D^0-$\overline{D^0}$ mixing and CP violation in neutral D-meson decays

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D^0-$\overline{D^0}$ mixing at the detectable level or significant CP violation in the charm system may strongly signify the existence of new physics. In view of the large discovery potential associated with the fixed target experiments, the B-meson factories and the τ-charm factories, we make a further study of the phenomenology of D^0-$\overline{D^0}$ mixing and CP violation in neutral D-meson decays. The generic formulas for the time-dependent and time-integrated decay rates of both coherent and incoherent $D^0\overline{D^0}$ events are derived, and their approximate expressions up to the second order of the mixing parameters x_D and y_D are presented. Explicitly we discuss D^0-$\overline{D^0}$ mixing and various CP-violating signals in neutral D decays to the semileptonic final states, the hadronic CP eigenstates, the hadronic non-CP eigenstates, and the CP-forbidden states. A few nontrivial approaches to the separate determination of x_D and y_D and to the demonstration of direct and indirect CP asymmetries in the charm sector are suggested. [S0556-2821(97)02401-6]

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I. INTRODUCTION

It is well known in particle physics that mixing between a neutral P^0 meson and its CP-conjugate counterpart $\overline{P^0}$ can arise if both of them couple to a subset of virtual and (or) real intermediate states. Such mixing effects provide a mechanism whereby interference in the transition amplitudes of P^0 and $\overline{P^0}$ mesons may occur, leading to the possibility of CP violation. Determining the magnitude of P^0-$\overline{P^0}$ mixing and probing possible CP-violating phenomena in the P^0-$\overline{P^0}$ system have been challenging tasks for particle physicists. To date, K^0-$\overline{K^0}$ and B^0_d-$\overline{B^0_d}$ mixing rates have been measured, and the CP-violating signal induced by K^0-$\overline{K^0}$ mixing has been unambiguously established [1]. Many sophisticated experimental efforts, such as the programs of ϕ factories, B factories, and high-luminosity hadron machines, are being made to discover new signals of CP asymmetries beyond the K^0-$\overline{K^0}$ system and to precisely measure the Kobayashi-Maskawa (KM) matrix elements.

The study of mixing and CP violation in the $Q=+2/3$ quark sector, particularly in the D^0-$\overline{D^0}$ system, is not only complementary to our knowledge of the K^0-$\overline{K^0}$ and B^0-$\overline{B^0}$ systems, but also important for exploring possible new physics that is out of reach of the standard model predictions. The rate of D^0-$\overline{D^0}$ mixing is commonly measured by two well-defined dimensionless parameters x_D and y_D, which correspond to the mass and width differences of D^0 and $\overline{D^0}$ mass eigenstates. The latest E691 data of Fermilab fixed target experiments only give an upper bound on D^0-$\overline{D^0}$ mixing [2]:

$$r_D \approx \frac{x_D^2+y_D^2}{2} < 3.7\times 10^{-3}. \tag{1.1}$$

In the standard model the short-distance contribution to D^0-$\overline{D^0}$ mixing is via box diagrams and its magnitude is expected to be negligibly small ($x_D^{\rm SD}\sim 10^{-5}$ and $y_D^{\rm SD}\leqslant x_D^{\rm SD}$ [3]). The long-distance effect on D^0-$\overline{D^0}$ mixing comes mainly from the real intermediate states of SU(3) multiplets, such as

$$D^0 \leftrightarrow \pi\pi, \pi K, \pi\overline{K}, K\overline{K} \leftrightarrow \overline{D^0}, \tag{1.2}$$

and is possible to be significant if the SU(3) symmetry is badly broken (e.g., $x_D^{\rm LD}\sim y_D^{\rm LD}\sim 10^{-3}$–$10^{-2}$ [4]). However, the dispersive approach [5] and heavy quark effective theory [6] seem to favor a much smaller result for the long-distance contribution: $x_D^{\rm LD}\sim 10\times x_D^{\rm SD}$ and $x_D^{\rm LD}\sim x_D^{\rm SD}$, respectively. Such theoretical discrepancies indicate our poor understanding of the dynamics for D^0-$\overline{D^0}$ mixing; hence, more efforts in both theory and experiments to better constrain the mixing rate are desirable. If calculations based on the standard model can reliably limit x_D and y_D to be well below 10^{-2}, then observation of r_D at the level of 10^{-4} or so will imply the existence of new physics. On the other hand, improved experimental knowledge of r_D, in particular the relative magnitude of x_D and y_D, can definitely clarify the ambiguities in current theoretical estimates and shed some light on both the dynamics of D^0-$\overline{D^0}$ mixing and possible sources of new physics beyond the standard model.

The phenomenology of CP violation in the D^0-$\overline{D^0}$ system was first developed by Bigi and Sanda [7], and further summarized by Bigi in Ref. [8]. These works have outlined the main features of D^0-$\overline{D^0}$ mixing and CP asymmetries anticipated to appear in neutral D-meson decays, although many of their formulas and results are approximate or just for illustrative purposes. The theoretical expectations on the magnitudes of various possible effects are also sketched in Refs. [7,8].

Recent experimental progress, particularly in observing the doubly Cabibbo-suppressed decay (DCSD) $D^0\rightarrow K^+\pi^-$ [9], constraining the D^0-$\overline{D^0}$ mixing rate [1,2] and searching for CP asymmetries in D decays to K^+K^-, etc. [10], are quite encouraging. Further experimental efforts, based mainly on the high-luminosity fixed target facilities [11], the

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forthcoming B-meson factories, and the proposed τ-charm factories [12], are underway to approach the above physical goals. In view of the large discovery potential associated with these experimental programs, a further study of the phenomenology of D^0-$\overline{D^0}$ mixing and CP violation in the charm system is no doubt necessary and important.

In this paper we shall, on the one hand, follow the pioneering work of Bigi and Sanda to refine upon the phenomenology of D^0-$\overline{D^0}$ mixing and CP violation in neutral D decays and, on the other hand, investigate some specific possibilities to separately determine x_D and y_D as well as to probe various CP-violating signals in the charm sector. A generic formulation for the time-dependent and time-integrated decay rates of both coherent and incoherent $D^0\overline{D^0}$ events is derived, and their approximate expressions up to $O(x_D^2)$ and $O(y_D^2)$ are presented. Systematically but explicitly, we discuss a variety of D^0-$\overline{D^0}$ mixing and CP-violating measurables in neutral D decays to the semileptonic final states, the hadronic CP eigenstates, the hadronic non-CP eigenstates, and the CP-forbidden states. We show that it is possible to determine the relative magnitude of x_D and y_D through observation of the dilepton events of coherent $D^0\overline{D^0}$ decays on the $\psi(4.14)$ resonance at a τ-charm factory. A model-independent constraint on D^0-$\overline{D^0}$ mixing can also be obtained by measuring the decay-time distributions of $D^0/\overline{D^0} \to K_{S,L} + \pi^0$, etc. By use of the isospin analysis and current data, we illustrate final-state interactions in $D \to KK$ and their influence on CP violation. The interplay of D^0-$\overline{D^0}$ mixing and DCSD effects in incoherent $D^0\overline{D^0}$ decays to $K^\pm\pi^\mp$ and in coherent $D^0\overline{D^0}$ decays to both $(l^\pm X^\mp, K^\pm\pi^\mp)$ and $(K^\pm\pi^\mp, K^\pm\pi^\mp)$ states is analyzed in the presence of CP violation and final-state interactions. We take a look at two types of CP-forbidden decays at the $\psi(3.77)$ and $\psi(4.14)$ resonances. Finally the possibility to test the $\Delta Q = \Delta C$ rule and CPT symmetry in the D^0-$\overline{D^0}$ system is briefly discussed.

This work is organized as follows. In Sec. II we derive the generic formulas for coherent and incoherent $D^0\overline{D^0}$ decays, and then make some analytical approximations for them. Secs. III, IV, V, and VI are devoted to D^0-$\overline{D^0}$ mixing and CP violation in neutral D decays to the semileptonic states, the hadronic CP eigenstates, the hadronic non-CP eigenstates, and the CP-forbidden states, respectively, where some distinctive approaches or examples are discussed for determining x_D and y_D or probing possible CP-violating effects. We summarize our main results in Sec. VII with some comments on tests of the $\Delta Q = \Delta C$ rule and CPT symmetry.

II. FUNDAMENTAL FORMULAS

We first develop a generic formulation for the time-dependent and time-integrated decays of neutral D mesons. Considering the smallness of D^0-$\overline{D^0}$ mixing indicated by both experimental searches and theoretical estimates, we then make some analytical approximations for the obtained decay rates up to the accuracy of $O(x_D^2)$ and $O(y_D^2)$.

A. Preliminaries

In the assumption of CPT invariance, the mass eigenstates of D^0 and $\overline{D^0}$ mesons can be written as

$$
\begin{aligned}
|D_L\rangle &= p|D^0\rangle + q|\overline{D^0}\rangle, \\
|D_H\rangle &= p|D^0\rangle - q|\overline{D^0}\rangle,
\end{aligned}
\tag{2.1}
$$

in which the subscripts ''L'' and ''H'' stand for light and heavy, respectively, and (p,q) are complex mixing parameters. Sometimes it is more convenient to use the notation

$$
\frac{q}{p} \equiv \left|\frac{q}{p}\right| \exp(i2\phi), \tag{2.2}
$$

where ϕ is a real CP-violating phase in D^0-$\overline{D^0}$ mixing. With the help of the conventions $CP|D^0\rangle = |\overline{D^0}\rangle$ and $CP|\overline{D^0}\rangle = |D^0\rangle$, the relations between the CP eigenstates

$$
|D_1\rangle \equiv \frac{|D^0\rangle + |\overline{D^0}\rangle}{\sqrt{2}}, \quad |D_2\rangle \equiv \frac{|D^0\rangle - |\overline{D^0}\rangle}{\sqrt{2}} \tag{2.3}
$$

and the mass eigenstates $|D_L\rangle$, $|D_H\rangle$ turn out to be

$$
\begin{aligned}
|D_L\rangle &= \frac{p+q}{\sqrt{2}}|D_1\rangle + \frac{p-q}{\sqrt{2}}|D_2\rangle, \\
|D_H\rangle &= \frac{p+q}{\sqrt{2}}|D_2\rangle + \frac{p-q}{\sqrt{2}}|D_1\rangle.
\end{aligned}
\tag{2.4}
$$

The proper-time evolution of an initially $(t=0)$ pure D^0 or $\overline{D^0}$ is given as

$$
\begin{aligned}
|D^0_{\rm phys}(t)\rangle &= g_+(t)|D^0\rangle + \frac{q}{p} g_-(t)|\overline{D^0}\rangle, \\
|\overline{D^0}_{\rm phys}(t)\rangle &= g_+(t)|\overline{D^0}\rangle + \frac{p}{q} g_-(t)|D^0\rangle,
\end{aligned}
\tag{2.5}
$$

where

$$
\begin{aligned}
g_+(t) &= \exp\left[-\left(im + \frac{\Gamma}{2}\right)t\right]\cosh\left[\left(i\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right], \\
g_-(t) &= \exp\left[-\left(im + \frac{\Gamma}{2}\right)t\right]\sinh\left[\left(i\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right],
\end{aligned}
\tag{2.6}
$$

with the definitions

$$
\begin{aligned}
m &\equiv \frac{m_L + m_H}{2}, \quad \Delta m \equiv m_H - m_L, \\
\Gamma &\equiv \frac{\Gamma_L + \Gamma_H}{2}, \quad \Delta\Gamma \equiv \Gamma_L - \Gamma_H.
\end{aligned}
\tag{2.7}
$$

Here $m_{L(H)}$ and $\Gamma_{L(H)}$ are the mass and width of $D_{L(H)}$, respectively. Note that the above definitions guarantee $\Delta m \geqslant 0$ and $\Delta\Gamma \geqslant 0$ in most cases. Practically, it is more popular to use the following two dimensionless parameters for describing D^0-$\overline{D^0}$ mixing:

$$x_D \equiv \frac{\Delta m}{\Gamma}, \quad y_D \equiv \frac{\Delta \Gamma}{2\Gamma}. \tag{2.8}$$

Certainly both x_D and y_D in most cases are positive (or vanishing).

B. Rates for incoherent D decays

The transition amplitude of a neutral D meson decaying to a semileptonic or nonleptonic state f can be obtained from Eq. (2.5) as

$$\langle f|\mathcal{H}|D^0_{\rm phys}(t)\rangle = g_+(t)A_f + \frac{q}{p}g_-(t)\overline{A}_f,$$

$$\langle f|\mathcal{H}|\overline{D^0_{\rm phys}}(t)\rangle = g_+(t)\overline{A}_f + \frac{p}{q}g_-(t)A_f, \tag{2.9}$$

where $A_f \equiv \langle f|\mathcal{H}|D^0\rangle$ and $\overline{A}_f \equiv \langle f|\mathcal{H}|\overline{D^0}\rangle$. For convenience, we also define the ratio of these two amplitudes:

$$\rho_f \equiv \frac{\overline{A}_f}{A_f}, \quad \lambda_f \equiv \frac{q}{p}\rho_f. \tag{2.10}$$

Then the time-dependent probabilities of such decay events are expressed as

$$\begin{aligned} R(D^0_{\rm phys}(t)\to f) \propto |A_f|^2 \exp(-\Gamma t)[&C_y \cosh(y_D\Gamma t) \\ &+ C_x \cos(x_D\Gamma t) + S_y \sinh(y_D\Gamma t) \\ &+ S_x \sin(x_D\Gamma t)], \end{aligned}$$

$$\begin{aligned} R(\overline{D^0_{\rm phys}}(t)\to f) \propto |A_f|^2 \exp(-\Gamma t)[&\overline{C_y} \cosh(y_D\Gamma t) \\ &+ \overline{C_x} \cos(x_D\Gamma t) + \overline{S_y} \sinh(y_D\Gamma t) \\ &+ \overline{S_x} \sin(x_D\Gamma t)], \end{aligned} \tag{2.11}$$

where

$$C_y \equiv \frac{1+|\lambda_f|^2}{2}, \quad S_y \equiv -\mathrm{Re}\lambda_f,$$

$$C_x \equiv \frac{1-|\lambda_f|^2}{2}, \quad S_x \equiv -\mathrm{Im}\lambda_f, \tag{2.12}$$

and

$$(\overline{C_y}, \overline{S_y}, \overline{C_x}, \overline{S_x}) = |p/q|^2 (C_y, S_y, -C_x, -S_x). \tag{2.13}$$

To obtain the time-independent decay rates, we integrate Eq. (2.11) over $t \in [0,\infty)$ and get

$$R(D^0_{\rm phys}\to f) \propto |A_f|^2 \left[\frac{1}{1-y_D^2}C_y + \frac{1}{1+x_D^2}C_x + \frac{y_D}{1-y_D^2}S_y + \frac{x_D}{1+x_D^2}S_x\right],$$

$$R(\overline{D^0_{\rm phys}}\to f) \propto |A_f|^2 \left[\frac{1}{1-y_D^2}\overline{C_y} + \frac{1}{1+x_D^2}\overline{C_x} + \frac{y_D}{1-y_D^2}\overline{S_y} + \frac{x_D}{1+x_D^2}\overline{S_x}\right]. \tag{2.14}$$

Equations (2.11) and (2.14) are the master formulas for incoherent D decays.

Following the same procedure one can calculate the decay rates of $D^0_{\rm phys}$ and $\overline{D^0_{\rm phys}}$ to $\overline{f}$, the CP-conjugate state of f. To express the relevant formulas in analogy with Eqs. (2.11) and (2.14), we define $A_{\overline{f}} \equiv \langle \overline{f}|\mathcal{H}|D^0\rangle$, $\overline{A}_{\overline{f}} \equiv \langle \overline{f}|\mathcal{H}|\overline{D^0}\rangle$, and

$$\overline{\rho}_{\overline{f}} \equiv \frac{A_{\overline{f}}}{\overline{A}_{\overline{f}}}, \quad \overline{\lambda}_{\overline{f}} \equiv \frac{p}{q}\overline{\rho}_{\overline{f}}. \tag{2.15}$$

Then $R(D^0_{\rm phys}(t)\to \overline{f})$, $R(\overline{D^0_{\rm phys}}(t)\to \overline{f})$ and $R(D^0_{\rm phys}\to \overline{f})$, $R(\overline{D^0_{\rm phys}}\to \overline{f})$ can be written out in terms of $\overline{A}_{\overline{f}}$, and $\overline{\lambda}_{\overline{f}}$. If f is a CP eigenstate (i.e., $|\overline{f}\rangle \equiv CP|f\rangle = \pm|f\rangle$), then we get $A_{\overline{f}} = \pm A_f$, $\overline{A}_{\overline{f}} = \pm \overline{A}_f$, $\overline{\rho}_{\overline{f}} = 1/\rho_f$, and $\overline{\lambda}_{\overline{f}} = 1/\lambda_f$.

C. Rates for coherent D decays

For a coherent $D^0_{\rm phys}\overline{D^0_{\rm phys}}$ pair at rest, its time-dependent wave function can be written as

$$\frac{1}{\sqrt{2}}[D^0_{\rm phys}(\mathbf{K},t)\rangle \otimes |\overline{D^0_{\rm phys}}(-\mathbf{K},t)\rangle + C|D^0_{\rm phys}(-\mathbf{K},t)\rangle \otimes |\overline{D^0_{\rm phys}}(\mathbf{K},t)\rangle], \tag{2.16}$$

where $\mathbf{K}$ is the three-momentum vector of the D mesons, and $C=\pm$ denotes the charge-conjugation parity of this coherent system. The formulas for the time evolution of $D^0_{\rm phys}$ and $\overline{D^0_{\rm phys}}$ mesons have been given in Eq. (2.5). Here we consider the case that one of the two D mesons (with momentum $\mathbf{K}$) decays to a final state f_1 at proper time t_1 and the other (with $-\mathbf{K}$) to f_2 at t_2. Here f_1 and f_2 may be either hadronic or semileptonic states. The amplitude of such a joint decay mode is given by

$$A(f_1,t_1;f_2,t_2)_C=\frac{1}{\sqrt{2}}A_{f_1}A_{f_2}\xi_C[g_+(t_1)g_-(t_2)+Cg_-(t_1)g_+(t_2)]+\frac{1}{\sqrt{2}}A_{f_1}A_{f_2}\zeta_C[g_+(t_1)g_+(t_2)+Cg_-(t_1)g_-(t_2)], \tag{2.17}$$

where $A_{f_i}\equiv\langle f_i|\mathcal{H}|D^0\rangle$ (with $i=1,2$), and

$$\xi_C\equiv\frac{p}{q}(1+C\lambda_{f_1}\lambda_{f_2}),$$

$$\zeta_C\equiv\frac{p}{q}(\lambda_{f_2}+C\lambda_{f_1}). \tag{2.18}$$

Here the definition of λ_{f_1} and λ_{f_2} is similar to that of λ_f in Eq. (2.10). After a lengthy calculation [13], we obtain the time-dependent decay rate as

$$R(f_1,t_1;f_2,t_2)_C\propto|A_{f_1}|^2|A_{f_2}|^2\exp(-\Gamma t_+)[(|\xi_C|^2+|\zeta_C|^2)\cosh(y_D\Gamma t_C)-2\mathrm{Re}(\xi_C^*\zeta_C)\sinh(y_D\Gamma t_C)-(|\xi_C|^2-|\zeta_C|^2)\cos(x_D\Gamma t_C)+2\mathrm{Im}(\xi_C^*\zeta_C)\sin(x_D\Gamma t_C)], \tag{2.19}$$

where

$$t_C\equiv t_2+Ct_1 \tag{2.20}$$

has been defined.

