Two effects in slowly evolving dissipative self-gravitating spheres

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We study the slow evolution of spherically symmetric fluid distributions undergoing dissipation in the form of a radial heat flow. We analyze three examples and show that, depending on the heat flow distribution, different signs in the velocity of fluid elements may appear, giving rise to the splitting of the configuration. It is also shown that whenever the slowly evolving regime is compatible with very strong gravitational fields, the heat flux becomes negative (directed inward), and the fluid will eventually expand. [S0556-2821(97)02004-3]

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I. INTRODUCTION

Dissipation due to the emission of massless particles (photons and/or neutrinos) is a characteristic process in the evolution of massive stars.

In fact, it seems that the only plausible mechanism to carry away the bulk of the binding energy of the collapsing star, leading to a neutron star or black hole is neutrino emission [1].

Usually, it is assumed that the energy flux of radiation (as that of thermal conduction) is proportional to the gradient of temperature. This assumption is in general very sensible, since the mean free path of particles responsible for the propagation of energy in stellar interiors is in general very small as compared with the typical length of the object.

Thus, for a main sequence star such as the sun, the mean free path of photons at the center is of the order of 2 cm. [2]. Also, the mean free path of trapped neurinos in compact cores of densities about 10^{12} g cm^{-3} becomes smaller than the size of the stellar core [3,4].

Furthermore, the observational data collected from supernova 1987A indicates that the regime of radiation transport prevailing during the emission process is closer to the diffusion approximation than to the streaming out limit [5].

In this work we study the slow evolution of spherically symmetric fluid distribution undergoing dissipation in the form of heat flow.

By slow evolution we mean that changes of the system take place on a time scale that is very long compared to the hydrostatic time scale. As a result of this assumption, physical variables are functions of time, but the system may be considered in hydrostatic equilibrium at any moment of its evolution.

The field equations for systems under consideration are given in Sec. II and will be used to examine three examples.

The first one corresponds to the shear-free fluid. It will be shown that in the Newtonian regime, sufficiently large (negative) temperature gradients and/or thermal conductivity might, in principle, lead to the occurrence of positive velocities (expansion) of outer shells and negative velocities (contraction) of the inner shells and this in turn leads to the splitting of the fluid distribution.

This effect, which we call ''thermal peeling,'' is also present in the relativistic regime. However, as we increase further and further the intensity of the field (keeping always the approximation of slow evolution), the gravitational term in the transport equation will eventually prevail and the heat flux becomes negative (directed inward), giving rise to positive velocities for all fluid elements.

These two effects are also exhibited in the two examples examined in Secs. IV and V.

II. FIELD EQUATIONS AND CONVENTIONS

We consider spherically symmetric distributions of collapsing anisotropic fluid, undergoing dissipation in the form of heat flow, bounded by a spherical surface Σ.

The line element is given in Schwarzschild-like coordinates by

$$ds^2=e^{\nu}dt^2-e^{\lambda}dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{1}$$

where $\nu(t,r)$ and $\lambda(t,r)$ are functions of their arguments. We number the coordinates: $x^0=t$; $x^1=r$; $x^2=\theta$; $x^3=\phi$.

The metric (1) has to satisfy Einstein field equations

$$G_\mu^\nu=-8\pi T_\mu^\nu, \tag{2}$$

which in our case read [6]

$$-8\pi T_0^0=-\frac{1}{r^2}+e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right), \tag{3}$$

$$-8\pi T_1^1=-\frac{1}{r^2}+e^{-\lambda}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right), \tag{4}$$

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$$-8\pi T_2^2 = -8\pi T_3^3 = -\frac{e^{-\nu}}{4}[2\ddot{\lambda}+\dot{\lambda}(\dot{\lambda}-\dot{\nu})] + \frac{e^{-\lambda}}{4}\left(2\nu''+\nu'^2-\lambda'\nu'+2\frac{\nu'-\lambda'}{r}\right), \tag{5}$$

$$-8\pi T_{01} = -\frac{\dot{\lambda}}{r}, \tag{6}$$

where dots and primes stand for differentiation with respect to t and r, respectively.

In order to give physical significance to the T_ν^μ components, we apply the Bondi approach [6].

