PHYSICAL REVIEW D VOLUME 55, NUMBER 4 15 FEBRUARY 1997

Inflation for Bianchi type IX models

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(Received 5 August 1996)

The influence of inflation on the initial (i.e., at Planck's epoch) large anisotropy of the Universe is studied. To this end we consider a more general metric than the isotropic one: the locally rotationally symmetric Bianchi type IX metric. We find, then, a large set of initial conditions of intrinsic curvature and shear allowing an inflationary epoch that make the anisotropy negligible. These are not trivial because of the nonlinearity of Einstein's equations. [S0556-2821(97)05204-1]

PACS number(s): 98.80.Cq, 04.20.Jb

I. INTRODUCTION

The observations of cosmic microwave background radiation can be utilized to give some constraints on the anisotropy of the Universe. These limits have been derived analytically by Hawkings and Collins [1], considering little deviations from isotropy, while Barrow, Juszkiewicz, and Sonoda [2] have computed the temperature pattern and the angular correlation function for the temperature perturbations expected in anisotropic models. Bunn, Ferreira, and Silk have used the theoretical temperature pattern of the Bianchi model of type VII$_h$ [3] to determine the values of shear and vorticity making a bestfit with cosmic microwave background radiation (CMBR) experimental data. A model-independent approach to the problem has been introduced by Maartens, Ellis, and Stoeger [4]. All these results are summarized in Table I and they show that, actually, the Universe is isotropic with a good approximation.

The set of initial conditions allowed from general relativity is much larger than isotropy; so we must look for a physical process making the Universe isotropic if, at Planck's epoch, it was highly anisotropic. The theory of inflation, as it leads to a ''natural'' prediction about the value of the curvature of the Universe, about the spectrum of scalar and tensor perturbation [12,13], and it solves the topological defects, flatness, and horizon problems [14], could be this physical process; even if it is not the only candidate [5–11].

To verify this hypothesis we assume an anisotropic metric, the Bianchi metric [15,16], and we introduce the stress-energy tensor of a scalar field minimally coupled to gravity that can give an inflationary epoch [17,18]. In this way, we are looking for the initial conditions allowing inflation and study the evolution of anisotropy, comparing final values with the observed one.

Between the different Bianchi types, we studied Bianchi type IX model, because it is the only one allowing positive intrinsic curvature R^3 [19].

Using the Raychaudhuri relation[1]

$$\frac{2}{3}\theta^2 = 2\frac{V(\phi)}{M_{\rm Pl}^2} + 2\sigma^2 - R^3, \qquad (1)$$

[where $V(\phi)$ is the potential energy of the scalar field, θ measures the rate of expansion of the Universe, and σ is the shear of the homogeneous hypersurfaces [20]], it is clear that, in the Bianchi type IX model, the positive value of R^3, describing a closed Universe, could cancel the expansion of the volume of the Universe, even with a scalar field acting as a cosmological constant [21,22]; whereas in the other Bianchi types, open or flat, shear and intrinsic curvature ''help'' the expansion.

The study of evolution of shear in the Bianchi type V model [23,24] shows that inflation leads this model to be completely isotropic. An attempt to study the dynamics of Bianchi models with a scalar field, using the Nöther symmetries in minisuperspace, has recently been performed [25], but there is no complete analysis yet.

II. INFLATION WITH A BIANCHI TYPE IX METRIC

Let us consider the metric of a locally rotationally symmetric (LRS) Bianchi type IX universe[2] [26–28]:

$$ds^2 = -M_{\rm Pl}^{-2}N^2(\lambda)d\lambda^2 + e^{-2\alpha}\{e^{-2\beta}(\omega^1)^2 + e^{\beta}[(\omega^2)^2 + (\omega^3)^2]\}, \qquad (2)$$

where N is the lapse function and the one-forms ω are defined by

$$d\omega^i = \frac{1}{2}\epsilon^i_{jk}\omega^j \wedge \omega^k. \qquad (3)$$

The evolution's rate of β is related to the shear:

$$\sigma^2 = 3\left(\frac{M_{\rm Pl}}{N}\beta'\right)^2. \qquad (4)$$

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[1] We will use such units that $c=h=k=1$, $8\pi G/3c^2 = M_{\rm Pl}^{-2}$.
[2] The Latin index i,j, etc., $=1,2,3$.