The time-independent decay rate is obtainable from Eq. (2.19) after the integration of $R(f_1,t_1;f_2,t_2)_C$ over $t_1\in[0,\infty)$ and $t_2\in[0,\infty)$:

$$R(f_1,f_2)_C\propto|A_{f_1}|^2|A_{f_2}|^2\left[\frac{1+Cy_D^2}{(1-y_D^2)^2}(|\xi_C|^2+|\zeta_C|^2)-\frac{2(1+C)y_D}{(1-y_D^2)^2}\mathrm{Re}(\xi_C^*\zeta_C)-\frac{1-Cx_D^2}{(1+x_D^2)^2}(|\xi_C|^2-|\zeta_C|^2)+\frac{2(1+C)x_D}{(1+x_D^2)^2}\mathrm{Im}(\xi_C^*\zeta_C)\right]. \tag{2.21}$$

We see that two interference terms $\mathrm{Re}(\xi_C^*\zeta_C)$ and $\mathrm{Im}(\xi_C^*\zeta_C)$ disappear in the case of $C=-1$, independent of the final states f_1 and f_2.

In a similar way, one can calculate the joint decay rates of $(D^0_{\mathrm{phys}}\overline{D^0_{\mathrm{phys}}})_C$ to $(f_1\overline{f_2})$, $(\overline{f_1}f_2)$, or $(\overline{f_1f_2})$, where $\overline{f_1}$ and $\overline{f_2}$ are CP-conjugate states of f_1 and f_2, respectively.

D. Analytical approximations

In the standard model, the magnitudes of x_D and y_D are expected to be very small, at most of the order 10^{-2} (see, e.g., Refs. [4–6]). The current experimental constraints on D^0-$\overline{D^0}$ mixing give $x_D^2+y_D^2<7.4\times10^{-3}$ [see Eq. (1.1)], which implies $x_D<0.086$ and $y_D<0.086$. Because of the smallness of x_D and y_D, the generic formulas obtained above can be approximately simplified to a good degree of accuracy.

Up to the accuracy of $O(x_D^2)$ and $O(y_D^2)$ for every distinctive term, the time-dependent decay rates in Eq. (2.11) are approximated as

$$R(D^0_{\mathrm{phys}}(t)\to f)\propto|A_f|^2\exp(-\Gamma t)[1+\tfrac{1}{4}(x_D^2+y_D^2)|\lambda_f|^2\Gamma^2t^2-\tfrac{1}{4}(x_D^2-y_D^2)\Gamma^2t^2-(y_D\mathrm{Re}\lambda_f+x_D\mathrm{Im}\lambda_f)\Gamma t],$$

$$R(\overline{D^0_{\mathrm{phys}}}(t)\to f)\propto|A_f|^2\left|\frac{p}{q}\right|^2\exp(-\Gamma t)[|\lambda_f|^2+\tfrac{1}{4}(x_D^2+y_D^2)\Gamma^2t^2-\tfrac{1}{4}(x_D^2-y_D^2)|\lambda_f|^2\Gamma^2t^2-(y_D\mathrm{Re}\lambda_f-x_D\mathrm{Im}\lambda_f)\Gamma t]. \tag{2.22}$$

Similarly we obtain the approximate decay rates for $D^0_{\mathrm{phys}}(t)\to\overline{f}$ and $\overline{D^0_{\mathrm{phys}}}(t)\to\overline{f}$:

$$R(D^0_{\mathrm{phys}}(t)\to\overline{f})\propto|\overline{A}_{\overline{f}}|^2\left|\frac{q}{p}\right|^2\exp(-\Gamma t)[|\overline{\lambda}_{\overline{f}}|^2+\tfrac{1}{4}(x_D^2+y_D^2)\Gamma^2t^2-\tfrac{1}{4}(x_D^2-y_D^2)|\overline{\lambda}_{\overline{f}}|^2\Gamma^2t^2-(y_D\mathrm{Re}\overline{\lambda}_{\overline{f}}-x_D\mathrm{Im}\overline{\lambda}_{\overline{f}})\Gamma t],$$

$$R(\overline{D^0_{\mathrm{phys}}}(t)\to\overline{f})\propto|\overline{A}_{\overline{f}}|^2\exp(-\Gamma t)[1+\tfrac{1}{4}(x_D^2+y_D^2)|\overline{\lambda}_{\overline{f}}|^2\Gamma^2t^2-\tfrac{1}{4}(x_D^2-y_D^2)\Gamma^2t^2-(y_D\mathrm{Re}\overline{\lambda}_{\overline{f}}+x_D\mathrm{Im}\overline{\lambda}_{\overline{f}})\Gamma t]. \tag{2.23}$$

The time-independent rates for these four processes turn out to be

$$R(D^0_{\mathrm{phys}}\to f)\propto|A_f|^2[1+\tfrac{1}{2}(x_D^2+y_D^2)|\lambda_f|^2-\tfrac{1}{2}(x_D^2-y_D^2)-(y_D\mathrm{Re}\lambda_f+x_D\mathrm{Im}\lambda_f)],$$

$$R(\overline{D}^0_{\rm phys}\to\overline{f})\propto|\overline{A}_{\overline{f}}|^2[1+\tfrac{1}{2}(x_D^2+y_D^2)|\overline{\lambda}_{\overline{f}}|^2-\tfrac{1}{2}(x_D^2-y_D^2)-(y_D{\rm Re}\overline{\lambda}_{\overline{f}}+x_D{\rm Im}\overline{\lambda}_{\overline{f}})] \tag{2.24}$$

and

$$R(\overline{D}^0_{\rm phys}\to f)\propto|A_f|^2\left|\frac{p}{q}\right|^2[|\lambda_f|^2+\tfrac{1}{2}(x_D^2+y_D^2)-\tfrac{1}{2}(x_D^2-y_D^2)|\lambda_f|^2-(y_D{\rm Re}\lambda_f-x_D{\rm Im}\lambda_f)],$$

$$R(D^0_{\rm phys}\to\overline{f})\propto|\overline{A}_{\overline{f}}|^2\left|\frac{q}{p}\right|^2[|\overline{\lambda}_{\overline{f}}|^2+\tfrac{1}{2}(x_D^2+y_D^2)-\tfrac{1}{2}(x_D^2-y_D^2)|\overline{\lambda}_{\overline{f}}|^2-(y_D{\rm Re}\overline{\lambda}_{\overline{f}}-x_D{\rm Im}\overline{\lambda}_{\overline{f}})]. \tag{2.25}$$

The formulas listed above are very useful for the study of neutral D decays in fixed target experiments or at B-meson factories. Here no assumption has been made for the magnitudes of $|\lambda_f|$ and $|\overline{\lambda}_{\overline{f}}|$. If they are considerably smaller than unity, e.g., in the DCSD's, then much simpler expressions can be drawn from Eqs. (2.22)–(2.25).

It is common knowledge that the decay-time distributions of coherent $(D^0_{\rm phys}\overline{D}^0_{\rm phys})_C$ pairs cannot be measured at a symmetric e^+e^- collider [14]. Since the presently proposed τ-charm factories are all based on symmetric e^+e^- colisions, it is more practical to study the time-integrated decays of $(D^0_{\rm phys}\overline{D}^0_{\rm phys})_C$ pairs. For completeness we shall present some important formulas for the decay-time distributions of $(D^0_{\rm phys}\overline{D}^0_{\rm phys})_C$ events, with the assumption of an asymmetric τ-charm factory, in Appendix A. Such a work might be of purely academic sense, but it could also be useful in the future experiments of charm physics.

In the approximations up to $O(x_D^2)$ and $O(y_D^2)$, the time-integrated rates for $(D^0_{\rm phys}\overline{D}^0_{\rm phys})_C$ decaying coherently to (f_1f_2), $(\overline{f}_1\overline{f}_2)$, $(f_1\overline{f}_2)$, and $(\overline{f}_1f_2)$ states are obtained from Eq. (2.21) as

$$\begin{aligned}R(f_1,f_2)_C\propto{}&|A_{f_1}|^2|A_{f_2}|^2\left|\frac{p}{q}\right|^2\{(2+C)(x_D^2+y_D^2)|1+C\lambda_{f_1}\lambda_{f_2}|^2+[2-(2+C)(x_D^2-y_D^2)]|\lambda_{f_2}+C\lambda_{f_1}|^2\\&-2(1+C)y_D[(1+|\lambda_{f_1}|^2){\rm Re}\lambda_{f_2}+C(1+|\lambda_{f_2}|^2){\rm Re}\lambda_{f_1}]+2(1+C)x_D[(1-|\lambda_{f_1}|^2){\rm Im}\lambda_{f_2}\\&+C(1-|\lambda_{f_2}|^2){\rm Im}\lambda_{f_1}]\},\end{aligned}$$

$$\begin{aligned}R(\overline{f}_1,\overline{f}_2)_C\propto{}&|\overline{A}_{\overline{f}_1}|^2|\overline{A}_{\overline{f}_2}|^2\left|\frac{q}{p}\right|^2\{(2+C)(x_D^2+y_D^2)|1+C\overline{\lambda}_{\overline{f}_1}\overline{\lambda}_{\overline{f}_2}|^2+[2-(2+C)(x_D^2-y_D^2)]|\overline{\lambda}_{\overline{f}_2}+C\overline{\lambda}_{\overline{f}_1}|^2\\&-2(1+C)y_D[(1+|\overline{\lambda}_{\overline{f}_1}|^2){\rm Re}\overline{\lambda}_{\overline{f}_2}+C(1+|\overline{\lambda}_{\overline{f}_2}|^2){\rm Re}\overline{\lambda}_{\overline{f}_1}]+2(1+C)x_D[(1-|\overline{\lambda}_{\overline{f}_1}|^2){\rm Im}\overline{\lambda}_{\overline{f}_2}\\&+C(1-|\overline{\lambda}_{\overline{f}_2}|^2){\rm Im}\overline{\lambda}_{\overline{f}_1}]\},\end{aligned} \tag{2.26}$$

and

$$\begin{aligned}R(f_1,\overline{f}_2)_C\propto{}&|A_{f_1}|^2|\overline{A}_{\overline{f}_2}|^2\{(2+C)(x_D^2+y_D^2)|\overline{\lambda}_{\overline{f}_2}+C\lambda_{f_1}|^2+[2-(2+C)(x_D^2-y_D^2)]|1+C\lambda_{f_1}\overline{\lambda}_{\overline{f}_2}|^2\\&-2(1+C)y_D[(1+|\lambda_{f_1}|^2){\rm Re}\overline{\lambda}_{\overline{f}_2}+C(1+|\overline{\lambda}_{\overline{f}_2}|^2){\rm Re}\lambda_{f_1}]-2(1+C)x_D[(1-|\lambda_{f_1}|^2){\rm Im}\overline{\lambda}_{\overline{f}_2}\\&+C(1-|\overline{\lambda}_{\overline{f}_2}|^2){\rm Im}\lambda_{f_1}]\},\end{aligned}$$

$$\begin{aligned}R(\overline{f}_1,f_2)_C\propto{}&|\overline{A}_{\overline{f}_1}|^2|A_{f_2}|^2\{(2+C)(x_D^2+y_D^2)|\lambda_{f_2}+C\overline{\lambda}_{\overline{f}_1}|^2+[2-(2+C)(x_D^2-y_D^2)]|1+C\overline{\lambda}_{\overline{f}_1}\lambda_{f_2}|^2\\&-2(1+C)y_D[(1+|\overline{\lambda}_{\overline{f}_1}|^2){\rm Re}\lambda_{f_2}+C(1+|\lambda_{f_2}|^2){\rm Re}\overline{\lambda}_{\overline{f}_1}]-2(1+C)x_D[(1-|\overline{\lambda}_{\overline{f}_1}|^2){\rm Im}\lambda_{f_2}\\&+C(1-|\lambda_{f_2}|^2){\rm Im}\overline{\lambda}_{\overline{f}_1}]\}.\end{aligned} \tag{2.27}$$

Taking $f_1 = K^+ l^- \overline{\nu_l}$ or $\overline{f_1} = K^- l^+ \nu_l$, for example, Eqs. (2.26) and (2.27) can be simplified significantly. Such semileptonic decay modes, which are flavor specific, play the role in identifying the flavor of the other D meson decaying to f_2 or $\overline{f_2}$.

III. SEMILEPTONIC *D* DECAYS

The manifestation of D^0-$\overline{D^0}$ mixing and CP violation in the semileptonic decays of neutral D mesons is relatively simple, since such transitions are flavor specific in the standard model or some of its extensions. Because of the flavor specification of $D^0 \to l^+ X^-$ and $\overline{D^0} \to l^- X^+$, it is not necessary to study the time dependence of $D^0_{\rm phys}$ and $\overline{D^0_{\rm phys}}$ decay modes.

A. D^0-$\overline{D^0}$ mixing and CP violation

For fixed target experiments or e^+e^- collisions at the $\Upsilon(4S)$ resonance, the produced D^0 and $\overline{D^0}$ mesons are incoherent. Knowledge of D^0-$\overline{D^0}$ mixing is expected to come from ratios of the wrong-sign to right-sign events of semileptonic D decays:

$$r \equiv \frac{R(D^0_{\rm phys} \to l^- X^+)}{R(D^0_{\rm phys} \to l^+ X^-)}, \quad \overline{r} \equiv \frac{R(\overline{D^0_{\rm phys}} \to l^+ X^-)}{R(\overline{D^0_{\rm phys}} \to l^- X^+)}. \quad (3.1)$$

By use of Eq. (2.14), we find

$$r = \left| \frac{q}{p} \right|^2 \frac{1-\alpha}{1+\alpha}, \quad \overline{r} = \left| \frac{p}{q} \right|^2 \frac{1-\alpha}{1+\alpha}, \quad (3.2)$$

where $\alpha = (1 - y_D^2)/(1 + x_D^2)$. Note that $|q/p| \neq 1$ signifies CP violation in D^0-$\overline{D^0}$ mixing. To fit more accurate data in the near future, we prefer the mixing parameter

$$r_D \equiv \frac{r + \overline{r}}{2} = w \frac{1-\alpha}{1+\alpha}, \quad (3.3)$$

with $w = (|q/p|^2 + |p/q|^2)/2$. For $|q/p| - 1 \sim \pm 1\%$, the value of w deviates less than 0.1% from unity. Thus this overall factor of r_D is safely negligible. In the approximation of $x_D \ll 1$ and $y_D \ll 1$, one obtains

$$r_D \approx \frac{x_D^2 + y_D^2}{2}. \quad (3.4)$$

The latest E691 data [2] give $r \approx \overline{r} \approx r_D < 0.37\%$ for small x_D and y_D, where $|q/p| \approx |p/q| \approx 1$, a worse approximation than $w \approx 1$, has been used.

The CP asymmetry between a semileptonic decay mode and its CP-conjugate counterpart is defined as

$$\Delta_D \equiv \frac{R(\overline{D^0_{\rm phys}} \to l^+ X^-) - R(D^0_{\rm phys} \to l^- X^+)}{R(\overline{D^0_{\rm phys}} \to l^+ X^-) + R(D^0_{\rm phys} \to l^- X^+)},$$

$$\overline{\Delta_D} \equiv \frac{R(\overline{D^0_{\rm phys}} \to l^- X^+) - R(D^0_{\rm phys} \to l^+ X^-)}{R(\overline{D^0_{\rm phys}} \to l^- X^+) + R(D^0_{\rm phys} \to l^+ X^-)}. \quad (3.5)$$

Straightforwardly, we get

$$\Delta_D = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4}, \quad \overline{\Delta_D} = 0. \quad (3.6)$$

If Δ_D is at the level of 10^{-3} or so, it can be measured to three standard deviations with about 10^7 wrong-sign events.

It should be noted that the asymmetry $\overline{\Delta_D}$ may be nonvanishing if there exists new physics affecting the semileptonic D decays. For example, either the violation of CPT symmetry or that of the $\Delta Q = \Delta C$ rule can lead to $\overline{\Delta_D} \neq 0$. Even if the $\Delta Q = \Delta C$ rule and CPT invariance hold, $\overline{\Delta_D} \neq 0$ is still possible in consequence of the phase shifts from final-state electromagnetic interactions or the CP-violating contributions of nonstandard electroweak models to the tree-level processes under discussion. Hence all such tiny effects should be kept in mind and carefully evaluated when one wants to isolate one of them from the others.

As pointed out by Bigi in Ref. [8], a nonvanishing value for r_D might only be a secondary signature of D^0-$\overline{D^0}$ mixing, because the presence of $\Delta Q = -\Delta C$ transitions would contribute to r_D in a significant and time-independent way. For the purpose of illustration, we shall specifically calculate this effect on the magnitudes of r_D and $\overline{\Delta_D}$ in the following.