Thus, following Bondi, let us introduce, purely locally, Minkowski coordinates (τ,x,y,z)

$$d\tau = e^{\nu/2}dt, \quad dx = e^{\lambda/2}dr, \quad dy = r d\theta, \quad dz = r\sin\theta d\phi.$$

Then, denoting the Minkowski components of the energy tensor by a bar, we have

$$\overline{T_0^0} = T_0^0, \quad \overline{T_1^1} = T_1^1, \quad \overline{T_2^2} = T_2^2, \quad \overline{T_3^3} = T_3^3,$$

$$\overline{T_{01}} = e^{-(\nu+\lambda)/2}T_{01}.$$

Next, we suppose that when viewed by an observer moving relative to these coordinates with velocity ω in the radial direction, the physical content of space consists of an anisotropic fluid of energy density ρ, radial pressure P_r, tangential pressure $P_\perp$, and radial heat flux $\hat{q}$. Thus, when viewed by this moving observer, the covariant tensor in Minkowski coordinates is

$$\begin{pmatrix} \rho & -\hat{q} & 0 & 0 \\ -\hat{q} & P_r & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\perp \end{pmatrix}.$$

Then, a Lorentz transformation readily shows that

$$T_0^0 = \overline{T_0^0} = \frac{\rho + P_r\omega^2}{1-\omega^2} + \frac{2Q\omega e^{\lambda/2}}{(1-\omega^2)^{1/2}}, \tag{7}$$

$$T_1^1 = \overline{T_1^1} = -\frac{P_r + \rho\omega^2}{1-\omega^2} - \frac{2Q\omega e^{\lambda/2}}{(1-\omega^2)^{1/2}}, \tag{8}$$

$$T_2^2 = T_3^3 = \overline{T_2^2} = \overline{T_3^3} = -P_\perp, \tag{9}$$

$$T_{01} = e^{(\nu+\lambda)/2}\overline{T_{01}} = -\frac{(\rho+P_r)\omega e^{(\nu+\lambda)/2}}{1-\omega^2} - \frac{Qe^{\nu/2}e^{\lambda}}{(1-\omega^2)^{1/2}}(1+\omega^2), \tag{10}$$

with

$$Q \equiv \frac{\hat{q}e^{-\lambda/2}}{(1-\omega^2)^{1/2}}. \tag{11}$$

Note that the velocity in the (t,r,θ,ϕ) system dr/dt is related to ω by

$$\omega = \frac{dr}{dt}e^{(\lambda-\nu)/2}. \tag{12}$$

At the outside of the fluid distribution, the spacetime is that of Vaidya, given by

$$ds^2 = \left(1 - \frac{2M(u)}{R}\right)du^2 + 2dudR - R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{13}$$

where u is a timelike coordinate such that $u=\text{const}$ is (asymptotically) a null cone open to the future and R is a null coordinate $(g_{RR}=0)$.

The two coordinate systems (t,r,θ,ϕ) and (u,R,θ,ϕ) are related at the boundary surface and outside it by

$$u = t - r - 2M\ln\left(\frac{r}{2M} - 1\right), \tag{14}$$

$$R = r. \tag{15}$$

In order to match smoothly the two metrics above on the boundary surface $r = r_\Sigma(t)$, we have to require the continuity of the first fundamental form across that surface.

Then,

$$[e^\nu - e^\lambda \dot{r}_\Sigma^2]_\Sigma dt^2 = \left[1 - \frac{2M}{R_b} + 2\frac{dR_b}{du}\right]_\Sigma du^2, \tag{16}$$

where $R = R_b(u)$ is the equation of the boundary surface in (u,R,θ,ϕ) coordinates.

From Eq. (16), using Eqs. (12), (14), and (15), it follows

$$e^{\nu_\Sigma} = 1 - \frac{2M}{R_b}, \tag{17}$$

$$e^{-\lambda_\Sigma} = 1 - \frac{2M}{R_b}, \tag{18}$$

where, from now on, subscript Σ indicates that the quantity is evaluated at the boundary surface Σ.