TABLE I. Limits on shear (σ) and vorticity (ω) of the Universe from CMBR observations. In Bianchi type VII_0 and VII_h models there is an adjustable parameter x; the ratio of the comoving length scale over which the orientation of the principal axes of shear change, to the present Hubble radius. In Bianchi type VII_h and Bianchi type V open models, we chose the limits for models with $\Omega_0=0.1$. Hakwing and Collins (HC) calculated the upper limits on anisotropy in 1973, when the upper limits on CMBR temperature anisotropy were approximately 3 orders of magnitude greater than values observed by the Cosmic Background Explorer (COBE); their results, after correction, become comparable with the other ones. Barrow, Juszkiewicz, and Sonoda (BJS) studied only the rotation, while Bunn, Ferreira, and Silk (BFS) limited to Bianchi type VII_h. Both studies do not give analytic expression of the limits with respect to parameter x, for the comparison we chose $x=1$ (the limits are those from the quadrupole anisotropy). The Maartens-Ellis-Stoeger (MES) analysis, instead, does not depend on the model, and they have the same order of magnitude for shear and vorticity.

Model		HC	BJS	BFS	MES
Bianchi type I	$\left(\frac{\sigma}{\theta}\right)_0$	6×10^{-8}			
Bianchi type V	$\left(\frac{\sigma}{\theta}\right)_0$	1.5×10^{-4}			
	$\left(\frac{\omega}{\theta}\right)_0$	4×10^{-4}	1.7×10^{-7}		
Bianchi type VII_0	$\left(\frac{\sigma}{\theta}\right)_0$	$\frac{1}{x}10^{-6}$			
	$\left(\frac{\omega}{\theta}\right)_0$	$\frac{2}{x}10^{-6}$	3.2×10^{-9}		10^{-5}
Bianchi type VII_h	$\left(\frac{\sigma}{\theta}\right)_0$	$\frac{2}{x}10^{-3}$		0.5×10^{-9}	
	$\left(\frac{\omega}{\theta}\right)_0$	$\frac{1}{x}10^{-3}$	1.2×10^{-7}	10^{-8}	
Bianchi type IX	$\left(\frac{\sigma}{\theta}\right)_0$	10^{-3}			
	$\left(\frac{\omega}{\theta}\right)_0$	10^{-11}	3.9×10^{-13}		

The general Bianchi type IX model is more complicated with respect to the LRS Bianchi type IX model; in fact, in that case Einstein's equations have not yet been solved. Our simplification does not affect the fundamental feature of the model that we want to study, the effect of positive curvature, and it allows us to compute the anisotropic initial conditions compatible with inflation.

The Lagrangian for gravitational and scalar field is [29–31]

$$L=M_{\mathrm{Pl}}^2\left[\frac{1}{2}w\left(-\alpha'^2+\beta'^2+\frac{1}{M_{\mathrm{Pl}}^2}\phi'^2\right)-\frac{12}{w}e^{-4\alpha}U(\beta) -\frac{24}{w}e^{-6\alpha}\frac{V(\phi)}{M_{\mathrm{Pl}}^4}\right], \tag{5}$$

$$U(\beta)=-2e^{-2\beta}+\frac{1}{2}e^{-8\beta}, \tag{6}$$

where the gauge freedom is hidden in the function $w\equiv 12e^{-3\alpha}/N(\lambda)$. The common feature to different theories of inflation is the so-called "slow-roll" approximation [32] in the description of the motion of the field. This implies that the Klein-Gordon equation reduces to

$$3\frac{M_{\mathrm{Pl}}^2}{N^2}\alpha'\phi'=\frac{\partial V}{\partial\phi}. \tag{7}$$

With this approximation, models of inflation satisfy two other conditions on the potential $V(\phi)$:

$$\epsilon\equiv\frac{M_{\mathrm{Pl}}^2}{2}\left(\frac{1}{V}\frac{\partial V}{\partial\phi}\right)^2\ll 1, \tag{8}$$

$$\eta\equiv M_{\mathrm{Pl}}^2\frac{1}{V}\frac{\partial^2 V}{\partial\phi^2}\ll 1. \tag{9}$$

As we are not interested in the length of inflation but in initial conditions causing inflation, it is enough to verify that the characteristic time in the evolution of the scalar field T_ϕ is much greater than gravitational characteristic time T_G.