B. Effect of $\Delta Q = -\Delta C$ transitions on r_D and $\overline{\Delta_D}$

Within the standard model the processes $D^0 \to l^- X^+$ and $\overline{D^0} \to l^+ X^-$ are forbidden according to the $\Delta Q = \Delta C$ rule. New physics beyond the standard model may allow $\Delta Q = -\Delta C$ transitions, which affect the parameters of D^0-$\overline{D^0}$ mixing and CP violation. In the assumption of CPT symmetry and the neglect of final-state electromagnetic interactions, the decay amplitudes of D^0 and $\overline{D^0}$ to $l^\pm X^\mp$ can be factorized as

$$\langle l^+ X^- | \mathcal{H} | D^0 \rangle = A_l, \quad \langle l^+ X^- | \mathcal{H} | \overline{D^0} \rangle = \sigma_l A_l,$$

$$\langle l^- X^+ | \mathcal{H} | \overline{D^0} \rangle = A_l^*, \quad \langle l^- X^+ | \mathcal{H} | D^0 \rangle = \sigma_l^* A_l^*, \quad (3.7)$$

where σ_l measures the $\Delta Q = -\Delta C$ transition amplitude. With the help of Eq. (2.14) and the notation

$$\lambda_+ \equiv \frac{q}{p} \sigma_l, \quad \lambda_- \equiv \frac{p}{q} \sigma_l^*, \quad (3.8)$$

we obtain

$$R(D^0_{\rm phys} \to l^+ X^-) \propto |A_l|^2 [(1+\alpha) + (1-\alpha)|\lambda_+|^2 - 2y_D {\rm Re}\lambda_+ - 2\alpha x_D {\rm Im}\lambda_+],$$

$$R(\overline{D^0_{\rm phys}} \to l^- X^+) \propto |A_l|^2 [(1+\alpha) + (1-\alpha)|\lambda_-|^2 - 2y_D {\rm Re}\lambda_- - 2\alpha x_D {\rm Im}\lambda_-], \quad (3.9)$$

and

$$R(\overline{D^0_{\rm phys}} \to l^+ X^-) \propto |A_l|^2 \left| \frac{p}{q} \right|^2 [(1-\alpha) + (1+\alpha)|\lambda_+|^2 - 2y_D {\rm Re}\lambda_+ + 2\alpha x_D {\rm Im}\lambda_+],$$

$$R(D^0_{\rm phys}\to l^- X^+)\propto |A_l|^2\left|\frac{q}{p}\right|^2[(1-\alpha)+(1+\alpha)|\lambda_-|^2 - 2y_D{\rm Re}\lambda_- + 2\alpha x_D{\rm Im}\lambda_-]. \tag{3.10}$$

For small $|\sigma_l|$ (e.g., $|\sigma_l|\sim x_D$ or y_D), the original mixing parameters r and $\bar{r}$ take the form

$$r\to r'\approx r+|\sigma_l|^2-\frac{2y_D}{1+\alpha}{\rm Re}\lambda_+ - \frac{2\alpha x_D}{1+\alpha}{\rm Im}\lambda_+ ,$$

$$\overline{r}\to\overline{r'}\approx\overline{r}+|\sigma_l|^2-\frac{2y_D}{1+\alpha}{\rm Re}\lambda_- - \frac{2\alpha x_D}{1+\alpha}{\rm Im}\lambda_- . \tag{3.11}$$

As a consequence,

$$r'_D\equiv\frac{r'+\overline{r'}}{2}\approx r_D+|\sigma_l|^2-\frac{y_D}{1+\alpha}{\rm Re}(\lambda_+ +\lambda_-) - \frac{\alpha x_D}{1+\alpha}{\rm Im}(\lambda_+ +\lambda_-). \tag{3.12}$$

In two extreme cases, $\sigma_l=0$ and $r_D=0$, we obtain $r'_D=r_D$ and $r'_D=|\sigma_l|^2$, respectively. This implies that a nonzero value for r'_D might not result exclusively from D^0-$\bar{D}^0$ mixing. For this reason, the study of D^0-$\bar{D}^0$ mixing in some other decay modes of neutral D mesons (e.g., $D^0/\bar{D}^0\to K^\pm\pi^\mp$) is necessary in order to pin down possible new physics in the charm sector.

The magnitudes of CP asymmetries Δ_D and $\bar{\Delta}_D$ might be affected by the $\Delta Q=-\Delta C$ transitions too. In the approximation of $|\sigma_l|\ll 1$ and $r_D\ll 1$, we find that $\bar{\Delta}_D$ becomes

$$\bar{\Delta}'_D\approx r_D|\sigma_l|^2\Delta_D-\frac{y_D}{1+\alpha}{\rm Re}(\lambda_- -\lambda_+) - \frac{\alpha x_D}{1+\alpha}{\rm Im}(\lambda_- -\lambda_+). \tag{3.13}$$

Note that nonvanishing $\bar{\Delta}'_D$ comes from the interference between the D^0-$\bar{D}^0$ mixing and $\Delta Q=-\Delta C$ amplitudes; i.e., either $r_D=0$ or $\sigma_l=0$ can give rise to $\bar{\Delta}'_D=\bar{\Delta}_D=0$. If $\Delta_D=0$ is assumed, then one obtains $\bar{\Delta}'_D\approx x_D{\rm Im}\lambda_+$. Since both x_D and $|\sigma_l|$ are expected to be very small (even vanishing), observation of the CP asymmetry $\bar{\Delta}'_D$ may be practically impossible.

C. Separate determination of x_D and y_D

Current theoretical estimates for the sizes of x_D and y_D have dramatic discrepancies due to the difficulty in dealing with the long-distance interactions [4–6]. Hence a separate determination of these two mixing parameters from direct measurements is very necessary [15,16]. Here we propose a time-independent method to probe the relative size of x_D and y_D in the dilepton events of coherent $D^0_{\rm phys}\bar{D}^0_{\rm phys}$ decays at the $\psi(4.14)$ resonance. In our calculations both CPT invariance and the $\Delta Q=\Delta C$ rule are assumed to hold exactly.

For a τ-charm factory running at the $\psi(4.14)$ resonance, the coherent $D^0\bar{D}^0$ events can be produced through $\psi(4.14)\to\gamma(D^0\bar{D}^0)_{C=+}$ or $\psi(4.14)\to\pi^0(D^0\bar{D}^0)_{C=-}$, where C stands for the charge-conjugation parity [12]. The generic formulas for the joint decay rates of two D mesons have been given in Eq. (2.21). For our present purpose, we only consider the primary dilepton events which are directly emitted from the coherent $(D^0_{\rm phys}\bar{D}^0_{\rm phys})_C$ decays. Let $N^{\pm\pm}_C$ and N^{+-}_C denote the time-integrated numbers of like-sign and opposite-sign dilepton events, respectively. By use of Eq. (2.21), we obtain

$$N^{++}_C=N_C\left|\frac{p}{q}\right|^2\left[\frac{1+Cy_D^2}{(1-y_D^2)^2}-\frac{1-Cx_D^2}{(1+x_D^2)^2}\right],$$

$$N^{--}_C=N_C\left|\frac{q}{p}\right|^2\left[\frac{1+Cy_D^2}{(1-y_D^2)^2}-\frac{1-Cx_D^2}{(1+x_D^2)^2}\right],$$

$$N^{+-}_C=2N_C\left[\frac{1+Cy_D^2}{(1-y_D^2)^2}+\frac{1-Cx_D^2}{(1+x_D^2)^2}\right], \tag{3.14}$$

where N_C is the normalization factor proportional to the rates of semileptonic D^0 and $\bar{D}^0$ decays. It is easy to check that the relation

$$N^{++}_- N^{--}_+ = N^{++}_+ N^{--}_- \tag{3.15}$$

holds stringently, and it is independent of the magnitudes of D^0-$\bar{D}^0$ mixing and CP violation.

Of course a coherent $D^0\bar{D}^0$ pair with $C=-$ can be straightforwardly produced from the decay of the $\psi(3.77)$ resonance. Its time-independent decay rates of the like-sign and opposite-sign dileptons obey Eq. (3.14) too. At a τ-charm factory the $(D^0\bar{D}^0)_{C=-}$ decays at both the $\psi(3.77)$ and $\psi(4.14)$ resonances will be measured, and a combination of them might increase the sensitiveness of our approach to probing D^0-$\bar{D}^0$ mixing.

Usually one is interested in the following two types of observables:

$$a_C\equiv\frac{N^{++}_C-N^{--}_C}{N^{++}_C+N^{--}_C},\quad r_C\equiv\frac{N^{++}_C+N^{--}_C}{N^{+-}_C}, \tag{3.16}$$

which signify nonvanishing CP violation and D^0-$\bar{D}^0$ mixing, respectively. Explicitly, we find

$$a_-=a_+=\Delta_D=\frac{|p|^4-|q|^4}{|p|^4+|q|^4}. \tag{3.17}$$

If a_- or a_+ is of the order 10^{-3}, it can be measured to three standard deviations at the second-round experiments of a τ-charm factory with about 10^7 like-sign dileptons (or, equivalently, about 10^{10} $D^0\bar{D}^0$ events). Furthermore,

$$r_-=w\frac{1-\alpha}{1+\alpha},\quad r_+=w\frac{\beta-\alpha^2}{\beta+\alpha^2}, \tag{3.18}$$

where $\beta=(1+y_D^2)/(1-x_D^2)$. One can see that $r_-=r_D$ holds without any approximation. For small x_D and y_D, we have

$$r_- \approx \frac{x_D^2 + y_D^2}{2}, \quad r_+ \approx 3r_- . \tag{3.19}$$

These two approximate results have been well known in the literature (see, e.g., Refs. [7,8]). In such an approximation, however, the relative size of x_D^2 and y_D^2 cannot be determined.

To distinguish between the different contributions of x_D and y_D to D^0-$\overline{D^0}$ mixing, one has to measure $r_\pm$ as precisely as possible. With the help of Eq. (3.18), we show that the magnitudes of x_D and y_D can be separately determined as

$$x_D^2 = \left(\frac{1+r_-}{1-r_-} \frac{1+3r_-}{1-r_-} - \frac{1+r_+}{1-r_+} \right) \left(\frac{1+r_-}{1-r_-} - \frac{1+r_+}{1-r_+} \right)^{-1},$$

$$y_D^2 = \left(\frac{1-r_-}{1+r_-} \frac{1-3r_-}{1+r_-} - \frac{1-r_+}{1+r_+} \right) \left(\frac{1-r_+}{1+r_+} - \frac{1-r_-}{1+r_-} \right)^{-1}. \tag{3.20}$$

Here it is worth emphasizing that w as the overall (and common) factor of r_D, r_-, and r_+ can be safely neglected. In the approximations up to $O(r_-^2)$ and $O(r_+^2)$, we obtain two simpler relations

$$x_D^2 - y_D^2 \approx 2\frac{r_+ - 3r_-}{r_+ - r_-}, \quad x_D^2 + y_D^2 \approx 4r_- \frac{r_+ - 2r_-}{r_+ - r_-}. \tag{3.21}$$

Thus it is crucial to examine the deviation of the ratio r_+/r_- from 3, in order to find the difference between x_D^2 and y_D^2. Instructively, we consider three special cases [16]

$$x_D \gg y_D \Rightarrow \frac{r_+}{r_-} \approx 3 + 2r_- > 3,$$

$$x_D \approx y_D \Rightarrow \frac{r_+}{r_-} \approx 3 - 9r_-^2 \approx 3,$$

$$x_D \ll y_D \Rightarrow \frac{r_+}{r_-} \approx 3 - 2r_- < 3. \tag{3.22}$$

These relations can be directly derived from Eq. (3.18) or (3.20). If r_- is close to the current experimental bound [i.e., $r_- = r_D \approx (x_D^2 + y_D^2)/2 < 0.37\%$], then measurements of r_+/r_- to the accuracy of 10^{-4} can definitely establish the relative magnitude of x_D and y_D. To this goal, about 10^8 like-sign dileptons [or, equivalently, about 10^{11} events of $(D^0\overline{D^0})_{C=-}$ and $(D^0\overline{D^0})_{C=+}$ pairs] are needed.

For illustration, we take a look at the changes of the measurable

$$\gamma \equiv \frac{r_+}{r_-} - 3, \tag{3.23}$$

with x_D, by fixing the value of y_D. Allowing $10^{-4} \leq r_- < 3.7\times 10^{-3}$ and taking $y_D = 0.001$, 0.04, and 0.08, respectively, we plot γ as the function of x_D in Fig. 1. It is clear that γ reflects the information about the relative magnitude of x_D and y_D and it can be detected if r_- is of the order 10^{-3} or so.

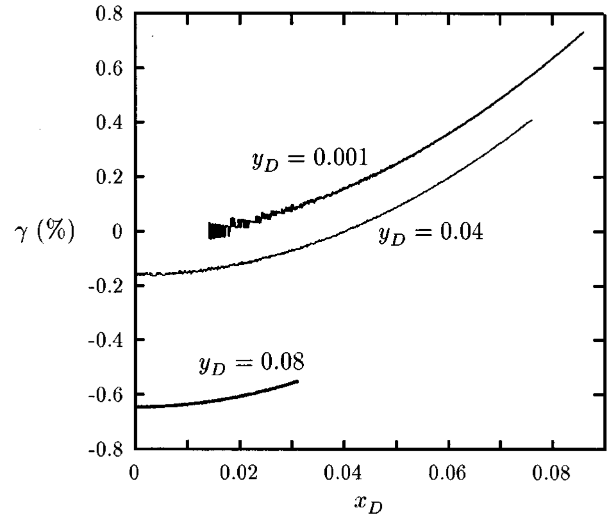

FIG. 1. The illustrative plot for the changes of γ with x_D, where the restriction $10^{-4} \leq r_- < 3.7\times 10^{-3}$ has been used.

In the assumption of a dedicated accelerator running for one year at an average luminosity of 10^{33} s^{-1}cm^{-2}, about 10^7 events of $\gamma(D^0\overline{D^0})_{C=+}$ and a similar number of $\pi^0(D^0\overline{D^0})_{C=-}$ are expected to be produced at the $\psi(4.14)$ resonance [12]. A precision of 10^{-4}–10^{-5} in measurements of r_- and r_+ is achievable if one assumes zero background and enough running time [12,17], and then similar precision can be obtained for the ratio r_+/r_- without much more experimental effort [see Eq. (3.22) for illustration]. If D^0-$\overline{D^0}$ mixing were at the level of $r_D \sim 10^{-3}$ (or at least $r_D \geq 10^{-4}$), then the relative magnitude of x_D and y_D should be detectable in the second-round experiments of a τ-charm factory (beyond the one under consideration at present).

IV. NEUTRAL D DECAYS TO CP EIGENSTATES

Neutral D-meson decays to hadronic CP eigenstates f (i.e., $|\bar{f}\rangle \equiv CP|f\rangle = \pm|f\rangle$), such as $f = \pi^+\pi^-$ and $K_S\pi^0$, are of particular interest for the study of CP violation in the charm sector. The formulas for their decay rates derived in Sec. II can be simplified because of the relations $A_{\bar{f}} = \pm A_f$, $\bar{A}_{\bar{f}} = \pm \bar{A}_f$, $\bar{\rho}_{\bar{f}} = 1/\rho_f$, and $\lambda_{\bar{f}} = 1/\lambda_f$. If one takes $|q/p| = 1$ in some cases, then $\bar{\lambda}_{\bar{f}} = \lambda_f^*$ is obtainable [18].

A. Three sources of CP violation

In the experimental analyses of incoherent D decays, the combined time-dependent rates

$$\mathcal{R}_\pm(t) \equiv R(D^0_{\rm phys}(t) \to f) \pm R(\overline{D}^0_{\rm phys}(t) \to f) \tag{4.1}$$

are commonly used. For convenience in expressing our analytical results, we first define

$$\mathcal{U}_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad \mathcal{V}_f \equiv \frac{-2\,\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}, \quad \mathcal{W}_f \equiv \frac{2\,\mathrm{Re}\lambda_f}{1+|\lambda_f|^2}, \tag{4.2}$$

which satisfy a concise sum rule

$$\mathcal{U}_f^2+\mathcal{V}_f^2+\mathcal{W}_f^2=1. \tag{4.3}$$

With the help of Eq. (2.11), we obtain

$$\begin{aligned}\mathcal{R}_+(t)=&\mathcal{R}_0\exp(-\Gamma t)[\cosh(y_D\Gamma t)-\mathcal{W}_f\sinh(y_D\Gamma t)\\ &-\hat{\Delta}_D\mathcal{U}_f\cos(x_D\Gamma t)-\hat{\Delta}_D\mathcal{V}_f\sin(x_D\Gamma t)],\\ \mathcal{R}_-(t)=&\mathcal{R}_0\exp(-\Gamma t)[-\hat{\Delta}_D\cosh(y_D\Gamma t)+\hat{\Delta}_D\mathcal{W}_f\sinh(y_D\Gamma t)\\ &+\mathcal{U}_f\cos(x_D\Gamma t)+\mathcal{V}_f\sin(x_D\Gamma t)],\end{aligned} \tag{4.4}$$

where

$$\mathcal{R}_0 \propto \tfrac{1}{2}|A_f|^2\left(1+\left|\frac{p}{q}\right|^2\right)(1+|\lambda_f|^2) \tag{4.5}$$

is a normalization factor, and $\hat{\Delta}_D \equiv (|p|^2-|q|^2)/(|p|^2+|q|^2)$ is related to Δ_D through

$$\Delta_D=\frac{2\hat{\Delta}_D}{1+\hat{\Delta}_D^2}. \tag{4.6}$$

To properly describe the signal of direct CP violation in neutral D decays, we further define

$$\mathcal{T}_f \equiv \frac{1-|\rho_f|^2}{1+|\rho_f|^2}. \tag{4.7}$$

By use of Eq. (2.10), we obtain the relation between $\mathcal{T}_f$ and $\mathcal{U}_f$:

$$\mathcal{U}_f=\frac{\mathcal{T}_f+\hat{\Delta}_D}{1+\hat{\Delta}_D\mathcal{T}_f}. \tag{4.8}$$

It is clear that $\hat{\Delta}_D$, $\mathcal{T}_f$, and $\mathcal{V}_f$ measure the CP asymmetry in D^0-$\bar{D}^0$ mixing, the direct CP asymmetry in the transition amplitudes of D decays, and the indirect CP asymmetry arising from the interplay of decay and D^0-$\bar{D}^0$ mixing, respectively. These sources of CP-violating effects appear in $\mathcal{R}_\pm(t)$ simultaneously, but they have different time distributions and can in principle be distinguished from one another [19]. The magnitudes of $\hat{\Delta}_D$, $\mathcal{T}_f$, and $\mathcal{V}_f$ are expected to be very small (e.g., at the percent level in some extensions of the standard electroweak model [20]). In contrast, the CP-conserving quantity $\mathcal{W}_f$ should be of order 1. Thus the $\cos(x_D\Gamma t)$ and $\sin(x_D\Gamma t)$ terms are considerably suppressed in $\mathcal{R}_+(t)$. This interesting feature implies that the mixing parameter y_D is possible to be constrained from the measurement of the flavor-untagged decay rate $\mathcal{R}_+(t)$. We shall discuss this possibility for some neutral D-meson decays in the next subsection.

