Radiation of extreme black holes

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The radiation from extreme Reissner-Nordström black holes is computed by considering the collapse of a spherical charged shell. No neutral radiation is found, and, therefore, the black hole temperature is zero, but there is emission of charged particles if the charge to mass ratio is greater than one. The absence of thermal effects is in accord with the predictions of Euclidean theory and it is argued why the radiation entropy should not be regarded as a black hole attribute. Rather, if any entropy is to be present in the future, this is interpreted as a loss of quantum coherence in the past by the presence of the timelike wormhole made by charges. The stability of the extreme black hole to emitting uncharged particles is consistent with the conjecture of cosmic censorship, which may thus be regarded as strengthened by the quantum theory. [S0556-2821(97)02104-8]

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I. INTRODUCTION

After Hawking's discovery [1] of the quantum emission of radiation from black holes, a large amount of investigation has been done to clarify the dynamical origin of black hole entropy. Recently, the particular case of extreme black holes (defined as black holes having zero surface gravity) has been investigated at length from many points of view, after the discovery that extreme black holes have zero entropy, but nonzero horizon area [2]. One argument runs as follows (see also [3]). According to finite temperature quantum field theory, the leading contribution to a partition function in a loop expansion, often called the tree approximation, is $\ln Z_\beta=-I(\beta,\widetilde{\phi})$, where $\widetilde{\phi}$ is the stationary point of the Euclidean action $I(\beta,\widetilde{\phi})$, and β is the period of the imaginary time, physically interpreted as the inverse temperature of the equilibrium state. For gravity in an asymptotically flat context, β is then the inverse temperature at infinity as the local equilibrium (Tolman) temperature is dependent on position. To get also a well-defined canonical ensemble, the system must be enclosed in a large box with fixed temperature near the boundary [4]. The Euclidean action for the Einstein-Maxwell system is [5,6] (we use dimensionless units $G=c=\hbar=1$)

$$I=-\frac{1}{16\pi}\int_{\mathcal{M}}R(g)^{1/2}d^4x-\frac{1}{8\pi}\int_{\partial\mathcal{M}}[K](h)^{1/2}d^3x+\frac{1}{16\pi}\int_{\mathcal{M}}F_{ab}F^{ab}(g)^{1/2}d^4x, \tag{1}$$

where (i) $\mathcal{M}$ is asymptotically flat with boundary $\partial M=S^2(r_0)\times S^1_\beta$ at some large radius r_0, (ii) the fields (g,F) are periodic with period β near the boundary, and (iii) K is the trace of the second fundamental form of the boundary and $[K]=K-K_0$, K_0 being the trace as if the boundary were embedded in flat Euclidean space. The subtraction of K_0 is to ensure that flat space has a vanishing partition function and that the arbitrary r_0 is at last removed by letting $r_0\to\infty$. In the case of nonextreme black hole configurations, it so happens that the unique β such that the Euclidean manifold is a stationary point of the action is the inverse Hawking temperature of the black hole, as any other choice yields a conical singularity in the space. For the extreme Reissner-Nordström black hole, the stationary point is represented by a radial electric field with charge $|Q|=M$ and the line element

$$ds^2=\widetilde{g}_{ab}dx^adx^b=\frac{(r-M)^2}{r^2}d\tau^2+\frac{r^2}{(r-M)^2}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2). \tag{2}$$

Under the Hodge duality on the field strength F_{ab}, it also describes a magnetically charged extreme black hole. Now this is a perfectly regular metric on the manifold $\mathcal{M}=\mathbb{R}\times\mathbb{R}\times S^2$, where τ ranges in $\mathbb{R}$. The topology is that of an annulus whose inner boundary at $r=M$ is at infinite distance from the interior points. As such, it can be identified to any period without danger of a conic singularity in the space. Therefore one considers the Euclidean theory on the manifold $S^1\times\mathbb{R}\times S^2$, where the circumference of the S^1 factor is any real β. The bulk term of the action is purely electromagnetic as $R=0$, and by adding the boundary term due to gravity one finds easily $\ln Z_\beta=-\beta M$, of which one half is the gravitational contribution and the other half the electromagnetic contribution (this is β times the electrostatic energy outside of a sphere with charge Q and radius $r=M$, in the extreme limit when $|Q|=M$). The partition function being linear in β, the canonical entropy vanishes rather than being one-quarter of the horizon area, as would be expected by continuity with nonextreme black holes. The essential reason for this difference is the crucial fact that the horizon of an extreme configuration is at infinite proper distance along spacelike directions from any point in the outer static region (but not along timelike or null directions). This might also imply that no information loss [7] exists for such bodies, although the hole can still swallow any matter configuration, as the information entropy is normally associated with regions whose boundary is at finite distances (a very different view has been expressed in [8]). In view of these facts, and the lack of continuity of the Bekenstein-Hawking geometric entropy, the question of whether extreme black holes radiate or not must be revisited. Indeed, in [2] it was thought that extreme black holes possibly radiate so as to keep the black hole extreme, if matter is sent down the hole, because the Euclidean solution can be identified to any β. This can be interpreted as the fact that unperturbed extreme black holes simply do not radiate. In two dimensions (in the sense of ignoring the two angular variables) they do not radiate in fact, as was noted by Davies [9]. As a related fact, the trace anomaly causes the stress tensor to be divergent on the horizon in dimension 2 [10], but in four dimensions things are different, as the stress tensor is regular in the zero temperature state and divergent near the horizon in any nonzero temperature state [11]. This would imply that an extreme black hole cannot be in equilibrium at any temperature but zero, which again is consistent with the idea that extreme black holes do not radiate. This is welcome perhaps, since extreme black holes are on the verge of developing a naked singularity, and any instability in this case would seem unhappy.

The question whether the temperature and entropy of an extreme charged black hole are really zero was then investigated by using the Haag-Hessling quantum equivalence principle [12–14] and Green functions [15], stress-energy calculations [16,17,11], and Hamiltonian methods [3,2]. New light was shed on these problems by the discovery that extreme black holes can be identified with elementary string excitations [18–21]. All such approaches were concerned with stationary extreme black holes and ignored the gravitational collapse bringing them to a quasistationary state. *A priori*, however, it seems obvious that a collapsing matter configuration, whether or not extreme ($M=|Q|$), coupled to a quantum field must radiate. The reason is twofold, one being the time-increasing redshift imparted to the field modes by the motion of the collapsing matter, the other being the presence of the Coulomb field which can easily reach the critical threshold of pair creation. Indeed, the lower the mass of the black hole, the higher the electric field intensity, suggesting that extreme black holes are indeed unstable objects. However, this is perhaps not so obvious, as in fact we obtain emission of charged particles if and only if the charge-to-mass ratio is greater than 1, but no emission at all of neutral particles. This is just what is needed to avoid naked singularities, since neutral particles would decrease the mass but not the charge of the black hole. As extreme black holes are clearly unstable under small variations of the mass and charge parameters, unless $\delta M=\delta|Q|$, this also means that extreme configurations are unstable to emitting particles. The quantum instability of extreme black holes to emitting charged particles has also been implicitly noted in [22], as when the temperature goes to zero by increasing the black hole mass, and therefore by reducing the surface gravity, the rate of charge loss remains finite. Such a continuity argument is not obvious because the Reissner-Nordström solution changes character abruptly in the extreme limit.