Next, the unit vector n_μ, normal to the boundary surface, has components

$$n_\mu^{(+)} = \left(-\beta\frac{dR_b}{du}, \beta, 0, 0\right), \tag{19}$$

where + indicates that the components are evaluated from the outside of Σ, and β is given by

$$\beta = \frac{1}{(1 - 2M(u)/R_b + 2dR_b/du)^{1/2}}. \tag{20}$$

The unit vector normal to Σ, evaluated from the inside, is given by

$$n_{\mu}^{(-)}=(-\dot{r}_{\Sigma}\gamma,\gamma,0,0), \tag{21}$$

with

$$\gamma=\frac{1}{(e^{-\lambda_{\Sigma}}-\dot{r}_{\Sigma}^{2}e^{-\nu_{\Sigma}})^{1/2}}. \tag{22}$$

Let us now define a timelike vector v^{μ} such that

$$v^{\mu(+)}=\beta\delta_{u}^{\mu}+\beta\frac{dR_{b}}{du}\delta_{R}^{\mu} \tag{23}$$

and

$$v^{\mu(-)}=\frac{e^{-\nu_{\Sigma}/2}}{(1-\omega_{\Sigma}^{2})^{1/2}}\delta_{t}^{\mu}+\frac{\omega_{\Sigma}e^{-\lambda_{\Sigma}/2}}{(1-\omega_{\Sigma}^{2})^{1/2}}\delta_{r}^{\mu}. \tag{24}$$

Then, junction conditions across Σ, require [in addition to Eq. (16)]

$$(T_{\mu\nu}n^{\mu}n^{\nu})_{\Sigma}^{(+)}=(T_{\mu\nu}n^{\mu}n^{\nu})_{\Sigma}^{(-)}, \tag{25}$$

$$(T_{\mu\nu}n^{\mu}v^{\nu})_{\Sigma}^{(+)}=(T_{\mu\nu}n^{\mu}v^{\nu})_{\Sigma}^{(-)}, \tag{26}$$

where the expressions for the energy-momentum tensor at both sides of the boundary surface are

$$T_{\mu\nu}^{(-)}=(\rho+P_{\perp})u_{\mu}u_{\nu}-P_{\perp}g_{\mu\nu}+(P_{r}-P_{\perp})s_{\mu}s_{\nu}+q_{\mu}u_{\nu}+q_{\nu}u_{\mu}, \tag{27}$$

and

$$T_{\mu\nu}^{(+)}=-\frac{1}{4\pi R^{2}}\frac{dM}{du}\delta_{\mu}^{0}\delta_{\nu}^{0}, \tag{28}$$

with

$$u^{\mu}=\left(\frac{e^{-\nu/2}}{(1-\omega^{2})^{1/2}},\frac{\omega e^{-\lambda/2}}{(1-\omega^{2})^{1/2}},\ 0,\ 0\right), \tag{29}$$

$$s^{\mu}=\left(\frac{\omega e^{-\nu/2}}{(1-\omega^{2})^{1/2}},\frac{e^{-\lambda/2}}{(1-\omega^{2})^{1/2}},\ 0,\ 0\right), \tag{30}$$

where u^{μ} denotes the four-velocity of the fluid and s^{μ} is a radially directed spacelike vector orthogonal to u^{μ}, and

$$q^{\mu}=Q(\omega e^{(\lambda-\nu)/2},\ 1,\ 0,\ 0). \tag{31}$$

Then it follows, from Eqs. (25) and (26),

$$[P_{r}]_{\Sigma}=-\left[\frac{1}{4\pi R^{2}}\frac{dM}{du}\beta^{2}\right]_{\Sigma}, \tag{32}$$

$$[Qe^{\lambda/2}(1-\omega^{2})^{1/2}]_{\Sigma}=-\left[\frac{1}{4\pi R^{2}}\frac{dM}{du}\beta^{2}\right]_{\Sigma}. \tag{33}$$

Equations (16), (32), and (33) are the necessary and sufficient conditions for a smooth matching of the two metrics (1) and (13) on Σ. Combining Eqs. (32) and (33), we get

$$[P_{r}]_{\Sigma}=[Qe^{\lambda/2}(1-\omega^{2})^{1/2}]_{\Sigma}, \tag{34}$$

expressing the discontinuity of the radial pressure in the presence of heat flow, which is a well-known result [7].

In this work we shall consider exclusively slowly evolving systems. That means that our sphere changes slowly on a time scale that is very long compared to the typical time in which it reacts on a slight perturbation of hydrostatic equilibrium, and this typical time is called hydrostatic time scale. Thus, our system is always in a hydrostatic equilibrium (very close to) and its evolution may be regarded as a sequence of static models linked by Eq. (6).