In lowest-order approximations, we keep only the leading terms of $\hat{\Delta}_D$, $\mathcal{T}_f$, and $\mathcal{V}_f$ in $\mathcal{R}_\pm(t)$. Then the CP-violating observable is given as

$$\mathcal{A}(t)\equiv\frac{\mathcal{R}_-(t)}{\mathcal{R}_+(t)}\approx-\hat{\Delta}_D+\mathcal{U}_f+x_D\mathcal{V}_f\Gamma t\approx\mathcal{T}_f+x_D\mathcal{V}_f\Gamma t. \tag{4.9}$$

One can see that $\hat{\Delta}_D$ has little contribution to $\mathcal{A}(t)$, and the term $\mathcal{T}_f$ is almost independent of the decay time t.

Integrating $\mathcal{R}_\pm(t)$ over $t\in[0,\infty)$, we obtain the time-independent decay rates as

$$\begin{aligned}\mathcal{R}_+&=\frac{\mathcal{R}_0}{1-y_D^2}[1-y_D\mathcal{W}_f-\alpha\hat{\Delta}_D(\mathcal{U}_f+x_D\mathcal{V}_f)],\\ \mathcal{R}_-&=\frac{\mathcal{R}_0}{1-y_D^2}[(y_D\mathcal{W}_f-1)\hat{\Delta}_D+\alpha(\mathcal{U}_f+x_D\mathcal{V}_f)],\end{aligned} \tag{4.10}$$

where α has been given in Sec. III A. The corresponding CP asymmetry turns out to be

$$\mathcal{A}\equiv\frac{\mathcal{R}_-}{\mathcal{R}_+}\approx-\hat{\Delta}_D+\mathcal{U}_f+x_D\mathcal{V}_f\approx\mathcal{T}_f+x_D\mathcal{V}_f \tag{4.11}$$

in the leading-order approximation.

At the $\psi(3.77)$ and $\psi(4.14)$ resonances, the produced $D^0\bar{D}^0$ pair may exist in a coherent state until one of them decays. Hence we can use the semileptonic decay of one D meson to tag the flavor of the other meson decaying to a flavor-nonspecific CP eigenstate f. The time-integrated rates of such joint decays can be read off from Eq. (2.21). We are more interested in the following combinations of decay rates:

$$\Omega_\pm(C)\equiv R(l^-X^+,f)_C\pm R(l^+X^-,f)_C. \tag{4.12}$$

After some straightforward calculations, we obtain

$$\begin{aligned}\Omega_+(C)=&\Omega_0[(1+Cy_D^2)-(1-Cx_D^2)\alpha^2\hat{\Delta}_D\mathcal{U}_f\\ &-(1+C)(y_D\mathcal{W}_f+x_D\alpha^2\hat{\Delta}_D\mathcal{V}_f)],\\ \Omega_-(C)=&\Omega_0[-(1+Cy_D^2)\hat{\Delta}_D+(1-Cx_D^2)\alpha^2\mathcal{U}_f\\ &+(1+C)(y_D\hat{\Delta}_D\mathcal{W}_f+x_D\alpha^2\mathcal{V}_f)],\end{aligned} \tag{4.13}$$

where

$$\Omega_0\propto\frac{2\mathcal{R}_0|A_l|^2}{(1-y_D^2)^2}, \tag{4.14}$$

and other quantities have been defined before. Keeping the leading terms of $\hat{\Delta}_D$, $\mathcal{T}_f$, and $\mathcal{V}_f$, we get the CP asymmetries for $C=\pm$ cases as

$$\begin{aligned}\mathcal{A}_-&\equiv\frac{\Omega_-(C=-)}{\Omega_+(C=-)}\approx-\hat{\Delta}_D+\mathcal{U}_f\approx\mathcal{T}_f,\\ \mathcal{A}_+&\equiv\frac{\Omega_-(C=+)}{\Omega_+(C=+)}\approx-\hat{\Delta}_D+\mathcal{U}_f+2x_D\mathcal{V}_f\approx\mathcal{T}_f+2x_D\mathcal{V}_f.\end{aligned} \tag{4.15}$$

Indeed, $\mathcal{A}_-$ is exactly independent of the indirect CP-violating term $\mathcal{V}_f$. The asymmetry $\mathcal{A}_+$ is mainly composed of two sources of CP violation. Comparing Eq. (4.15)

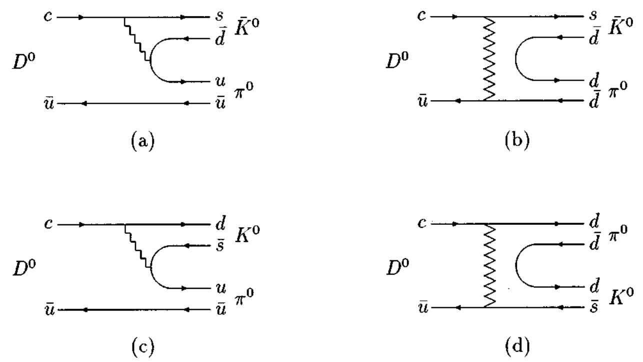

FIG. 2. Quark diagrams for the Cabibbo-allowed decay $D^0 \rightarrow \overline{K^0}\pi^0$ and the doubly Cabibbo-suppressed decay $D^0 \rightarrow K^0\pi^0$.

with Eq. (4.11), one can see that there exists an interesting relation among three time-independent CP measurables:

$$\mathcal{A}_- + \mathcal{A}_+ \approx 2\mathcal{A}. \tag{4.16}$$

This result should be testable in a variety of neutral D decays to CP eigenstates.

B. Approach to constrain y_D and x_D

It has been pointed out that y_D might be probed through measurements of the singly Cabibbo-suppressed decays $D^0_{\rm phys}(t) \rightarrow K^+K^-$ and $\pi^+\pi^-$ if CP conservation could hold in them [15]. This idea can be straightforwardly understood from the combined decay rates $\mathcal{R}_+(t)$ in Eq. (4.4). Assuming CP invariance, i.e., $\hat{\Delta}_D = \mathcal{T}_f = \mathcal{V}_f = 0$ and $\mathcal{W}_f = 1$ (or $\mathcal{W}_f = -1$ for CP-odd final states), we find

$$\begin{aligned}\mathcal{R}_+(t) &= \mathcal{R}_0 \exp(-\Gamma t)[\cosh(y_D \Gamma t) - \sinh(y_D \Gamma t)] \\ &= \mathcal{R}_0 \exp[-(1+y_D)\Gamma t],\end{aligned} \tag{4.17}$$

with $\mathcal{R}_0 \propto 2|A_f|^2$. Because $(1+y_D)\Gamma = \Gamma_L$, the signature of D^0-$\overline{D^0}$ mixing is indeed a deviation of the slope of $\mathcal{R}_+(t)$ from $\exp(-\Gamma t)$. Since Γ can be measured via other approaches, one is then able to constrain the magnitude of y_D. The above method depends strongly upon the assumption of CP conservation in D decays; hence, it may not work well in practice. Subsequently we shall show that a model-independent constraint on y_D (or x_D) is indeed achievable, without any special assumption, from measuring the decay-time distributions of $D^0_{\rm phys}(t)/\overline{D^0_{\rm phys}}(t) \rightarrow K_{S,L} + \pi^0$, etc.

Exactly speaking, $K_S\pi^0$ and $K_L\pi^0$ are not CP eigenstates due to the existence of small CP violation in K^0-$\overline{K^0}$ mixing. Here we want to keep this CP-violating contribution to D decays (measured by ϵ), but it can be safely neglected in most cases.

In the standard model the transitions $D^0 \rightarrow \overline{K^0}\pi^0$ and $D^0 \rightarrow K^0\pi^0$ (and their CP-conjugate processes) are Cabibbo allowed and doubly Cabibbo suppressed, respectively. Both of them occur only through the tree-level quark diagrams, as illustrated in Fig. 2. Since any new physics cannot significantly affect the direct decays of charm quark via the tree-level W-mediated graphs [21], one expects that Fig. 2 remains to be a valid quark-diagram description of the above-mentioned decay modes even beyond the standard model. Indeed significant new physics may exist in D^0-$\overline{D^0}$ mixing and the loop-induced penguin transitions of D mesons [22,23]. The processes $D^0 \rightarrow K_{S,L} + \pi^0$ and $\overline{D^0} \rightarrow K_{S,L} + \pi^0$ take place through Fig. 2 with K^0-$\overline{K^0}$ mixing in the final states. The mass eigenstates of K^0 and $\overline{K^0}$ mesons can be written as[1]

$$\begin{aligned}|K_S\rangle &= (1+\epsilon)|K^0\rangle + (1-\epsilon)|\overline{K^0}\rangle, \\ |K_L\rangle &= (1+\epsilon)|K^0\rangle - (1-\epsilon)|\overline{K^0}\rangle,\end{aligned} \tag{4.18}$$

where the complex parameter ϵ has been unambiguously measured ($|\epsilon| \approx 2.27 \times 10^{-3}$ and $\phi_\epsilon \approx 43.6°$ [1]). Note again that we do not assume $K_S\pi^0$ and $K_L\pi^0$ to be the exact CP eigenstates, although such an assumption is safely allowed by our main results presented later on. The overall decay amplitudes of $D^0/\overline{D^0} \rightarrow K_{S,L} + \pi^0$ are then given by

$$\begin{aligned}A(D^0 \rightarrow K_{S,L} + \pi^0) &= (1+\epsilon^*)A_{K^0\pi^0} \pm (1-\epsilon^*)A_{\overline{K^0}\pi^0}, \\ A(\overline{D^0} \rightarrow K_{S,L} + \pi^0) &= (1+\epsilon^*)\overline{A}_{K^0\pi^0} \pm (1-\epsilon^*)\overline{A}_{\overline{K^0}\pi^0}.\end{aligned} \tag{4.19}$$

Here $A_{K^0\pi^0}$, etc. can be factorized as

$$\begin{aligned}A_{K^0\pi^0} &= (V_{cd}V^*_{us})T_1 \exp(i\delta_1), \\ A_{\overline{K^0}\pi^0} &= (V_{cs}V^*_{ud})T_2 \exp(i\delta_2), \\ \overline{A}_{\overline{K^0}\pi^0} &= (V^*_{cd}V_{us})T_1 \exp(i\delta_1), \\ \overline{A}_{K^0\pi^0} &= (V^*_{cs}V_{ud})T_2 \exp(i\delta_2),\end{aligned} \tag{4.20}$$

where V_{us}, etc., are the KM matrix elements, T_1 and T_2 stand for the real (positive) hadronic matrix elements, and δ_1 and δ_2 are the corresponding strong phases. Denoting $h \equiv T_2/T_1$ and $\delta \equiv \delta_2 - \delta_1$, we obtain

$$\begin{aligned}\rho_{K_S\pi^0} &= + \frac{(1+\epsilon^*)(V^*_{cs}V_{ud})h\exp(i\delta) + (1-\epsilon^*)(V^*_{cd}V_{us})}{(1-\epsilon^*)(V_{cs}V^*_{ud})h\exp(i\delta) + (1+\epsilon^*)(V_{cd}V^*_{us})}, \\ \rho_{K_L\pi^0} &= - \frac{(1+\epsilon^*)(V^*_{cs}V_{ud})h\exp(i\delta) - (1-\epsilon^*)(V^*_{cd}V_{us})}{(1-\epsilon^*)(V_{cs}V^*_{ud})h\exp(i\delta) - (1+\epsilon^*)(V_{cd}V^*_{us})}.\end{aligned} \tag{4.21}$$

By use of the Wolfenstein parameter $\lambda \approx 0.22$, we have $|\epsilon| \approx \lambda^4$, $V^*_{cs}V_{ud} \approx 1$, and $V^*_{cd}V_{us} \approx -\lambda^2$. Furthermore, $h \approx 1$ is anticipated in the factorization approximation [24]. As a consequence,

$$\rho_{K_S\pi^0} \approx 1 + 2\epsilon^*, \quad \rho_{K_L\pi^0} \approx -\rho_{K_S\pi^0} \tag{4.22}$$

hold to a good degree of accuracy. This result implies that the direct CP asymmetries in $D^0_{\rm phys}(t)/\overline{D^0_{\rm phys}}(t) \rightarrow K_{S,L} + \pi^0$ are dominated by K^0-$\overline{K^0}$ mixing [25]:

[1]For simplicity, we neglect the common normalization factor $1/\sqrt{2(1+|\epsilon|^2)}$ for $|K_S\rangle$ and $|K_L\rangle$.

$$\mathcal{T}_{K_S\pi^0}\approx\mathcal{T}_{K_L\pi^0}\approx-2\,\mathrm{Re}\epsilon\approx-2|\epsilon|\cos\phi_\epsilon. \qquad (4.23)$$

Explicitly, we get $\mathcal{T}_{K_S\pi^0}\approx\mathcal{T}_{K_L\pi^0}\approx-3.3\times10^{-3}$.

For simplicity, we shall use the notation $q/p=|q/p|\exp(\mathrm{i}2\phi)$ [see Eq. (2.2)] later on. With the help of Eqs. (4.22) and (4.23) as well as the reasonable assumption $|\hat{\Delta}_D|\leqslant10^{-2}$, we obtain

$$\mathcal{U}_{K_S\pi^0}\approx+\mathcal{U}_{K_L\pi^0}\approx\hat{\Delta}_D-2|\epsilon|\cos\phi_\epsilon,$$

$$\mathcal{V}_{K_S\pi^0}\approx-\mathcal{V}_{K_L\pi^0}\approx2|\epsilon|\sin\phi_\epsilon\cos(2\phi)-\sin(2\phi),$$

$$\mathcal{W}_{K_S\pi^0}\approx-\mathcal{W}_{K_L\pi^0}\approx2|\epsilon|\sin\phi_\epsilon\sin(2\phi)+\cos(2\phi), \qquad (4.24)$$

in good approximations. Clearly the unknown new physics may enter $\mathcal{V}_{K_S\pi^0}$ and $\mathcal{W}_{K_S\pi^0}$ through the D^0-$\overline{D^0}$ mixing phase ϕ. Within the standard model one expects $\phi\sim0$; thus, $\mathcal{V}_{K_S\pi^0}\approx3.1\times10^{-3}$ and $\mathcal{W}_{K_S\pi^0}\approx1$. Beyond the standard model it is possible that the magnitudes of $\mathcal{V}_{K_S\pi^0}$ and $\mathcal{W}_{K_S\pi^0}$ are dominated by $\sin(2\phi)$ and $\cos(2\phi)$, respectively. The quantities $\mathcal{V}_{K_L\pi^0}$ and $\mathcal{W}_{K_L\pi^0}$ are in the similar situation.

Because of the smallness of x_D and y_D, some analytical approximations can be made for $\mathcal{R}_\pm(t)$ in Eq. (4.4) up to $O(x_D^2)$ and $O(y_D^2)$. Taking Eqs. (4.23) and (4.24) into account, we find

$$\mathcal{R}_+^{K_S\pi^0}(t)\approx\mathcal{R}_0^{K_S\pi^0}\exp(-\Gamma t)[1+X\Gamma^2t^2-Y\Gamma t],$$

$$\mathcal{R}_+^{K_L\pi^0}(t)\approx\mathcal{R}_0^{K_L\pi^0}\exp(-\Gamma t)[1+X\Gamma^2t^2+Y\Gamma t], \qquad (4.25)$$

where X and Y are functions of x_D and y_D:

$$X\approx\tfrac{1}{2}[y_D^2+x_D^2\hat{\Delta}_D(\hat{\Delta}_D-2|\epsilon|\cos\phi_\epsilon)],$$

$$Y\approx2|\epsilon|\sin\phi_\epsilon[y_D\sin(2\phi)+x_D\hat{\Delta}_D\cos(2\phi)]$$
$$+y_D\cos(2\phi)-x_D\hat{\Delta}_D\sin(2\phi). \qquad (4.26)$$

We can see that X and Y vanish in the absence of D^0-$\overline{D^0}$ mixing, and the contribution of x_D to them is significantly suppressed by $\hat{\Delta}_D$. Naively one might expect to measure the deviations of $\mathcal{R}_+^{K_S\pi^0}(t)$ and $\mathcal{R}_+^{K_L\pi^0}(t)$ from $\mathcal{R}_0^{K_S\pi^0}\exp(-\Gamma t)$ and $\mathcal{R}_0^{K_L\pi^0}\exp(-\Gamma t)$, respectively, in order to determine the sizes of X and Y. However, this is very difficult, if not even practically impossible, because of the smallness of X and Y. The interesting point here is that a comparison between the time distributions of $\mathcal{R}_+^{K_S\pi^0}(t)$ and $\mathcal{R}_+^{K_L\pi^0}(t)$ can definitely constrain the magnitude of Y. In view of $|\epsilon|\sim10^{-3}$, $x_D<0.086$, $y_D<0.086$, and $|\hat{\Delta}_D|\leqslant10^{-2}$ from our present experimental knowledge (and theoretical expectation), only the $y_D\cos(2\phi)$ term of Y is possible to be at the percent level (magnitudes of the other three terms in Y are all below 10^{-3}). One can conclude that the detectable signal of Y has to be at the percent level and it must come mainly from the width difference of D^0 and $\overline{D^0}$ mass eigenstates. For illustration, the time distributions of $\mathcal{R}_+^{K_S\pi^0}(t)$ and $\mathcal{R}_+^{K_L\pi^0}(t)$ are depicted in Fig. 3 by taking $y_D\approx0.08$ and $\phi\approx0$. We see that around $\Gamma t=2$ the difference between $\mathcal{R}_+^{K_S\pi^0}(t)/\mathcal{R}_0^{K_S\pi^0}$ and $\mathcal{R}_+^{K_L\pi^0}(t)/\mathcal{R}_0^{K_L\pi^0}$ can be as large as 5%, allowing us to extract a signal of D^0-$\overline{D^0}$ mixing provided that the accuracy of practical measurements is good enough.

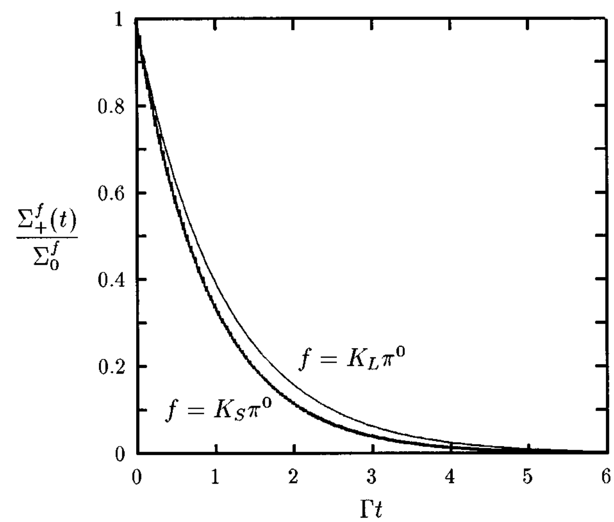

FIG. 3. The illustrative plot for the time distributions of $\Sigma_+^{K_S\pi^0}(t)$ and $\Sigma_+^{K_L\pi^0}(t)$, where $y_D\approx0.08$ and $\phi\approx0$ have been taken.