Here we compute the particle emission at infinity by taking into account the collapse process of an extremal charged shell, as has been described in [23]. The Hawking radiation from the collapse of a thin uncharged shell was investigated earlier by Boulware [24]. After discussing the asymptotic behavior of the shell, we pass to the quantum theory by following the traditional Hawking's scheme for handling problems involving horizons [1,25]. The implications of the result and some comments about black hole entropy will be postponed to the discussion. The calculation shows that one may forget about the collapse process and work equivalently with the maximally extended solution, by taking the modes emerging from the past horizon to be positive frequency with respect to the retarded time $u=t-r^*$, where r^* is the tortoise coordinate in the extreme Reissner-Nordström metric, and the incoming modes to be positive frequency with respect to the advanced time, $v=t+r^*$ (these are the conditions defining the Boulware vacuum in the Schwarzschild background). Only the radiation at infinity will be clearly displayed, as the particle spectrum near the black hole can be subject to many nontrivial phenomena, depending on the existence of bound or resonance states in the Coulomb and gravitational field of the shell. The result does not seem to depend on the fact that matter is in a thin shell and holds for an extreme ball as well.

II. ASYMPTOTIC BEHAVIOR

As was explained in [23] (see also Refs. [26–29]), the equations of motion of a collapsing charged spherical shell can be deduced by relating the discontinuity of the extrinsic curvature across the shell with the surface energy density in the shell, the Israel-Kuchar equations. Let ρ be the surface energy density of the shell in the comoving frame and $\mathcal{M}=4\pi R^2\rho$ the total proper mass, R being the shell radius. For an extreme shell, we introduce the abbreviation

$$a=\frac{\mathcal{M}^2-M^2}{2\mathcal{M}},$$

where M is the total gravitational mass of the shell. The exterior metric is the extremal Reissner-Nordström metric

$$ds^2=g_{ab}dx^a dx^b=-\frac{(r-M)^2}{r^2}dt^2+\frac{r^2}{(r-M)^2}dr^2 + r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{3}$$

and the interior is part of flat Minkowski space. The shell position is specified by $r=R(\tau)$, $t=T(\tau)$, where τ is the proper time along the shell history. The equations of motion are then

$$\dot{R}=-\sqrt{\left[\frac{M}{\mathcal{M}}+\frac{a}{R}\right]^2-1}, \quad \dot{T}=\frac{R}{R-M}\sqrt{1+\frac{R^2\dot{R}^2}{(R-M)^2}}. \tag{4}$$

For such an extreme shell, one may expect the Coulomb repulsion of like charges to stop the collapse by turning in a bounce. In fact, a simple consequence of the equations of motion is that if $M>\mathcal{M}>0$, then the shell radius has a turning point at $R_T=(M+\mathcal{M})/2<M$ which is a minimum radius. Therefore the shell reaches the horizon value at $R=M$, and disappears from sight by forming a future event horizon at some advanced time v_0 (see Fig. 1).

As Fig. 1 shows, the shell subsequently bounces and expands by forming another asymptotically flat region with a past horizon, and singularities are not formed. The space outside the shell in between the two horizons is a closed

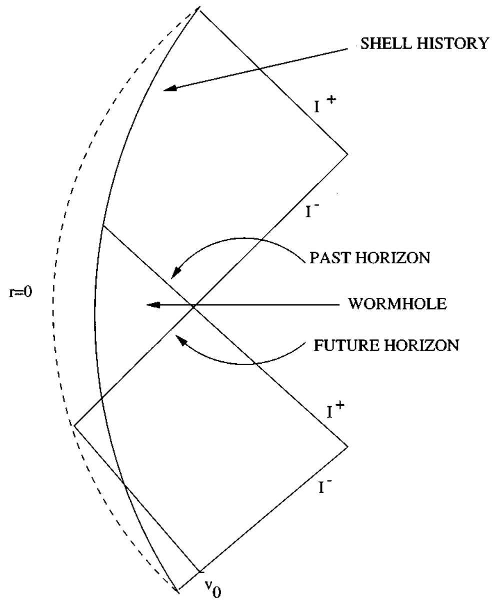

FIG. 1. The Penrose diagram of an extreme bouncing shell. In the bottom region the shell is collapsing, in the top region the shell is expanding, and each point represents a two-sphere. The region to the left of the shell history is a flat space of radius R and to the right is part of the extended Reissner-Nordström metric. I^+ (I^-) denotes future (past) null infinity.

universe, and a topology change takes place. We shall focus on the bottom region containing the future event horizon, i.e., the one where the shell appears collapsing (the top region will be important later).

Although the equations of motion are exact integrables, we only need the asymptotic expansion of the functions $R(\tau)$ and $T(\tau)$ near the horizon. More precisely we need the advanced time v of the shell as a function of the retarded time u, namely, the relation $v(u)$ near the horizon to leading nontrivial order in $\varepsilon = R - M$. From the equations of motion we find

$$\dot{R} = a_0 + a_1\varepsilon + a_2\varepsilon^2 + O(\varepsilon^3), \tag{5}$$

$$\dot{T} = b_{-2}\varepsilon^{-2} + b_{-1}\varepsilon^{-1} + b_0 + O(\varepsilon), \tag{6}$$

where the exact form of the coefficients is left apart, as the really important fact is that both a_0 and b_{-2} are nonvanishing. The advanced and retarded coordinate times are, respectively, $t \pm r^*$, where

$$r^* = r + 2M\ln\left(\frac{r}{M} - 1\right) - \frac{M^2}{r-M}, \tag{7}$$

shows the characteristic first order pole of extreme black holes. From the equations it now follows that $v(u)$ remains finite on the horizon while u diverges to infinity. More exactly,

$$v(u) = v_1 - Cu^{-1} + O(u^{-2}\ln u), \quad u \to \infty, \tag{8}$$

where v_1 is the value of advanced time that marks the birth of the black hole and C is a positive constant depending on M and $\mathcal{M}$. This behavior may be compared with the nonextreme black hole, where $v(u)$ approaches v_1 exponentially fast in retarded time, the exact formula being

$$v(u) = v_1 - Ce^{-ku} + O(e^{-2ku}), \quad k = \frac{r_+ - r_-}{2r_+^2}, \tag{9}$$

where k is the surface gravity and $r_\pm = M \pm \sqrt{M^2 - Q^2}$. This can be traced back to the first order pole singularity that controls the behavior of r^* near the horizon in contrast with the logarithmic singularity of the nonextreme case. The asymptotic behavior (8) on the horizon is not restricted to a thin shell. In fact, the exterior geometry is the Reissner-Nordström metric even for a collapsing, charged extreme ball. It is then easy to show that v finite on the horizon implies the behavior (8), which is almost all one needs to compute the radiation at infinity.

