$.

Since

$$\mathrm{Det}^{-1/2}H=\exp(-\tfrac{1}{2}\mathrm{Trln}H), \tag{7.17}$$

we receive an additional contribution to the effective action of the remaining (physical) degrees of freedom, which represents an action barrier to keep the system out of the singular field configurations. Such (energy) barriers have been also found in alternative formulations of gauge theory in terms of gauge-invariant variables [24,25].

Finally, let us make a few comments concerning the relation of the present approach with those interpreting confinement as a dual Meissner effect arising from monopole condensation. In fact, the gauge defined by Eqs. (3.8) and (3.17) is a variant of maximal Abelian gauge [15,16]. In these gauges monopoles arise at those singular points in configuration space where the gauge fixing is not unique. The gauge (3.8) is not unique at those singular points $x=x_S$, where the field $a_3(x)$ is degenerate; i.e., two eigenvalues of $a_3(x)$ coincide. It is straightforward to show [15,16] that near these singular points the gauge transformation $\Omega(\mathbf{x})$ necessary to fulfill the gauge (3.8) is such that the Abelian part of $\Omega\partial_\mu\Omega^\dagger(x)$ develops a ''magnetic'' monopole in $\mu=0,1,2$ space. Field configurations for which the gauge fixing is not unique give rise to zeros of the Faddeev-Popov determinant. In fact in the present case the Faddeev-Popov determinant

$$\mathrm{Det}iD_3\sim J\sim\sqrt{\mathrm{Det}\,K} \tag{7.18}$$

vanishes at the singular points of degenerate $a_3(\overline{x})$ field configurations. The field configurations of vanishing Faddeev-Popov determinants define the Gribov horizon, which in the present context is therefore built up from monopoles.

VIII. CONCLUDING REMARKS

In this paper we have considered $D=3+1$ dimensional Yang-Mills theory defined on a spatial torus. Using a variant of 't Hooft's maximum Abelian gauge [see Eqs. (3.8), (3.17), and (4.16)] we have performed a complete resolution of Gauss' law. This has resulted in a functional integral representation of Yang-Mills theory which is entirely defined in terms of unconstrained, gauge-fixed variables. These are the spatial gauge fields A'_i, $i=1,2$, defined by Eqs. (4.2), (4.14), and (4.18), the neutral x_3-independent part of which is transverse in the 1-2 plane [see Eq. (4.16)]. In addition an Abelian x_3-independent field $a_3(\overline{x})$, defined by Eq. (3.16), arises, so that the total number of degrees of freedom is that of $2(N^2-1)$ fields. This is the correct number of unconstrained degrees of freedom of massless Yang-Mills theory with gauge group SU(N). In this respect the present approach is more efficient than the pioneering works of Refs. [19,20] where twice as much integration variables remain, since in that case the electric field variables cannot be integrated out in closed form.

We have worked here in the Hamilton functional integral formulation which obviously violates Lorentz covariance. Furthermore, the adopted gauge also violates spatial SO(3) invariance but preserves axial symmetry. In this respect the present gauge is advantageous over the gauge used in Ref. [43] which violates also axial symmetry. Of course a complete elimination of all gauge degrees of freedom without violating any space-time symmetry would be preferable. This has been partly achieved in Refs. [24,25] for SU(2) in $D=3,4$. This appraoch works in the canonical Hamilton (operator) approach, which obviously violates Lorentz covariance but preserves all spatial symmetries. Unfortunately there is no direct way to extend this approach to higher gauge groups SU($N>2$), although some attempts have been undertaken for SU(3). For SU(2) in $D=3$ a complete covariant, gauge-invariant description has been achieved in the so-called field strength approach [40] at the expense of a doubling of the degrees of freedom [28,29]. We consider, however, the violation of a global symmetry, which can easily be restored, a minor problem. Of course, the exact Green functions preserve all space-time symmetries even in gauges which violate these symmetries.

As was illustrated in Sec. VI the field $a_3(\overline{x})$ represents the dominant infrared degrees of freedom, which in particular are responsible for the emergence of the area law. As a first step one might include only this Abelian field for a study of the infrared sector of QCD. In fact, because of our adopted gauge, this is in the spirit of the Abelian dominance observed in lattice calculations performed in maximum Abelian types of gauge [30].