This assumption is very sensible because the hydrostatic time scale is very small for almost any phase of the life of a star. It is of the order of 27 minutes for the Sun, 4.5 seconds for a white dwarf, and 10^{-4} seconds for a neutron star of one solar mass and 10 Km radius [2].

Let us now express this assumption through conditions for ω and metric functions.

First of all, slow contraction (or expansion) means that the radial velocity ω, measured by the Minkowski observer, as well as time derivatives are so small that their products as well as second time derivatives may be neglected. Thus, we shall assume

$$\ddot{\nu}\approx\ddot{\lambda}\approx\dot{\lambda}\dot{\nu}\approx\dot{\lambda}^{2}\approx\dot{\nu}^{2}\approx\omega^{2}\approx\dot{\omega}=0. \tag{35}$$

Then, it follows from Eqs. (6) and (10) that Q is of order $O(\omega)$, a result that might be deduced from purely physical considerations, since intense emission is not expected to be compatible with slow evolution.

Next, taking r derivatives of Eq. (4) and using Eqs. (3) and (5), we obtain in the approximation of slow evolution

$$P_{r}'+(\rho+P_{r})\frac{\nu'}{2}-2\frac{P_{\perp}-P_{r}}{r}=0, \tag{36}$$

which is the equation of hydrostatic equilibrium for an anisotropic fluid.

Thus, as mentioned before, the system, although evolving, is in a hydrostatic equilibrium [up to order $O(\omega)$]; this allows for a very simple extension of any static solution to the slowly evolving case.

Finally, we calculate the shear tensor in the slow motion approximation. As usual, the components of the shear tensor are given by

$$\sigma_{\mu\nu}=u_{\mu;\nu}+u_{\nu;\mu}-u_{\mu}a_{\nu}-u_{\nu}a_{\mu}-\frac{2}{3}\Theta P_{\mu\nu}, \tag{37}$$

where

$$P_{\mu\nu}=g_{\mu\nu}-u_{\mu}u_{\nu},\quad \Theta=u_{;\mu}^{\mu},\quad a_{\mu}=u^{\nu}u_{\mu;\nu} \tag{38}$$

are, respectively, the projection tensor, the expansion scalar, and the four acceleration.

After tedious but simple calculations, we get (always in the slow motion approximation)

$$\Theta=\omega' e^{-\lambda/2}+\frac{\dot{\lambda}e^{-\nu/2}}{2}+\frac{2\omega e^{-\lambda/2}}{r}+\frac{\nu'\omega e^{-\lambda/2}}{2}, \tag{39}$$

$$\sigma_{11}=-\frac{2\sigma_{22}}{r^2}e^{\lambda}=-\frac{2\sigma_{33}}{r^2\sin^2\theta}e^{\lambda}$$
$$=-\frac{4}{3}e^{\lambda/2}\left(\omega'-\frac{\omega\lambda'}{2}-\frac{\omega}{r}-4\pi rQe^{3\lambda/2}\right), \qquad (40)$$

where Eqs. (3)–(6) have been used.

We can solve Eq. (40) for ω, to obtain

$$\omega=\omega_\Sigma\left(\frac{r}{r_\Sigma}\right)e^{(\lambda-\lambda_\Sigma)/2}-4\pi re^{\lambda/2}$$
$$\times\int_r^{r_\Sigma}\left(Qe^{\lambda}-\frac{3}{16\pi}e^{-\lambda}\frac{\sigma_{11}}{r}\right)dr. \qquad (41)$$

III. THE SHEAR-FREE CASE

The first model we shall consider corresponds to the shear-free case.

Then, it follows from Eq. (41) that

$$\omega=\left[\frac{\omega_\Sigma}{r_\Sigma}e^{-\lambda_\Sigma/2}-\int_r^{r_\Sigma}4\pi Qe^{\lambda}dr\right]e^{\lambda/2}r. \qquad (42)$$

Now, from the relativistic Maxwell-Fourier law [8,9], we have

$$q^{\mu}=\kappa P^{\mu\nu}(T_{,\nu}-Ta_{\nu}) \qquad (43)$$

or

$$q^1=Q=-\kappa e^{-\lambda}\left(T'+\frac{T\nu'}{2}\right), \qquad (44)$$

where T is the temperature and κ denotes the coefficient of conduction.