III. MINIMAL QUANTUM THEORY

The asymptotic behavior just discussed has the consequence that the phase of a positive frequency ingoing mode from past infinity will be redshifted at a much lower rate than for nonextreme black holes. Indeed, it is fairly straightforward to deduce the asymptotic form of such a mode near the event horizon, when the black hole is about to form. Precisely we have the following. Let $P_{j\omega,e}$ be a positive frequency solution of the charged Klein-Gordon equation in the collapse geometry of the infalling shell,

$$g^{ab}(\nabla_a + ieA_a)(\nabla_b + ieA_b)P_{j\omega,e} - \mu^2 P_{j\omega,e} = 0,$$
$$A_a = Qr^{-1}\delta_a^0, \tag{10}$$

where e is the charge of the field, which is ingoing from past infinity, and therefore [j standing for (l,m)]

$$P_{j\omega,e} = (4\pi p)^{-1/2} r^{-1} Y_j(\theta,\phi) R_{j\omega}(r) e^{-i\omega t},$$
$$\omega = \sqrt{p^2 + \mu^2}, \tag{11}$$

where $p \geqslant 0$ is the radial momentum, $Y_j(\theta,\phi)$ are the spherical harmonics, and the modes are normalized to a δ function of ω. Then, asymptotically for large u, for $\omega \gg \mu$, and for $r \to M$, the outgoing part of $P_{j\omega,e}$ after reflection through the shell is

$$P_{j\omega,e} \simeq (4\pi p)^{-1/2} r^{-1} Y_j(\theta,\phi) \exp(iCpu^{-1}) e^{-i\sigma e t - i\omega v_1},$$
$$\sigma = \frac{Q}{|Q|}. \tag{12}$$

Because of the ultrarelativistic condition $\omega \gg \mu$, we may neglect the mass μ from now on, its only role being to set the lower bound for integration over energy. The quantum field

$$\phi = \sum_j \int_\mu^\infty d\omega [A_{j\omega} P_{j\omega,e} + B_{j\omega}^\dagger P_{j\omega,-e}^*] \tag{13}$$

is assumed to be in the $|\mathrm{in}\rangle$ vacuum state relative to the above decomposition. In order to obtain the particle content at future infinity, we have to expand the in mode into positive and negative frequency waves of the kind $\exp(\pm i\omega u)$, namely, waves which are purely outgoing to infinity. This is a rather delicate matter. Considering Eq. (12) as an holomorphic function of u in the lower half complex u plane, we argue that the Fourier expansion of $P_{j\omega,e}$ can only contain the factors $\exp(-i\omega u)$, which just share with $P_{j\omega,e}$ the same holomorphic property. We immediately conclude that *there is no neutral scalar radiation at infinity from an extremal black hole*. However, for charged fields things are different due to the superradiant phenomenon. Indeed, in the extended Reissner-Nordström geometry the radial outgoing waves from the past horizon have the asymptotic behavior [22] (we are neglecting the logarithmic phase due to the Coulomb field as it is unnecessary to our aims)

$$\vec{R}_{j\omega}\simeq\begin{cases} e^{i(\omega-\sigma e)r^*}+\vec{A}_{j,e}(\omega)e^{-i(\omega-\sigma e)r^*}, & r^*\to-\infty,\\ \vec{B}_{j,e}(\omega)e^{ipr^*}, & r^*\to\infty, \end{cases} \tag{14}$$

which define the reflection and transmission coefficients of the potential barrier surrounding the black hole. One defines similar reflection-transmission coefficients for ingoing waves from past infinity (with arrows pointing to the left). One has then the unitarity relations [22]

$$p\vec{B}_{j,e}(\omega)=(\omega-\sigma e)\overleftarrow{B}_{j,e}(\omega), \tag{15}$$

$$(\omega-\sigma e)|\overleftarrow{B}_{j,e}(\omega)|^2=p[1-|\overleftarrow{A}_{j,e}(\omega)|^2]. \tag{16}$$

From Eq. (14), a positive frequency wave packet at the horizon containing the factor $\exp(-i\omega u)$ arrives at infinity with the time dependence $\exp[-i(\omega+\sigma e)u]$. This is holomorphic in the lower half complex u plane if $\omega+\sigma e>0$, but is holomorphic in the upper half complex u plane if $\omega+\sigma e<0$, which is just one superradiance condition. Therefore the wave packet can have a negative frequency component at infinity, corresponding to particle emission in the quantum theory.

We now show that this is actually the case. Let $\nu\in\mathbb{C}$ such that $-1<\mathrm{Re}\,\nu<1/2$ and $J_\nu(z)$ the Bessel function of order ν. Then

$$\int_0^\infty e^{-i\omega' u}(\omega')^{\nu/2}J_\nu(2\sqrt{\omega\omega'})d\omega' =\frac{\omega^{\nu/2}e^{-i\pi\nu/2}}{iu^{\nu+1}}\exp\left(\frac{i\omega}{u}\right)\theta(u). \tag{17}$$

By forming a wave packet one can easily show that the limit $\nu\to-1$ can be safety taken. This means that in a distribution sense we have

$$\int_{-\infty}^\infty e^{-i\omega' u}(\omega')^{-1/2}J_{-1}(2\sqrt{\omega\omega'})\theta(\omega')d\omega' =\omega^{-1/2}\exp\left(\frac{i\omega}{u}\right)\theta(u). \tag{18}$$