The asymmetry between $\mathcal{R}_+^{K_S\pi^0}(t)$ and $\mathcal{R}_+^{K_L\pi^0}(t)$ can be given as

$$\mathcal{A}_{LS}(t)\equiv\frac{\mathcal{R}_+^{K_S\pi^0}(t)-\mathcal{R}_+^{K_L\pi^0}(t)}{\mathcal{R}_+^{K_S\pi^0}(t)+\mathcal{R}_+^{K_L\pi^0}(t)}$$
$$\approx-2\lambda^2\frac{\cos\delta}{h}(1+X\Gamma^2t^2)-Y\Gamma t. \qquad (4.27)$$

Indeed the coefficient $-2\lambda^2\cos\delta/h$ measures the decay-rate asymmetry between $D^0\to K_S+\pi^0$ and $D^0\to K_L+\pi^0$ (or their flavor-conjugate processes) [19,24]. The measurement of $\mathcal{A}_{LS}(t)$ allows us to extract the magnitude of Y. To give one a numerical feeling, the changes of $\mathcal{A}_{LS}(t)$ with t are illustrated in Fig. 4 by assuming $X\approx y_D^2/2$, $Y\approx y_D\cos(2\phi)$, $h\approx1$, and $\delta\approx0$ and taking $y_D\approx0.08$, $|\cos(2\phi)|\approx1$. It is clear that a large signal of y_D should be detectable from $\mathcal{A}_{LS}(t)$.

The effects of D^0-$\overline{D^0}$ mixing and CP violation also manifest themselves in the combined rates $\mathcal{R}_-^{K_S\pi^0}(t)$ and $\mathcal{R}_-^{K_L\pi^0}(t)$:

$$\mathcal{R}_-^{K_S\pi^0}(t)\approx\mathcal{R}_0^{K_S\pi^0}\exp(-\Gamma t)[-2|\epsilon|\cos\phi_\epsilon$$
$$+X'\Gamma^2t^2+Y'\Gamma t],$$

$$\mathcal{R}_-^{K_L\pi^0}(t)\approx\mathcal{R}_0^{K_L\pi^0}\exp(-\Gamma t)[-2|\epsilon|\cos\phi_\epsilon$$
$$+X'\Gamma^2t^2-Y'\Gamma t], \qquad (4.28)$$

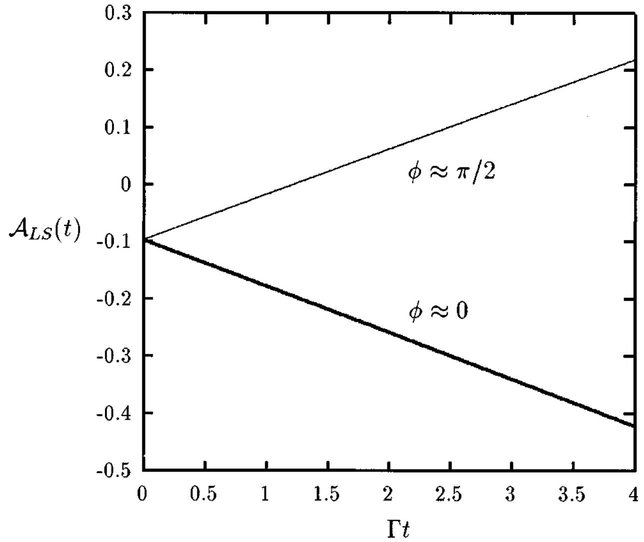

FIG. 4. The illustrative plot for the time distribution of $\mathcal{A}_{LS}(t)$, where $y_D \approx 0.08$ and $|\cos(2\phi)| \approx 1$ have been taken.

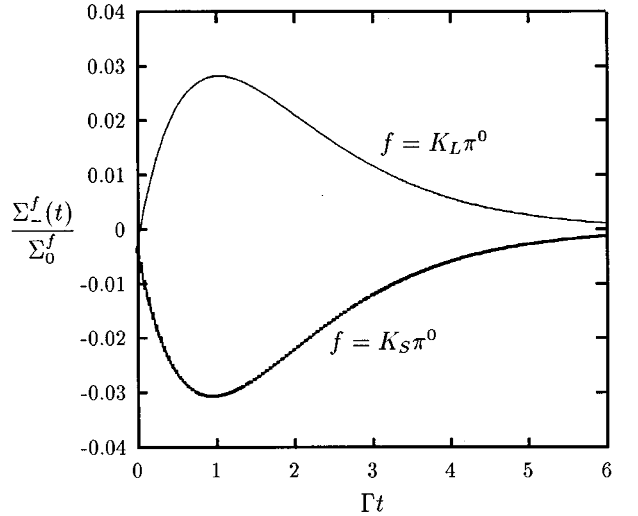

FIG. 5. The illustrative plot for the time distributions of $\Sigma_-^{K_S\pi^0}(t)$ and $\Sigma_-^{K_L\pi^0}(t)$, where $x_D \approx 0.08$, $\phi \approx \pi/4$, and $\hat{\Delta}_D \approx 0$ have been taken.

where

$$X' \approx x_D^2 |\epsilon| \cos\phi_\epsilon - r_D \hat{\Delta}_D,$$

$$Y' \approx 2|\epsilon| \sin\phi_\epsilon [y_D \hat{\Delta}_D \sin(2\phi) + x_D \cos(2\phi)] + y_D \hat{\Delta}_D \cos(2\phi) - x_D \sin(2\phi) \tag{4.29}$$

with $r_D \approx (x_D^2 + y_D^2)/2$. Obviously the CP asymmetry induced by K^0-$\overline{K^0}$ mixing (i.e., $\mathrm{Re}\epsilon$) plays an important role in the decay modes under discussion. The contribution of CP violation in D^0-$\overline{D^0}$ mixing (i.e., $\hat{\Delta}_D$) to Y' is not significant even if $\phi \sim 0$. If new physics considerably enhances x_D and ϕ, e.g., $x_D \sim 10^{-2}$ and $|\sin(2\phi)| \sim 1$, then $\mathcal{R}_-^{K_S\pi^0}(t)$ and $\mathcal{R}_-^{K_L\pi^0}(t)$ will be dominated by the CP asymmetry arising from the interplay of decay and D^0-$\overline{D^0}$ mixing. In other words, the signals of CP asymmetries

$$\mathcal{A}_{K_S\pi^0}(t) \equiv \frac{\mathcal{R}_-^{K_S\pi^0}(t)}{\mathcal{R}_+^{K_S\pi^0}(t)} \approx -2|\epsilon|\cos\phi_\epsilon + X'\Gamma^2 t^2 + Y'\Gamma t,$$

$$\mathcal{A}_{K_L\pi^0}(t) \equiv \frac{\mathcal{R}_-^{K_L\pi^0}(t)}{\mathcal{R}_+^{K_L\pi^0}(t)} \approx -2|\epsilon|\cos\phi_\epsilon + X'\Gamma^2 t^2 - Y'\Gamma t \tag{4.30}$$

at the percent level will indicate that new physics is definitely present in D^0-$\overline{D^0}$ mixing [e.g., $|\sin(2\phi)| \geqslant 0.5$] and the magnitude of x_D must be of order 10^{-2}. Taking $x_D \approx 0.08$, $\phi \approx \pi/4$, and $\hat{\Delta}_D \approx 0$, for example, we illustrate the time distributions of $\mathcal{R}_-^{K_S\pi^0}(t)$ and $\mathcal{R}_-^{K_L\pi^0}(t)$ in Fig. 5. We find that around $\Gamma t = 1$ the magnitudes of the decay-rate differences between $D^0_{\rm phys}(t) \to K_{S,L} + \pi^0$ and $\overline{D^0}_{\rm phys}(t) \to K_{S,L} + \pi^0$ can be as large as 3%. Since $\mathcal{R}_-^{K_S\pi^0}(1/\Gamma) \sim -\mathcal{R}_-^{K_L\pi^0}(1/\Gamma)$, it is possible to extract the rough size of $Y' \approx -x_D \sin(2\phi)$. Clearly the measurements of $\mathcal{R}_+^{K_S\pi^0}(t)$, $\mathcal{R}_+^{K_L\pi^0}(t)$ and $\mathcal{R}_-^{K_S\pi^0}(t)$, $\mathcal{R}_-^{K_L\pi^0}(t)$ are complementary to one another and can shed some light on the mixing parameters x_D and y_D as well as the possible new physics hidden in D^0-$\overline{D^0}$ mixing.

Note that the above discussions can be directly extended to neutral D decays to the final states such as $K_{S,L} + \rho^0$, $K_{S,L} + a_1^0$, and $K_{S,L} + \omega$, which occur through the same quark diagrams as $D^0/\overline{D^0} \to K_{S,L} + \pi^0$ (see Fig. 2). Because $X^{(\prime)}$ and $Y^{(\prime)}$ depend only upon the D^0-$\overline{D^0}$ and K^0-$\overline{K^0}$ mixing parameters, a sum over the above modes is possible, without any dilution effect on the signals of D^0-$\overline{D^0}$ mixing and CP violation, to increase the number of decay events in statistics.

C. Final-state interactions in $D \to K\overline{K}$

Recently the CLEO Collaboration has searched for CP violation in neutral D decays to the CP eigenstates K^+K^-, $K_S\phi$, and $K_S\pi^0$. The confidence intervals (90%) on CP asymmetries in these three modes were found to be $-0.020 < \mathcal{A}_{K\overline{K}} < 0.180$, $-0.182 < \mathcal{A}_{K_S\phi} < 0.126$, and $-0.067 < \mathcal{A}_{K_S\pi^0} < 0.031$, respectively [10]. Although a definite signal of CP violation was not established from the data above, the possibility that these decays may accommodate CP asymmetries at the percent level could not be ruled out. In the following we shall concentrate on the final-state interactions in $D^0/\overline{D^0} \to K^+K^-$ and $K^0\overline{K^0}$, since they may affect the magnitudes of CP asymmetries significantly. Similar discussions can be extended to some other decay modes such as $D^0/\overline{D^0} \to \pi^+\pi^-$ and $K_S\pi^0$.

We begin with an isospin analysis of $D^0 \to K^+K^-$, $D^+ \to K^+\overline{K^0}$, and $D^0 \to K^0\overline{K^0}$. To do this, we assume that there is no mixture of $D \to KK$ with other channels. In the language of quark diagrams [26], these modes can occur through both tree-level and penguin diagrams. However, such a naive description is problematic due to the presence of final-state rescattering effects [27]. The final states $K\overline{K}$ may contain $I=1$ and $I=0$ isospin configurations, and the overall decay amplitudes of $D \to K\overline{K}$ can be written as

$$A_{+-} \equiv \langle K^+K^-|\mathcal{H}|D^0\rangle = \tfrac{1}{2}(A_1 + A_0),$$

$$A_{00} \equiv \langle K^0\overline{K^0}|\mathcal{H}|D^0\rangle = \tfrac{1}{2}(A_1 - A_0),$$

$$A_{+0} \equiv \langle K^+\overline{K^0}|\mathcal{H}|D^+\rangle = A_1, \tag{4.31}$$

where A_1 and A_0 are two isospin amplitudes. Clearly three decay amplitudes can form an isospin triangle in the complex plane: $A_{+-} + A_{00} = A_{+0}$. Since the branching ratios of $D \to KK$ have been measured, one is able to determine the relevant isospin amplitudes from the relations above. For our purpose, we are more interested in the ratio of two isospin amplitudes: $A_0/A_1 \equiv Z\exp(i\varphi)$. It is straightforward to obtain

$$Z = \sqrt{2(R_{+-} + R_{00}) - 1}, \quad \cos\varphi = \frac{R_{+-} - R_{00}}{Z}, \tag{4.32}$$

where $R_{+-} \equiv |A_{+-}/A_{+0}|^2$ and $R_{00} = |A_{00}/A_{+0}|^2$ are two observables. If the annihilation diagrams and penguin effects in $D \to KK$ are negligible, then A_{+-}, A_{00}, and A_{+0} have a common Kobayashi-Maskawa (KM) factor (i.e., $V_{cs}V_{us}^*$) from the dominant tree-level (spectator) quark transitions. In this case, φ is purely a strong phase shift and the magnitude of Z is independent of the KM matrix elements.

Current experimental data give $B(D^0 \to K^+K^-) = (4.54 \pm 0.29) \times 10^{-3}$, $B(D^0 \to K^0\overline{K^0}) = (1.1 \pm 0.4) \times 10^{-3}$, and $B(D^+ \to K^+\overline{K^0}) = (7.8 \pm 1.7) \times 10^{-3}$ [1]. The lifetimes of D^0 and D^+ mesons are $\tau_{D^0} = (0.415 \pm 0.004) \times 10^{-12}$ s and $\tau_{D^+} = (1.057 \pm 0.015) \times 10^{-12}$ s, respectively. In the neglect of small phase space differences of three decay modes, we obtain $R_{+-} = 1.48 \pm 0.45$ and $R_{00} = 0.36 \pm 0.22$. The sizes of Z and φ can be solved from Eq. (4.32) with the inputs of R_{+-} and R_{00}, but there is large error propagation in this procedure, particularly for $\cos\varphi$ which is bounded by unity. For simplicity and illustration, we plot the allowed regions of Z and $\cos\varphi$ in Fig. 6. One can observe that $1.7 \leqslant Z \leqslant 2.0$ and $0.3 \leqslant \cos\varphi \leqslant 1.0$ (the central values of R_{+-} and R_{00} lead to $Z \approx 1.6$ and $\cos\varphi \approx 0.68$). This implies that significant final-state interactions may exist in the processes $D \to KK$.

The isospin amplitudes A_1 and A_0 can be expanded in terms of the tree-level and penguin transition amplitudes [28]. Without loss of generality, we write[2]

$$A_1 = A_{1T}\exp[i(\phi_T + \delta_{1T})] + A_{1P}\exp[i(\phi_P + \delta_{1P})],$$

$$A_0 = A_{0T}\exp[i(\phi_T + \delta_{0T})] + A_{0P}\exp[i(\phi_P + \delta_{0P})], \tag{4.33}$$

where ϕ_T and ϕ_P are the overall weak phases of tree-level and penguin diagrams, respectively, and δ_{nT} and δ_{nP} (with $n = 1,0$) denote the corresponding strong phases. Hence Z and φ defined above are complicated functions of A_{nT}, A_{nP}, ϕ_T, ϕ_P, δ_{nT}, and δ_{nP}. Since new physics may significantly affect the penguin amplitudes, direct CP violation is possible to appear in $D \to KK$. A constraint on the $I = 1$ penguin contribution to $D \to KK$ can be obtained by observing the decay-rate asymmetry between $D^+ \to K^+\overline{K^0}$ and $D^- \to K^-K^0$:

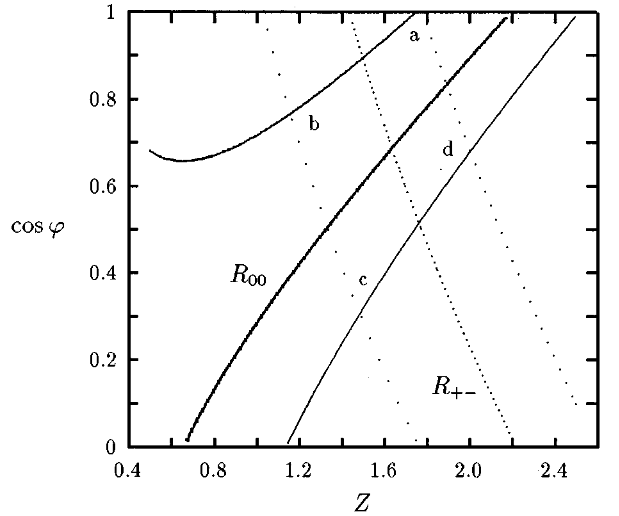

FIG. 6. Possible regions of Z and $\cos\varphi$ for $D \to K\overline{K}$ (i.e., the quadrangle $\langle abcd\rangle$) allowed by current data: $R_{+-} = 1.48 \pm 0.45$ and $R_{00} = 0.36 \pm 0.22$.

$$\frac{|\langle K^+\overline{K^0}|\mathcal{H}|D^+\rangle|^2 - |\langle K^-K^0|\mathcal{H}|D^-\rangle|^2}{|\langle K^+\overline{K^0}|\mathcal{H}|D^+\rangle|^2 + |\langle K^-K^0|\mathcal{H}|D^-\rangle|^2} = \frac{-2A_{1T}A_{1P}\sin(\phi_P - \phi_T)\sin(\delta_{1P} - \delta_{1T})}{A_{1T}^2 + A_{1P}^2 + 2A_{1T}A_{1P}\cos(\phi_P - \phi_T)\cos(\delta_{1P} - \delta_{1T})}. \tag{4.34}$$

Note that the weak phase difference $|\phi_P - \phi_T|$ may be rather small within the standard model, but some sources of new physics (e.g., the existence of the fourth quark family or an isosinglet up-type quark [29]) can significantly enhance it through the breakdown of unitarity of the 3×3 KM matrix in the penguin loops [30]. The direct CP asymmetries in $D^0/\overline{D^0} \to K^+K^-$ and $K^0\overline{K^0}$ contain both $I = 1$ and $I = 0$ penguin contributions, and the latter can in principle be distinguished from the former with the help of Eq. (4.34). In practical experiments, $\mathcal{T}_{K^+K^-}$ and $\mathcal{T}_{K^0\overline{K^0}}$ are cleanly detectable on the $\psi(3.77)$ resonance [see Eq. (4.15) for illustration]. If one wants to calculate the decay-rate asymmetries between $D^\pm \to K^\pm + K_{S,L}$ or between $D^0/\overline{D^0} \to K_{S,L} + K_{S,L}$, then the CP violation induced by K^0-$\overline{K^0}$ mixing in the final states has to be taken into account.

It is also argued that inelastic final-state interactions may affect $D \to KK$ [27]. This kind of effect is possible to yield a nonvanishing rate asymmetry between the charged D decays to $K^+\overline{K^0}$ and K^-K^0, even though the penguin contributions are negligibly small. To justify the role of penguin transi-

[2]Here we have neglected the contributions of tree-level annihilation diagrams to $D^0 \to K^0\overline{K^0}$, which involve both $V_{cs}V_{us}^*$ and $V_{cd}V_{ud}^*$. These two graph amplitudes are expected to have large cancellation with each other due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [27].

tions and inelastic final-state interactions, one has to rely on future data on direct CP asymmetries in the decay modes under discussion.