Then, feeding back Eq. (44) into Eq. (42) and using Eq. (18), we obtain

$$\omega=\left[\frac{\omega_\Sigma}{r_\Sigma}\left(1-\frac{2M(u)}{r_\Sigma}\right)^{1/2}+4\pi\kappa(T_\Sigma-T)\right.$$
$$\left.+2\pi\kappa\int_r^{r_\Sigma}T\nu' dr\right]e^{\lambda/2}r. \qquad (45)$$

In the Newtonian limit we have $M(u)\approx\lambda\approx\nu\approx 0$ and, therefore, we get the following expression for ω:

$$\omega_{\text{Newt}}=\frac{\omega_\Sigma}{r_\Sigma}r+4\pi\kappa(T_\Sigma-T)r. \qquad (46)$$

Thus, unlike the nondissipative case [10], the shear-free collapse in the Newtonian limit does not yield the linear law of homologous contraction.

Furthermore, the sign of ω for any value of r is not necessarily the same as that of ω_Σ (as is the case in the nondissipative evolution).

In particular, for sufficiently large (negative) gradient of temperature, we may have $\omega_\Sigma>0$ and $\omega<0$.

In other words, the system may be evolving in such a way that inner shells collapse, whereas outer ones expand.

This effect, which we call "thermal peeling," is also present in the relativistic regime, provided the third term in the right-hand side of Eq. (45) is not too large.

Two comments are in order at this point. First, observe that going from geometric to cgs units, one sees that

$$\kappa T\sim 10^{-59}[\kappa][T] \ \text{cm}^{-1},$$

where $[\kappa]$ and $[T]$ denote the numerical values of these quantities as measured in erg s^{-1} cm^{-1} K^{-1} and K, respectively. Therefore, extremely high conductivities and/or $\triangle T$ are required for thermal peeling to be observed in Newtonian regime. However, such high thermal conductivities are associated with highly compact, degenerate objects where Newtonian limit is not reliable. In general, it should be clear that we are only indicating the possibility of thermal peeling to occur. The appearance of such an effect could only be inferred from a specific temperature distribution.

On the other hand, it should be noticed that in Eq. (46) it has been assumed that terms of order $O(M/r_\Sigma)$ and higher are negligible with respect to $\kappa(T_\Sigma-T)$. This, of course, is not always true, as commented above, in which case Eq. (46) is not valid.

If the gravitational field becomes too strong (always keeping the slowly evolving regime), then the third term in Eq. (45) will prevail over the other two. Since it is positive defined, that means that ω, for any value of r, will be positive, i.e., the fluid is expanding.

The same conclusion may be obtained by inspection of Eqs. (42) and (44).

In fact, if the field becomes very strong, then the heat flow becomes negative and so the second term on the right-hand side of Eq. (42) becomes positive. Since $e^{-\lambda_\Sigma/2}$ is small in the limit of strong field, it is clear that ω will be positive for any value of r. Of course, all this will be valid only if the slowly evolving regime may be made compatible with the presence of strong gravitational fields. Also, it could be argued that these results depend on the shear-free condition.

However, in the next two sections we shall work out two examples with shear, which lead to the same conclusions as the shear-free case above. Furthermore, in the example examined in Sec. V, the radius of the configuration may be arbitrarily close to $2M(u)$, without the system leaving the slowly evolving regime.

IV. THE HOMOGENEOUS AND LOCALLY ISOTROPIC SPHERE

Let us now consider a homogeneous ($\rho=\rho(t)$) and locally isotropic ($P_\perp=P_r=P$) fluid sphere undergoing dissipation, in slow contraction.