This equation, combined with Eq. (14), can be used to propagate the in mode $P_{j\omega,e}$ to future infinity. Indeed, each monochromatic component in Eq. (18) tunnels through the potential barrier surrounding the black hole, by carrying the amplitude $\vec{B}_{j,e}(\omega')$ to infinity [see Eq. (14)]. In this way we find the mode at future infinity to be (we omitted the spherical harmonics on the right-hand side for simplicity)

$$P_{j\omega,e}\simeq\frac{\sqrt{C}}{\sqrt{4\pi}}\int_\mu^\infty e^{-i\omega' t+ip'r^*}J_{-1}(2\sqrt{Cp(\omega'-\sigma e)})\vec{B}_{j,e}(\omega')\frac{\theta(\omega'-\sigma e)d\omega'}{\sqrt{(\omega'-\sigma e)}}+\frac{\sqrt{C}}{\sqrt{4\pi}}\int_\mu^\infty e^{i\omega' t-ip'r^*}J_{-1}(2\sqrt{Cp(-\sigma e-\omega')})\vec{B}^*_{j,-e}(\omega')\frac{\theta(-\sigma e-\omega')d\omega'}{\sqrt{(-\sigma e-\omega')}}, \tag{19}$$

and we can see the presence of a negative frequency component, provided $\sigma e<0$. Similarly, a negative frequency in mode will have a positive frequency component at infinity if $\sigma e>0$. By following the usual procedure [1,25], the Bogoljubov coefficient responsible for particle creation can be determined easily from Eq. (19). We find

$$\beta^{(+)}_{j\omega'\omega,e}=-\frac{\sqrt{Cp'}}{\sqrt{\sigma e-\omega'}}J_{-1}(2\sqrt{Cp(\sigma e-\omega')})\vec{B}^*_{j,e}(\omega')\,\theta(\sigma e-\omega') =\frac{\sqrt{C(\sigma e-\omega')}}{\sqrt{p'}}J_{-1}(2\sqrt{Cp(\sigma e-\omega')})\overleftarrow{B}^*_{j,e}(\omega')\times\theta(\sigma e-\omega') \tag{20}$$

by using Eq. (15). We now see that the same result could have been obtained by working in the extended Reissner-Nordström solution and by taking the modes emerging from the past horizon to be positive frequency with respect to $u=t-r^*$, as the holomorphy argument applies exactly in the same way. The Bogoljubov coefficients are then as in Eq. (20), except that the Bessel function gets replaced by $\delta(\omega+\omega'-\sigma e)$. In this "Unruh-like picture" there is no redshift, only superradiance as the basis of the emission.

Among the consequences of Eq. (20), the following emerge. Because σ is the sign of Q, only charges having the sign of Q will appear in the outer region and the black hole will discharge. These are the particles associated with incoming modes which started at past infinity at advanced time v for which $v-v_0$ was small and negative (see Fig. 1). There are also particles associated with modes for which $v-v_0$ was small and positive. These are created in the wormhole region and will escape in the top region of the figure by crossing the past horizon of that region. To the author it is not obvious how to define an $|\mathrm{in}\rangle$ vacuum state in the top region. Presumably an observer at past infinity in this region would require that no particles be crossing the past horizon, and then a flux of particles will be present at future infinity, with the sign of the black hole charge. Thus it would be of great concern for him to observe an even larger flux than expected (due to particles that crossed the wormhole). Instead, there is little doubt as to how defining the $|\mathrm{out}\rangle$ vacuum state in the top region as $\mathcal{I}^+$ alone, for example, is a

Cauchy surface for massless fields. The Bogoljubov coefficient of the partner particles can be found by the same type of arguments as used above, i.e., by expressing the phase of the mode after reflection through the shell in terms of the retarded time u of the wormhole region, which provides a past extension of the top universe where the shell is expanding. It is given once more by Eq. (20), the basic reason being that the coordinate t retains its timelike character in the wormhole (in Schwarzschild geometry it is r that enjoys the timelike character).

Again from Eq. (20), there are no charges of opposite sign in the outgoing modes, but as charge conservation requires them, these particles must be created too. The detailed fate of such particles is less clear, because the Klein-Gordon equation possibly admits bound or quasibound states around the black hole, in the potential well due to the Coulomb binding energy. If resonances indeed exist, they should be included among a complete set of horizon states which are needed to describe the ingoing particles. The continuum states can be taken to have a time dependence of the form $\exp(\pm i\omega v)$ on the horizon. They are then spread over the entire future horizon, while the resonances or the bound states will decay exponentially at infinity. The Bogoljubov coefficient of the ingoing particles is then determined by

$$\beta^{(-)}_{j\omega'\omega,e}=\frac{\sqrt{(\sigma e-\omega')}}{\sqrt{p'}}\,\delta(\omega+\omega'-\sigma e)\overleftarrow{B}_{j,e}(\omega')\,\theta(\sigma e-\omega'), \tag{21}$$

where now $j=(l,-m)$. Notice that in the Schwarzschild solution only ingoing modes for which $|v-v_0|$ was small were actually important. In the present case the entire future horizon is involved in particle creation, as the electric field is everywhere constant over there. To fix ideas let us suppose $Q>0$. The calculation shows that to each positive charge emitted to infinity with quantum numbers (l,m) a negatively charged particle having $(l,-m)$ falls into the black hole. The particles may then be captured in the quasibound states by forming a negatively charged cloud around the black hole (but this we cannot prove at present even if resonances exist) or be emitted into ingoing modes and crossing then the horizon, but none of them escape to infinity. This is similar to certain effects which are predicted to arise close to very heavy nuclei. In the field of overcritical nuclei with atomic number Z greater than some critical Z_c (for a point nuclei is $Z_c\simeq 137$) a pion condensate is expected from a proper quantum field theory calculation [30], leading to the concept of a charged vacuum. For the black hole it is Q that is the relevant parameter and overcriticality is achieved roughly for $|Q|>e/\mu^2$, which means that the Coulomb field at the horizon exceed the critical field for pair production.