V. NEUTRAL D DECAYS TO NON-CP EIGENSTATES

We proceed to consider the case that both D^0 and $\overline{D^0}$ mesons decay to a common non-CP eigenstate. Most of such decay modes occur through quark transitions of the types $c \to s(u\bar{d})$ and $c \to d(u\bar{s})$ or their flavor-conjugate counterparts, and typical examples are the Cabibbo-allowed decay $D^0 \to K^- \pi^+$ and the doubly Cabibbo-suppressed process $D^0 \to K^+ \pi^-$. Because neutral D decays to $K^\pm \pi^\mp$ are of particular interest for the study of D^0-$\overline{D^0}$ mixing and DCSD's in charm physics, we shall concentrate on them in this section. Of course similar discussions can be extended to other non-CP eigenstates.

Note that $D^0 \to K^\pm \pi^\mp$ and their CP-conjugate processes take place only via the tree-level quark diagrams, on which no new physics can have a significant effect [14,23]. Thus the four transition amplitudes are factorized as

$$
\begin{aligned}
A_{K^-\pi^+} &= (V_{cs}V_{ud}^*) T_a \exp(i\delta_a),\\
A_{K^+\pi^-} &= (V_{cd}V_{us}^*) T_b \exp(i\delta_b),\\
\overline{A}_{K^+\pi^-} &= (V_{cs}^*V_{ud}) T_a \exp(i\delta_a),\\
\overline{A}_{K^-\pi^+} &= (V_{cd}^*V_{us}) T_b \exp(i\delta_b),
\end{aligned}
\tag{5.1}
$$

where T_a and T_b denote the real (positive) hadronic matrix elements, and δ_a and δ_b are the corresponding strong phases. Defining $h_{K\pi} \equiv T_b / T_a$ and $\delta_{K\pi} \equiv \delta_b - \delta_a$, we obtain

$$
\rho_{K^-\pi^+} \approx \overline{\rho}_{K^+\pi^-} \approx -\lambda^2 h_{K\pi} \exp(i\delta_{K\pi}),
\tag{5.2}
$$

to a good degree of accuracy, where $\lambda \approx 0.22$ is the Wolfenstein parameter. In the factorization approximation, the magnitude of $h_{K\pi}$ is expected to be of order 1. The strong phase shift $\delta_{K\pi}$ vanishes only in the limit of SU(3) symmetry [31]. To fit the recent CLEO result for $D^0 \to K^\pm \pi^\mp$ [9], which gives $|\rho_{K^-\pi^+}|^2 = (0.77 \pm 0.25 \pm 0.25)\%$, one finds $\delta_{K\pi} \sim 5°-13°$ from a few phenomenological models [32–34]. Of course a larger value for $\delta_{K\pi}$ cannot be absolutely ruled out from current experimental data because of the many uncertainties associated with the empirical models used to analyze nonleptonic D decays. Finally, the expressions for $\lambda_{K^-\pi^+}$ and $\overline{\lambda}_{K^+\pi^-}$ are obtainable from Eq. (5.2) as

$$
\begin{aligned}
\lambda_{K^-\pi^+} &\approx -\lambda^2 h_{K\pi} \left|\frac{q}{p}\right| \exp[i(\delta_{K\pi} + 2\phi)],\\
\overline{\lambda}_{K^+\pi^-} &\approx -\lambda^2 h_{K\pi} \left|\frac{p}{q}\right| \exp[i(\delta_{K\pi} - 2\phi)],
\end{aligned}
\tag{5.3}
$$

where we have used the notation of q/p given in Eq. (2.2).

A. Incoherent D decays to $K^\pm \pi^\mp$

A lot of attention has been paid to the time distributions of incoherent D decays to $K^\pm \pi^\mp$ (see, e.g., Refs. [15,23,34]). In particular, Browder and Pakvasa have given a quite detailed analysis of the implications of CP violation and final-state interactions in the search for D^0-$\overline{D^0}$ mixing from $D^0_{\rm phys}(t) \to K^+ \pi^-$ and $\overline{D^0_{\rm phys}}(t) \to K^- \pi^+$ [34]. Our subsequent discussions are complementary to their work on three points: (a) The CP-violating asymmetry between $D^0_{\rm phys}(t) \to K^- \pi^+$ and $\overline{D^0_{\rm phys}}(t) \to K^+ \pi^-$ is analyzed, (b) the different effects of x_D and y_D on $D^0_{\rm phys}(t) \to K^\pm \pi^\mp$ and $\overline{D^0_{\rm phys}}(t) \to K^\pm \pi^\mp$ are explored in detail, and (c) the time-independent measurements of these decay modes are considered.

Up to $O(x_D^2)$, $O(y_D^2)$, and $O(\lambda^4)$ for every distinctive term, the decay rates of D to $K^\pm \pi^\mp$ can be directly read off from Eqs. (2.22) and (2.23):

$$
\begin{aligned}
R(D^0_{\rm phys}(t) \to K^- \pi^+) &\propto |A_{K^-\pi^+}|^2 \exp(-\Gamma t) \left\{ 1 + \lambda^2 h_{K\pi} \left|\frac{q}{p}\right| [y_D \cos(\delta_{K\pi} + 2\phi) + x_D \sin(\delta_{K\pi} + 2\phi)] \Gamma t - \tfrac{1}{4}(x_D^2 - y_D^2) \Gamma^2 t^2 \right\},\\
R(\overline{D^0_{\rm phys}}(t) \to K^+ \pi^-) &\propto |A_{K^-\pi^+}|^2 \exp(-\Gamma t) \left\{ 1 + \lambda^2 h_{K\pi} \left|\frac{p}{q}\right| [y_D \cos(\delta_{K\pi} - 2\phi) + x_D \sin(\delta_{K\pi} - 2\phi)] \Gamma t - \tfrac{1}{4}(x_D^2 - y_D^2) \Gamma^2 t^2 \right\}
\end{aligned}
\tag{5.4}
$$

and

$$
\begin{aligned}
R(D^0_{\rm phys}(t) \to K^+ \pi^-) &\propto |A_{K^-\pi^+}|^2 \exp(-\Gamma t) \left\{ \lambda^4 h_{K\pi}^2 + \lambda^2 h_{K\pi} \left|\frac{q}{p}\right| [y_D \cos(\delta_{K\pi} - 2\phi) - x_D \sin(\delta_{K\pi} - 2\phi)] \Gamma t + \frac{r}{2} \Gamma^2 t^2 \right\},\\
R(\overline{D^0_{\rm phys}}(t) \to K^- \pi^+) &\propto |A_{K^-\pi^+}|^2 \exp(-\Gamma t) \left\{ \lambda^4 h_{K\pi}^2 + \lambda^2 h_{K\pi} \left|\frac{p}{q}\right| [y_D \cos(\delta_{K\pi} + 2\phi) - x_D \sin(\delta_{K\pi} + 2\phi)] \Gamma t + \frac{\overline{r}}{2} \Gamma^2 t^2 \right\},
\end{aligned}
\tag{5.5}
$$

where r and $\overline{r}$ have been presented in Eq. (3.2). To probe CP violation and D^0-$\overline{D^0}$ mixing, the following two types of measurables can be analyzed in experiments:

(1) The CP-violating asymmetry

$$\mathcal{A}_{K\pi}(t) \equiv \frac{R(D^0_{\rm phys}(t) \rightarrow K^-\pi^+) - R(\overline{D^0_{\rm phys}}(t) \rightarrow K^+\pi^-)}{R(D^0_{\rm phys}(t) \rightarrow K^-\pi^+) + R(\overline{D^0_{\rm phys}}(t) \rightarrow K^+\pi^-)}. \tag{5.6}$$

Explicitly, we get

$$\mathcal{A}_{K\pi}(t) \approx -\lambda^2 h_{K\pi}[\hat{\Delta}_D \cos(2\phi)(y_D \cos\delta_{K\pi} + x_D \sin\delta_{K\pi}) + \sin(2\phi)(y_D \sin\delta_{K\pi} - x_D \cos\delta_{K\pi})]\Gamma t, \tag{5.7}$$

where the observable $\hat{\Delta}_D$ has been defined before [see Eq. (4.6)]. One can see that $\mathcal{A}_{K\pi}(t)$ are composed of two sources of CP-violating effects, that in D^0-$\overline{D^0}$ mixing [proportional to $\hat{\Delta}_D$] and that from the interplay of decay and mixing [proportional to $\sin(2\phi)$]. The magnitude of $\mathcal{A}_{K\pi}(t)$ is constrained by both the DCSD amplitude $\lambda^2 h_{K\pi}$ and the D^0-$\overline{D^0}$ mixing parameters x_D and y_D. Since $\hat{\Delta}_D \leqslant 10^{-2}$ is expected, $\mathcal{A}_{K\pi}(t)$ can reach the percent level only when the $\sin(2\phi)$ term is significantly enhanced by new physics. In new physics scenarios with $y_D \ll x_D$ [23,24], we get $\mathcal{A}_{K\pi}(t) \approx \lambda^2 h_{K\pi} x_D \sin(2\phi)\cos\delta_{K\pi}\Gamma t$ as a safe approximation. Taking $h_{K\pi} \approx 1.8$ (implied by $|\rho_{K^-\pi^+}|^2 \approx 7.7 \times 10^{-3}$ [9]), $x_D < 0.086$, and $\delta_{K\pi} \geqslant 5^0$, one finds the restriction $\mathcal{A}_{K\pi}(t) < 7.5 \times 10^{-3}\Gamma t$.

(2) The combined decay rate

$$R_{K\pi}(t) \equiv R(D^0_{\rm phys}(t) \rightarrow K^+\pi^-) + R(\overline{D^0_{\rm phys}}(t) \rightarrow K^-\pi^+). \tag{5.8}$$

By use of Eq. (5.5), we obtain

$$R_{K\pi}(t) \propto |A_{K^-\pi^+}|^2 \exp(-\Gamma t)\{2\lambda^4 h^2_{K\pi} + r_D \Gamma^2 t^2 + 2\lambda^2 h_{K\pi}[\cos(2\phi)(y_D \cos\delta_{K\pi} - x_D \sin\delta_{K\pi}) - \hat{\Delta}_D \sin(2\phi)(y_D \sin\delta_{K\pi} + x_D \cos\delta_{K\pi})]\Gamma t\}, \tag{5.9}$$

where $r_D = (r + \overline{r})/2$ defined in Eq. (3.3) has been used. The three terms of $R_{K\pi}(t)$, which have different time distributions, come, respectively, from DCSD, D^0-$\overline{D^0}$ mixing, and the interplay of these two effects. Thus the detection of $R_{K\pi}(t)$ can determine the D^0-$\overline{D^0}$ mixing rate r_D and distinguish it from the DCSD contribution. If $|\phi|$ is not large (e.g., in the standard model), the interference term will be dominated by $\cos(2\phi)(y_D \cos\delta_{K\pi} - x_D \sin\delta_{K\pi})$ due to the smallness of $\hat{\Delta}_D$. In this case, information about y_D might be obtainable if the contribution of x_D to the interference term is suppressed by small $\delta_{K\pi}$. To justify the possible magnitude of ϕ, however, one has to combine the measurements of $\mathcal{A}_{K\pi}(t)$ and $R_{K\pi}(t)$.

Now we take a brief look at the time-independent decay rates of $D^0_{\rm phys}/\overline{D^0_{\rm phys}} \rightarrow K^\pm \pi^\mp$. With the help of Eqs. (2.24) and (2.25), we obtain

$$\mathcal{A}_{K\pi} \equiv \frac{R(D^0_{\rm phys} \rightarrow K^-\pi^+) - R(\overline{D^0_{\rm phys}} \rightarrow K^+\pi^-)}{R(D^0_{\rm phys} \rightarrow K^-\pi^+) + R(\overline{D^0_{\rm phys}} \rightarrow K^+\pi^-)} \approx -\lambda^2 h_{K\pi}[\hat{\Delta}_D \cos(2\phi)(y_D \cos\delta_{K\pi} + x_D \sin\delta_{K\pi}) + \sin(2\phi)(y_D \sin\delta_{K\pi} - x_D \cos\delta_{K\pi})]; \tag{5.10}$$

i.e., $\mathcal{A}_{K\pi}$ is approximately equal to the value of $\mathcal{A}_{K\pi}(t)$ at $t = 1/\Gamma$. Similarly, one can calculate another CP-violating asymmetry

$$\overline{\mathcal{A}_{K\pi}} \equiv \frac{R(D^0_{\rm phys} \rightarrow K^+\pi^-) - R(\overline{D^0_{\rm phys}} \rightarrow K^-\pi^+)}{R(D^0_{\rm phys} \rightarrow K^+\pi^-) + R(\overline{D^0_{\rm phys}} \rightarrow K^-\pi^+)} \approx \lambda^2 h_{K\pi}[-\hat{\Delta}_D \cos(2\phi)(y_D \cos\delta_{K\pi} - x_D \sin\delta_{K\pi}) + \sin(2\phi)(y_D \sin\delta_{K\pi} + x_D \cos\delta_{K\pi})] \times \{\lambda^4 h^2_{K\pi} + r_D + \lambda^2 h_{K\pi}[\cos(2\phi)(y_D \cos\delta_{K\pi} - x_D \sin\delta_{K\pi}) - \hat{\Delta}_D \sin(2\phi)(y_D \sin\delta_{K\pi} + x_D \cos\delta_{K\pi})]\}^{-1}. \tag{5.11}$$

Taking $\hat{\Delta}_D \approx 0$ and $\sin(2\phi) \approx \pm 1$, for example, we find that $\overline{\mathcal{A}_{K\pi}}$ may be significant:

$$\overline{\mathcal{A}_{K\pi}} \approx \pm \frac{\lambda^2 h_{K\pi}(y_D \sin\delta_{K\pi} + x_D \cos\delta_{K\pi})}{\lambda^4 h^2_{K\pi} + r_D}. \tag{5.12}$$

Because of the suppressed rates of $D^0_{\rm phys} \rightarrow K^+\pi^-$ and $\overline{D^0_{\rm phys}} \rightarrow K^-\pi^+$, however, the measurement of $\overline{\mathcal{A}_{K\pi}}$ will be a stiff experimental challenge.

B. Coherent D decays to $K^{\pm}\pi^{\mp}$

At the $\psi(3.77)$ and $\psi(4.14)$ resonances, the $K^{\pm}\pi^{\mp}$ events may come from the coherent decays of $(D^0_{\rm phys}\overline{D^0_{\rm phys}})_C$ pairs. The flavor of one D meson decaying to $K^{\pm}\pi^{\mp}$ can be tagged by detecting the other D decaying to the semileptonic states $l^{\pm}X^{\mp}$. The overall rates for such joint decay events, up to $O(x_D^2)$, $O(y_D^2)$, or $O(\lambda^4)$ for every distinctive term, are obtainable from Eqs. (2.26) and (2.27) as[3]

$$R(l^-,K^-\pi^+)_-\propto|A_l|^2|A_{K^-\pi^+}|^2(2-x_D^2+y_D^2),$$

$$R(l^+,K^+\pi^-)_-\propto|A_l|^2|A_{K^-\pi^+}|^2(2-x_D^2+y_D^2),$$

$$R(l^-,K^+\pi^-)_-\propto|A_l|^2|A_{K^-\pi^+}|^2\left[2\lambda^4h_{K\pi}^2+(x_D^2+y_D^2)\left|\frac{q}{p}\right|^2\right],$$

$$R(l^+,K^-\pi^+)_-\propto|A_l|^2|A_{K^-\pi^+}|^2\left[2\lambda^4h_{K\pi}^2+(x_D^2+y_D^2)\left|\frac{p}{q}\right|^2\right] \tag{5.13}$$

and

$$R(l^-,K^-\pi^+)_+\propto|A_l|^2|A_{K^-\pi^+}|^2\left\{2-3(x_D^2-y_D^2)+4\lambda^2h_{K\pi}\left|\frac{q}{p}\right|[y_D\cos(\delta_{K\pi}+2\phi)+x_D\sin(\delta_{K\pi}+2\phi)]\right\},$$

$$R(l^+,K^+\pi^-)_+\propto|A_l|^2|A_{K^-\pi^+}|^2\left\{2-3(x_D^2-y_D^2)+4\lambda^2h_{K\pi}\left|\frac{p}{q}\right|[y_D\cos(\delta_{K\pi}-2\phi)+x_D\sin(\delta_{K\pi}-2\phi)]\right\},$$

$$R(l^-,K^+\pi^-)_+\propto|A_l|^2|A_{K^-\pi^+}|^2\left\{2\lambda^4h_{K\pi}^2+3(x_D^2+y_D^2)\left|\frac{q}{p}\right|^2+4\lambda^2h_{K\pi}\left|\frac{q}{p}\right|[y_D\cos(\delta_{K\pi}-2\phi)-x_D\sin(\delta_{K\pi}-2\phi)]\right\},$$

$$R(l^+,K^-\pi^+)_+\propto|A_l|^2|A_{K^-\pi^+}|^2\left\{2\lambda^4h_{K\pi}^2+3(x_D^2+y_D^2)\left|\frac{p}{q}\right|^2+4\lambda^2h_{K\pi}\left|\frac{p}{q}\right|[y_D\cos(\delta_{K\pi}+2\phi)-x_D\sin(\delta_{K\pi}+2\phi)]\right\}. \tag{5.14}$$

Some discussion of these results is in order.

(1) To an excellent degree of accuracy, we have

$$R(l^-,K^-\pi^+)_-\approx R(l^+,K^+\pi^-)_-. \tag{5.15}$$

The joint decay rates $R(l^{\mp},K^{\pm}\pi^{\mp})_-$ can be normalized by $R(l^{\mp},K^{\mp}\pi^{\pm})_-$, and the resultant rate difference or sum reads

$$S_-^{(-)}\equiv\frac{R(l^-,K^+\pi^-)_-}{R(l^-,K^-\pi^+)_-}-\frac{R(l^+,K^-\pi^+)_-}{R(l^+,K^+\pi^-)_-}\approx-2r_D\Delta_D,$$

$$S_-^{(+)}\equiv\frac{R(l^-,K^+\pi^-)_-}{R(l^-,K^-\pi^+)_-}+\frac{R(l^+,K^-\pi^+)_-}{R(l^+,K^+\pi^-)_-}\approx2\lambda^4h_{K\pi}^2+2r_D, \tag{5.16}$$

where r_D and Δ_D have been given in Eqs. (3.3) and (3.6), respectively. Observation of the CP-violating asymmetry $S_-^{(-)}$ may be practically impossible due to the smallness of Δ_D and r_D. However, $S_-^{(+)}$ is expected to be measurable at a τ-charm factory running on the $\psi(3.77)$ resonance. As we shall show in the next subsection, r_D can be extracted from the joint decay rates $R(K^+\pi^-,K^+\pi^-)_-$ and $R(K^-\pi^+,K^+\pi^-)_-$; thus, a comparison of this measurement with that for $S_-^{(+)}$ will separately determine the magnitudes of D^0-$\overline{D^0}$ mixing and DCSD. This idea is interesting in that the relevant measurements are time independent and the involved decay modes are only $D^0/\overline{D^0}\rightarrow K^{\pm}\pi^{\mp}$.