In our model $T_G\propto M_{\mathrm{Pl}}^{-1}$, whereas from assumptions (7) and (8) we obtain $T_\phi\propto(M_{\mathrm{Pl}}^3/\epsilon V(\phi))$. We shall assume a value of about M_{Pl}^4 for the energy potential density at Planck's epoch in the "chaotic" inflation theory. This assumption is justified by quantum effects giving radiative corrections to the effective potential [33] of this typical order of magnitude [34]; then

$$T_\phi=\frac{1}{\epsilon}T_G. \tag{10}$$

We can neglect, then, the "kinetic" term of the scalar field and assume that the potential term acts as a cosmological constant.

Hence, the contribution to the Lagrangian due to the scalar field becomes

$$L_\phi\approx M_{\mathrm{Pl}}^2\frac{24}{w}e^{-6\alpha}. \tag{11}$$

The solutions of Einstein's equations in this case have been found by Cahen and Defrise [35]; following the work of Uggla, Jantzen, and Rosquist, we made a different choice of slicing gauge and we have found the solutions of Einstein's equations for the new functions:

$$W=e^{-3\alpha-3\beta},\quad Z=e^{2\alpha-2\beta}. \tag{12}$$

They can be written as

$$Z(\tau)=\Gamma(1-\tau^2), \tag{13}$$

$$W(\tau)=\Gamma^{-5/2}\left\{\left(\Gamma-\frac{4}{3}\right)[8(1-\tau^2)^{1/2}-4(1-\tau^2)^{-1/2}]+\frac{4}{3}(1-\tau^2)^{-3/2}+\Sigma\tau\right\}+\Delta, \tag{14}$$

where Γ, Σ, and Δ are constants of integration.

Inserting solutions (13) and (14) into Eq. (1), we obtain the constraint $\Delta=0$. The new coordinate time τ is related to comoving time t by

$$dt=-2M_{\rm Pl}^{-1}\Gamma^{1/2}W^{-1/2}Z^{-7/4}d\tau. \tag{15}$$

We see that when $\tau\rightarrow 1$ then $t\rightarrow-\infty$, and when $\tau\rightarrow-1$ then $t\rightarrow\infty$.

We choose as an initial comoving time $t_{\rm PL}=10^{-43}$ sec; the correspondence with initial coordinate time τ_0, for a given solution of Einstein equations, depends on the choice of integration constant in Eq. (15). The physically important part of the solutions is limited to the interval $[\tau_0,-1[$, i.e., $[t_{\rm Pl},+\infty[$. The physical quantities θ and σ can be expressed as functions of W and Z:

$$\theta=M_{\rm Pl}\Gamma^{-1/2}\left(\frac{3}{8}W^{1/2}Z^{3/4}\frac{dZ}{d\tau}-\frac{1}{4}W^{-1/2}Z^{7/4}\frac{dW}{d\tau}\right), \tag{16}$$

$$\sigma=M_{\rm Pl}\Gamma^{-1/2}\left|\frac{\sqrt{3}}{8}W^{1/2}Z^{3/4}\frac{dZ}{d\tau}+\frac{\sqrt{3}}{12}W^{-1/2}Z^{7/4}\frac{dW}{d\tau}\right|. \tag{17}$$

III. INITIAL CONDITIONS OF THE UNIVERSE

The initial coordinate time chosen is $\tau_0=0$; in this way by Eqs. (12) and (16) we can write explicitly the relation between integration constants and the quantities β_0 and θ_0:

$$\Gamma=\frac{4}{4-e^{-6\beta_0}}, \tag{18}$$

$$\Sigma=-\frac{8\theta_0}{M_{\rm Pl}}\sqrt{\Gamma-1}. \tag{19}$$

Because of Eqs. (13) and (12), Γ must be positive.