From Eq. (20) we also obtain the equation

$$\int_0^\infty \beta^{(+)*}_{j\omega'\omega,e}\beta^{(+)}_{j\omega''\omega,e}dp=-[1-|\overleftarrow{A}_{j,e}(\omega')|^2]\times\theta(\sigma e-\omega')\,\delta(\omega'-\omega'') \tag{22}$$

by using Eq. (16) and the completeness relation for Bessel functions [replacing the $(+)$ with the $(-)$ and e with $-e$, it holds for the ingoing particles too]. The expected number of particles at infinity is given by Eq. (22) for $\omega'=\omega''$, and therefore it is infinite, being proportional to $\delta(0)$. This reflects the fact that there is a steady rate of emission lasting an infinite time, as judged from infinity. To see this one writes $\delta(0)=T/2\pi$, for a large time T, as follows from

$$\delta(\omega-\omega')\simeq\frac{1}{2\pi}\int_{-T/2}^{T/2}e^{i(\omega-\omega')t}dt, \tag{23}$$

with increasing accuracy the larger is T. Hence Eq. (22) really gives the expected number of particles emitted during a large interval of time. Dividing by T gives the luminosity of the black hole as

$$\begin{aligned}L&=-\frac{1}{2\pi}\sum_j\int_\mu^{\sigma e}[1-|\overleftarrow{A}_{j,e}(\omega)|^2]\omega d\omega\\&=-\frac{1}{2\pi}\sum_{l\geqslant 0}(2l+1)\int_\mu^{\sigma e}[1-|\overleftarrow{A}_{l,e}(\omega)|^2]\omega d\omega,\end{aligned} \tag{24}$$

since $|\overleftarrow{A}_{j,e}(\omega)|$ does not depend on the azimuth quantum number m (the centrifugal barrier depends on l but not on m). One may note that $L>0$ because the absorbitivity of the black hole, $\Gamma_l(\omega)=1-|\overleftarrow{A}_{l,e}(\omega)|^2$, is negative in the superradiance region, and also that the range of integration in L is very large for all known elementary particles, as for them $e/\mu\gg 1$. Similarly, one gets the rate of charge loss,

$$\dot{Q}=\frac{e}{2\pi}\sum_{l\geqslant 0}(2l+1)\int_\mu^{\sigma e}[1-|\overleftarrow{A}_{l,e}(\omega)|^2]d\omega, \tag{25}$$

which may be compared with [22]. One expects that only Fourier components of the field having wavelength smaller than M interact significantly with the black hole; i.e., only frequencies greater than M^{-1} will be important. If the stronger condition $\omega\gg l(l+1)/M$ holds and also $\mu^{-1}\ll M$, then the absorbtivity of the black hole may be computed in the WKB approximation [22], which gives

$$\Gamma_j(\omega)=-\exp\left(-\frac{\pi\mu^2M^2}{eQ}\right),\quad |Q|=M, \tag{26}$$

and the rate of charge loss is the Schwinger result

$$\dot{Q}\simeq-\frac{e^4Q^3}{M}\exp\left(-\frac{\pi\mu^2M^2}{eQ}\right). \tag{27}$$

The expected number of particles in the outgoing wave packet Q_i, with $i=(l,m,\omega)$, carrying angular momentum $j=(l,m)$ and sharply peaked around ω, is

$$\langle\mathrm{in}|N_i|\mathrm{in}\rangle=n_i=-\Gamma_j(\omega)=|\overleftarrow{A}_{l,e}(\omega)|^2-1, \tag{28}$$

where N_i is the number operator for that mode. Knowing the Bogoljubov coefficients, the higher moments can likewise be computed. For example, a simple calculation gives

$$\langle\mathrm{in}|(N_i)^2|\mathrm{in}\rangle=n_i+2n_i^2, \tag{29}$$

and this can be used to find $\langle \text{in}|(N_i)^k|\text{in}\rangle$ by induction over k. They are all derivable from the following probability distribution for N particles in the given mode (the moments determine the distribution):

$$P(N_i)=\frac{n_i^N}{(1+n_i)^{N+1}}. \tag{30}$$

It is perhaps remarkable that this exhibits the same relationship among probability and mean particle numbers as in pair creation by a pure electric field in flat space [31,32].

From the above derivation it is clear that the emission vanishes not only for neutral particles, but for charged also if $|e|\leqslant\mu$, which again is consistent with cosmic censorship. The results are consistent if the back reaction of the created particles on the shell is negligible, which will be the case if the typical energy of the quanta is much smaller than the black hole mass. This then requires $\sigma e \ll M$, which also implies $\mu \ll M$ (for the existing charged particles).

A similar analysis can be done for an extreme, rotating shell. This time we do not have at our disposal exact equations since the exterior geometry is not given by the Kerr metric all the time (a solution which is valid to first order in the angular velocity of the shell but to all orders for the mass and radial velocity has been worked out in [33]). Hence one assumes that the Kerr geometry settles asymptotically, when the event horizon is about to form. It is a result in the theory of gravitational collapse that asymptotically $\dot\phi \simeq \Omega \dot T$, where Ω is the stationary value of the angular velocity of the horizon and $\dot\phi$ the angular velocity of the shell (this can be seen explicitly in [33]). From the timelike condition on the four-velocity of the shell, $g_{ab}u^a u^b=-1$, and from the Kerr metric, it then follows that

$$\dot T=\frac{2M^2}{(R-M)^2}\sqrt{\frac{(R-M)^2}{M^2(1+\cos^2\theta)}+\dot R^2}. \tag{31}$$

Therefore, since $\dot R$ converges to a finite value on the horizon, one gets

$$\dot R=a_0+a_1\varepsilon+O(\varepsilon^2),$$

$$T=b_{-2}\varepsilon^{-2}+b_{-1}\varepsilon^{-1}+b_0+O(\varepsilon), \tag{32}$$

with θ-independent leading coefficients and $\varepsilon=R-M$. This is the behavior that occurred in the Reissner-Nordström case, and accordingly it yields a quantum emission with luminosity (see [34] for nonextreme black holes)

$$L=-\frac{1}{2\pi}\sum_{lm}\int_{\mu}^{m\Omega}[1-|\overleftarrow{A}_{lm,e}(\omega)|^2]\omega d\omega, \tag{33}$$

where $\Omega=(2M)^{-1}$ is the angular velocity of the horizon for the extreme Kerr metric. Hence there is no emission of s-wave radiation, a characteristic feature of superradiance phenomena.