(2) It is easy to obtain the rate asymmetry

$$\frac{R(l^-,K^-\pi^+)_+-R(l^+,K^+\pi^-)_+}{R(l^-,K^-\pi^+)_++R(l^+,K^+\pi^-)_+}\approx2\mathcal{A}_{K\pi}, \tag{5.17}$$

where $\mathcal{A}_{K\pi}$ has been given in Eq. (5.10). Normalizing the joint decay rates $R(l^{\mp},K^{\pm}\pi^{\mp})_+$ by $R(l^{\mp},K^{\mp}\pi^{\pm})_+$, we get

$$S_+^{(-)}\equiv\frac{R(l^-,K^+\pi^-)_+}{R(l^-,K^-\pi^+)_+}-\frac{R(l^+,K^-\pi^+)_+}{R(l^+,K^+\pi^-)_+}$$
$$\approx-6r_D\Delta_D-4\lambda^2h_{K\pi}$$
$$\times[\hat{\Delta}_D\cos(2\phi)(y_D\cos\delta_{K\pi}-x_D\sin\delta_{K\pi})-\sin(2\phi)$$
$$\times(y_D\sin\delta_{K\pi}+x_D\cos\delta_{K\pi})],$$

$$S_+^{(+)}\equiv\frac{R(l^-,K^+\pi^-)_+}{R(l^-,K^-\pi^+)_+}+\frac{R(l^+,K^-\pi^+)_+}{R(l^+,K^+\pi^-)_+}$$
$$\approx2\lambda^4h_{K\pi}^2+6r_D+4\lambda^2h_{K\pi}$$

[3]The formulas with the assumption of $y_D\ll x_D$ and $|q/p|=1$ have been given in Ref. [14].

$$\times[\cos(2\phi)(y_D\cos\delta_{K\pi}-x_D\sin\delta_{K\pi})-\hat{\Delta}_D\sin(2\phi)$$
$$\times(y_D\sin\delta_{K\pi}+x_D\cos\delta_{K\pi})]. \quad (5.18)$$

From Eqs. (5.11), (5.16), and (5.18), one can see the following relation among $S_{\pm}^{(\pm)}$ and $\mathcal{A}_{K\pi}$:

$$\overline{\mathcal{A}_{K\pi}}\approx\frac{S_{+}^{(-)}-3S_{-}^{(-)}}{S_{+}^{(+)}-4r_D+S_{-}^{(+)}}. \quad (5.19)$$

This result could be tested if the data on all six measurables were available.

(3) To give one a feeling of the approximate numbers to be expected, we roughly estimate the magnitudes of the above-mentioned observables by assuming $\Delta_D=\hat{\Delta}_D=0$ and $y_D\ll x_D$. Taking the semileptonic decay mode serving for flavor tagging to be $D^0\to K^- e^+\nu_e$, we have its branching ratio $B(D^0\to K^- e^+\nu_e)\approx 3.8\%$ [1]. In addition, the current data give $B(D^0\to K^-\pi^+)\approx 4.01\%$ [1]. Then $R(l^+,K^+\pi^-)_\pm$ and $R(l^-,K^-\pi^+)_\pm$ are at the level 10^{-3} or so, while $R(l^-,K^+\pi^-)_\pm$ and $R(l^+,K^-\pi^+)_\pm$ may be of the order 10^{-5} if we input $x_D\sim 0.06$. Within the experimental capabilities of a τ-charm factory, it is possible to measure the latter four decay rates to an acceptable degree of accuracy with about 10^7 $D^0\bar{D}^0$ events [14]. Furthermore, the upper bounds of the CP asymmetries $\mathcal{A}_{K\pi}$ and $S_{+}^{(-)}$ can be obtained by use of the experimental results $x_D<0.086$ and $|\rho_{K^-\pi^+}|^2\approx 0.77\%$ [1,9]. Taking $\cos\delta_{K\pi}=1$ and $\sin(2\phi)=\pm 1$, we get $|\mathcal{A}_{K\pi}|<0.008$ and $|S_{+}^{(-)}|<0.03$. In the assumption of perfect detectors or 100% tagging efficiencies, one needs about 10^8 $D^0\bar{D}^0$ events to uncover $|S_{+}^{(-)}|\sim 0.01$ at the level of three standard deviations or to measure $2|\mathcal{A}_{K\pi}|\sim 0.005$ in Eq. (5.17) at the level of one standard deviation. Accumulation of so many events is of course a serious challenge to all types of experimental facilities for charm physics, but it should be achievable in the second-round experiments of a τ-charm factory.

C. Ratios of $R(K^\pm\pi^\mp,K^\pm\pi^\mp)_C$ to $R(K^\pm\pi^\mp,K^\mp\pi^\pm)_C$

It has been pointed out that the coherent decays $(D^0_{\rm phys}\bar{D}^0_{\rm phys})_C\to(K^\pm\pi^\mp)(K^\pm\pi^\mp)$ can be used to search for D^0-$\bar{D}^0$ mixing and to separate it from the DCSD effect [35]. The relevant measurables are

$$r_C^{+-}\equiv\frac{R(K^+\pi^-,K^+\pi^-)_C}{R(K^-\pi^+,K^+\pi^-)_C},\qquad r_C^{-+}\equiv\frac{R(K^-\pi^+,K^-\pi^+)_C}{R(K^-\pi^+,K^+\pi^-)_C}. \quad (5.20)$$

Since in previous calculations the effects of CP violation or nonvanishing $\delta_{K\pi}$ on $r_C^{\pm\mp}$ were neglected, it is worth having a recalculation for these observables without special approximations.

By use of Eqs. (2.26), (2.27), and (5.3), we obtain

$$R(K^-\pi^+,K^+\pi^-)_-\propto|A_{K^-\pi^+}|^4[2-x_D^2+y_D^2-4\lambda^4h_{K\pi}^2\cos(2\delta_{K\pi})],$$

$$R(K^+\pi^-,K^-\pi^+)_-\propto|A_{K^-\pi^+}|^4[2-x_D^2+y_D^2-4\lambda^4h_{K\pi}^2\cos(2\delta_{K\pi})],$$

$$R(K^-\pi^+,K^-\pi^+)_-\propto|A_{K^-\pi^+}|^4(x_D^2+y_D^2)\left|\frac{p}{q}\right|^2,$$

$$R(K^+\pi^-,K^+\pi^-)_-\propto|A_{K^-\pi^+}|^4(x_D^2+y_D^2)\left|\frac{q}{p}\right|^2, \quad (5.21)$$

and

$$R(K^-\pi^+,K^+\pi^-)_+\propto|A_{K^-\pi^+}|^4\left\{2-3x_D^2+3y_D^2+4\lambda^4h_{K\pi}^2\cos(2\delta_{K\pi})+4\lambda^2h_{K\pi}\left|\frac{q}{p}\right|[y_D\cos(\delta_{K\pi}+2\phi)+x_D\sin(\delta_{K\pi}+2\phi)]\right.$$
$$\left.+4\lambda^2h_{K\pi}\left|\frac{p}{q}\right|[y_D\cos(\delta_{K\pi}-2\phi)+x_D\sin(\delta_{K\pi}-2\phi)]\right\},$$

$$R(K^+\pi^-,K^-\pi^+)_+\propto|A_{K^-\pi^+}|^4\left\{2-3x_D^2+3y_D^2+4\lambda^4h_{K\pi}^2\cos(2\delta_{K\pi})+4\lambda^2h_{K\pi}\left|\frac{q}{p}\right|[y_D\cos(\delta_{K\pi}+2\phi)+x_D\sin(\delta_{K\pi}+2\phi)]\right.$$
$$\left.+4\lambda^2h_{K\pi}\left|\frac{p}{q}\right|[y_D\cos(\delta_{K\pi}-2\phi)+x_D\sin(\delta_{K\pi}-2\phi)]\right\},$$

$$R(K^-\pi^+,K^-\pi^+)_+\propto|A_{K^-\pi^+}|^4\left\{3(x_D^2+y_D^2)\left|\frac{p}{q}\right|^2+8\lambda^4h_{K\pi}^2+8\lambda^2h_{K\pi}\left|\frac{p}{q}\right|[y_D\cos(\delta_{K\pi}+2\phi)-x_D\sin(\delta_{K\pi}+2\phi)]\right\},$$

$$R(K^+\pi^-,K^+\pi^-)_+ \propto |A_{K^-\pi^+}|^4 \left\{ 3(x_D^2+y_D^2)\left|\frac{q}{p}\right|^2 + 8\lambda^4 h_{K\pi}^2 + 8\lambda^2 h_{K\pi}\left|\frac{q}{p}\right| [y_D\cos(\delta_{K\pi}-2\phi) - x_D\sin(\delta_{K\pi}-2\phi)] \right\} \tag{5.22}$$

up to $O(x_D^2)$, $O(y_D^2)$, or $O(\lambda^4)$. Clearly $R(K^-\pi^+,K^+\pi^-)_C \approx R(K^+\pi^-,K^-\pi^+)_C$ holds to an excellent degree of accuracy. As a consequence, the ratios $r_C^{\pm\mp}$ are given by

$$r_-^{+-} \approx \frac{x_D^2+y_D^2}{2}\left|\frac{q}{p}\right|^2, \quad r_-^{-+} \approx \frac{x_D^2+y_D^2}{2}\left|\frac{p}{q}\right|^2 \tag{5.23}$$

and

$$\begin{aligned} r_+^{+-} &\approx 3r_-^{+-} + 4\lambda^4 h_{K\pi}^2 + 4\lambda^2 h_{K\pi}\left|\frac{q}{p}\right| \\ &\quad \times [y_D\cos(\delta_{K\pi}-2\phi) - x_D\sin(\delta_{K\pi}-2\phi)], \\ r_+^{-+} &\approx 3r_-^{-+} + 4\lambda^4 h_{K\pi}^2 + 4\lambda^2 h_{K\pi}\left|\frac{p}{q}\right| \\ &\quad \times [y_D\cos(\delta_{K\pi}+2\phi) - x_D\sin(\delta_{K\pi}+2\phi)]. \end{aligned} \tag{5.24}$$

One can see that r_-^{+-} and r_-^{-+} are approximately equivalent to r and $\bar{r}$ obtained in Eq. (3.2). The difference between r_-^{+-} and r_-^{-+} measures CP violation in D^0-$\overline{D^0}$ mixing, and the sum of them amounts approximately to r_D given in Eq. (3.3). The DCSD effect on r_+^{+-} and r_+^{-+} is significant and nonnegligible, but its magnitude can be isolated from the difference $r_+^{+-}-3r_-^{+-}$ or $r_+^{-+}-3r_-^{-+}$. In addition, we find

$$\begin{aligned} r_+^{-+} - r_+^{+-} &\approx 8\lambda^2 h_{K\pi}[\hat{\Delta}_D\cos(2\phi)(y_D\cos\delta_{K\pi} - x_D\sin\delta_{K\pi}) \\ &\quad - \sin(2\phi)(y_D\sin\delta_{K\pi} + x_D\cos\delta_{K\pi})]. \end{aligned} \tag{5.25}$$

Comparing this result with those derived in Eq. (5.18), one gets

$$r_+^{-+} - r_+^{+-} \approx -2(6r_D\Delta_D + S_+^{(-)}). \tag{5.26}$$

Such a CP-violating signal might be detectable at a τ-charm factory running on the $\psi(4.14)$ resonance.

Although the above discussions concentrate only on $D^0/\overline{D^0} \to K^\pm\pi^\mp$, similar results can be obtained for some other decay modes taking place via the same quark diagrams, such as $D^0 \to K^\pm\rho^\mp$, $K^{*\pm}\pi^\mp$ and their flavor-conjugate processes. All these channels are expected to have the same weak interactions, but their final-state interactions may be different from one another (e.g., $\delta_{K\pi} \neq \delta_{K\rho}$). If the SU(3)-breaking effects in $D^0/\overline{D^0} \to (K^\pm,K^{*\pm}) + (\pi^\mp,\rho^\mp,a_1^\mp,\text{etc.})$ are not so significant that all the strong phase shifts lie in the same quadrant as $\delta_{K\pi}$, then a sum over these modes is possible to increase the number of decay events in statistics, with few dilution effects on the signal of D^0-$\overline{D^0}$ mixing and CP violation.

VI. *CP*-FORBIDDEN DECAYS

We now consider CP-forbidden transitions of the type

$$(D^0_{\rm phys}\overline{D^0_{\rm phys}})_\pm \to (f_1 f_2)_\mp , \tag{6.1}$$

where the $D^0\overline{D^0}$ pair with definite CP parity can be coherently produced on the $\psi(3.77)$ or $\psi(4.14)$ resonance, and f_1 and f_2 denote the CP eigenstates with the same or opposite CP parity. It is worth remarking that for such decay modes the CP-violating signals can be established by detecting the joint decay rates other than the decay-rate asymmetries. In practice, this implies that neither flavor tagging for the initial D mesons nor time-dependent measurements of the whole decay processes are necessary. The joint decay rate $R(f_1,f_2)_C$ and its analytical approximation have been presented in Eqs. (2.21) and (2.26). For simplicity and illustration, here we concentrate mainly on the CP-forbidden decays $(D^0_{\rm phys}\overline{D^0_{\rm phys}})_- \to (f_1 f_2)_+$, such as $(f_1 f_2)_+ = (K^+K^-)(\pi^+\pi^-)$ and $(K^+K^-)(K^+K^-)$. The case $(D^0_{\rm phys}\overline{D^0_{\rm phys}})_+ \to (f_1 f_2)_-$ will be briefly discussed by taking $f_1 = K_S\pi^0$ and $f_2 = K_L\pi^0$, for example.

By use of the quantities $\mathcal{U}_f$, $\mathcal{V}_f$, and $\mathcal{W}_f$ defined in Eq. (4.2), the joint decay rate $R(f_1,f_2)_-$ can be written as

$$\begin{aligned} R(f_1,f_2)_- &\propto |A_{f_1}|^2|A_{f_2}|^2(1+|\lambda_{f_1}|^2)(1+|\lambda_{f_2}|^2)\left|\frac{p}{q}\right|^2 \\ &\quad \times \left[\frac{1}{1-y_D^2}(1-\mathcal{W}_{f_1}\mathcal{W}_{f_2}) \right. \\ &\quad \left. - \frac{1}{1+x_D^2}(\mathcal{U}_{f_1}\mathcal{U}_{f_2} + \mathcal{V}_{f_1}\mathcal{V}_{f_2}) \right]. \end{aligned} \tag{6.2}$$

Here we assume f_1 and f_2 to be two CP eigenstates with the same CP parity. CP conservation requires $\mathcal{V}_{f_1} = \mathcal{V}_{f_2} = 0$, $\mathcal{U}_{f_1} = \mathcal{U}_{f_2} = 0$, and $\mathcal{W}_{f_1} = \mathcal{W}_{f_2} = \pm 1$; then, we get $R(f_1,f_2)_- = 0$. Thus nonvanishing $R(f_1,f_2)_-$ is a clean signal of CP violation. In the special case $f_1 = f_2 \equiv f$, one finds

$$\begin{aligned} R(f,f)_- &\propto |A_f|^4(1+|\lambda_f|^2)^2\left|\frac{p}{q}\right|^2\left(\frac{1}{1-y_D^2} - \frac{1}{1+x_D^2}\right) \\ &\quad \times (\mathcal{U}_f^2 + \mathcal{V}_f^2). \end{aligned} \tag{6.3}$$

This result can be straightforwardly obtained from Eq. (6.2) with the help of Eq. (4.3). As discussed before, $\mathcal{U}_f$ is composed of the CP asymmetry in D^0-$\overline{D^0}$ mixing and that in the direct transition amplitudes of D decays, while $\mathcal{V}_f$ signifies the CP asymmetry induced by the interplay of decay and D^0-$\overline{D^0}$ mixing. Because of the smallness of $\mathcal{U}_f$, $\mathcal{V}_f$, x_D, and y_D, we believe that $R(f,f)_-$ must be significantly suppressed.

In comparison with $R(f,f)_-$, the joint decay rate $R(f,f)_+$ is not CP forbidden:

$$R(f,f)_+ \propto |A_f|^4(1+|\lambda_f|^2)^2\left|\frac{p}{q}\right|^2\left[\frac{1+y_D^2}{(1-y_D^2)^2}(1+\mathcal{W}_f)^2 - \frac{4y_D}{(1-y_D^2)^2}\mathcal{W}_f - \frac{1-x_D^2}{(1+x_D^2)^2}(\mathcal{U}_f^2-\mathcal{V}_f^2) - \frac{4x_D}{(1+x_D^2)^2}\mathcal{U}_f\mathcal{V}_f\right]. \tag{6.4}$$

Approximately, we obtain

$$\frac{R(f,f)_-}{R(f,f)_+} \approx \frac{(x_D^2+y_D^2)(\mathcal{U}_f^2+\mathcal{V}_f^2)}{1+\mathcal{W}_f^2-4y_D\mathcal{W}_f}. \tag{6.5}$$

This relation can in principle be tested for $f=K^+K^-$, etc., at the $\psi(4.14)$ resonance in the second-round experiments of a τ-charm factory, if the rate of D^0-$\overline{D^0}$ mixing is at the detectable level.