Equation (18) implies

$$\beta_0>-\frac{1}{6}\ln(4)\equiv\beta_{\rm lim} \tag{20}$$

that, for the equality

$$R^3=-M_{\rm Pl}^2e^{2\alpha}U(\beta), \tag{21}$$

is equivalent to

$$R^3{}_0>0. \tag{22}$$

The volume of the Universe is given by

$$\text{Vol}=e^{-3\alpha}=\frac{W^{1/2}}{Z^{3/4}}=\frac{\sqrt{(\Gamma-4/3)[8(1-\tau^2)^{1/2}-4(1-\tau^2)^{-1/2}]+(4/3)(1-\tau^2)^{-3/2}+\Sigma\tau}}{\Gamma^2(1-\tau^2)^{3/4}}. \tag{23}$$

The behavior of this function depends on the two independent initial conditions β_0 and θ_0. The other two initial conditions α_0 and σ_0 are related to β_0 and θ_0 by

$$-\alpha_0=-\beta_0+\frac{1}{2}\ln\Gamma, \tag{24}$$

$$\sigma_0=\frac{\sqrt{3}}{3}\theta_0. \tag{25}$$

The former is not physically relevant because it simply fixes the length scales at $\tau=0$. The latter, instead, is the consequence of the Hamiltonian constraint (1), and it means that (with the choice $\tau_0=0$) our model describes an initial ''equipartition'' of energy among the different gravitational degrees of freedom α and β. This is a more general and ''natural'' initial condition than isotropy, and it has been recently analyzed by Barrow [36].

The function (23), for different choices of free parameters Γ and Σ, gives two possible evolution of the Universe's volume. In the first case, the anisotropy cannot stop expansion, and we have

$$\text{Vol}=e^{-3\alpha}\rightarrow+\infty \quad \text{when} \quad \tau\rightarrow-1. \tag{26}$$

In the second case, the volume reaches a maximum and then it shrinks again, while shear and intrinsic curvature diverge; this happens at a coordinate time $\tau_C\in[0,-1[$, corresponding always to a finite comoving time t_C. Integrating numerically Eq. (15), we can show that $t_C\approx$ a few $t_{\rm Pl}$, and the exact value depends on initial condition Γ and Σ.

The former possibility occurs if $\theta_0>\theta_\star$, where

$$\theta_\star\approx 54.60M_{\rm Pl}, \quad \beta_0\in]\beta_{\rm lim},0.996\beta_{\rm lim}],$$
$$\theta_\star\approx 0.05M_{\rm Pl}\left(\ln\frac{\beta_{\rm lim}}{\beta_{\rm lim}-\beta_0}\right)^{3.80},$$
$$\beta_0\in[0.996\beta_{\rm lim},0.792\beta_{\rm lim}], \tag{27}$$
$$\theta_\star\approx 0, \quad \beta_0\in[0.792\beta_{\rm lim},+\infty],$$

otherwise the Universe does not inflate.

Hence, an initial value of $\beta_0=0.792\beta_{\rm lim}\equiv\beta_\star$ exists distinguishing two different behaviors.

(a) $\beta_0 > \beta_\star$. There is inflation with any initial value of expansion θ_0 and shear σ_0, even if their values are much greater than $M_{\rm Pl}$. To explain this unexpected result, let us come back to the Lagrangian (5), obtaining the differential equation for the evolution of β with respect to comoving time t:

$$\frac{d^2\beta}{dt^2} - 3\frac{d\alpha}{dt}\frac{d\beta}{dt} + \frac{1}{12}M_{\rm Pl}^2 e^{2\alpha}\frac{\partial}{\partial\beta}U(\beta) = 0; \tag{28}$$

then, we can show that, with high values of θ_0 and σ_0, the ''friction'' term

$$-3\frac{d\alpha}{dt}\frac{d\beta}{dt} \tag{29}$$

due to a combined action of expansion and shear is more relevant than the ''forcing'' term:

$$+\frac{1}{12}M_{\rm Pl}^2 e^{2\alpha}\frac{\partial}{\partial\beta}U(\beta) \tag{30}$$

because the potential $U(\beta)$ is not enough steep, i.e., the value of the derivative of a potential with respect to β is not enough large; moreover the large value of θ implies that $e^{2\alpha}$ becomes ''small'' very quickly.

After a few Planck's times, the intrinsic curvature and the shear become dynamically negligible and they decay:

$$\sigma^2 = \mathrm{Vol}^{-2}, \tag{31}$$

$$R^3 = \mathrm{Vol}^{-2/3}, \tag{32}$$

whereas expansion θ reaches its isotropic value $\sqrt{3}M_{\rm Pl}$ [Eq. (16)].