IV. FERMIONS

The calculation with fermions is more complicated but the basic principle is the same as for the scalar case. Namely, the phase of an ultrarelativistic ingoing mode near the event horizon after reflection through the shell will be the same as for the scalar field. However, the amplitude will be different because it must be propagated along the radially ingoing null geodesics by parallel transport, in the geometric optics approximation. Precisely we have the following. Let $P^{(+)}_{\lambda\omega}$ be a positive frequency solution of the charged Dirac equation

$$i\gamma^a(\nabla_a+ieA_a)P^{(+)}_{\lambda\omega}-\mu P^{(+)}_{\lambda\omega}=0 \tag{34}$$

in the collapse geometry of the infalling shell, which is ingoing from past infinity and normalized to a δ function of ω, and therefore

$$P^{(+)}_{\lambda\omega}=(2\pi p)^{-1/2}r^{-1}N^{-1/2}\begin{pmatrix} f_1(r)\chi_\lambda \\ f_2(r)\phi_\lambda\end{pmatrix}e^{-i\omega t},$$

$$\omega=\sqrt{p^2+\mu^2}, \tag{35}$$

where $p\geqslant 0$ is the radial momentum, $N=\sqrt{-g_{00}}$, and the two-spinors χ_λ and ϕ_λ are eigenspinors of the Dirac operator on the two-sphere corresponding to the eigenvalue λ (see below). Then, asymptotically for large u, for $\omega\gg\mu$, and for $r\to M$ just outside the shell, the outgoing part of $P^{(+)}_{\lambda\omega}$ after reflection through the shell is

$$P^{(+)}_{\lambda\omega}\simeq(2\pi p)^{-1/2}r^{-1}Du^{-1/2}\begin{pmatrix}\chi_\lambda\\ \phi_\lambda\end{pmatrix}$$

$$\times\exp(iCpu^{-1})e^{-i\sigma et-i\omega v_1+i\alpha}, \tag{36}$$

where C, D, and α are constants with $C>0$. The constant D can be fixed by requiring the normal component of the Dirac current to be continuous across the shell [35]. More directly, it can be found by requiring the Bogoljubov coefficients below to represent a unitary transformation between initial and final states. As for the scalar case we may now conclude that there is *no emission of neutral fermions (neutrinos) from an extreme black hole*. For charged fermions things are different but, this time, not as a consequence of superradiance since there is not, in fact, superradiance for fermions. To investigate this we need the asymptotic behavior of spinors near the horizon and infinity. In the Schwarzschild background, the Dirac equation has been given a complete treatment in [35,24]. The neutrino equations in the Kerr geometry were solved and quantized in [34], where the absence of superradiance for fermions was noted. A lot of material in this case can also be found in the Chandrasekhar monograph [36], where the theory of separability of the Dirac equation is discussed at length. In the Reissner-Nordström geometry, the Dirac equation can likewise be separated. In the standard representation of Dirac matrices, for a positive frequency solution of the form (35), one obtains for the radial functions the two coupled equations

$$iN^2f_1'+i\frac{\lambda N}{r}f_1+[(\omega-eA_0)+N\mu]f_2=0, \tag{37}$$

$$iN^2f_2'-i\frac{\lambda N}{r}f_2+[(\omega-eA_0)-N\mu]f_1=0, \tag{38}$$

where a prime denotes a derivative with respect to r, and the two-component spinors χ_λ and ϕ_λ are regular solutions of the equations

$$\left[i\sigma^2\left(\partial_\theta+\frac{1}{2}\cot\theta\right)+\frac{i}{\sin\theta}\sigma^3\partial_\phi\right]\chi_\lambda=i\lambda\phi_\lambda, \quad \sigma^1\chi_\lambda=\phi_\lambda, \tag{39}$$

which implies also

$$\left[i\sigma^2\left(\partial_\theta+\frac{1}{2}\cot\theta\right)+\frac{i}{\sin\theta}\sigma^3\partial_\phi\right]\chi_\lambda=-i\lambda\chi_\lambda, \tag{40}$$

where σ^a are the standard Pauli matrices. The eigenvalue λ takes then all nonzero integer values and there are $4|\lambda|$ states for each given λ [37,38]. From the radial equations, the positive frequency outgoing modes from the past horizon are defined by the asymptotic behavior (we are neglecting the logarithmic phase due to the Coulomb field as is unnecessary to our aims)

$$\vec{f}_1\simeq[e^{i(\omega-\sigma e)r^*}+\vec{A}_\lambda(\omega)e^{-i(\omega-\sigma e)r^*}], \quad r^*\to-\infty, \tag{41}$$

$$\vec{f}_2\simeq[e^{i(\omega-\sigma e)r^*}-\vec{A}_\lambda(\omega)e^{-i(\omega-\sigma e)r^*}], \quad r^*\to-\infty, \tag{42}$$

$$\vec{f}_1\simeq\sqrt{\frac{\omega+\mu}{p}}\vec{B}_\lambda(\omega)e^{ipr^*}, \quad r^*\to\infty, \tag{43}$$

$$\vec{f}_2\simeq\sqrt{\frac{\omega-\mu}{p}}\vec{B}_\lambda(\omega)e^{ipr^*}, \quad r^*\to\infty, \tag{44}$$

where a normalization to a δ function of ω is understood. For two solutions of the radial equations, (f_1,f_2) and (g_1,g_2), the quantity $W=f_1^*g_2+f_2^*g_1$ is a constant. This then implies the unitarity equations

$$|\vec{B}_\lambda(\omega)|^2=1-|\vec{A}_\lambda(\omega)|^2, \tag{45}$$

showing that no superradiance exists for fermions. Similarly, one defines an ingoing solution with corresponding transmission and reflection amplitudes $\overleftarrow{B}_\lambda(\omega)$, $\overleftarrow{A}_\lambda(\omega)$ and obtains the relation $\overleftarrow{B}_\lambda(\omega)=-\vec{B}_\lambda(\omega)$.

To obtain the particle content at future infinity, we have to expand the mode in Eq. (36) into outgoing solutions, whose behavior near the horizon just outside the shell is like $u^{-1/2}\exp[-i(\omega-\sigma e)u-i\sigma et]$. This is because $N\simeq u^{-1/2}$ in this region. Using then Eq. (17) for $\nu=0$, we find the mode at future infinity to be

$$\begin{aligned}P^{(+)}_{\lambda\omega}\simeq{}& i\int_\mu^\infty\frac{\sqrt{C}}{\sqrt{2\pi p'}}\begin{pmatrix}\chi_\lambda\sqrt{\omega'+\mu}\\ \phi_\lambda\sqrt{\omega'-\mu}\end{pmatrix}e^{-i\omega' t+ip'r^*}J_0\\ &\times(2\sqrt{Cp(\omega'-\sigma e)})\vec{B}_\lambda(\omega')\theta(\omega'-\sigma e)d\omega'\\ &-i\int_\mu^\infty\frac{\sqrt{C}}{\sqrt{2\pi p'}}\begin{pmatrix}\chi_\lambda\sqrt{\omega'-\mu}\\ \phi_\lambda\sqrt{\omega'+\mu}\end{pmatrix}e^{i\omega' t-ip'r^*}J_0\\ &\times(2\sqrt{Cp(-\sigma e-\omega')})\vec{B}^*_\lambda(\omega')\theta(-\sigma e-\omega')d\omega',\end{aligned} \tag{46}$$