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APPENDIX A: EVALUATION OF THE FADDEEV-POPOV DETERMINANT

Below, we evaluate the functional determinant of $i\hat{d}_3$. Consider the eigenvalue equation

$$i\hat{D}_3'^{ab}\varphi_\nu^b=\mu_\nu\varphi_\nu^a, \tag{A1}$$

where φ_ν has to satisfy periodic boundary conditions in x_3. Multiplying this equation with the generators in the fundamental representation T^a and using

$$T^a\hat{D}_\mu^{ab}\varphi^b=[D_\mu,\varphi],\quad \varphi=\varphi^aT^a, \tag{A2}$$

the eigenvalue equation becomes

$$i\partial_3\varphi^{(\nu)}+[ia_3,\varphi^{(\nu)}]=\mu_\nu\varphi^{(\nu)}. \tag{A3}$$

This equation is easily solved, since $ia_3=ia_3^{c_0}T^{c_0}$ is a (traceless) Hermitian diagonal matrix, with real elements $\alpha_k\equiv(a_3)_{kk}$ satisfying $\Sigma_{k=1}^N\alpha_k=0$. Hence, the eigenvalue equation reads, explicitly,

$$i\partial_3\varphi_{kl}^{(\nu)}+(\alpha_k-\alpha_l)\varphi_{kl}^{(\nu)}=\mu_\nu\varphi_{kl}^{(\nu)}. \tag{A4}$$

The periodic eigenfunctions are given by $[\nu=(c,n)]$

$$\varphi_{kl}^{(\nu)}=\eta_{kl}^c e^{i\omega_n x_3},\quad \omega_n=\frac{2\pi n}{L},\quad n=0,\pm1,\pm2,\ldots, \tag{A5}$$

where the η_{kl}^c denote the vectors of the Weyl basis of SU(N), which with $c=(r,s)$, $r,s=1,2,\ldots,N$, is defined by

$$\eta_{kl}^c=\frac{1}{\sqrt{2}}\delta_{kr}\delta_{ls}. \tag{A6}$$

The corresponding eigenvalues read

$$\mu_\nu=\mu_{c,n}=\omega_n+\alpha_r-\alpha_s\quad c=(r,s). \tag{A7}$$

For fixed n, there are $2\binom{N}{2}$ eigenvectors $\varphi^{\nu=(r,s)}$ corresponding to the off-diagonal elements $r\neq s$. The corresponding eigenvalues come in pairs $\omega_n\pm|\alpha_r-\alpha_s|$. Since $\mathrm{tr}\varphi^{(\nu)}=0$, there are only $N-1$ independent eigenvectors $\varphi^{\nu=(r,s)}$ with $r=s$, being degenerate with eigenvalue ω_n. The total number of independent eigenvalues (for fixed N) is of course N^2-1.

We therefore obtain, for the determinant under consideration,

$$\begin{aligned}\mathrm{Det}(i\hat{D}_3')&=\prod_\nu\mu_\nu=\prod_{c^0}\prod_{n=-\infty}^{\infty}\mu_{c^0,n}\\&=\left[\prod_{c^0}\prod_{n\neq0}\omega_n\right]\left[\prod_{r\neq s}\prod_{n=-\infty}^{\infty}(\omega_n+\alpha_r-\alpha_s)\right]\\&=\mathrm{Det}(i\partial_3')\mathrm{Det}(\hat{d}_3).\end{aligned} \tag{A8}$$

Note that by definition of $\hat{D}_3'$ the mode $n=0$ has to be excluded for $r=s$, which is indicated by the prime. The expression in the first bracket yields an irrelevant (diverging) constant, which can be absorbed into the renormalization of the functional integral.