(b) $\beta_0 < \beta_\star$. This case is more complicated; in fact, $\theta_\star \neq 0$, and it reaches a value of a few tens of Planck's mass when $\beta_0 \to \beta_{\lim}$. The Universe inflates if $\theta_0 > \theta_\star$, or, because of Eq. (25), if $\sigma_0 > (\sqrt{3}/3)\theta_\star$. Hence, with respect to a previous case, inflation can be avoided; but, surprisingly, only with the smallest value of shear.

If there is inflation, Eqs. (31) and (32) remain valid; again the Universe becomes isotropic. If, instead, there is not inflation, we have that

$$\beta_+ \to \infty. \tag{33}$$

Equations (6), (17), (21), and (33), then, show that

$$\sigma^2 \to \infty, \tag{34}$$

$$R^3 \to \infty. \tag{35}$$

This behavior is due to initial large ''steepness'' of the potential $U(\beta)$ when $\beta_0 < \beta_\star$. This gives to β an initial ''acceleration'' enough to win the ''friction'' term (29) and reach the values $\beta \gg 1$.

In our model, the dependence of evolution of the volume on the initial value of the intrinsic curvature, R_0^3 is nontrivial. For example, relation (21) shows that when $\beta_0 \to \beta_{\lim}$ the curvature $R_0^3 \to 0$, whereas if $\beta_0 \to \infty$ then $R_0^3 \to 2M_{\rm Pl}^2$. In the first case, where the Universe is nearly flat at the beginning, there is inflation only if $\theta_0 \gg M_{\rm Pl}$, that is, because of Eq. (25), if its shear is very large. In the second case, where the energy tied to the curvature is comparable with the scalar field potential energy, the Universe inflates for any value of θ_0 and σ_0.

We must note that $\theta_\star$ falls suddenly at $\beta_0 \approx \beta_\star$. In fact, for $\beta_0 \leqslant \beta_\star$, the value of $\theta_\star$ is small ($\approx 10^{-3}$) but finite, whereas for $\beta_0 > \beta_\star$, $\theta_\star$ is null. This is a consequence of the strong nonlinearity of Einstein's equations. Slightly different initial conditions can evolve in a completely different way.

IV. CONCLUSIONS

A strongly anisotropic Universe ($\theta \sim \sigma$) at early times can go through an inflationary period, because of a scalar field minimally coupled to gravity. In this case, Eqs. (31) and (32) show that shear and curvature became negligible with respect to the expansion. In fact, the inflation causes the growth of 10^{40} times, at least, of linear dimension of the Universe; this assures that σ^2 and R^3 decrease, respectively, 10^{240} and 10^{80} times. The expansion θ, instead, remains nearly constant. In this case, at the end of inflation, the Universe can be described with the flat Friedmann-Robertson-Walker (FRW) metric.

For $\beta_0 \geqslant \beta_\star$ this situation happens for *any* initial value of σ, θ, and R^3; hence, even if the anisotropy and curvature ''energy'' are much larger than the scalar field ''energy.'' For $\beta_0 \leqslant \beta_\star$, instead, the Universe does not inflate only if $\theta_0 \leqslant (\sqrt{3}/3)\theta_\star$.

The fundamental feature to underline is that the set of initial conditions that do not allow inflation is *small* but *finite*. In the spirit of chaotic inflation, we could assume that any form of energy at Planck's epoch is of Planck's energy order, $\sim M_{\rm Pl}$ (i.e., the energy ''equipartition'' that Barrow has proposed). This would imply that inflation would be avoided for $\beta_0 \leqslant 0.883\beta_{\lim}$, because $\theta_\star \sim M_{\rm Pl}$ at $\beta_0 \sim 0.883\beta_{\lim}$ and when $\beta_0 \to \beta_{\lim}$, it reaches $54.60M_{\rm Pl}$.

In this simple model, then, neither the initial values of dynamical quantities θ, σ nor the geometrical quantity R^3, would determine the evolution; the only important quantity would be β.

ACKNOWLEDGMENTS

The authors are very grateful to Francesco Chierici and Carlo Bertoni for useful comments and suggestions.

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