From the definition of the mass function,

$$m(r,t)=4\pi\int_0^r T_0^0 r^2 dr, \qquad (47)$$

we obtain, for this case (in our approximation),

$$m=\frac{4\pi}{3}\rho r^3 \qquad (48)$$

and, for the total mass,

$$M=\frac{4\pi}{3}\rho r_\Sigma^3. \quad (49)$$

Also, from Eqs. (3) and (36),

$$e^{-\lambda}=1-\frac{2m}{r}, \quad (50)$$

$$e^{\nu/2}=\frac{1}{2}\left[3\left(1-\frac{2M}{r_\Sigma}\right)^{1/2}-\left(1-\frac{2m}{r}\right)^{1/2}\right]. \quad (51)$$

Next, taking time derivative of Eq. (50) and using Eqs. (6) and (12), we find

$$\frac{\dot{m}}{4\pi r^2}=-\rho\frac{dr}{dt}-P\frac{dr}{dt}-Qe^{\nu/2}, \quad (52)$$

or using Eq. (48),

$$\frac{\dot{m}}{4\pi r^2}+\frac{dr}{dt}\frac{m'}{4\pi r^2}=-\frac{dr}{dt}P-Qe^{\nu/2}. \quad (53)$$

Denoting the convective (comoving) time differentiation by D/Dt, we write Eq. (53) as

$$\frac{Dm}{Dt}=-4\pi r^2\left(P\frac{dr}{dt}+Qe^{\nu/2}\right) \quad (54)$$

which shows how the mass inside a particular shell of particles changes with time. The two terms on the right-hand side of Eq. (54) have a simple physical meaning, so we shall not comment on them [6]. The change of the total mass becomes, using Eqs. (17), (34), and (54),

$$\frac{DM}{Dt}=-4\pi r_\Sigma^2 Q_\Sigma\left(1-\frac{2M}{r_\Sigma}\right)^{1/2}+O(\omega^2). \quad (55)$$

On the other hand, we obtain, from Eq. (48),

$$\frac{Dm}{Dt}\equiv\dot{m}+m'\frac{dr}{dt}=\frac{4\pi}{3}\dot{\rho}r^3+4\pi\rho r^2\frac{dr}{dt}. \quad (56)$$

Equating Eqs. (54) and (56), we get

$$\dot{\rho}=-\frac{3}{r}\left[\frac{dr}{dt}(\rho+P)+Qe^{\nu/2}\right]. \quad (57)$$

Or, evaluating at Σ,

$$\dot{\rho}=-\frac{3}{r_\Sigma}[Q_\Sigma e^{\nu_\Sigma/2}+(\rho+P_\Sigma)\dot{r}_\Sigma]. \quad (58)$$

Now, the pressure is obtained from Eq. (36) with condition (34). It is a simple matter to see that

$$P=P(Q=0)+O(\omega), \quad (59)$$

where $P(Q=0)$ is the pressure for the dissipationless case [10].

Since P appears in Eqs. (57) and (58), multiplying terms of order $O(\omega)$, we can neglect the last term on the right-hand side of Eq. (59). Thus

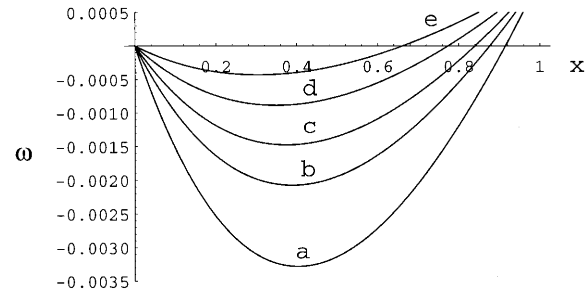

FIG. 1. ω as function of $x\equiv r/r_\Sigma$ for the locally isotropic case $\Omega=10^{-3}$ and different values of surface gravitational potential. Curves a, b, c, d, e correspond to M/r_Σ=1/90, 1/60, 1/45, 1/30, 1/18, respectively.

$$P=\rho\frac{(1-2m/r)^{1/2}-(1-2M/r_\Sigma)^{1/2}}{3(1-2M/r_\Sigma)^{1/2}-(1-2m/r)^{1/2}}. \quad (60)$$

Feeding back Eq. (60) into Eqs. (57) and (58), we obtain, using Eq. (12),

$$\omega=\omega_\Sigma\left(\frac{r}{r_\Sigma}\right)e^{(\lambda-\lambda_\Sigma)/2}+\frac{Q_\Sigma}{r_\Sigma}\frac{r}{\rho}e^{\lambda/2}-\frac{Q}{\rho}e^{\nu/2}e^{(\lambda+\lambda_\Sigma)/2}. \quad (61)$$

Thus, as in the shear-free case, the last term in Eq. (61) will prevail in the case of very intense gravitational field. However, we cannot reach arbitarily strong fields in this model, since already for $2M/r_\Sigma=8/9$ the pressure given by Eq. (60) becomes singular at the center. Obviously, for this critical value of the surface potential, the heat flux does not necessarily become negative and, therefore, it is not necessarily true that, as we approach the critical value 8/9, the velocity of any fluid element becomes positive.