Using

$$\sin x=x\prod_{n=1}^{\infty}\left[1-\left(\frac{x}{\pi n}\right)^2\right], \tag{A9}$$

the expression in the second set of brackets can be transformed to

$$\begin{aligned}\mathrm{Det}i\hat{d}_3&=\prod_{r\neq s}(\alpha_r-\alpha_s)\prod_{n=1}^{\infty}[(\alpha_r-\alpha_s)^2-\omega_n^2]\\&=\mathrm{const}'\times\prod_{r\neq s}(\alpha_r-\alpha_s)\prod_{n=1}^{\infty}\left[1-\left(\frac{\alpha_r-\alpha_s}{\omega_n}\right)^2\right]\\&=\mathrm{const}\times\left[\prod_{r\neq s}\frac{2}{L}\sin L\,\frac{\alpha_r-\alpha_s}{2}\right]\\&=\mathrm{const}\times\prod_{r>s}\left[\frac{2}{L}\sin L\,\frac{\alpha_r-\alpha_s}{2}\right]^2.\end{aligned} \tag{A10}$$

Thus up to an irrelevant constant this determinant agrees for $\Sigma_{r=1}^N\alpha_r=0$ with the Haar measure of the group SU(N):

$$J(L\alpha)=\prod_{k>l}\sin^2L\frac{(\alpha_k-\alpha_l)}{2}. \tag{A11}$$

APPENDIX B: EVALUATION OF Det H

In what follows, we work out the functional determinant Det H for $A_\perp'=0$. In this limit H is block diagonal in color space; i.e., it has no matrix elements between the neutral and charged color space:

$$H^{ab}=\begin{pmatrix}H^{a_0b_0} & 0\\ 0 & H^{\bar{a}\bar{b}}\end{pmatrix}. \tag{B1}$$

This is because the same is true for the matrix K, Eq. (7.1). Therefore, the functional determinant Det H factorizes as

$$\mathrm{Det}\,H=\mathrm{Det}^{(n)}H\mathrm{Det}^{(\mathrm{ch})}H. \tag{B2}$$

Since the color neutral part of H is given by

$$H_{\underline{i}\,\underline{j}}^{a_0b_0}=\delta^{a_0b_0}\left(\delta_{\underline{i}\,\underline{j}}+\nabla_{\underline{i}}'\frac{1}{(\partial_3')^2}\nabla_{\underline{j}}'\right). \tag{B3}$$

$\mathrm{Det}^{(n)}H$ is an irrelevant constant, which will be ignored in the following.

For the evaluation of $\mathrm{Det}^{(\mathrm{ch})}H$ we consider the corresponding eigenvalue equation

$$H^{\overline{a}\,\overline{b}}_{\underline{i}\,\underline{j}}\phi^{\overline{b}}_{\underline{j}}(x)=\mu\phi^{\overline{a}}_{\underline{i}}(x), \tag{B4}$$

which with the explicit form of $H^{\overline{a}\,\overline{b}}$ reads

$$-\nabla'_i(\overline{K^{-1}})^{\overline{a}\,\overline{b}}\nabla'_{\underline{j}}\phi^{\overline{b}}_{\underline{j}}(x)=(\mu-1)\phi^{\overline{a}}_{\underline{i}}(x). \tag{B5}$$

Here, the eigenfunctions have to satisfy periodic boundary conditions and we have introduced the abbreviation $\overline{K}=-\hat{d}_3\hat{d}_3$, which is the matrix K, Eq. (7.1), in the charged subspace.

The eigenfunctions $\phi^{\overline{b}}_{\underline{i}}(x)$ represent two-dimensional spatial vectors, which we can split into longitudinal and transverse parts,

$$\phi^{\overline{b}}_{\underline{i}}(x)=\phi^{\overline{b}}_{\underline{i}}(x)^T+\phi^{\overline{b}}_{\underline{i}}(x)^L, \tag{B6}$$

satisfying

$$\partial_{\underline{i}}\phi^{\overline{b}}_{\underline{i}}(x)^T=0,\quad \partial_{\underline{i}}\phi^{\overline{b}}_{\underline{i}}(x)^L=\partial_{\underline{i}}\phi^{\overline{b}}_{\underline{i}}(x). \tag{B7}$$

Obviously any (spatially) transverse vector function $\phi^{\overline{b}}_{\underline{i}}(x)^T$ gives rise to a zero eigenvalue of $\nabla_{\underline{i}}K^{-1}\nabla_{\underline{j}}$ and hence to an eigenvalue $\mu=1$ of H.

The longitudinal part $\phi^{\overline{a}}_{\underline{i}}(x)^L$ gives rise to a nontrivial eigenvalue $\mu\neq 1$. For these eigenvalues Eq. (B5) can be simplified.