In the neglect of CP violation in K^0-$\overline{K^0}$ mixing, the states $K_S\pi^0$ and $K_L\pi^0$ are two CP eigenstates with opposite CP parity. Thus the process $(D^0_{\rm phys}\overline{D^0_{\rm phys}})_+ \rightarrow (K_S\pi^0)(K_L\pi^0)$ should be CP forbidden. As a good approximation, we have $|A_{K_L\pi^0}| \approx |A_{K_S\pi^0}|$ and $\rho_{K_L\pi^0} \approx -\rho_{K_S\pi^0}$ [see Eq. (4.22)]. Then the joint decay rate with $C=+$ turns out to be

$$R(K_L\pi^0,K_S\pi^0)_+ \propto |A_{K_S\pi^0}|^4(1+|\lambda_{K_S\pi^0}|^2)^2\left|\frac{p}{q}\right|^2\left[\frac{1+y_D^2}{(1-y_D^2)^2} - \frac{1-x_D^2}{(1+x_D^2)^2}\right](\mathcal{U}_{K_S\pi^0}^2+\mathcal{V}_{K_S\pi^0}^2). \tag{6.6}$$

Using the approximate results in Eq. (4.24) and taking $|\epsilon|\approx 0$, we obtain a simpler expression for the equation above:

$$R(K_L\pi^0,K_S\pi^0)_+ \propto 6|A_{K_S\pi^0}|^4(1+w)(x_D^2+y_D^2) \times[\hat{\Delta}_D^2+\sin^2(2\phi)], \tag{6.7}$$

where w has been defined in Eq. (3.3). In contrast, it is easy to check from Eq. (6.3) that

$$R(K_L\pi^0,K_L\pi^0)_- \approx R(K_S\pi^0,K_S\pi^0)_- \approx \tfrac{1}{3}R(K_L\pi^0,K_S\pi^0)_+ . \tag{6.8}$$

Note that CP violation in D^0-$\overline{D^0}$ mixing (i.e., $\hat{\Delta}_D$) might be negligibly small; thus, the dominant signal of CP violation in $R(K_S\pi^0,K_S\pi^0)_-$ or $R(K_L\pi^0,K_S\pi^0)_+$ could come from the mixing phase ϕ enhanced by new physics. In this sense, it is worthwhile to experimentally search for the above-mentioned CP-forbidden transitions.

VII. SUMMARY

To meet various delicate experiments in the near future at fixed target machines, B-meson factories, and τ-charm factories, we have made a further study of the phenomenology of D^0-$\overline{D^0}$ mixing and CP violation in neutral D-meson decays. The generic formulas for the time-dependent and time-integrated decay rates of both coherent and incoherent $D^0\overline{D^0}$ events were derived, and their approximate expressions up to the accuracy of $O(x_D^2)$ and $O(y_D^2)$ were presented. A variety of D^0-$\overline{D^0}$ mixing and CP-violating signals was analyzed in detail for neutral D decays to the semileptonic states, the nonleptonic CP eigenstates, the nonleptonic non-CP eigenstates, and the CP-forbidden states.

In particular, we have shown that it is possible to separately determine the magnitudes of x_D and y_D through precise measurements of the dilepton events of coherent $D^0\overline{D^0}$ decays on the $\psi(4.14)$ resonance at a τ-charm factory. We gave a detailed analysis of D^0-$\overline{D^0}$ mixing signals and DCSD effects in the time-dependent and time-independent decays $D^0/\overline{D^0}\rightarrow K^\pm\pi^\mp$. It is found that some constraints on x_D and y_D can be achieved in both fixed target and τ-charm factory experiments, and the mixing and DCSD effects are distinguishable from each other. Taking CP violation and final-state interactions into account, we recalculated the joint decay rates of coherent $D^0\overline{D^0}$ pairs to $(K^\pm\pi^\mp)(K^\pm\pi^\mp)$, which are useful for the time-independent determination of r_D and DCSD amplitudes. Special attention has been paid to the D^0-$\overline{D^0}$ mixing signals in the decay modes $D^0/\overline{D^0}\rightarrow K_{S,L}+\pi^0$, etc. We pointed out that a model-independent restriction on x_D and y_D should be obtainable from the time distributions of such decay modes.

CP violation in D^0-$\overline{D^0}$ mixing can be well constrained in the semileptonic decays of coherent or incoherent $D^0\overline{D^0}$ events. In addition to this source of CP asymmetry, we have shown that both direct CP asymmetry in the transition amplitudes of D decays and the indirect CP asymmetry arising from the interplay of decay and D^0-$\overline{D^0}$ mixing can also manifest themselves in neutral D decays to hadronic CP eigenstates. These different CP-violating signals usually have different time distributions in the decay rates; thus, they are possible to be distinguished from one another. In particular, direct CP violation can be cleanly probed in the coherent $(D^0\overline{D^0})_-$ decays to a CP eigenstate plus a semileptonic state on the $\psi(3.77)$ or $\psi(4.14)$ resonance. For the decay modes with K^0-$\overline{K^0}$ mixing in the final states, however, the CP asymmetry induced by the mixing parameter ϵ may be non-negligible and even dominant over the direct CP-violating signal from the charm quark transitions. Taking $D\rightarrow KK$, for example, we illustrated the significant effects of final-state interactions on CP violation. Different from those neutral D decays to CP eigenstates, $D^0/\overline{D^0}\rightarrow K^\pm\pi^\mp$ are expected to have no direct CP asymmetries. Although indirect CP-violating effects exist in such processes, they are suppressed to some extent by the DCSD amplitudes. We also discussed the CP-forbidden transitions on the $\psi(3.77)$ and $\psi(4.14)$ resonances. A search for CP-forbidden modes such as $(D^0\overline{D^0})_-\rightarrow(K^+K^-)(\pi^+\pi^-)$ and $(D^0\overline{D^0})_+\rightarrow(K_S\pi^0)(K_L\pi^0)$ is worthwhile in future experiments of charm physics.

Throughout our calculations CPT symmetry in the D^0-$\overline{D^0}$ mixing matrix has been assumed. Also the $\Delta Q=\Delta C$ rule was assumed to hold in most cases, but the effects of

$\Delta Q=-\Delta C$ transitions on D^0-$\overline{D^0}$ mixing and CP violation were briefly discussed in Sec. III B. Because of the smallness of x_D and y_D, it will be very difficult to accurately test the $\Delta Q=\Delta C$ rule and CPT invariance in the D^0-$\overline{D^0}$ system. Recently Colladay and Kostelecký have studied a few possibilities to examine CPT symmetry in neutral D decays on the basis of future fixed target and τ-charm factory experiments [36]. Considering this work and some other works on tests of CPT symmetry in the B^0-$\overline{B^0}$ system [37,38], we want to remark that one of the most sensitive signals for CPT violation or $\Delta Q=-\Delta C$ transitions should be the nonvanishing asymmetry Δ_D defined in Eq. (3.5). However, one should keep in mind that $\Delta_D\neq 0$ might also come from the phase shifts of final-state electromagnetic interactions or the CP-violating contributions of nonstandard electroweak models to tree-level W-mediated semileptonic D decays. Another possible way to test CPT invariance in D^0-$\overline{D^0}$ mixing, which in principle works, is to measure the time distributions of opposite-sign dilepton events at an asymmetric τ-charm factory (see Appendix B).

Of course, much more theoretical effort should be made to give reliable numerical predicitions for the magnitudes of various D^0-$\overline{D^0}$ mixing and CP-violating phenomena.

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APPENDIX A

This appendix is devoted to giving some generic formulas for the time-dependent D decays at an assumed asymmetric τ-charm factory. The asymmetric e^+e^- collisions just above the production threshold of $(D^0_{\rm phys}\overline{D^0_{\rm phys}})_C$ pairs will offer the possibility to measure the decay-time difference $t_-=(t_2-t_1)$ between $D^0_{\rm phys}\to f_1$ and $\overline{D^0_{\rm phys}}\to f_2$. Usually it is difficult to measure the $t_+=(t_2+t_1)$ distribution in either linacs or storage rings, unless the bunch lengths are much shorter than the decay lengths [39]. Here we calculate the t_- distributions of joint decay rates starting from the master formula in Eq. (2.19). For simplicity, we use t to denote t_- in the following. Integrating $R(f_1,t_1;f_2,t_2)_C$ over t_+, we obtain the decay rates (for $C=\pm$) as

$$R(f_1,f_2;t)_-\propto|A_{f_1}|^2|A_{f_2}|^2\exp(-\Gamma|t|)[(|\xi_-|^2+|\zeta_-|^2)\cosh(y_D\Gamma t)-2\mathrm{Re}(\xi_-^*\zeta_-)\sinh(y_D\Gamma t)$$
$$-(|\xi_-|^2-|\zeta_-|^2)\cos(x_D\Gamma t)+2\mathrm{Im}(\xi_-^*\zeta_-)\sin(x_D\Gamma t)] \tag{A1}$$

and

$$R(f_1,f_2;t)_+\propto|A_{f_1}|^2|A_{f_2}|^2\exp(-\Gamma|t|)\left[\frac{|\xi_+|^2+|\zeta_+|^2}{\sqrt{1-y_D^2}}\cosh(y_D\Gamma|t|+\phi_y)-\frac{2\mathrm{Re}(\xi_+^*\zeta_+)}{\sqrt{1-y_D^2}}\sinh(y_D\Gamma|t|+\phi_y)\right.$$
$$\left.-\frac{|\xi_+|^2-|\zeta_+|^2}{\sqrt{1+x_D^2}}\cos(x_D\Gamma|t|+\phi_x)+\frac{2\mathrm{Im}(\xi_+^*\zeta_+)}{\sqrt{1+x_D^2}}\sin(x_D\Gamma|t|+\phi_x)\right], \tag{A2}$$

where the phase shifts ϕ_x and ϕ_y are defined by $\tan\phi_x=x_D$ and $\tanh\phi_y=y_D$, respectively. One can check that integrating $R(f_1,f_2;t)_C$ over t, where $t\in(-\infty,+\infty)$, will lead to the time-independent decay rate $R(f_1,f_2)_C$ given in Eq. (2.21). Equations (A1) and (A2) are two basic formulas for investigating coherent $D^0\overline{D^0}$ decays at asymmetric τ-charm factories.

Another possibility is to measure the time-integrated decay rates of $(D^0_{\rm phys}\overline{D^0_{\rm phys}})_C$ with a proper time cut, which can sometimes increase the sizes of CP asymmetries [13]. In practice, appropriate time cuts can also suppress background and improve the statistic accuracy of signals. If the decay events in the time region $t\in[+t_0,+\infty)$ or $t\in(-\infty,-t_0]$ are used, where $t_0\geqslant 0$, the respective decay rates can be defined by

$$\hat{R}(f_1,f_2;+t_0)_C\equiv\int_{+t_0}^{+\infty}R(f_1,f_2;t)_C dt,$$
$$\hat{R}(f_1,f_2;-t_0)_C\equiv\int_{-\infty}^{-t_0}R(f_1,f_2;t)_C dt. \tag{A3}$$

By use of Eqs. (A1) and (A2), we obtain

$$\hat{R}(f_1,f_2;\pm t_0)_- \propto |A_{f_1}|^2|A_{f_2}|^2\exp(-\Gamma t_0)\left[\frac{|\xi_-|^2+|\zeta_-|^2}{2\sqrt{1-y_D^2}}\cosh(y_D\Gamma t_0+\phi_y)\mp\frac{\mathrm{Re}(\xi_-^*\zeta_-)}{\sqrt{1-y_D^2}}\sinh(y_D\Gamma t_0+\phi_y)\right.$$
$$\left.-\frac{|\xi_-|^2-|\zeta_-|^2}{2\sqrt{1+x_D^2}}\cos(x_D\Gamma t_0+\phi_x)\pm\frac{\mathrm{Im}(\xi_-^*\zeta_-)}{\sqrt{1+x_D^2}}\sin(x_D\Gamma t_0+\phi_x)\right] \tag{A4}$$

and

$$\hat{R}(f_1,f_2;\pm t_0)_+ \propto |A_{f_1}|^2|A_{f_2}|^2\exp(-\Gamma t_0)\left[\frac{|\xi_+|^2+|\zeta_+|^2}{2(1-y_D^2)}\cosh(y_D\Gamma t_0+2\phi_y)-\frac{\mathrm{Re}(\xi_+^*\zeta_+)}{1-y_D^2}\sinh(y_D\Gamma t_0+2\phi_y)\right.$$
$$\left.-\frac{|\xi_+|^2-|\zeta_+|^2}{2(1+x_D^2)}\cos(x_D\Gamma t_0+2\phi_x)+\frac{\mathrm{Im}(\xi_+^*\zeta_+)}{1+x_D^2}\sin(x_D\Gamma t_0+2\phi_x)\right]. \tag{A5}$$

It is easy to check that

$$\hat{R}(f_1,f_2;+0)_C+\hat{R}(f_1,f_2;-0)_C=R(f_1,f_2)_C. \tag{A6}$$

One can observe that in $\hat{R}(f_1,f_2;\pm t_0)_C$ different terms are sensitive to the time cut t_0 in different ways. Thus it is possible to enhance a CP-violating term (and suppress the others) via a suitable cut t_0.

APPENDIX B

In this appendix we take a brief look at the possible effect of CPT violation in D^0-$\overline{D^0}$ mixing on the decay rates of semileptonic D decays. For simplicity, we assume the $\Delta Q=\Delta C$ rule and direct CPT invariance in D decays to hold exactly. We also assume the absence of final-state electromagnetic interactions and other sources of new physics that could affect the tree-level W-mediated D decays. Because of the presence of CPT violation, the mass eigenstates $|D_L\rangle$ and $|D_H\rangle$ can now be expressed as

$$|D_L\rangle=\cos\frac{\theta}{2}p|D^0\rangle+\sin\frac{\theta}{2}q|\overline{D^0}\rangle,$$
$$|D_H\rangle=\sin\frac{\theta}{2}p|D^0\rangle-\cos\frac{\theta}{2}q|\overline{D^0}\rangle, \tag{B1}$$

where θ is in general complex. Note that CPT invariance requires $\cos\theta=0$, while CP conservation requires both $\cos\theta=0$ and $p=q=1$ [40]. Taking $\theta=\pi/2$, i.e., CPT symmetry, one can reproduce Eq. (2.1) from Eq. (B1). The proper-time evolution of an initially $(t=0)$ pure D^0 or $\overline{D^0}$ turns out to be

$$|D^0_{\rm phys}(t)\rangle=[g_+(t)+g_-(t)\cos\theta]|D^0\rangle+\frac{q}{p}[g_-(t)\sin\theta]|\overline{D^0}\rangle,$$
$$|\overline{D^0}_{\rm phys}(t)\rangle=[g_+(t)-g_-(t)\cos\theta]|\overline{D^0}\rangle+\frac{p}{q}[g_-(t)\sin\theta]|D^0\rangle, \tag{B2}$$

where $g_\pm(t)$ have been given in Eq. (2.6).

Starting from Eq. (B2), one can calculate the CP asymmetry $\overline{\Delta}_D$ defined in Eq. (3.5) for semileptonic D transitions. We find

$$\overline{\Delta}_D=\frac{2x_D\alpha\mathrm{Im}(\cos\theta)+2y_D\mathrm{Re}(\cos\theta)}{(1+\alpha)+(1-\alpha)|\cos\theta|^2}, \tag{B3}$$

where $\alpha=(1-y_D^2)/(1+x_D^2)$ has been defined before. Clearly $\overline{\Delta}_D=0$, if there is no CPT violation in D^0-$\overline{D^0}$ mixing (i.e., $\cos\theta=0$). Since $|\cos\theta|$ must be a small quantity, the $|\cos\theta|^2$ term in the denominator of $\overline{\Delta}_D$ is negligible. Anyway observation of the signal $\overline{\Delta}_D$ will be greatly difficult in practice, since its magnitude is suppressed by both the small mixing rate and the small CPT asymmetry.

Next let us assume the experimental scenario to be an asymmetric τ-charm factory, in which $D^0\overline{D^0}$ pairs can be coherently produced at the $\psi(3.77)$ or $\psi(4.14)$ resonance and the time-dependent measurements of their decays are available. To probe possible CPT violation in D^0-$\overline{D^0}$ mixing, we consider the case that one D meson decays to the semileptonic state $e^\pm X_e^\mp$ at (proper) time t_e and the other to the semileptonic state $\mu^\mp X_\mu^\pm$ at t_μ. The joint decay rate for having such an event can be given as a function of the decay-time difference $t\equiv t_\mu-t_e$. For simplicity and definition, we choose $t>0$ by convention. This implies that $e^\pm X_e^\mp$ events may serve for flavor-tagging of $\mu^\mp X_\mu^\pm$ events. After a lengthy calculation, we obtain

$$R(e^\pm X_e^\mp,\mu^\mp X_\mu^\pm;t)_- \propto |A_e|^2|A_\mu|^2\exp(-\Gamma t)[\cosh(y_D\Gamma t)+\cos(x_D\Gamma t)\pm2\mathrm{Re}(\cos\theta)\sinh(y_D\Gamma t)\pm2\mathrm{Im}(\cos\theta)\sin(x_D\Gamma t)] \tag{B4}$$

and

$$R(e^{\pm}X_e^{\mp},\mu^{\mp}X_\mu^{\pm};t)_+ \propto |A_e|^2|A_\mu|^2\exp(-\Gamma t)\left\{\frac{\cosh(y_D\Gamma t+\phi_y)}{\sqrt{1-y_D^2}}+\frac{\cos(x_D\Gamma t+\phi_x)}{\sqrt{1+x_D^2}}\right.$$
$$\pm\frac{2|\cos\theta|}{\sqrt{x_D^2+(2-y_D)^2}}[\cos(\Theta+\omega_-)\exp(+y_D\Gamma t)-\cos(\Theta+\omega_+ + x_D\Gamma t)]$$
$$\left.\mp\frac{2|\cos\theta|}{\sqrt{x_D^2+(2+y_D)^2}}[\cos(\Theta-\omega_+)\exp(-y_D\Gamma t)-\cos(\Theta-\omega_+ - x_D\Gamma t)]\right\}, \tag{B5}$$

where ϕ_x and ϕ_y have been defined in Appendix A, and the phase shifts $\omega_\pm$ and Θ are defined by $\tan\omega_\pm \equiv x_D/(2\pm y_D)$ and $\tan\Theta \equiv \mathrm{Im}(\cos\theta)/\mathrm{Re}(\cos\theta)$, respectively. In obtaining Eqs. (B4) and (B5), we have neglected those higher-order terms of $\cos\theta$. It is clear that the opposite-sign dilepton events $R(l^{\pm}X_l^{\mp},l^{\mp}X_l^{\pm};t)_C$ cannot be used to explore possible CPT violation in D^0-$\overline{D^0}$ mixing, because the time order of l^+ and l^- is hardly distinguishable in practical experiments. In addition, the signal of CPT violation cannot manifest itself in the time-integrated decay rates of $(D^0_{\rm phys}\overline{D^0_{\rm phys}})_C \rightarrow (e^{\pm}X^{\mp})(\mu^{\mp}X^{\pm})$, as obviously shown by the equations above. That is why we need an asymmetric τ-charm factory to test CPT symmetry in D^0-$\overline{D^0}$ mixing.

Of course, CPT violation can appear in many other decay modes of neutral D mesons. The semileptonic processes discussed above are more attractive to us for the study of CPT violation, since they do not involve CP asymmetry in D^0-$\overline{D^0}$ mixing (measured by $|q/p|\neq 1$) and other CP-violating signals. In general, however, both direct and indirect CPT asymmetries as well as $\Delta Q=-\Delta C$ transitions (and other sources of new physics) are possible to affect the decay modes in question [41].

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