On the other hand, the possibility of thermal peeling for this model is suggested by Eq. (61). In order to exhibit this effect, let us assume the following trial heat function

$$4\pi r^2 Qe^{\nu/2}=\alpha m, \quad (62)$$

where the ''opacity'' factor α is given by

$$\alpha=\frac{\Omega e^{-r/r_\Sigma}}{M} \quad (63)$$

and Ω is a numerical factor of order $O(\omega)$. Although the luminosity implied by Eq. (62) may be too large to accommodate in any physically reasonable scenario, we present it just to illustrate the effect.

Then, Eq. (61) becomes

$$\omega=\left(\frac{r}{r_\Sigma}\right)e^{\lambda/2}\frac{1}{(1-2M/r_\Sigma)^{1/2}}\left[\omega_\Sigma\left(1-\frac{2M}{r_\Sigma}\right)-\frac{r_\Sigma\Omega}{3eM}(e^{1-(r/r_\Sigma)}-1)\right]. \quad (64)$$

Figures 1 and 2 show ω as function of r/r_Σ for different surface potentials. We shall comment on them in the last section.

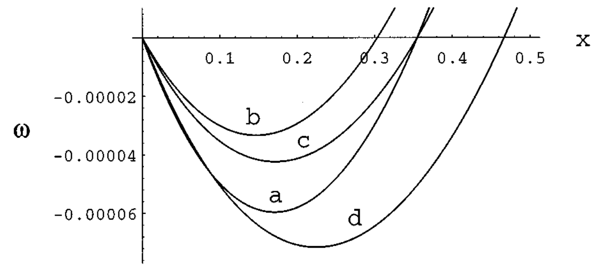

FIG. 2. Same as Fig. 1, but now curves a, b, c, d correspond to M/r_Σ=3/18, 5/18, 6/18, 7/18.

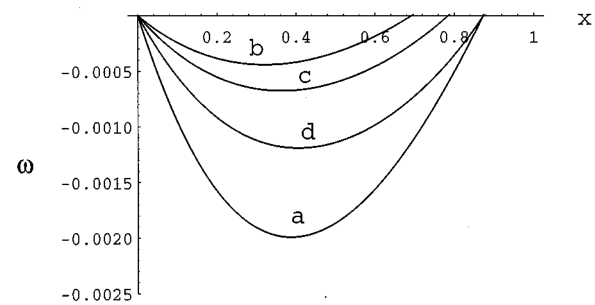

FIG. 3. ω as function of x for the third model ($P_r=0$) and $\Omega=10^{-3}$. Curves a, b, c, d correspond to M/r_Σ=1/20, 6/20, 8/20, 9/20.

V. HOMOGENEOUS SPHERE SUSTAINED ONLY BY TANGENTIAL STRESSES

Let us now consider a homogeneous sphere $[\rho=\rho(t)]$ sustained only by tangential stresses ($P_r=0$) undergoing slow dissipative evolution (the dissipationless version of this model has been studied in [10]).

From Eqs. (3), (4), and (36), it is not difficult to find that

$$e^{\nu}=\frac{(1-2M/r_\Sigma)^{3/2}}{(1-2m/r)^{1/2}}, \tag{65}$$

$$e^{-\lambda}=1-\frac{2m}{r}, \tag{66}$$

$$P_\perp=\frac{2\pi\rho^2r^2}{3\left(1-\frac{8\pi}{3}\rho r^2\right)}, \tag{67}$$

$$m=\frac{4\pi}{3}\rho r^3. \tag{68}$$

In this case, Eqs. (57) and (58) are valid with $P=0$.