Operating on Eq. (B5) from the left with ∇'_i and defining

$$\varphi^{\overline{a}}(x)=\nabla'_{\underline{i}}\phi^{\overline{a}}_{\underline{i}}(x), \tag{B8}$$

the eigenvalue equation becomes

$$-\Delta'_\perp(K^{-1})^{\overline{a}\,\overline{b}}\varphi^{\overline{b}}=(\mu-1)\ \ \varphi^{\overline{a}}. \tag{B9}$$

Therefore the nontrivial part of the determinant of H is given by

$$\mathrm{Det}H=\mathrm{Det}(1-\Delta'_\perp K^{-1}). \tag{B10}$$

Note that the kernel of the right-hand side (RHS) is a matrix in color and functional space but a scalar in ordinary space, contrary to H which is also a matrix in the two-dimensional Euclidean space spanned by the $x_{i=1,2}$ axis. The missing dimension on the RHS is due to eigenvalues $\mu=1$ of H. For later use it will be convenient to separate off K^{-1}, yielding

$$\mathrm{Det}H=\frac{\mathrm{Det}(\overline{K}-\Delta'_\perp)}{\mathrm{Det}\overline{K}}. \tag{B11}$$

Since $i\hat{d}_3$ is the Hermitian operator, $K=-\hat{d}_3^2$ is positive semidefinite, while $(-\Delta'_\perp)$ is strictly positive definite. Therefore, $(\overline{K}-\Delta'_\perp)$ is a positive definite operator and consequently $\mathrm{Det}^{-1/2}(\overline{K}-\Delta'_\perp)$ is nonsingular, even for field configurations $a_3(\overline{x})$ for which K has zero eigenvalues. These field configurations do not contribute to the transition amplitude due to the presence of $\mathrm{Det}^{1/2}\,\overline{K}$, see Eq. (7.9). Note that this determinant agrees with the Haar measure of SU(N) given in Appendix A.

APPENDIX C: RESIDUAL GAUGE INVARIANCE

The spatially periodic boundary conditions to the gauge fields $A_i(x)$ restrict the gauge transformations $A_i\to A_i^\Omega$ to those gauge functions $\Omega(x)$, which satisfy the equation

$$[D_i,\widetilde{\Omega}_k(x)]=0, \tag{C1}$$

where

$$\widetilde{\Omega}_k(x)=\Omega^\dagger(x+L\mathbf{e}^k)\Omega(x). \tag{C2}$$

For simplicity we have set here $L_1=L_2=L_3=L$. This equation, which has to be fulfilled for all $A_i(x)$ and all $k=1,2,3$, is solved for

$$\widetilde{\Omega}_k(x)=Z_{n_k}, \tag{C3}$$

with

$$Z_n=e^{-\phi_n}=e^{i(2\pi n/N)},\quad n=0,1,2,\ldots,N-1, \tag{C4}$$

being an element of the center of the gauge group. The center of the group is defined by the set of elements commuting with all elements of the group. Here ϕ_n is an element of the Cartan algebra $\mathcal{H}$. For SU(2) the center of the group is given by

$$\phi_{n=0}=0,\quad Z_0=1,$$

$$\phi_{n=1}=-i\pi\tau_3,\quad Z_1=-1. \tag{C5}$$

Equations (C2) and (C3) imply

$$\Omega(x+\mathbf{e}^kL)=Z_n^*\Omega(x). \tag{C6}$$

The gauge conditions chosen above [see Eqs. (3.8), (3.17), and (4.16)] do not yet fix the gauge completely. There is a residual gauge symmetry left, which will be exhibited below.

First we note that all three gauge-fixing conditions are left unchanged under permutations of the color indices k,l of the fundamental representation T^a_{kl} for the gauge group SU(N). Such permutations of the basis color vectors are generated by those global gauge transformations $S\in\mathrm{SU}(N)$, which are N-dimensional matrix representations of the symmetry group S_N (group of permutations of N elements). These transformations form the Weyl group. For SU(2) the Weyl group consists of two elements $S_0=1$ and $S_1=-i\tau^1=2T^1$, which correspond, respectively, to the trivial permutation and to an exchange of the two color indices. The nontrivial permutation in fact represents a rotation in color space around the one-axis through an angle π:

$$S_1=-i\tau^1=e^{-i\pi(\tau^1/2)}=e^{\pi T^1}. \tag{C7}$$

Note that S_1 is not an element of the Cartan algebra. These considerations can obviously be extended to larger gauge groups SU($N>2$).