and we can see the presence of a negative frequency component if $\sigma e<0$. A similar formula can be written for a negative frequency ingoing mode $P^{(-)}_{\lambda\omega}$, and then a positive frequency component will be present if $\sigma e>0$. The Bogoljubov coefficients describing particle creation are defined by the expansion of the outgoing solutions defining the out-vacuum state, here denoted by $Q^{(\pm)}_{\lambda\omega}$, in terms of the ingoing solutions which define the initial vacuum state at past infinity: i.e.,

$$Q^{(\pm)}_{\lambda\omega}=\int_\mu^\infty[\alpha^{(\pm)}_{\lambda\omega\omega'}P^{(\pm)}_{\lambda\omega'}+\beta^{(\pm)}_{\lambda\omega\omega'}P^{(\mp)}_{\lambda\omega'}]d\omega', \tag{47}$$

the $+$ $(-)$ standing for positive (negative) frequency. The asymptotic behavior of the functions $Q^{(\pm)}_{\lambda\omega}$ at future infinity is

$$Q^{(\pm)}_{\lambda\omega}\simeq(2\pi p)^{-1/2}r^{-1}\begin{pmatrix}\chi_\lambda\sqrt{\omega\pm\mu}\\ \phi_\lambda\sqrt{\omega\mp\mu}\end{pmatrix}e^{\mp i(\omega' t-ip'r^*)}. \tag{48}$$

We then obtain, for large ω,

$$\beta^{(+)}_{\lambda\omega'\omega}=i\sqrt{C}J_0(2\sqrt{Cp(\sigma e-\omega')})\vec{B}^*_\lambda(\omega')\theta(\sigma e-\omega'), \tag{49}$$

$$\beta^{(-)}_{\lambda\omega'\omega}=i\sqrt{C}J_0(2\sqrt{Cp(-\sigma e-\omega')})\vec{B}_\lambda(\omega')\theta(-\sigma e-\omega'), \tag{50}$$

from which we get also

$$\begin{aligned}\int_0^\infty\beta^{(\pm)*}_{\lambda\omega'\omega}\beta^{(\pm)}_{\lambda\omega''\omega}dp={}&[1-|\vec{A}_\lambda(\omega')|^2]\theta(\pm\sigma e-\omega')\\ &\times\delta(\omega'-\omega'').\end{aligned} \tag{51}$$

The conclusions that can be drawn from these results follow the same pattern as for bosons. To fix ideas we take $Q>0$: A flux of positron radiation is then emitted to infinity. Each positron may be regarded as one member of a pair, the other member being emitted just inside the future horizon. As there are no electrons in the outgoing modes, they are mostly created and captured near the shell. The electron pairs may then occupy the bound states or the resonances possibly admitted by the Dirac equation around the black hole (in the Schwarzschild case there exist indeed resonance states [39]), or be captured in the shell by crossing the event horizon in an ingoing mode. This is only a pictorial description, of course, as we have not chosen a complete set of horizon states to describe mathematically the ingoing particles (this could be done along the lines of [25]). Moreover, there is not a unique set available, and so the physics near the horizon is better described by stress tensor considerations. But it seems clear that complicated and interesting effects can arise near the black hole and ones which require much more detailed calculations.

Apart from the horizon, the picture is quite similar to the phenomenon of positron creation by heavy nuclei. When the charge number $Z>137$ (for a pointlike nucleus), the electric field at the first Bohr radius exceeds the critical field for pair creation. If the lower energy levels around the nucleus are empty, then a flux of positrons, created in pairs, is emitted, escaping to infinity while the electrons fill the available energy levels [40,30]. The analogous condition for the black hole is $M \leqslant e/\mu^2$, which can be a very large mass if μ is the electron mass (see discussion below). However, in the gravitational case a flux is present even if there are not bound states.

From Eq. (51) the luminosity can likewise be determined. As there are $4|\lambda|$ states available to each λ, and $\vec{A}_\lambda(\omega)=\vec{A}_{-\lambda}(\omega)$, the luminosity of the black hole will be

$$L=\frac{4}{\pi}\sum_{\lambda>0}\lambda\int_\mu^{\sigma e}[1-|\vec{A}_\lambda(\omega)|^2]\omega d\omega, \tag{52}$$

and the rate of charge loss is

$$\dot{Q}=-\frac{4e}{\pi}\sum_{\lambda>0}\lambda\int_\mu^{\sigma e}[1-|\vec{A}_\lambda(\omega)|^2]d\omega. \tag{53}$$

Both are what one would obtain from the Hawking radiation formula by taking the surface gravity approaching zero. For neutrinos, both $L=\dot{Q}=0$, which is again what one would expect from the emission of nonextreme black holes in the same limit. The probability to have zero or one fermion at infinity in a wave packet Q_i, with $i=(\lambda,\omega)$, sharply peaked around ω, can likewise be computed. These are, respectively, $P_0=1-n_i$ and $P_1=n_i$, where $n_i=\Gamma_{\lambda\omega}=1-|\vec{A}_\lambda(\omega)|^2$ is the mean number of fermions in the given mode (it is also the escape probability of a fermion from near the shell to reach infinity). This can be loosely interpreted in terms of a degenerate Fermi gas near the horizon with $\sigma e>0$ in the role of Fermi energy, all states outside this range being empty.

V. DISCUSSION

We have shown that extreme charged black holes formed by collapse radiate steadily charged particles for which $e>\mu$, the bosons being emitted in the superradiance region $\omega-\sigma e<0$, and likewise the fermions. For both fermions and bosons, the emission is dominated by a Schwinger process of pair creation in a Coulomb field, enhanced by the shell because as it collapses the volume filled by the electric field increases. The critical field for pair creation to be significant is of order μ^2/e, and the electric field of the shell near the horizon is $Q/M^2=\pm M^{-1}$. Therefore there is a smallest mass $M_c=e/\mu^2$ needed to have significant emission, in the sense that black holes with larger masses are effectively stable to emitting charged particles. As the lightest charged particles are the electron-positron system, this gives a critical $M_c\simeq 10^{38}g=10^5 M_\odot$. Note that the above statements have an invariant meaning as the electric field intensity is the square root of the invariant $F_{ab}F^{ab}/2$.