Then, we get for ω in this case

$$\omega=\left[\frac{Q_\Sigma}{\rho}e^{\nu_\Sigma/2}\left(\frac{r}{r_\Sigma}\right)-\frac{Q}{\rho}e^{\nu/2}+\omega_\Sigma\left(\frac{r}{r_\Sigma}\right)e^{(\nu_\Sigma-\lambda_\Sigma)/2}\right]e^{(\lambda-\nu)/2}. \tag{69}$$

However, since in this model the radial pressure vanishes everywhere, junction condition (34) implies

$$Q_\Sigma=0. \tag{70}$$

Thus, the heat flow vanishes at the boundary surface and, therefore, the total mass remains constant.

Using Eqs. (65), (66), and (70), we may write, for ω,

$$\omega=\omega_\Sigma\left(\frac{r}{r_\Sigma}\right)\frac{(1-2M/r_\Sigma)^{1/4}}{(1-2m/r)^{1/4}}-\frac{Q}{\rho}\frac{1}{(1-2m/r)^{1/2}}. \tag{71}$$

The important point to stress here is that, in this model, r_Σ may take values arbitrarily close to $2M$, without leaving the slowly evolving regime [11]. On the other hand, since ν' is given by

$$\nu'=\frac{2m}{r^2(1-2m/r)}, \tag{72}$$

it is clear that the "gravitational term" in Eq. (44) will prevail over the temperature gradient, at some stage of the evolution, leading to a negative heat flux.

This, in turn would imply that ω would be positive for any fluid element, since the second term on the right-hand side of Eq. (71) would clearly prevail over the first one, as r_Σ approaches to $2M$.

Just for the illustration of the occurrence of thermal peeling in this model, let us consider the following trial heat function:

$$4\pi r^2Qe^{\nu/2}=\frac{\Omega}{M}(e^{1-(r/r_\Sigma)}-1), \tag{73}$$

where as before, Ω is a numerical factor of order $O(\omega)$.

Then, ω becomes

$$\omega=\frac{(r/r_\Sigma)}{(1-2m/r)^{1/4}(1-2M/r_\Sigma)^{3/4}}\left[\omega_\Sigma\left(1-\frac{2M}{r_\Sigma}\right)+\frac{\Omega}{3(M/r_\Sigma)}(1-e^{1-(r/r_\Sigma)})\right]. \tag{74}$$

Figure 3 exhibits the peelings for different surface gravitational potentials. We shall comment on this in the next section.

VI. CONCLUSIONS

We have seen that, in the regime of slow evolution, dissipation in the form of heat flow may affect drastically the evolution of self-gravitating objects, even though Q is of order $O(\omega)$.

The thermal peeling defined above may be present even in the Newtonian regime.

Figures 1 and 2 give the profiles of ω as function of r/r_Σ for different values of M/r_Σ, for the locally isotropic case. Higher potentials involve more layers in the peeling, until the value 5/18 is reached. For this and higher potentials, the tendency reverses.

The same is true for the third example ($P_r=0$), as shown in Fig. 3

The appearance of negative heat flux due to gravitational term in Eq. (44) may or may not occur, depending on the value of temperature gradients and on how strong the gravitational field can be, without destroying the slowly evolving regime.

In particular, in the locally isotropic model we know that the surface gravitational potential cannot be larger than (or even equal to) 8/9. However, in our last example ($P_r=0$), we know that r_Σ can continuously approach $2M$, without the system leaving the slowly evolving regime. In this case, negative heat flux is unavoidable at some stage of evolution and the fluid will necessarily expand.

In general, all locally isotropic models have more or less stringent restrictions in values of their surface gravitational potentials [12] and, therefore, negative heat fluxes are more likely to appear in anisotropic fluids.

We would like to conclude with the following three observations.

(1) We are aware of the fact that the Maxwell-Fourier type law [Eq.(43)] leads to a parabolic equation for temperature, a fact which is at the origin of pathologies [13] found in Eckart [8] and Landau [9] approaches. However, it is clear that all pertinent relaxation terms appearing in causal dissipative theories [14–17] are of the order $O(\dot{Q})\approx O(\dot{\omega})$ and, therefore, have to be neglected in our approximation.

(2) The two effects mentioned above are predicted under the strict verification of the slowly evolving regime. Whether or not they, or some similar effects, appear in the ''dynamic'' regime (far from hydrostatic equilibrium) remains to be seen.

(3) The appearance of negative heat flux has been already exhibited in some models obtained in the general (''dynamic'') case [18].

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