Besides the above discussed global residual gauge symmetry the gauge conditions (3.8), (3.17), and (4.16) are also invariant under the so-called displacement symmetry [41,31]. Let us show how this comes about. The gauge condition $A_3^{\rm ch}=0$ leaves residual gauge transformations of the form

$$\Omega = U(x)S, \quad U(x)=e^{-\omega(x)}, \quad \omega(x)\in\mathcal{H}, \tag{C8}$$

where $S\in S_N$ and $U(x)$ is a gauge transformation in the maximal Abelian subgroup (invariant torus).

The second gauge condition $\partial_3 A_3^n=0$ is left invariant by gauge transformations of the form (C8) provided that

$$\partial_3\partial_3\omega(x)=0. \tag{C9}$$

This equation is satisfied if $\omega(x)$ is of the form

$$\omega(x)=\omega^{(0)}(\overline{x})+\omega^{(1)}(\overline{x})x_3, \quad \omega^{(0)}(\overline{x}),\omega^{(1)}(\overline{x})\in\mathcal{H}. \tag{C10}$$

The quasiperiodic boundary condition to the gauge function (C6) requires

$$e^{-\omega^{(1)}(\overline{x})L}=Z_{n_3}^*=e^{\phi_{n_3}}. \tag{C11}$$

This equation has to be satisfied for all $\overline{x}$ and is solved for

$$\omega^{(1)}(\overline{x})=\frac{1}{L}(2\pi i k_3-\phi_{n_3}):=\alpha_3, \tag{C12}$$

with $k_3=\mathrm{dia}(k_3^{(1)},k_3^{(2)},\ldots,k_3^{(N)})$ being a N-dimensional traceless diagonal matrix, $\Sigma_{i=1}^N k_3^{(i)}=0$, with integer entries $k_3^{(i)}$.

Finally, the third gauge constraint restricts the residual gauge transformations to such a function, satisfying

$$\nabla_\perp\cdot\nabla_\perp[\omega^{(0)}(\overline{x})+\tfrac{1}{2}L\,\omega^{(1)}(\overline{x})]=0, \tag{C13}$$

which in view of Eq. (C12) reduces to

$$\nabla_\perp\nabla_\perp\omega^{(0)}(\overline{x})=0, \tag{C14}$$

which implies

$$\omega^{(0)}(\overline{x})=\beta(x^0)+\boldsymbol{\alpha}_\perp(x^0)\mathbf{x}_\perp, \quad \beta,\boldsymbol{\alpha}_\perp\in\mathcal{H}. \tag{C15}$$

The quasiperiodic boundary condition (C6) requires the $\underline{\alpha}_i(x^0)$ to satisfy the relation

$$e^{-\underline{\alpha}_i L}=Z_{\underline{n}_i}^*=e^{\underline{\phi}_{n_i}}, \tag{C16}$$

which is satisfied for

$$\boldsymbol{\alpha}_\perp=\frac{1}{L}(2\pi\mathbf{k}_\perp i-\boldsymbol{\phi}_{n_\perp}), \tag{C17}$$

with $\mathbf{k}_\perp=\mathrm{dia}(\mathbf{k}_\perp^{(1)},\mathbf{k}_\perp^{(2)},\ldots,\mathbf{k}_\perp^{(N)})$ being again a traceless diagonal matrix with integer entries $\mathbf{k}_\perp^{(i)}=(\mathbf{k}_1^{(i)},\mathbf{k}_2^{(ii)})$ satisfying $\Sigma_{i=1}^N\mathbf{k}_\perp^{(i)}=0$.

The residual gauge symmetry left after the three gauge constraints have been implemented is therefore given by

$$\Omega(x)=Se^{-(\beta+\boldsymbol{\alpha}\mathbf{x})}. \tag{C18}$$

Besides the discrete symmetry S generating the permutation of color indices the residual gauge symmetry consists of a global Abelian gauge transformation $e^{-\beta}$ and the so-called displacement symmetry $e^{-\boldsymbol{\alpha}\mathbf{x}}$.

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