However, there is no emission of uncharged particles of either statistics. Therefore, the temperature of such black holes must be zero and extreme black holes with larger masses than M_c can exist as effectively stable objects (assuming that the lightest charged boson is the pion). Moreover, the particles in the superradiance modes are emitted in pure states which is still another reason for the temperature to vanish. This is what we know about temperature and we come now to what we believe about entropy. For this we have not such a sharp conclusion since we have not followed the evolution of the in vacuum with the required details (which also depends on the fate of the shell). As at future infinity one can observe only particles of one type (positively charged if $\sigma>0$, negatively charged if $\sigma<0$) with probability $P(N_i)$ given by Eq. (30), the radiation can be described by the density matrix (one can easily shows that these particles are uncorrelated)

$$\rho=\sum_{N_1,\ldots,N_j}P(N_1,\ldots,N_j)|N_1,\ldots,N_j\rangle\langle N_1,\ldots,N_j|, \tag{54}$$

where the sum is over all many-particles states with N_1 particles in a mode $Q_1,\ldots,N_j$ particles in a mode Q_j and

$$P(N_1,\ldots,N_j)=\prod_k P(N_k). \tag{55}$$

As $\mathrm{Tr}\rho^2<1$, this is a mixed state and one can pose the problem: Is the density matrix description fundamentally related to the formation of a black hole? In flat space the entropy associated with ρ is the von Neumann entropy arising from the fact that only particles of one type are chosen to be detected, and in fact the final state in the Schwinger process is a pure state [32]. Hence the density matrix description is not a fundamental one in this case (that a subsystem of a system in a pure state is described by a density matrix is a well-known discovery due to Landau and von Neumann; see [41]). For the collapsing shell, however, the case may be that the emission is unable to turn the bounce into a collapse. Then a second asymptotically flat region forms, a white hole indeed, which is connected to the first by a traverse timelike wormhole (the region bounded by the two horizons and the shell in Fig. 1), and the oppositely charged particles captured by the shell are effectively lost to outside observers. However, the entropy associated with ρ is not related to that loss as it increases with time while the ρ entropy is a stationary quantity—i.e., we do not have an entropy rate—with a corresponding entropy loss from the black hole as in the nonextreme case. Thus the ρ description does not seem fundamentally related to the formation of a black hole. The entanglement entropy [42–44] of the $|\mathrm{in}\rangle$ state associated with any Cauchy surface intersecting the horizon must also be zero because the horizon is at infinite proper distance along spacelike directions (this has not been verified though).

In the top region, a flux of particles seems to emerge from the past event horizon. These are the partner particles of those considered up to now, created just inside the future horizon (see also [24] for a noncharged shell). This radiation is also described by a density matrix and will have a corresponding entropy. We noted above that for an observer in the top region this radiation would be perhaps surprising as it seems to emerge from the past horizon of the shell (now expanding). In fact, the top region can be interpreted as an extreme white hole. Thus, if a wormhole really forms, it is unclear how to define the final state (from the viewpoint of scattering theory, this should be the one at future infinity in the top region in Fig. 1). For an observer in the top region it is hardly possible to measure correlations that had been created before the wormhole formed, and a mixed final state is a real possibility. In this case, one has a loss of quantum coherence in the past, because part of a pure state failed to traverse the wormhole made by charges, but the radiation entropy has nothing to do with the entropy of a black hole, at least if this is interpreted as the number of different configurations that a black hole can have for given values of its macroscopic parameters.

It must be said that the above discussion really ignored the back reaction of the emission on the shell. As a matter of fact, the emission of charged particles with $e > \mu$ necessarily evolves the body toward nonextreme states. There is then another likely outcome. The shell may bounce by forming a timelike wormhole with a hidden singularity and expand in another asymptotically flat region, because this is a possible solution of Einstein field equations (Fig. 2). The partner particles then end up in the singularity, the horizon is pushed at finite spacelike distances, and there is the usual geometric entropy of black holes.

The issue of extreme black hole entropy loses its meaning in the present context; i.e., it makes sense to say that the black hole has no entropy only over periods where we may neglect the charge and mass loss. Thus, if by careful operations one prepares a pure state containing an extreme charged shell and let it collapse, the future evolution is likely to be similar to that of nonextreme configurations, having nonzero entropy.

However, one can devise processes (like black hole evaporation) that can yield a state arbitrarily close to the extreme state [45], and the entropy of such states should be nonzero by continuity (but by the third law of black hole mechanics [46,47] the extreme state may be physically inaccessible). Arguments have also been proposed according to which the extreme state may have an entropy proportional to its mass [48]. On the other side, for eternal or stable (over long periods) extreme black holes, the Euclidean theory predicts zero entropy [2,49] (but at the same time, the thermodynamic description seems to break down in the case of extreme black holes [50]). This includes those with mass greater than M_c or magnetically charged black holes [45]. Then it seems that we have systems with the same macroscopic parameters (mass and charge or magnetic charge) and several possible values for the entropy, as may happen for certain systems with broken symmetries and a degenerate ground state (e.g., the realization of a charged vacuum). The possibility of stable extreme black holes is thus a very interesting fact. They can link white-to-black holes and their wormhole character can in principle activate processes whereby information is lost, in agreement with the view expressed in [8]. But if the mass required for stability is, as we argued, really greater than M_c, then the question may be only a matter of principle.

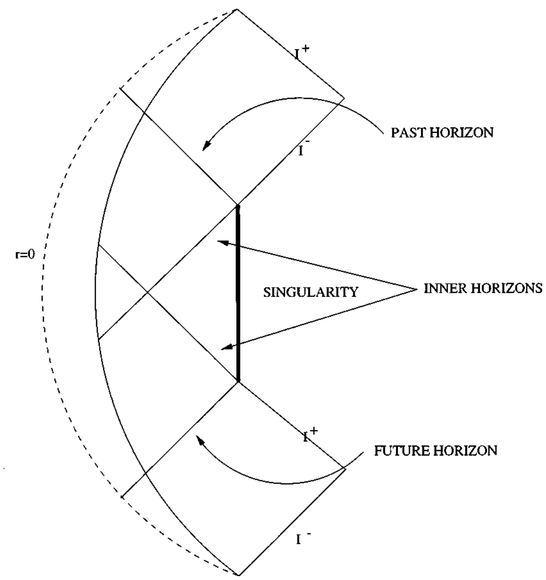

FIG. 2. Penrose's diagram of a bouncing nonextreme shell. To the left of the shell history is part of the maximally extended Reissner-Nordström geometry.

As our last point, since e/μ is much greater than 1 for all known elementary particles (about 10^{21} for electrons), the instability caused by the quantum emission drives the black hole away from the possibility to form a naked singularity, which may be regarded as cosmic censorship at work. On the other hand, the nonextreme charged shell almost invariably forms a singularity and there is both thermal emission and geometric entropy. The entropy problem may thus be connected with the formation of singularities, rather than with the formation of horizons, in accordance with certain ideas of Penrose [51].

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